DETUNED MULTI-MODE CAVITIES FOR A TWO-BEAM HIGH-GRADIENT ACCELERATOR STRUCTURE *, **

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*Research supported in part by US DoE - Office of High Energy Physics

The basic idea (S. Kuzikov) is to try to increase the breakdown limit by decreasing the exposure time to high electric fields. It may be possible to do this by using cavities that can be excited in several harmonically-related modes. The idea is clear from pictures below:

RF energy is concentrated on the bunch for only short times during bunch transit.
Motivation - I

To reduce exposure time $\tau$ of structure surfaces to peak E- and H- fields, for increasing accelerating gradient.

- Peak surface field limit $E_s$ is often modeled by the relationship $\tau E_s^n = \text{const.}$, so reducing $\tau$ by a factor-of-2 could increase $E_s$ by a factor of $2^{1/n}$. *Good if $n$ not too big!*

- In "classical" RF breakdown models, $\tau$ is usually taken to be the ($\mu$s--ish) RF pulse width, even though this is much longer than the (ps--ish) $\mu$m-scale mechanisms that may govern breakdown.

- So maybe reducing exposure time during each RF cycle, rather than reducing the pulse width, is the key for raising the breakdown threshold? *This can be done using a multi-mode, harmonically-related cavity; see U.Md. HG Workshop.*
Motivation - II

We previously analyzed the multi-mode cavity idea in the context of a two-beam accelerator scheme, involving cavity detuning to obtain significant transformer ratio. [See Proc.13th AAC, pp. 439-444 (2009).]

If multi-mode cavities will sustain increased RF breakdown thresholds, further development of the two-beam concept will build upon that advantage. But if multi-mode cavities do not prove beneficial, the two-beam concept will employ only single-mode cavities.
Harmonically-related, multi-mode cavity

$TM_{nn0}$ modes in a square box have eigenfrequencies

$\omega_{nn} = \sqrt{2\pi}nc/L$, where $L$ is the box width. Modes with odd $n$ have their peak fields at the cavity center, and thus couple well to a beam passing thru the box center.

Composite $E$- and $H$-fields in a box excited simultaneously in $TM_{110}$, $TM_{330}$, and $TM_{550}$ modes at times of peak values (top) and $1/20^{th}$ of a period later (bottom). See rapid collapse of fields.
Reduction in exposure time when three modes are excited, as compared with one

For the same peak fields, 95%, 90%, and 80% of the peak field is present 20%, 29%, and 41% of the time for excitation in a single mode; while only 6%, 8.4%, and 10% of the time when three harmonic modes are excited.
More realistic cavity geometry

with resonances in $TM_{110}$ mode at 3.0 GHz, $TM_{330}$ mode at 9.0 GHz, and $TM_{550}$ & $TM_{710}$ & $TM_{170}$ modes at 15.0 GHz

Outline of multi-mode square box cavity that includes beam tunnels (left), and dimensions in mm for 1/8th section model (right).
Field patterns

3 GHz mode

9 GHz mode

15 GHz mode 1

15 GHz mode 2

E_field

E_field

E_field

E_field

E_field

E_field

H_field

H_field

H_field

H_field
Peak surface E-field along cavity perimeter due to passage along cavity axis of a 1-nC, 3-GHz bunch train.

Red curve is for excitation of the 3.0 GHz mode.
Blue curve is for excitation of the 3.0 and 9.0 GHz modes.
Green curve is for excitation of 3.0, 9.0, and three 15.0 GHz modes.
Summary - I

• Simultaneous excitation of several harmonically-related $\text{TM}_{nn0}$ modes of a square box cavity:
  - can reduce surface exposure time to high fields during each RF cycle; and
  - can move peak surface field along the cavity wall during collapse of fields, thus reducing local exposure time.

• Cavity with beam tunnel can be tuned for desired modes, and detuned for undesired modes:
  - by selective wall deformations; or
  - by selective absorbing protrusions.

• Multi-mode excitation can be achieved using:
  - several phase-locked high-power RF amplifiers; or
  - 3 GHz beam, e.g. as CLIC drive beam (2.4 GeV, 25-100 A).
• Measurements of breakdown probability under similar conditions for single- and multi-mode harmonically-related cavities may determine if the critical exposure time of structure surfaces to high fields is at the ps level (i.e., a fraction of an RF cycle), or if it is at the $\mu$s level (cumulative over many RF cycles). This could help illuminate the basic nature of RF breakdown.

• Experimental proof that multi-mode cavities can sustain lower breakdown rates than can single-mode cavities could lead to improved designs for a high-gradient two-beam accelerator for a future multi-TeV collider with acceleration gradient $> 100$ MeV/m. *We are now discussing possible experiments using the CTF-3 drive beam with our CERN friends.*

• But how can one build an accelerator using such cavities?
The answer is to use two beams in the same cavity. If we reflect the square cavity about any e-wall, we will get a “two-box” cavity with the same spectral properties as one box.

But what good is this, since it appears that transformer ratio $= 1$. The solution is *detune*.

For a cavity without loss $\chi = \frac{I_{\text{drive}}}{I_{\text{acc}}}$

For a cavity with loss $\chi = \eta \frac{I_{\text{drive}}}{I_{\text{acc}}}$
Transformer ratio $\chi$ for a detuned mode with quality factor $Q$ is

$$\chi = \frac{2Q \frac{\Delta \omega}{\omega} \sin(\varphi) - \alpha - \cos(\varphi)}{2\alpha Q \frac{\Delta \omega}{\omega} \sin(\varphi) + 1 + \alpha \cos(\varphi)}$$

$$\alpha = \frac{I_{acc}}{I_{drive}}$$

$\varphi$ - angle between $I_{drive}$ and $I_{acc}$

Accelerating with one mode

Accelerating with two modes
Acceleration is possible in either direction:

\[ V_p = \frac{c}{1 \pm n \frac{\lambda}{L}} \]

\[ V_p = \frac{c}{1 \pm \frac{(2n+1)\lambda}{2L}} \]

\[ n = 0 \]
\[ L = \frac{\lambda}{4} \]
\[ V_p = -c \]

\[ n = 0 \]
\[ L \ll \lambda \]
\[ V_p \approx \frac{2L}{\lambda} \]
Parameters for two-beam alternate detuned cavity accelerator structure

Current ratio is \( \xi = I_t/I_d \)

Transformer ratio is \( \chi = E_t/E_d \), with \( E_t = \Xi \epsilon \)

and normalization factor is \( \Xi = \kappa \omega_o Q I_d \Theta/8\pi c \), where

\[ \kappa = 4\sqrt{\mu_o/\epsilon_o} x_{01}^2 J_1^2(x_{01}) = 966.8 \Omega. \]

Thus \( \Xi[V/m] = 726 f[GHz] Q \Theta I_d[A] \), with the transit-time factor

\( \Theta = (2c/\omega_o g) \sin(\omega_o g/2c) \).

Efficiency of power transfer between beams is \( \eta = -I_t E_t/I_d E_d = -\xi \chi \).

Normalized detuning is \( \Delta = Q (\Delta \omega/\omega_o) \).

Normalized accelerating field on test particle is \( \epsilon = \frac{4\Delta - 2\xi}{1 + 4\Delta^2} \).

Power transfer efficiency between beams is \( \eta = \frac{2\xi \Delta - \xi^2}{1 + 2\xi \Delta} = -\xi \chi \).

Parameter space is constrained as follows:

\[ \epsilon + \eta \leq 1; \]

\[ 2\Delta > \xi; \]

\[ \epsilon (\epsilon + 2\xi) \leq 1; \]

and \( \epsilon \Delta < 1 \).
Efficiency, transformer ratio, and acceleration gradient, versus detuning. $I_d = 25.2$ A, $I_t = 1.2$ A.
Efficiency, transformer ratio, and acceleration gradient, versus detuning. $I_d = 50.4 \, \text{A}, \, I_t = 1.2 \, \text{A}$. 

$I_d=50.4\,\text{A},\,I_t=1.2\,\text{A},\,f=3\,\text{GHz},\,g/\Lambda=0.9,\,\beta_t=\beta_d=1$
Efficiency, transformer ratio, and acceleration gradient, versus detuning. $I_d = 100.8 \text{ A, } I_t = 1.2 \text{ A.}$
Suggested construction: accelerating structure—a series of cavities—could be milled from six copper blocks, assembled with slots (similar to CLIC structure), to suppress spurious modes and wake-fields.
In this talk we will consider only the growth of horizontal emittance. It supposed that cavity placed horizontally and beams with ideal alignment do not exists vertical wake fields. At the same time horizontal wake fields are existed because the beams shifted from centre of cavity horizontally.

Criterion:

**Estimation of emittance growth:**

\[
\Delta(\gamma\varepsilon_x) \approx \frac{Q^2 \sigma^2 \left( \frac{dW}{ds} \right)^2 \beta^3 \gamma_0}{U_0^2}
\]

(V.Yakovlev)

- \(Q\) – drive bunch charge
- \(\sigma\) – r.m.s. bunch length (140\(\mu\)m)
- \(W_{\perp}\) – transverse wake potential per length
- \(s\) – distance from the bunch center
- \(\beta\) – beta-function (8m)
- \(\gamma_0\) – initial relativistic factor (10\(^4\))
- \(U_0\) – initial energy in eV (10\(^{10}\))

\[
\Delta(\varepsilon_x) < 600\, nm
\]
Closed copper cavity

We need a damping!
Lossy cavity, $\sigma = 1e-5*\text{cooper}$

$\Delta(\gamma \varepsilon_x) \equiv 1800nm$ for $Q = 33.6nC$ (transformer ratio = 31)

$\Delta(\gamma \varepsilon_x) \equiv 100nm$ for $Q = 8.4nC$ (transformer ratio = 7.5)
Copper cavity with slots

\[ \Delta(\gamma \varepsilon_x) \equiv 1315 \text{nm} \quad \text{for } Q = 30 \text{nC (transformer ratio = 31)} \]

\[ \Delta(\gamma \varepsilon_x) \equiv 82 \text{nm} \quad \text{for } Q = 8.4 \text{nC (transformer ratio = 7.5)} \]
9 cell copper structure with slots

Δ(γεx) = 360nm for Q = 30nC (transformer ratio = 31)

Δ(γεx) = 23nm for Q = 8.4nC (transformer ratio = 7.5)
Collider considerations (per Workshop charge) for detuned multi-mode cavity two-beam accelerator

Drive beam parameters: 5 GeV, 6 nC, 40 MW, $I_{av} = 8$ mA, 1.3x10^6 bunches/sec, or at 50 Hz prf, 26.7x10^3 bunches/pulse, which at 3 GHz gives a pulse length of 8.9 μs, and $I_{pulse} = 18.0$ A.

Accelerated beam parameters: $\Delta E = 1500$ GeV for $E_{cm} = 3$ TeV. $G = 150$ MeV/m, so active length $L_{ac} = 10$ km. With transformer ratio $T = 25$, number of sections $N = 1500/5x25 = 12$, and energy gain/section = 12.5 GeV. So $L_{section} = 833$ m, divided into small ~3-10 m sub-sections. With beam-to-beam efficiency of 35%, one has beam power = 14 MW, $I_{av} = 9.3$ μA, $I_{pulse} = 12$ mA, or 4.06 pC/bunch.

*Gives overall machine length 2x10 + 2x3 (ip) = 26 km.
FINAL SUMMARY

• Since a decision on technology for the next linear collider (after LHC) will probably not be made within the next few years, we radicals have the luxury of time to suggest way-out ideas that are off the beaten path.

• Two-beam concepts (including CLIC) already depart from the conventional RF accelerator that has many discrete power feeds from individual klystrons.

• Smooth-bore dielectric-lined two-channel symmetric (i.e., coaxial) structures hold promise; proof-of-principle experiments to demonstrate high transformer ratio are being built. See WG #2.

• Assemblies of detuned cavities also hold promise, but considerable analysis and testing is needed to confirm this promise, either for a future collider or (maybe) for a multi-MW proton driver for a sub-critical reactor, or even for a 2-beam, proton-driven, electron accelerator (!!).