

The Strong RF Focusing: a Possible Approach to Get Short Bunches at the IP

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Short colliding bunches are required in the next generation particle factories to increase the luminosity by reducing the transverse beta functions at the interaction point (IP). The strong RF focusing consists in obtaining short bunches by substantially increasing the lattice momentum compaction and the RF gradient. In this regime the bunch length is modulated along the ring and could be minimized at the IP. If the principal impedance generating elements of the ring are located where the bunch is long (in the region near the RF cavities) it is possible to avoid microwave instability and excessive bunch lengthening due to the potential well distortion. By properly choosing the machine design parameters, 2 mm rms bunch length at the IP seems to be a realistic goal at the energy of the Φ resonance (1 GeV in the center of mass).

1. INTRODUCTION

The minimum value of the vertical beta-function β_y at the IP in a collider is set by the hourglass effect [1] and it is almost equal to the bunch length σ_z . Reduction of the bunch length is an obvious approach to increase the luminosity. By scaling the horizontal and vertical beta functions β_x and β_y at the IP as the bunch length σ_z , the linear tune shift parameters $\xi_{x,y}$ remain unchanged while the luminosity scales as $1/\sigma_z$ [2].

A natural way to decrease the bunch length is to decrease the storage ring momentum compaction and/or to increase the RF voltage. However, in such a way the wakefields are likely to generate potential well distortion and microwave instability above some bunch current threshold, causing bunch distortion and elongation.

In this paper we consider an alternative strategy to get short bunches at the IP. In particular, we propose to use strong RF focusing [3] (i.e. high RF voltage and high momentum compaction) to modulate the bunch length along the ring in such a way that the bunches are very short at the IP and progressive elongate toward the RF cavity.

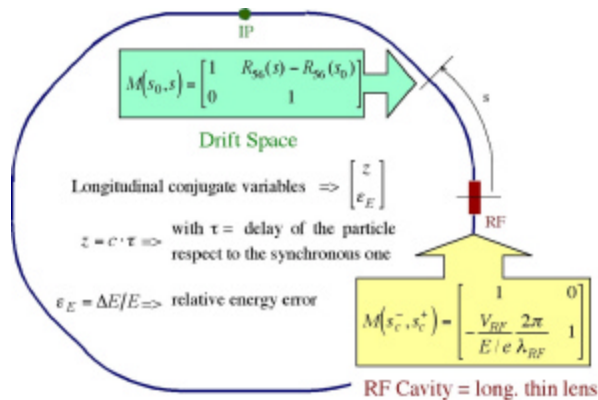


Figure 1: Longitudinal dynamics linear model

With respect to the case of short bunches with constant length all along the ring, the situation seems less critical since the average charge density driving the Touschek scattering is smaller. Besides, this allows placing the most important impedance generating devices near the RF cavity

where the bunch is longest thus minimizing the effect of the wakefields.

The basics of the strong RF focusing concept are summarized in paragraph 2. Considerations on RF acceptance and bunch lengthening are reported in paragraphs 3 and 4 respectively. Finally, criteria for optimal choice of the machine parameters to design a collider implementing the strong RF focusing scheme are given in paragraph 5.

2. STRONG RF FOCUSING BASICS

2.1. One-turn Transfer Matrix and Longitudinal Twiss Parameters

In order to compress the bunch at the IP in a collider a strong RF focusing can be applied. For this purpose high values of the momentum compaction factor α_c and extremely high values of the RF gradient are required. It is estimated that, for a Φ -factory collider, an RF voltage V_{RF} of the order of 10 MV is necessary provided that the α_c value is of the order of 0.2. Such a parameter choice produces large values of the synchrotron tune ν_s (of the order of 0.1 or larger) and the commonly used "smooth approximation" in the analysis of the longitudinal dynamics is no longer valid. Instead, the longitudinal dynamics is much more similar to the transverse one, and can be analyzed on the base of transfer matrices of the simple linear model reported in Fig. 1. In this model the cavity behaves like a thin focusing lens in the longitudinal phase space, while the rest of the machine is a drift space, where the "drifting" variable is the $R_{s6}(s)$, which is defined as:

$$R_{s6}(s) = \int_0^s \frac{\eta(\tilde{s})}{\rho(\tilde{s})} d\tilde{s} \quad (1)$$

where $\rho(s)$ is the local bending radius and $\eta(s)$ is the ring dispersion function.

In Fig. 1 $\lambda_{RF} = c/f_{RF}$ is the RF wavelength, E/e is the particle energy in voltage units, while L is the total ring length.

Taking the cavity position as the reference point $s=0$, the one-turn transfer matrix $M(s, s+L)$ of this system starting from the generic azimuth s is given by:

$$M(s, s+L) = \begin{bmatrix} M_{11}(s) & M_{12}(s) \\ M_{21}(s) & M_{22}(s) \end{bmatrix} \quad (2)$$

with

$$\begin{aligned} M_{11}(s) &= 1 - 2\pi \frac{R_{56}(s)}{\lambda_{RF}} \frac{V_{RF}}{E/e} \\ M_{12}(s) &= \alpha_c L \left(1 - 2\pi \frac{R_{56}(s)}{\lambda_{RF}} \left(1 - \frac{R_{56}(s)}{\alpha_c L} \right) \frac{V_{RF}}{E/e} \right) \\ M_{21}(s) &= -\frac{V_{RF}}{E/e} \frac{2\pi}{\lambda_{RF}} \\ M_{22}(s) &= 1 + 2\pi \frac{R_{56}(s)}{\lambda_{RF}} \left(1 - \frac{\alpha_c L}{R_{56}(s)} \right) \frac{V_{RF}}{E/e} \end{aligned} \quad (3)$$

The one turn synchrotron phase advance is given by:

$$\cos \mu = \frac{1}{2} \text{Tr}[M(s, s+L)] = 1 - \pi \frac{\alpha_c L}{\lambda_{RF}} \frac{V_{RF}}{E/e} \quad (4)$$

leading to the following stability condition:

$$\begin{aligned} |\cos \mu| \leq 1 &\Rightarrow \mu \leq \pi \Rightarrow \nu_s \leq 1/2 \Rightarrow \\ &\Rightarrow V_{RF} \leq \frac{2}{\pi} \frac{\lambda_{RF}}{\alpha_c L} E/e = V_{RF_{Max}} \end{aligned} \quad (5)$$

which shows that there is a constraint in the choice of the values of V_{RF} and α_c .

The longitudinal Twiss parameters, as derived from the one-turn transfer matrix, are given by:

$$\begin{aligned} \alpha_l(s) &= \frac{1 - \cos \mu}{\sin \mu} \left[1 - \frac{2R_{56}(s)}{\alpha_c L} \right] \\ \beta_l(s) &= \frac{\alpha_c L}{\sin \mu} \left[1 - (1 - \cos \mu) \frac{2R_{56}(s)}{\alpha_c L} \left(1 - \frac{R_{56}(s)}{\alpha_c L} \right) \right] \\ \gamma_l(s) &= \frac{1 - \cos \mu}{\sin \mu} \frac{2}{\alpha_c L} \end{aligned} \quad (6)$$

Just as in the description of the transverse motion, as long as the radiation process is ignored the trajectories of the synchrotron motion of a particle in the longitudinal phase space are ellipses. The aspect ratio and the orientation of the ellipse vary with the azimuthal position, but the ellipse area is a constant of the motion, being the particle longitudinal emittance.

For a given longitudinal emittance value ϵ_l , the amplitude of the oscillation of the normalized energy deviation of the particle $\Delta E_{\max}/E$ is given by:

$$\Delta E_{\max}/E = \sqrt{\epsilon_l \gamma_l} \quad (7)$$

which is independent on the observation azimuth since γ_l does not depend on s . If ϵ_l represents the equilibrium rms longitudinal emittance of the bunch, expression (7) gives its normalized energy spread σ_E/E :

$$\sigma_E/E = \sqrt{\epsilon_l \gamma_l} \Rightarrow \epsilon_l = (\sigma_E/E)^2 \frac{\sin \mu}{1 - \cos \mu} \frac{\alpha_c L}{2} \quad (8)$$

On the contrary, since β_l does depend upon s , the horizontal size of the ellipse (i.e. the bunch length σ_z) varies along the ring according to:

$$\begin{aligned} \sigma_z(s) &= \sqrt{\epsilon_l \beta_l(s)} = (\sigma_E/E) \alpha_c L \cdot \\ &\cdot \sqrt{\frac{1}{2} \left[\frac{1}{1 - \cos \mu} - \frac{2R_{56}(s)}{\alpha_c L} \left(1 - \frac{R_{56}(s)}{\alpha_c L} \right) \right]} = \\ &= \sigma_z(0) \sqrt{1 - (1 - \cos \mu) \frac{2R_{56}(s)}{\alpha_c L} \left(1 - \frac{R_{56}(s)}{\alpha_c L} \right)} \end{aligned} \quad (9)$$

where $\sigma_z(0)$ is the bunch length at $s=0$ (i.e. at the cavity position). It may be noticed that $\sigma_z(0) = \sigma_{z_{\max}}$ is the maximum value of the bunch length along the ring. On the other hand, the minimum value $\sigma_{z_{\min}}$ corresponds to the s_{\min} position where $R_{56}(s_{\min}) = \alpha_c L/2$. If the position of the minimum corresponds to the IP one gets:

$$\sigma_z(IP) = \sigma_{z_{\max}} \sqrt{1 - \frac{\pi}{2} \frac{\alpha_c L}{\lambda_{RF}} \frac{V_{RF}}{E/e}} = \sigma_{z_{\max}} \sqrt{\frac{1 + \cos \mu}{2}} \quad (10)$$

At phase advance values μ close to 180° , the ratio between the bunch lengths at the IP and at the RF goes to zero. This result is of great interest since it allows designing a ring where the bunch is short at the IP and progressively elongates moving toward the RF position.

2.2. Equilibrium Energy Spread

In order to compute exactly the bunch size along the ring one needs to know the equilibrium rms longitudinal emittance value (or, according to (8), the value of the equilibrium energy spread). These values can be worked out from a rigorous analysis of the longitudinal dynamics (abandoning the smooth approximation) or from a multi-particle tracking simulation including the distributed emission process along the machine. We follow an analytical approach based on the computation of the second momenta of the bunch equilibrium distributions using the eigenvectors

of the longitudinal one-turn transfer matrix [4] that gives the following result:

$$(\sigma_E/E)^2 = \frac{1}{1+\cos\mu} \frac{55}{48\sqrt{3}} \frac{r_e \hbar}{m_e} \frac{\gamma^5 \tau_d}{L} \oint \left[1 - \frac{2\pi R_{56}(s)}{\lambda_{RF}} \left(1 - \frac{R_{56}(s)}{\alpha_c L} \right) \frac{V_{RF}}{E/e} \right] \frac{ds}{|\rho(s)|^3} \quad (11)$$

where r_e and m_e are the electron classical radius and rest mass, τ_d is the longitudinal damping time and $\gamma = E/(m_e c^2)$ is the relativistic factor. It may be noticed that the equilibrium energy spread σ_E/E diverges as μ tends to π , while at low tunes it tends to the value $(\sigma_E/E)_0$:

$$(\sigma_E/E)_0^2 = \frac{55}{96\sqrt{3}} \frac{r_e \hbar}{m_e} \frac{\gamma^5 \tau_d}{L} \oint \frac{ds}{|\rho(s)|^3} \quad (12)$$

which is the expression reported in literature [5].

Expression (12) can be also conveniently rewritten in the following forms:

$$\begin{aligned} (\sigma_E/E)^2 &= \frac{1}{1+\cos\mu} \frac{55}{48\sqrt{3}} \frac{r_e \hbar}{m_e} \frac{\gamma^5 \tau_d}{L} \oint \frac{\beta_I(s)}{\beta_I(0)|\rho(s)|^3} ds = \\ &= (\sigma_E/E)_0^2 \frac{2}{1+\cos\mu} \frac{\oint \frac{\beta_I(s)}{\beta_I(0)|\rho(s)|^3} ds}{\oint \frac{ds}{|\rho(s)|^3}} = \\ &= (\sigma_E/E)_0^2 G^2[\mu, \rho(s), \beta_I(s)] \end{aligned} \quad (13)$$

where the function $G[\mu, \rho(s), \beta_I(s)]$ represents the energy spread magnification due to the strong RF focusing, which is given by:

$$G[\mu, \rho(s), \beta_I(s)] = \sqrt{\frac{2}{1+\cos\mu} \frac{\oint \frac{\beta_I(s)}{\beta_I(0)|\rho(s)|^3} ds}{\oint \frac{ds}{|\rho(s)|^3}}} \quad (14)$$

In general, for a given longitudinal phase advance μ , the magnification factor G is large if the curvature radius ρ is small where the β_I function is large (synchrotron emission more concentrated near the RF section) and vice-versa. In the simplified assumption of constant a bending radius ρ and an $R_{56}(s)$ function linearly growing in the arcs, expression (14) reduces to:

$$G^2 = \frac{2}{3} \frac{2+\cos\mu}{1+\cos\mu} \quad (15)$$

In this special case, that we call “smooth longitudinal lattice”, the longitudinal emittance ϵ_l and the bunch lengths at the RF cavity and IP are given by:

$$\begin{aligned} \epsilon_l &= \frac{\alpha_c L}{3} (\sigma_E/E)_0^2 \frac{2+\cos\mu}{\sin\mu}; \\ \sigma_z(\text{Cav}) &= \frac{\alpha_c L}{\sin\mu} (\sigma_E/E)_0 \sqrt{\frac{2+\cos\mu}{3}}; \\ \sigma_z(\text{IP}) &= \alpha_c L (\sigma_E/E)_0 \sqrt{\frac{2+\cos\mu}{6(1-\cos\mu)}} \end{aligned} \quad (16)$$

The emittance and the bunch length at the RF cavity, as well as the energy spread, diverge as μ approaches π , while the bunch length at the IP remains finite. Figure 2 shows the longitudinal emittance and the equilibrium energy spread as a function of the phase advance μ . The lines correspond to the analytical expressions (16), while dots represent the results of the multi-particle tracking simulations. The bunch length dependences on μ (both analytical and numerical) are reported in Fig. 3.

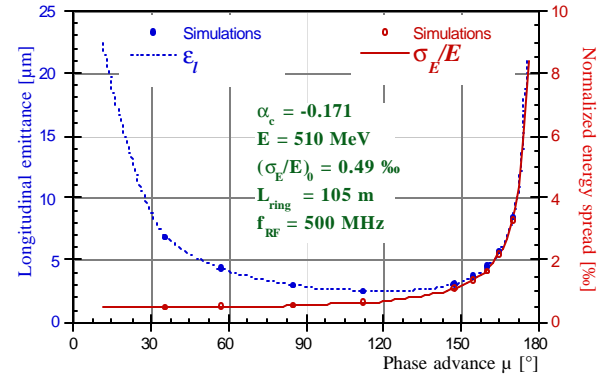


Figure 2: Longitudinal emittance and energy spread vs. longitudinal phase advance

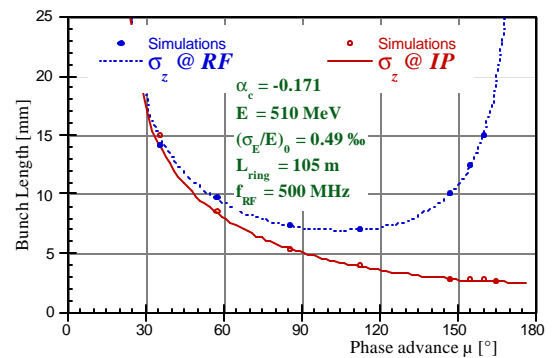


Figure 3: Bunch length @ RF and IP vs. phase advance

As shown, the longitudinal emittance exhibits a minimum at $\mu = 120^\circ$. The analogy with the transverse case is quite evident [6]. Being the momentum compaction fixed, the various phase advances correspond to different values of the RF voltages. The voltage required to approach the limit

phase advance value of 180° exceeds 10 MV. The use of superconducting cavities is mandatory in this case.

3. RF ACCEPTANCE

In a storage ring the RF acceptance is defined as the maximum energy deviation corresponding to a stable particle trajectory in the longitudinal phase space. The trajectory associated to the maximum energy deviation is the so-called “separatrix” since it separates the stable (closed) to the unstable (open) trajectories in the longitudinal phase space.

In a standard, low v_s storage ring the longitudinal phase space trajectories are independent on the particular observation abscissa s . The RF acceptance is computed as the separatrix half-height [5].

In the strong RF focusing case, the longitudinal phase space trajectories configuration changes along the ring, and the same does the separatrix, as shown in Figure 4.

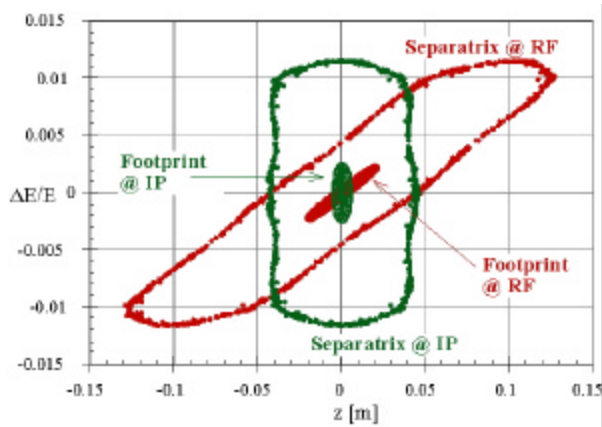


Figure 4: Long. Phase space separatrices at IP and RF

In order to compute the particle loss rate caused by the Touschek scattering process, the relevant RF acceptance is the half-height of the separatrix section at $z = 0$, which is a function of the azimuth s in this case. In fact, the maximum acceptable energy variation for a particle starting from about the origin of the phase space is smaller in the RF cavity region (where the bunch is longer); this must be taken into account in lifetime evaluations.

Concerning the loss rate caused by statistical fluctuations of the particle energy deviation (quantum lifetime) the relevant parameter is the separatrix area, which is independent on the azimuth s as a consequence of the Liouville theorem.

4. BUNCH LENGTHENING

In a collider based on the strong RF focusing concept the bunch at the IP has to remain short up to the design bunch current value. This is a critical point, since the effect of the machine wakefields is strongly dependent on the bunch length. In the case of pure inductive impedance, for

example, the wake potential scales as $1/\sigma_z^3$. On the other hand, the strong RF focusing produces a modulation of the bunch length along the machine. To minimize the bunch lengthening effect it is worth to design a storage ring with all the most dangerous impedance generating components (such as injection kickers, longitudinal and transverse kickers of the fast feedback systems, monitors and striplines, ...) placed where the bunch is longer, i.e. near the RF cavity section.

To estimate the criticality of the impedance location we have performed multi-particle tracking simulation of a bunch in the strong RF focusing regime using the DAΦNE wake (see, for example, [7]) in the two extreme (and unrealistic) cases of impedance completely centred near the RF or near the IP. In the first case the simulation results reported in Figure 5 show that with the chosen parameters ($\alpha_c = -0.17$, $\mu = 165^\circ$, $3 \cdot 10^{10}$ particles $\equiv I_b = 15$ mA) the bunch is about 3 mm long at the IP, and 11 mm long at the RF location, with no significant degradation with respect to the bunch length at zero current.

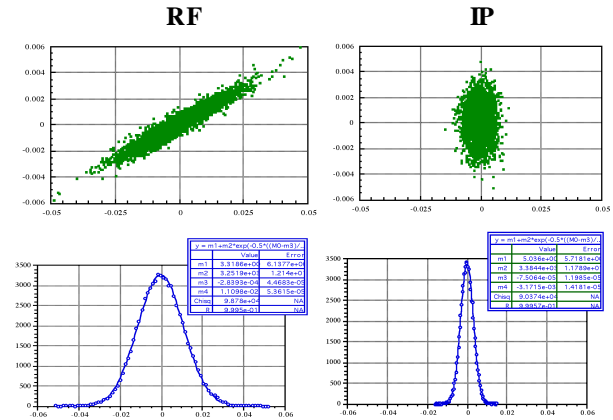


Figure 5: Particle distribution at RF and IP position in the case of a wake located near the RF section

In the second case, where the wake is centred near the IP, the results of the tracking simulations are reported in Figure 6 and show a strongly deformed bunch profile as a consequence of a microwave instability.

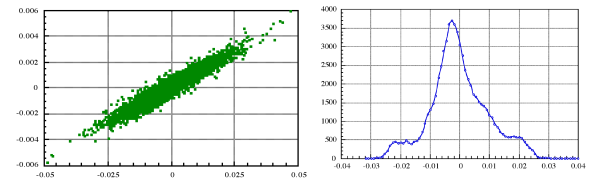


Figure 6: Particle distribution at RF position in the case of a wake located near the IP section

We have also considered the case of positive momentum compaction ($\alpha_c = +0.17$) and wake centred near the RF.

The simulations show that the bunch is stable with a very moderate lengthening.

5. DESIGN PARAMETERS OPTIMIZATION

In this paragraph we present the guidelines for choosing the main parameters of a strong RF focusing system to be implemented in a collider working at the Φ resonance (1020 MeV in the center of mass).

5.1. Longitudinal Phase Advance

The longitudinal phase advance μ is immediately determined once the desired minimum and maximum bunch length values $\sigma_z(IP)$ and $\sigma_z(RF)$ are defined. To limit the lengthening effect of the short-range wakefields it is worth to have a maximum bunch length of about 10 mm. The plots of the required phase advance as function of the IP bunch length $\sigma_z(IP)$ for 3 different values of $\sigma_z(RF)$ (namely 8, 10 and 12 mm) are reported in Fig. 7. Phase advance values in the $150^\circ \div 160^\circ$ and $165^\circ \div 170^\circ$ range are required to get $\sigma_z(IP)$ values of 2 and 1 mm respectively.

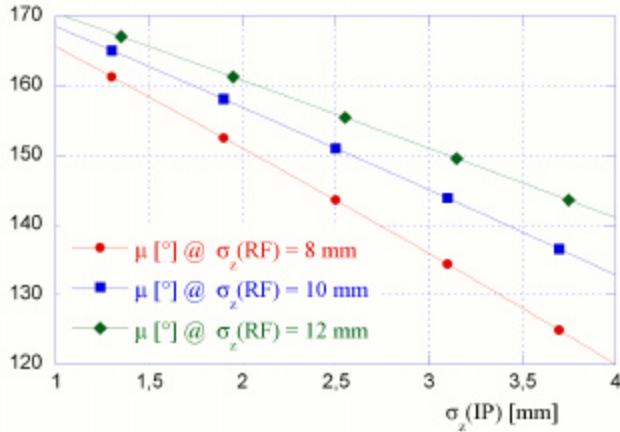


Figure 7: Required phase advance vs. IP bunch length

5.2. Energy Spread Magnification

The bunch energy spread is enlarged in the strong RF focusing scheme. The natural energy spread in a Φ -factory collider is typically $(\sigma_E/E)_0 \approx 4 \div 5 \cdot 10^{-4}$. The energy acceptance must be at least 6 times larger than the energy spread to have an acceptable quantum lifetime but, as it will be shown later on, its actual value can hardly exceed 1%. This means that the acceptable energy spread is limited to $(\sigma_E/E) \leq 1.7 \cdot 10^{-3}$ and implies that the magnification factor introduced in eq. (14) is limited to about $G \leq 3.5$. The analytical expression of G is particularly simple in a “smooth longitudinal lattice” structure (see eq. (15)). The plots of the factor G as function of the IP bunch length $\sigma_z(IP)$ for the 3 different values of $\sigma_z(RF)$ are reported in Fig. 8. At low $\sigma_z(IP)$ values smooth longitudinal lattices (dashed lines) may produce excessive magnification of the energy spread. In this case the only way to limit the G value

is to implement longitudinal lattices where the synchrotron emission is more concentrated in the low β_l region.

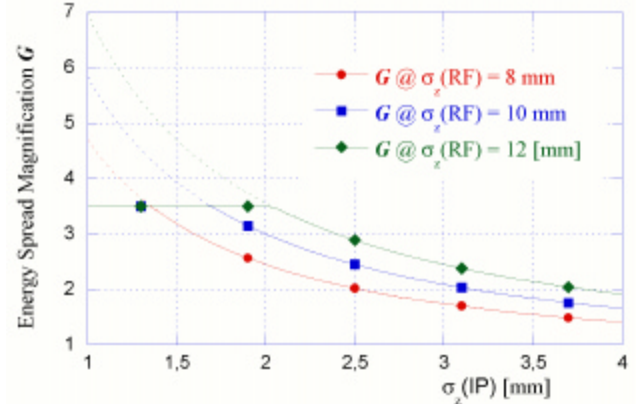


Figure 8: Energy spread magnification vs. IP bunch length

It is also important to notice that, being the width of the Φ resonance cross section $\sigma_\Phi \approx 1.9$ MeV, the energy spread of the colliding bunches reduces the rate of Φ meson production according to:

$$L_\Phi/L = \sigma_\Phi / \sqrt{\sigma_\Phi^2 + \sigma_{e^+}^2 + \sigma_{e^-}^2} \quad (17)$$

This effect is not negligible. For instance assuming $(\sigma_E/E) = 1.7 \cdot 10^{-3}$, that means $\sigma_{e^+} = \sigma_{e^-} \approx 0.87$ MeV, the “effective” luminosity (in terms of Φ production rate) is reduced by $\approx 16\%$.

5.3. One-Turn Normalized Path Elongation

The one-turn path elongation $R_{56}(L) = \alpha_c L$ is another fundamental design parameter of the strong RF focusing scheme. It is related to the previously defined parameters according to:

$$R_{56}(L) = \alpha_c L = 2 \frac{\sigma_z(RF)}{\sigma_E/E} \sqrt{\frac{1 - \cos \mu}{2}} \quad (17)$$

The plots of the $R_{56}(L)$ values as function of the IP bunch length $\sigma_z(IP)$ for the 3 different values of $\sigma_z(RF)$ are shown in Fig. 9. It is clearly shown that the optimal $R_{56}(L)$ value strongly depends on the target $\sigma_z(IP)$ value.

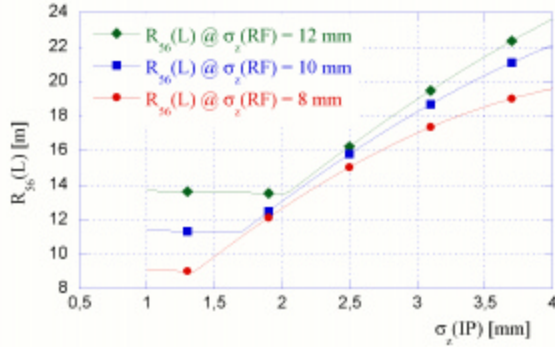


Figure 9: One-turn path elongation vs. IP bunch length

5.4. RF Voltage and Wavelength

Directly from eq. (4) it is possible to calculate the required RF gradient, i.e. the ratio between the RF voltage and the RF wavelength:

$$V_{RF}/\lambda_{RF} = \frac{1 - \cos\mu}{\pi} \frac{E/e}{\alpha_c L} \quad (18)$$

If harmonic voltages are used together with the fundamental one, eq. (18) has to be rewritten in the following form:

$$\frac{1}{\lambda_{RF}} \sum_{n=1}^N n V_{RF_n} = \frac{1 - \cos\mu}{\pi} \frac{E/e}{\alpha_c L} \quad (19)$$

The plots of the RF gradient values as function of the IP bunch length $\sigma_z(IP)$ for the 3 different values of $\sigma_z(RF)$ are shown in Fig. 10. It may be noticed that the 3 plots almost coincide in the region where the smooth longitudinal lattice is applicable (the required gradient is almost independent on the chosen value of $\sigma_z(RF)$).

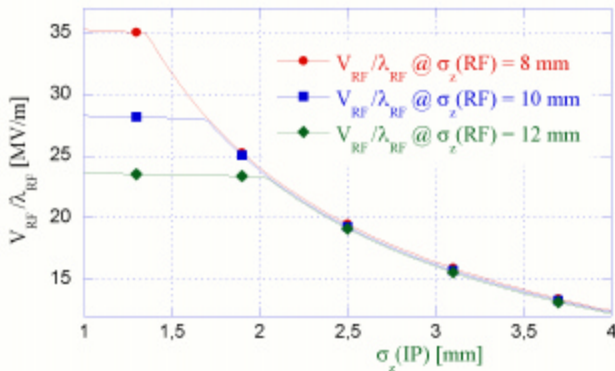
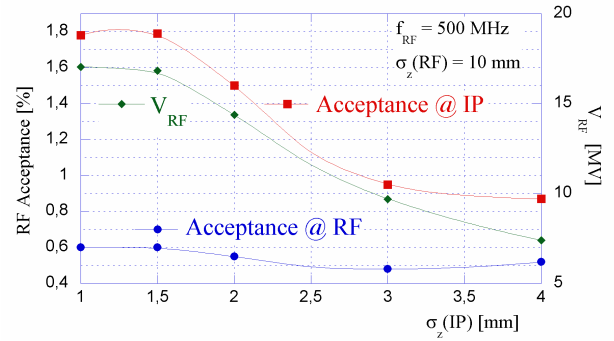


Figure 10: Required RF gradient vs. IP bunch length

High RF frequencies (small wavelength) are more efficient in producing high gradients. However, the frequency choice also defines the RF acceptance, and in this case lower frequencies are preferred. The choice of the RF frequency is a compromise between these different needs.

The use of harmonic cavities adds one degree of freedom and may help in finding a good trade-off.

The only realistic way to generate such high RF gradients is to use superconducting RF cavities. In particular, there are already SC RF cavities equipped with all the ancillary systems (such as high power input coupler, wide range tuners, efficient HOM suppressors,...) operating successfully in high current regime [8, 9]. The typical operating frequency of such cavities is 500 MHz . That is why this frequency is our primary considered option. Being $\lambda_{RF} = 0.6\text{ m}$ in that case, the gradients of Fig. 10 correspond to peak accelerating voltages in the range $V_{RF} = 7 \div 21\text{ MV}$.


 Fig. 11: RF Acceptances and RF Voltage vs. $\sigma_z(IP)$

The needed RF voltage and the RF acceptances at the IP and RF cavity regions as function of the desired IP bunch length are reported in Fig. 11, assuming $f_{RF} = 500\text{ MHz}$ and $\sigma_z(RF) = 10\text{ mm}$. For $\sigma_z(IP)$ values larger than $\approx 2.5\text{ mm}$ the required RF voltage is limited to 10 MV , which is a large but still manageable figure, resulting in a sufficient RF acceptance value ($\approx 1\%$ or slightly less at the IP). More ambitious designs ($\sigma_z(IP) < 2.5\text{ mm}$) require larger RF voltages that are probably unmanageably high, even if better RF acceptances would result. In this case a higher RF frequency is more suitable, or the use of harmonic cavities should be foreseen.

6. CONCLUSIONS

In this paper the motion of particles in a strong longitudinal focusing storage ring is described by means of the linear matrices formalism. Longitudinal optical functions are derived, showing that the bunch length varies along the ring and may be minimized at the IP.

Analytical expressions for the longitudinal emittance and the energy spread of the bunch equilibrium distribution have been obtained and validated by comparison with results from multiparticle tracking simulations. It has been shown that the longitudinal emittance and the energy spread, as well as the bunch length at the RF cavity position diverge as the synchrotron phase advance approaches 180° per turn, while the bunch length at the IP tends to a minimum value which is finite.

The RF acceptance concept in the strong RF focusing regime has been revised, together with the implications on Touschek and quantum lifetime evaluations.

Very preliminary multiparticle tracking simulations based on the DAΦNE short range wake show that the short bunch length at the IP can be preserved up to relatively high bunch current (> 10 mA) provided that all the wake is concentrated near the RF cavity, the position where the bunch is longest.

The basic criteria to optimally design a strong RF focusing collider at the energy of the Φ resonance have been presented. Bunch length down to ≈ 2.5 mm at the IP can be obtained by using 500 MHz superconducting RF technology already developed for high luminosity colliders. More ambitious design goals may require additional features, such as higher RF frequency, harmonic cavities and lattice optimization for reducing the energy spread growth

Many aspects of beam physics need to be studied to establish whether or not a collider may efficiently work in the strong longitudinal focusing regime. The most relevant issues are bunch lengthening due to the wakefields, Touschek lifetime, dynamic aperture and beam-beam effect.

References

- [1] G. E. Fisher, "A Brief Note on the Effect of Bunch Length on the Luminosity of a Storage Ring with Low Beta at the Interaction Point", SPEAR Note 154, SLAC, December 1972.
- [2] S. Bartalucci et al., "DAΦNE Design Criteria" DAΦNE Note G-2, Frascati, Nov. 12, 1990.
- [3] V. N. Litvinenko, "On a possibility to Suppress Microwave Instability in Storage Rings Using Strong Longitudinal Focusing", AIP Conference Proc. 395: 275-283, 1996.
- [4] A. W. Chao, "Evaluation of Beam Distribution Parameters in an Electron Storage Ring", Journal of Applied Physics 50: 595-598, 1979.
- [5] M. Sands, "The Physics of Electron Storage Rings. An Introduction", SLAC pub 121, November 1970.
- [6] H. Wiedemann, "Brightness of Synchrotron Radiation from Electron Storage Rings", NIM 172 (1980), 33-37.
- [7] M. Zobov et al., "Bunch Lengthening and Microwave Instability in the DAΦNE Positron Ring", e-Print Archive: Physics/0312072.
- [8] T. Furuya, "Update on the Performance of Superconducting Cavities in KEKB", WGB07, this workshop
- [9] S. Belomestnykh et al., "Running CESR at High Luminosity and Beam Current with Superconducting RF System", Proc. of EPAC 2000, Vienna, p. 2025