Beam-beam limit estimated by a quasi-strong-strong simulation

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We have discussed that a kind of beam-beam limit was caused by distortion of particle distribution of colliding beams. The luminosity was determined by equilibrium distributions of the two beams in some cases. A quasi-strong-strong simulation method is proposed to get the distributions. The simulation is regarded as an iteration of weak-strong simulation. Macro-particles in a beam are tracked interacting with electron-magnetic field formed by another beam. The field is given by the particle in cell (PIC) method to deal with an arbitrary distribution of the beam. We discuss the beam-beam limit estimated by the simulation.

A quasi-strong-strong simulation had proposed to study equilibrium distribution of colliding beams with self-consistency. This simulation method is useful to predict the beam-beam limit caused by distortion of equilibrium particle distributions of colliding beams. In the first study [1], Gaussian approximation was used, therefore it was impossible to predict the beam-beam limit related to distortion of particle distribution deviated from Gaussian. We are interested in the beam-beam limit caused by distortion of beam distribution at an equilibrium stage. The quasi-strong-strong method is extended by using the particle in cell method, which permits to treat an arbitrary distribution.

In the simulation, a number of macro-particles, which are less than those in the strong-strong simulation, are tracked with experience of electro-magnetic field due to the beam-beam interaction. The tracking is done by similar manner with the weak-strong simulation, and the coordinates given by tracking are accumulated in every revolutions. An average distribution, which is obtained by the accumulation, is gives electro-magnetic field of the beam-beam interaction in next period.

One turn map for both beams, $\boldsymbol{x} = (\boldsymbol{x}_+, \boldsymbol{x}_-)$, including the beam-beam interaction, is expressed by

$$\boldsymbol{x}(C) = S\exp\left[- : \int_0^C (H_0 + \phi_{bb})ds :\right]\boldsymbol{x}(0) \tag{1}$$

$$= V_0(C) S\exp\left[- : \int_{-\Delta}^{\Delta} V_0^{-1}(s)\phi_{bb}(s)V_0(s)ds :\right]\boldsymbol{x}(0),$$

where $\phi_\pm(\boldsymbol{x}_\mp, s_\mp)$ is potential due to $e_\pm$ beam at the coordinates of macro-particles in $e^\mp$ beam,

$$\phi_{bb}(s) = \phi_+(\boldsymbol{x}_-) + \phi_-(\boldsymbol{x}_+). \tag{2}$$

$V_0(C)$, which is the transfer map of the lattice, is expressed by

$$V_0(C) = S\exp\left[- : \int_0^C (H_{0,+} + H_{0,-}(s))ds :\right]. \tag{3}$$

$V_0(s)$ is the transfer map between the collision point $(s = 0)$ to the interaction point of a longitudinal part (slice) of bunch, $H_{0,\pm} = (p_{x,\pm}^2 + p_{y,\pm}^2)s/2$.

The procedures of the quasi-strong-strong simulation are summarized by

$$\boldsymbol{x}(nC) = S\exp\left[- : \int_0^{nC} (H_0 + \bar{\phi}_{bb}(\boldsymbol{x}))ds :\right]\boldsymbol{x}(0). \tag{4}$$

n, which is period of accumulation, is chosen to be less than the radiation damping time. The particle coordinates at $s = iC$ are accumulated and averaged at every n revolutions,

$$\bar{\rho}(\boldsymbol{x}) = \frac{1}{n}\sum_{i=0}^{n-1}\rho(\boldsymbol{x}, s = (i-n)C). \tag{5}$$

$\bar{\phi}$ is determined by solving Poisson equation using the averaged distribution,

$$\bar{\phi}_\pm(\boldsymbol{x}) = \frac{r_e}{\gamma_\mp}\triangle\bar{\rho}_\pm(\boldsymbol{x}). \tag{6}$$

The simulation is regarded as an iteration process to obtain the equilibrium distributions. Potential calculation, which is heavy job in the simulation, is thinned out every period of renewal of the beam distribution. Taking average of the distribution turn by turn gains for statistics of the number of the macro-particles. Coherent motion is suppressed in the quasi-strong-strong simulation, because the beam distribution is averaged over turn by turn. This is a trade-off due to the simplification for the simulation method. We show simulation results for two-dimensional $(x - y)$ quasi-strong-strong simulation in this paper. Three dimensional code is under construction.

The quasi-strong-strong simulation is executed using parameter of the super KEKB [2]. Figure 1 shows evolution of luminosity given by the strong-strong and the quasi-strong-strong simulation. The strong-strong simulation was performed in three dimensional space, while the quasi-strong-strong is in two dimensional space (x-y). Two-dimensional strong-strong simulation gave a coherent instability in this condition [2]. The coherent motion is suppressed by algorithm of the quasi-strong-strong simulation as is mentioned before. The accumulation period n is chosen to be 500, since the damping time is 4,000 and 8,000 turns for

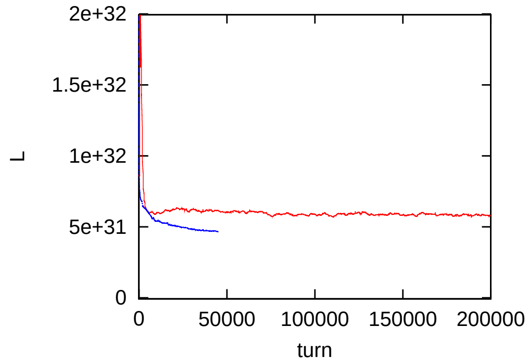

Figure 1: Evolution of luminosity given by quasi-strong-strong (2D) and strong-strong simulations (3D).

electron and positron beams, respectively. Number of the macro-particle is 10,000, while it was 100,000 for the strong-strong simulation. Number of particle to determine the potential using PIC method is $500 \times 10,000 = 5,000,000$. The luminosity goes down around 5×10^{31} cm^{-2}s^{-1} for the both simulations from the geometrical value of 1×10^{32} cm^{-2}s^{-1}. This value, 5×10^{31} cm^{-2}s^{-1}, corresponds to the beam-beam parameter of 0.1.

Figure 2 shows evolution of the vertical beam size given by the quasi-strong-strong and strong-strong simulations. The behaviors in the beginning and middle stages are different between these two simulations, but the finial sizes at the equilibrium are almost consistent each other. Positron beam has slower damping time, therefore its size is larger than that of electron beam. Figure 3 shows the equilibrium distribution of two beams. Tail part of the distribution is distorted from Gaussian.

We study the beam-beam limit depending on the radiation damping time. People believe that faster damping time is efficient for higher beam-beam parameters. The quasi-strong-strong simulation is executed for the radiation damping times, 4,000, 10,000, and 40,000 turns using the parameter of the super KEKB. Figure 4 shows the beam-beam parameter depending on the operating current. Two tune operating points, (0.518,0.58) and (0.508,0.55), which are those for HER and LER of present KEKB, respectively, are examined. Faster damping time shows higher beam-beam limit in the figure, though not quite satisfactorily. The beam-beam parameter depending on the current did not have simple behavior: i.e., it was not function with monotonically increasing. The beam-beam parameters at their limits were 0.08-0.11, 0.07-0.08 and 0.06-0.07, for the radiation damping time of 4000, 10000 and 40000 turns, respectively, at a rough estimate.

We decreased the number of the macro-particle because of the accumulation. Actually since the macro-particles accumulated are not independent, it is not clear whether the accumulation helps statistics. The simulation should be extended to three dimensional one. More studies needs to get final results.

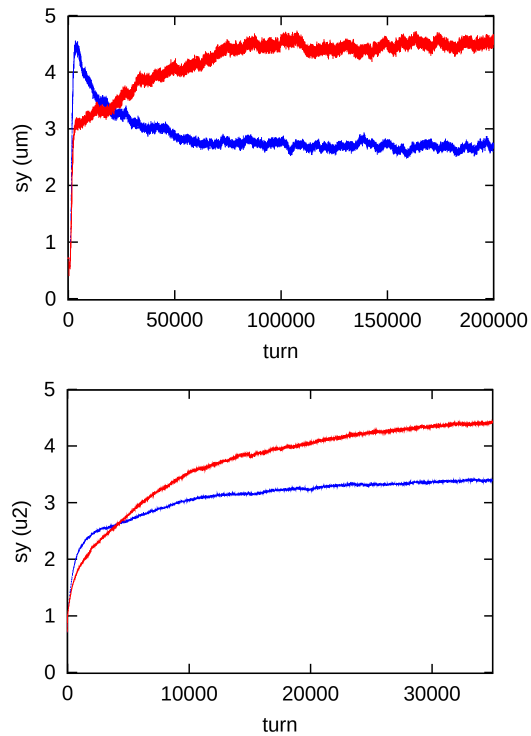

Figure 2: Evolution of vertical beam size given by the quasi-strong-strong simulation (upper) and the strong-strong simulation (lower). Red and blue lines depict evolution of positron and electron sizes, respectively.

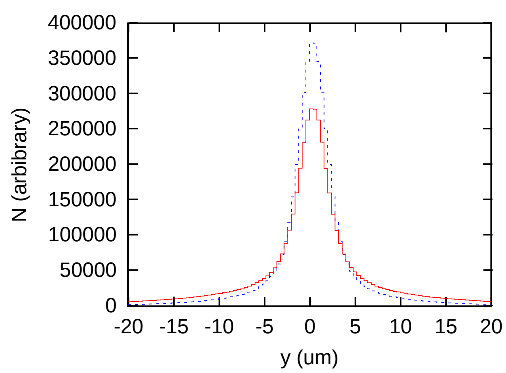

Figure 3: Equilibrium distribution of two beams obtained by the quasi-strong-strong simulation.

The simulation method permits to study the beam-beam limit for circular accelerators with very slow damping time. It may be possible to study even for proton beam, for example the revolution of 1M turn corresponds to 10-100 sec for the circumference of 3,000-30,000 m, which is not so short time. Fig-

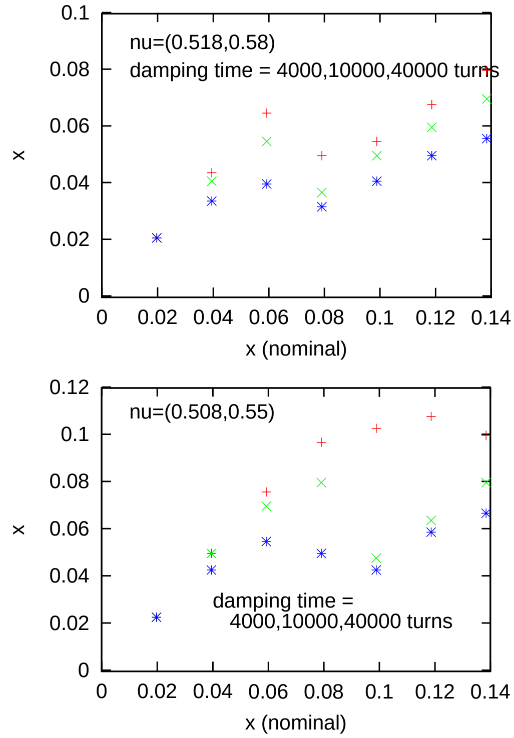

Figure 4: Beam-beam parameter for various damping time at two operating points. Red, green and blue points depict the beam-beam parameters for the radiation damping times of 4000, 10000 and 40000 turns, respectively.

ure 5 shows evolution of the beam-beam parameter for very long term. We again use parameters of the super KEKB except for the damping time. The simulations were performed for the damping time of 1M turn and infinity. Eight lines with four color depict the beam-beam parameters with/without radiation for four nominal beam-beam parameters, 0.02, 0.04, 0.06 and 0.08. The figure shows the synchrotron radiation makes worse the beam-beam parameters. This fact may contradict our experience in which the beam-beam limit in proton colliders is much lower than that of electron-positron colliders. The degradation of the beam-beam parameter seems to be due to radiation excitation. Diffusion due to nonlinear beam-beam interaction is very weak in the symplectic beam-beam system, therefore the degradation of the beam-beam parameter is weak or slow. The synchrotron radiation gives an external diffusion into the symplectic beam-beam system, with the result that the beam-beam parameter is degraded. Similar behavior, which shows no degradation of beam-beam parameter for the beam-beam system, is seen for higher nominal beam-beam parameter around 0.1 [2].

These results show that diffusion plays an important role for the beam-beam limit. Even in symplectic system, diffusion due to nonlinear interaction is

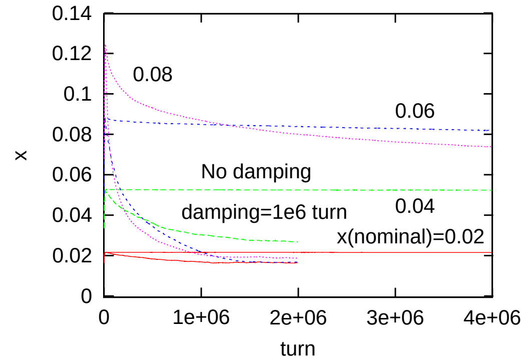

Figure 5: Evolution of the beam-beam parameter for very slow damping. Eight lines with four color depict the beam-beam parameters with/without radiation for four nominal beam-beam parameters, 0.02, 0.04, 0.06 and 0.08.

enhanced by coupling of degree of freedom, i.e., x-y or x-z (crossing angle and dispersion) [2]. There may be external diffusions concerning an actual operation, ripple of electricity, ground oscillation or some other sources. Such the diffusion may be fatal for systems without any damping mechanism. However, if we can realize ideal condition, the potential for the beam-beam limit of proton colliders may be higher than our impression.

Acknowledgments

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