Brief Review of Super-Bunches for Hadron Colliders

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Cancellations in the beam-beam force (1) between two collision points with alternating planes of beam crossing, (2) for a uniform longitudinal bunch profile, and (3) when operating in a regime of large Piwinski angle or, alternatively, with long super-bunches allow for significant increases in hadrom-collider luminosity. The relevant expressions for beam-beam tune shifts and luminosity are revisited. The results are illustrated by considering possible upgrades of the Large Hadron Collider.

1. HISTORY AND MOTIVATION

The CERN Intersecting Storage Rings (ISR) were the first hadron collider. It stored coasting (unbunched) proton beams with currents up to 50 A and it reached a peak luminosity of 1.4x10^{32} cm^{-2}s^{-1}. Despite of 25 years which have passed since, this performance has not yet been achieved by any other hadron collider, as illustrated in Table 1.

Table 1: Commissioning year, beam energy and peak luminosity of all hadron colliders. The Large Hadron Collider (LHC) is still under construction and the LHC upgrade (LHC-II) in the early design phase.

Collider	Commissioning year	Energy [GeV]	Peak Luminosity [10^{32} cm^{-2}s^{-1}]
ISR	1970	31	1.4
SPPS	1980	316	0.06
Tevatron	1987	980	0.4
RHIC	2000	100	0.02 (polarized)
LHC	2007	7000	100
LHC-II?	2017?	>7000?	>1000?

ISR-style quasi-coasting beams were recently identified as a key ingredient for future highest-energy high-luminosity proton colliders (see, e.g., Ref. [1]). In particular, a possible upgrade of the CERN Large Hadron Collider (LHC) can be considered by operating not exactly with coasting beams, but with long "super-bunches", which are confined, e.g., by a barrier bucket rf system [2]. As an illustration, Table 2 compares the nominal and ultimate LHC design parameters with three possible upgrades. The last column shows the pure super-bunch upgrade, while the second-to-last column displays an alternative parameter set, where the number of bunches remains nominal, and only the bunch line density and crossing angle are increased (the second set of numbers in this row refer to 'hollow' or 'flat' bunches with a uniform longitudinal profile, which, for the same beam-beam tune shift, offer 30-40% higher luminosity than bunches of Gaussian shape).

The main advantages of long or "super-bunches" are (1) a cancellation between the head-on and 'long-range' components of the beam-beam tune shift, which is realized by colliding the beams at two interaction points with alternating planes of crossing, (2) the absence of PACMAN bunches otherwise existent at the end of a bunch train, and (3) the avoidance of beam-induced multipacting and electron-cloud build up.

Table 2: Parameters for the nominal and the ultimate LHC, and for three possible upgrades [2].

	nominal	ultimate	upgrade	upgrade
#bunches	2808	2808	2808	1
rms bunch length [cm]	7.6	7.6	7.6, 4.2	7500
rms energy spread [10-4]	1.1	1.1	1.1, 3.7	5.8
beta at IP [m]	0.5	0.5	0.25	0.25
crossing angle[μrad]	300	315	485	1000
beam current [A]	0.56	0.86	1.3,1.3	1.0
luminosity [10^{34} cm^{-2} s^{-1}]	1	2.3	7.3, 9.7	9.0

2. TUNE SHIFT

The cancellation between head-on and 'long-range' components is evident from the formulae for the horizontal and vertical tune shifts for a single interaction point (IP) with horizontal crossing angle [3]:

$$\Delta Q_x = \frac{\lambda r_p \beta^*}{\pi\gamma}\left(\frac{1+\cos\theta}{2}\right)\cos\theta \int_{-l/2}^{l/2}\left(1+\frac{s^2}{\beta^{*2}}\right)\times \left(\frac{1}{s^2\sin^2\theta}\left[1-\exp\left(-\frac{s^2\sin^2\theta}{2\sigma^2}\right)\right] - \exp\left(-\frac{s^2\sin^2\theta}{2\sigma^2}\right)\frac{1}{\sigma^2}\right)ds \quad (1)$$

$$\Delta Q_y = -\frac{\lambda r_p \beta^*}{\pi\gamma}\left(\frac{1+\cos\theta}{2}\right)\int_{-l/2}^{l/2}\left(1+\frac{s^2}{\beta^{*2}}\right)\times \left(\frac{1}{s^2\sin^2\theta}\left[1-\exp\left(-\frac{s^2\sin^2\theta}{2\sigma^2}\right)\right]\right)ds \quad (2)$$

where $\sigma = \sigma(s) = \sigma^*\sqrt{1+s^2/\beta^{*2}}$,

$\sigma^* = \sqrt{\beta^* \varepsilon} = \sqrt{\beta^* \varepsilon_N / \gamma}$, and ε denotes the geometric transverse emittance. Except for the last term in (1), the expressions (1) and (2) are nearly identical and of opposite sign. In case of a vertical crossing, the expressions for the two planes are interchanged. Therefore, by colliding the two beams at two interaction points with alternating crossing, the absolute value of the tune shift in both planes is identical and much smaller than that for a single IP. The same cancellation is also achieved, already for a single IP, if the beams are collided in a crossing plane tilted at 45^o or 135^o [4,5], which, however, may introduce betatron coupling. Figure 1 shows a schematic of a super-bunch hadron collider with alternating crossing at two IPs.

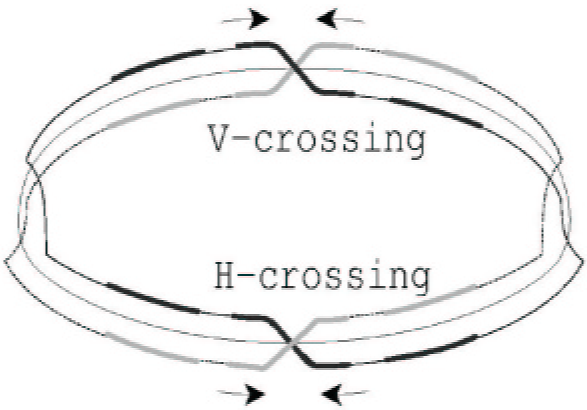

Figure 1: Schematic of a super-bunch hadron collider with alternating crossing at two collision points [6].

3. OPTIMUM LUMINOSITY

As shown in Ref. [3], the luminosity of a conventional hadron collider operating with round bunched Gaussian beams can be increased in proportion to the bunch current, while keeping a constant beam-beam tune shift, by increasing the product of bunch length and crossing angle. If the longitudinal profile is made uniform instead of Gaussian an additional factor of $\sqrt{2}$ is gained [5,7]. Indeed, the total tune shift for flat (uniform) profiles is given by [5]

$$\Delta Q_{tot}^{(flat)} = \Delta Q_x + \Delta Q_y$$
$$= \frac{\lambda \beta^* r_p}{\pi\gamma} \frac{1+\cos\theta}{2} \int_{-l/2}^{l/2} \left(1 + \frac{s^2}{\beta^{*2}}\right) \times$$
$$\left[\frac{\cos\theta - 1}{s^2 \sin^2\theta} \left(1 - \exp\left(-\frac{s^2 \sin^2\theta}{2\sigma^2}\right)\right) \right. \qquad (3)$$
$$\left. - \exp\left(-\frac{s^2 \sin^2\theta}{2\sigma^2}\right) \frac{\cos\theta}{\sigma^2} \right] ds$$

In the case of Gaussian bunches we define the form factor

$$g(s) = \exp\left(-\frac{s^2 (1+\cos\theta)^2}{2\sigma_z}\right),$$

and can then express the total tune shift for two IPs as [5]

$$\Delta Q_{tot}^{(Gauss)} = \Delta Q_x + \Delta Q_y$$
$$= -\frac{\lambda \beta^* r_p}{\pi\gamma} \frac{1+\cos\theta}{2} \int_{-l/2}^{l/2} ds \left(1 + \frac{s^2}{\beta^{*2}}\right)$$
$$\left[\frac{1-\cos\theta}{s^2 \sin^2\theta} \left(1 - \exp\left(-\frac{s^2 \sin^2\theta}{2\sigma^2}\right)\right) \right. \qquad (4)$$
$$+ \exp\left(-\frac{s^2 \sin^2\theta}{2\sigma^2}\right) \frac{\cos\theta}{\sigma^2}$$
$$\left. + \frac{1+\cos\theta}{\sigma_z^2} \left(1 - \exp\left(-\frac{s^2 \sin^2\theta}{2\sigma^2}\right)\right) \right] g(s),$$

The expressions (3) and (4) simplify greatly, if we make a number of simplifying assumptions.

For the case of Gaussian bunches, we consider the limit of a small crossing angle, i.e., $\theta << 1$, we assume that the rms bunch length is larger than the transverse IP beam size, but smaller than the IP beta function, i.e., $\sigma^* < \sigma_z < \beta^*$, and we are interested in the regime of a large Piwinski angle: $\theta\sigma_z/(2\sigma^*) >> 1$ [3]. Under the above assumptions, the tune shift for Gaussian bunches, Eq. (4), can be written as

$$\Delta Q_{tot}^{(Gauss)} \approx \frac{N_b r_p \beta^*}{2\pi\gamma\sigma^* \sqrt{\sigma^{*2} + \theta^2 \sigma_z^2 / 4}} \approx \sqrt{\frac{2}{\pi}} \frac{r_p \beta^*}{\gamma\sigma^* \theta} \hat{\lambda}, \qquad (5)$$

where $\hat{\lambda}$ denotes the peak line density, and the luminosity of Gaussian bunches simplifies to

$$L^{Gauss} \approx \frac{f_{coll} N_b^2}{4\pi\sigma^* \sqrt{\sigma^{*2} + \theta^2 \sigma_z^2 / 4}} \approx \frac{f_{coll} N_b^2}{2\pi\sigma^* \theta\sigma_z} \qquad (6)$$

Combining the last two expressions, the luminosity for Gaussian bunches becomes [3,5,6]

$$L^{Gauss} \approx \frac{f_{coll} \gamma \varepsilon_N}{r_p^2 \beta^*} \Delta Q_{tot}^2 \pi \sqrt{1 + \frac{\theta^2 \sigma_z^2}{4\sigma^{*2}}} \approx \frac{f_{coll} \gamma \varepsilon_N}{r_p^2 \beta^*} \Delta Q_{tot}^2 \frac{\pi\theta\sigma_z}{2\sigma^*} \qquad (7)$$

which explicitly shows that, for a constant total tune shift, the luminosity increases in proportion to $\theta\sigma_z$.

For the case of uniform super-bunches, we also consider a small crossing angle, $\theta << 1$, and in addition, we demand that the crossing angle is larger than the rms beam divergence, i.e., $\theta >> \sqrt{\varepsilon / \beta^*}$, and that the total bunch length l_{flat} is larger than the effective length of the interaction region, or $l_{flat} > 10\sigma^*/\theta$.

The total tune shift for super-bunches, Eq. (3), then reduces to

$$\Delta Q_{tot}^{(flat)} \approx -\frac{\lambda r_p \sqrt{2}^{*}}{\gamma\sqrt{\pi}}\sqrt{\frac{\beta^*\gamma}{\varepsilon_n}}\frac{1}{\theta}\mathrm{Erf}\left(\frac{l\theta}{2\sqrt{2}\sigma^*}\right)$$
$$\approx -\frac{\lambda r_p\sqrt{2}}{\sqrt{\pi}\gamma}\sqrt{\frac{\beta^*\gamma}{\varepsilon_n}}\frac{1}{\theta} \approx \sqrt{\frac{2}{\pi}}\frac{r_p\beta^*}{\gamma\sigma^*\theta}\lambda \quad (8)$$

and the luminosity to

$$L^{flat} \approx \frac{f_{coll} l_{flat}\lambda\gamma}{2\pi}\left(\frac{\lambda}{\varepsilon_n}\right)\times$$
$$\int_{-l_{\det}/(2\beta^*)}^{l_{\det}/(2\beta^*)} \frac{1}{1+u^2}\exp\left[-\frac{\beta^{*2}\theta^2}{4\sigma^{*2}}\frac{u^2}{1+u^2}\right]du \quad (9)$$
$$\approx \frac{f_{coll} l_{flat}\lambda^2}{\sqrt{\pi}}\frac{1}{\theta}\frac{1}{\sqrt{\beta^*\varepsilon_n/\gamma}}.$$

Combining the last two equations, we get an expression similar to (7) for Gaussian bunches, namely [5,7]

$$L^{flat} \approx \frac{f_{coll}\gamma\varepsilon_n}{r_p^2\beta^*}\Delta Q_{tot}^2\frac{\sqrt{\pi}\theta l_{\mathrm{flat}}}{2\sigma^*} \quad (10)$$

which is proportional to the crossing angle θ and to the total length of the super-bunch l_{flat}.

More succinctly, for the purpose of comparison, we can also express the luminosities in terms of the total tune shift and the total bunch population N_b. For Gaussian bunches we obtain [5,7]

$$L^{Gauss} \approx \frac{1}{2}\frac{f_{coll}\gamma}{r_p\beta^*}\Delta Q_{tot}N_b \quad (11)$$

and for super-bunches with a uniform profile

$$L^{flat} \approx \frac{1}{\sqrt{2}}\frac{f_{coll}\gamma}{r_p\beta^*}\Delta Q_{tot}N_b \quad (12)$$

As indicated already in Table 2, a factor of about $\sqrt{2}$ increase in luminosity is gained as well, if we simply replace the Gaussian profile of a bunched beam by a flat uniform longitudinal shape, as long as we operate in the regime of a large Piwinski angle. Flat or 'hollow' bunches can be generated by radiofrequency gymnastics in the CERN PS booster and they are already available from the LHC injector chain [8].

As an example for the usefulness of the above formulae, Fig. 2 shows the possible luminosity gain in the LHC that can be achieved by increasing the Piwinski angle with bunches of either Gaussian or uniform profile.

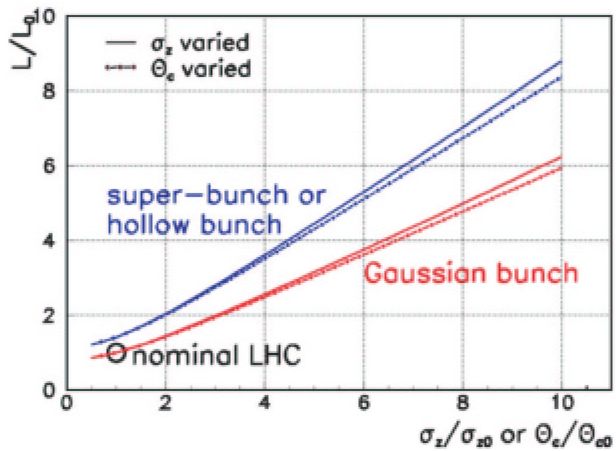

Figure 2: Relative luminosity gain for the Large Hadron Collider as a function of relative increase in crossing angle or bunch length for a uniform bunch profile or super-bunches (top) and for regular Gaussian bunches (bottom). The vertical axis is normalized to a base luminosity at the beam-beam limit with two IPs of L_0=2.3x10^{34} cm^{-2}s^{-1} and the horizontal axis to an rms bunch length of σ_{z0}=7.6 cm or to a crossing angle θ_0=300 μrad [5,7].

4. CONCLUSIONS

By colliding either bunches with a large Piwinski angle or long super-bunches, the LHC luminosity can be increased about 10 times to 10^{35} cm^{-2}s^{-1} for the same total tune shift as in the 'ultimate' design and only moderately increased total beam current. Problems expected to arise in the nominal LHC due to PACMAN bunches [9] and electron cloud would be solved simultaneously, if super-bunches are employed [5,7].

Acknowledgement

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