”CSR Effects in e+e- Storage Rings”

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Outline of the Talk

- Introduction and motivation
- CSR radiation, wake, and impedance
- CSR instability: linear theory and simulation of nonlinear regime
- Experimental results
- CSR instability in rings with wigglers, shielding and single mode regime
- Summary

Introduction

- Over the last several years, there have been observations of bursts of radiation in light source storage rings, with $\lambda < \sigma_z$ (NSLS VUV, SURF at NIST, ALS at LBL, BESSY). Intensity $\propto N^2$, polarization of synchrotron radiation.
- Those observations renewed interest and initiated intensive theoretical and computational studies of the instability driven by CSR.
- In numerical simulations of bunch compressors for next generation FELs, it was found that a CSR instability may amplify initial small-scale density perturbations in the bunch.

CSR Workshops

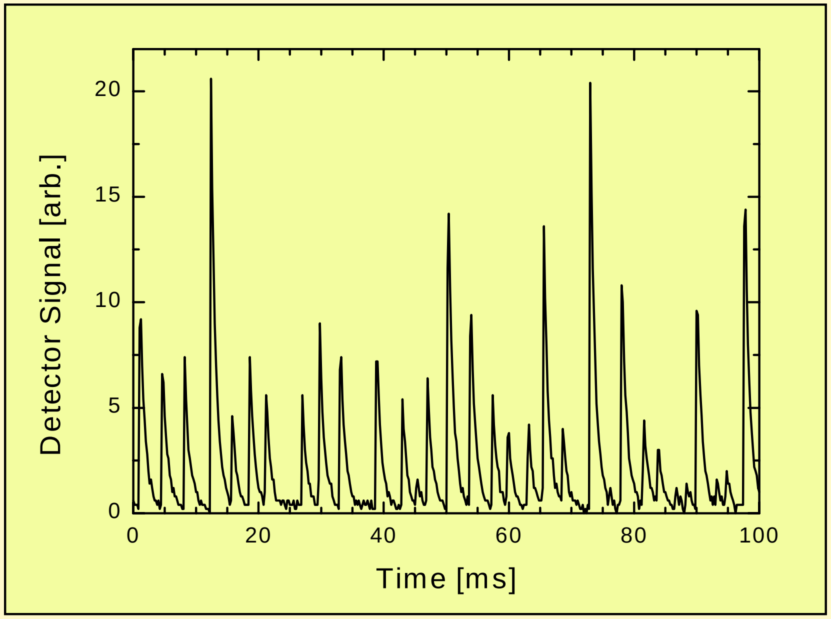
Berlin, Feb. 2002:

http://www.desy.de/csr/csr_workshop_2002/csr_workshop_2002_index.html

Napa, CA, Oct. 2002: http://www-als.lbl.gov/LSWorkshop/index.html

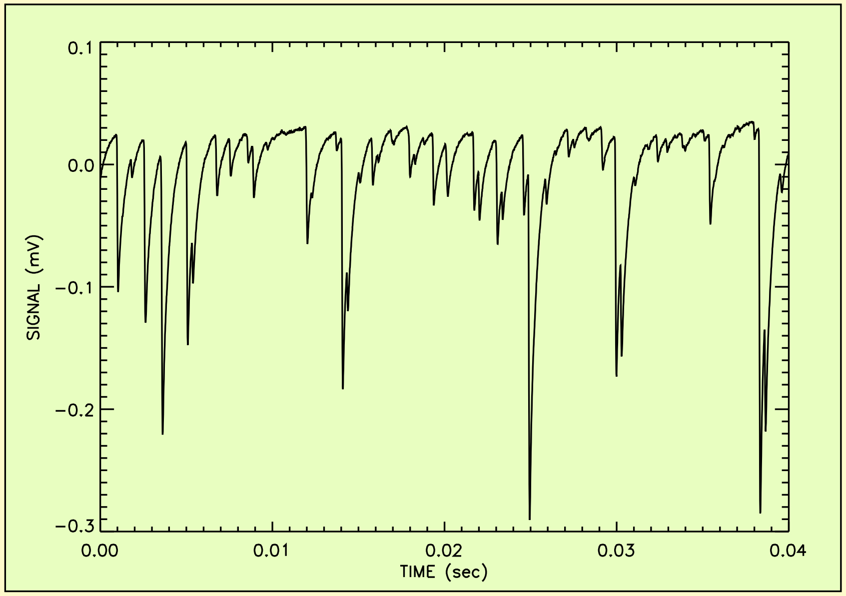
CSR satellite meeting at PAC03.

Observations

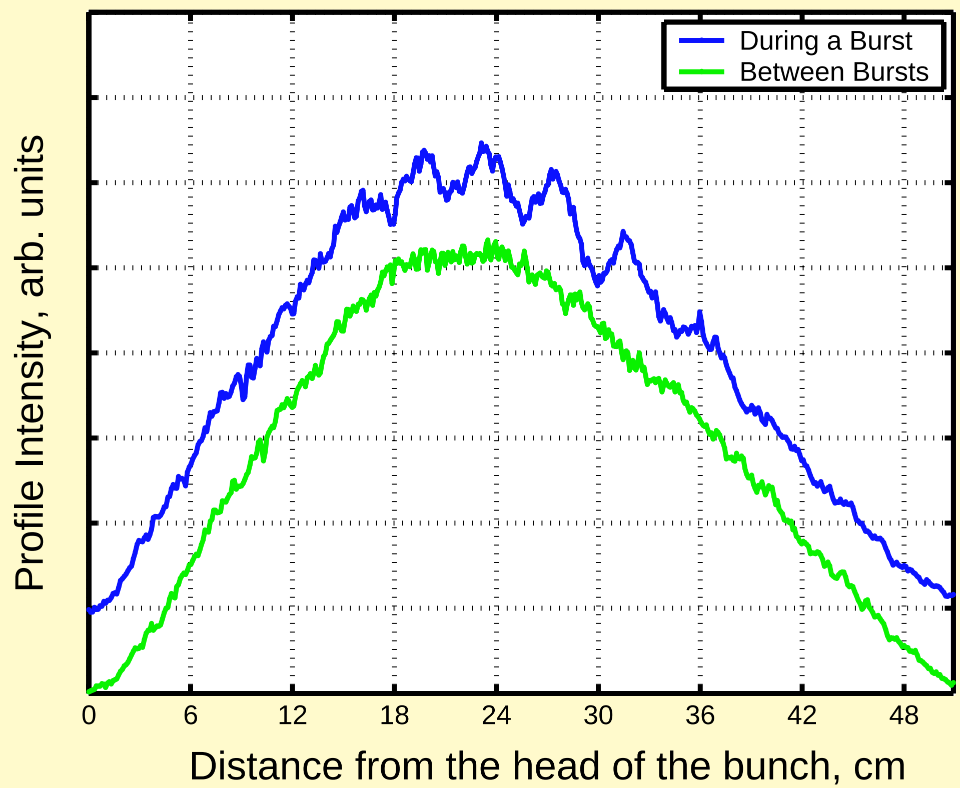


Bursts of CSR (far infrared) in NSLS VUV ring [Carr et al., NIM, 387, (2001)]. Frequency range from $\sim$ 6 to $\sim$ 60 GHz.

Bursts of CSR in BESSY II [G. Wustefeld et al.]. Typical wavelength $\sim$ 0.5 mm.



Observations



Typical bursting and non-bursting beam profiles in NSLS-VUV ring [Podobedov et al., PAC 2001]. Modulation corresponds to $f = 6 - 7$ GHz.

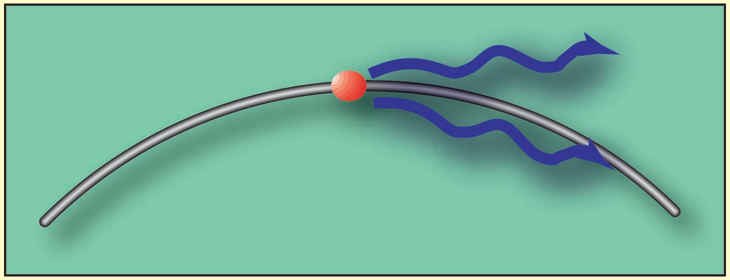
Microwave Instability

It was assumed that those observations are an evidence of microwave instability caused by some kind of impedance in the machine. Experimentally it is difficult to figure out the origin of this impedance.

“Traditional” geometric impedance decreases with frequency, and unlikely to be a source of the instability with wavelength of a fraction of mm.

CSR is a universal source of impedance at high frequency. This impedance remains even in a machine with ideally smooth perfectly conducting vacuum chamber.

CSR – Review of Theory



Low-frequency radiation, $\omega \ll$ critical frequency ($\omega_H = eB/\gamma mc$)

$$\frac{dP}{d\omega} = \frac{3^{1/6}}{\pi}\Gamma\left(\frac{2}{3}\right)\left(\frac{\omega}{\omega_H}\right)^{1/3}\frac{e^2\omega_H}{c^2}$$

- Angular spread: $\theta \sim (\lambda/R)^{1/3}$. ($\lambda = c/\omega$, R - bending radius)
- Formation length: $l_f \sim \lambda/\theta^2 \sim (\lambda R^2)^{1/3}$ – the length after which the field “disconnects” from the particle. “Overtaking” distance.
- Transverse size: $l_\perp \sim \lambda/\theta \sim (\lambda^2 R)^{1/3}$. This is the minimal spot size to which the radiation can be focused. It is also a transverse coherence size: electrons separated by $l_\perp$ in transverse direction, radiate incoherently.

CSR – Review of Theory

- Shielding by conducting walls: if the walls are closer than $l_\perp$, the field lines during circular motion close onto the conducting walls, rather than disconnect from the charge.

Numerical example: $R = 10$ m, $\lambda = 1$ mm.

$\theta \sim 25$ mrad, $l_f \sim 25$ cm, $l_\perp \sim 6$ mm.

Steady state radiation occurs for $l_{bend} \gg l_f$. Transient effects are important if $l_f \gtrsim l_{bend}$.

Coherent vs Incoherent Radiation

Typically, there are $N \sim 10^{10}$ electrons in the bunch, and radiation of each electron interferes with others. Assuming transverse coherence (1D model for the beam),

$$\left.\frac{dP}{d\omega}\right|_{\text{bunch}} = \frac{dP}{d\omega}\left(N + N^2|\hat{f}(\omega)|^2\right)$$

where $\hat{f}(\omega) = \int_{-\infty}^{\infty} dz f(z) e^{i\omega z/c}$.

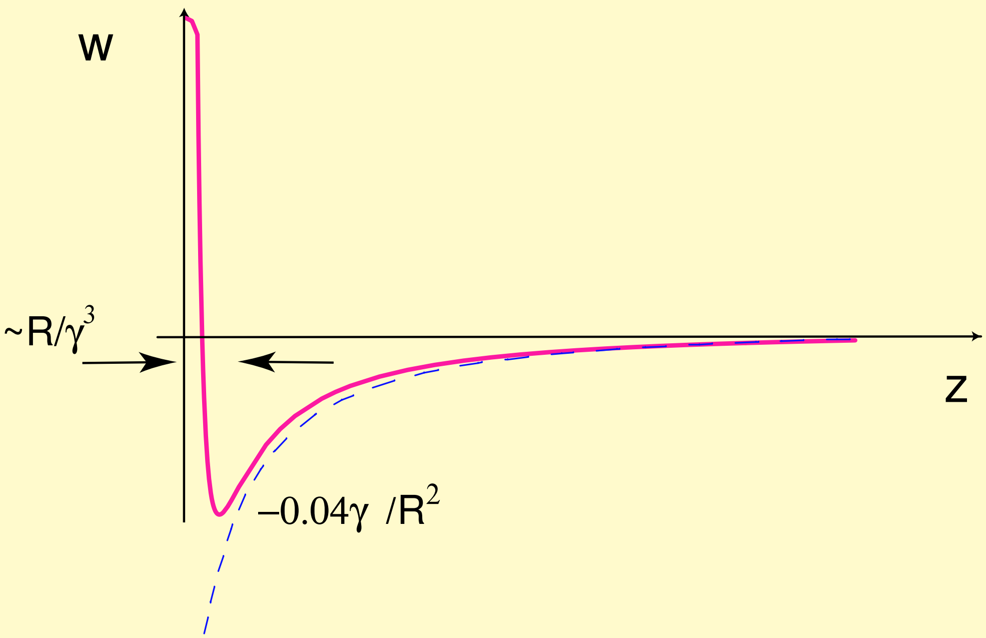
For a smooth distribution function, $\hat{f}(\omega)$ vanishes for $\lambda \lesssim \sigma_z$. However, an initial beam density modulation with $\lambda \lesssim \sigma_z$ would radiate coherently! If the radiation reaction force drives the growth of the initial fluctuation, one can expect an instability which leads to micro-bunching of the beam and increased coherent radiation at short wavelengths.

Earlier Papers

- Goldreich and Keeley, 1971—CSR instability in astrophysics, neglected “momentum compaction”
- Warnock and Bane, 1995—numerical simulation
- J.-M. Wang, 1998

Radiation Reaction Force—CSR Wake

A relativistic particle moving in vacuum in a circular orbit of radius R, in steady state, generates a CSR wake (per unit length of path) (Ioganssen and Rabinovich, 1960; Murphy, Krinsky and Gluckstern, 1995; Derbenev et al., 1995).



The wake is localized in front of the particle. For $z \ll R$ and $z \gg R/\gamma^3$,

$$w(z) \approx -\frac{E_\parallel}{q} = -\frac{2}{3^{4/3} R^{2/3} z^{4/3}}$$

Area under $w(z)$ is zero.

Radiation Reaction Force—CSR Wake

Approximations:

- Shielding is neglected
- Small transverse beam size, $\sigma_\perp \lesssim l_\perp \ll (\lambda^2 R)^{1/3}$.
- Transient effects at the entrance to and exit from the magnet are neglected:

$$l_{\text{magnet}} > l_f$$

  Transient effects in a short magnet—Saldin et al., 1997, Emma and Stupakov 2002.

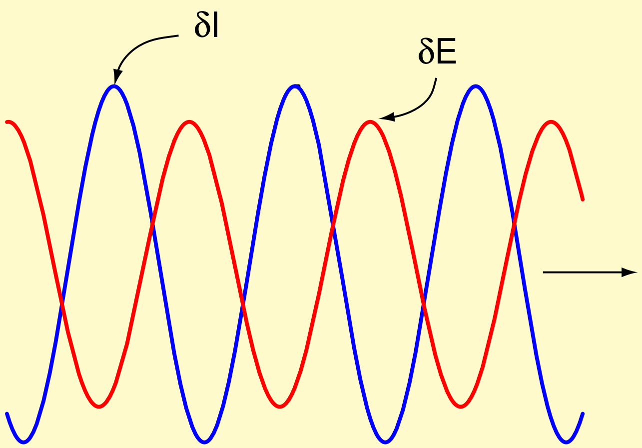
CSR Impedance Drives Instability

Longitudinal CSR impedance

$$Z(k) = \frac{1}{c}\int_0^\infty dz w(z) e^{-ikz} = \frac{2}{3^{1/3}}\Gamma\left(\frac{2}{3}\right) e^{i\pi/6}\frac{k^{1/3}}{cR^{2/3}}.$$

$\mathrm{Re}\,Z$ is related to the energy loss of a charge—due to radiation:

$$\frac{dP}{d\omega} = \frac{e^2}{\pi}\mathrm{Re}Z$$



- Due to the CSR wake, δI induces energy modulation in the beam, $\delta E = Z \times \delta I$
- Momentum compaction of the ring translates δE into $\delta n \to \delta I$. Under certain conditions, the final δI is greater than the initial one.

Cold Beam Approximation

Assume that the wavelength is much shorter than the bunch length, $\lambda \ll \sigma_z$—one can use coasting beam approximation. Neglect energy spread in the beam. Then the theory gives $\omega^2 \propto ikZ(k)$.

$$\mathrm{Im}\,\omega = Ac\left(\frac{I}{I_A}\right)^{1/2}\left(\frac{|\eta|}{\gamma}\right)^{1/2}\frac{k^{2/3}}{R^{1/3}}$$

$A = 1.2$ for positive η, and 0.7 for negative η, I – peak current, $I_A = 17$ kA.

Example: ”ALS type” ring, $I = 300$ A, $R = 10$ m, $\eta = 10^{-3}$, 1 mm wavelength:

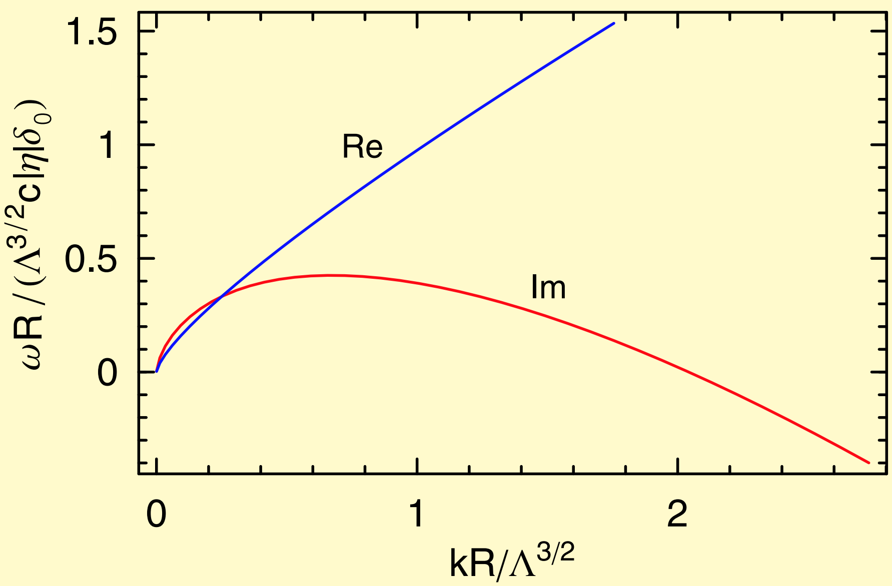
$\mathrm{Im}\,\omega \approx 0.23\ \mu s\,,$ Gain length ≈ 70 m

Finite Energy Spread of the Beam

Use Keil-Schnell theory (Heifets and Stupakov, 2002). The dispersion relation

$$\frac{i r_0 c^2 Z(k)}{\gamma} \int \frac{d\delta\,(df/d\delta)}{\omega + ck\eta\delta} = 1\,,$$

$r_0 = e^2/mc^2$, $f(\delta)$ – energy distribution function.



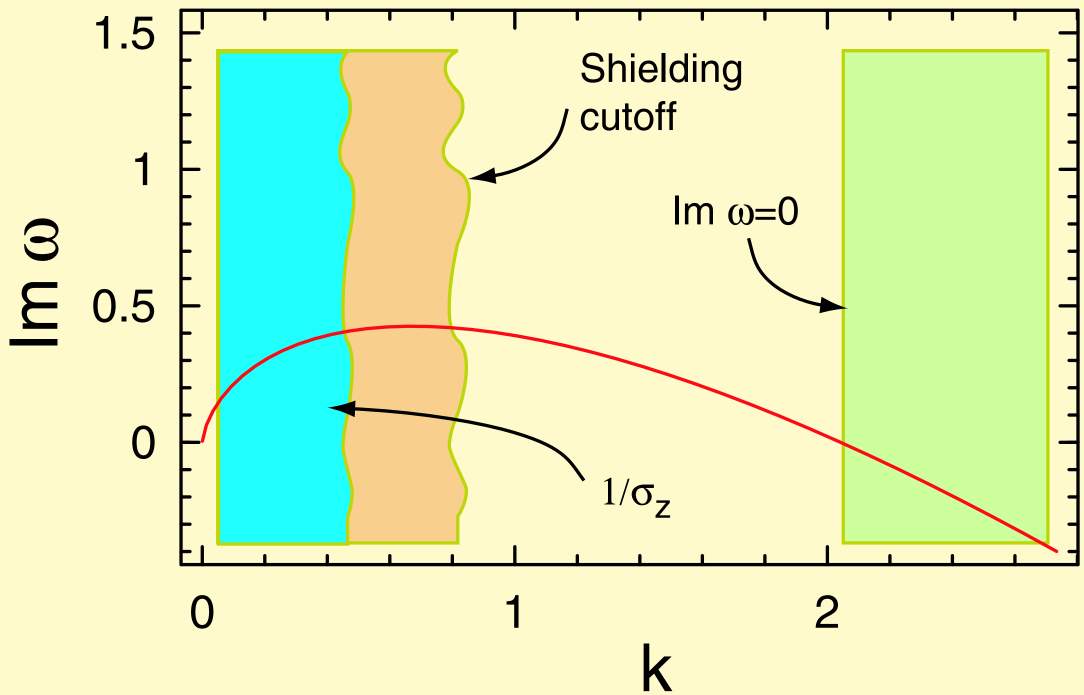
The beam is unstable for such wavelengths that

$$kR < 2.0\Lambda^{3/2}.$$

$$\Lambda = \frac{1}{|\eta|\gamma\delta_0^2}\frac{I}{I_A}\frac{R}{\langle R\rangle}$$

Energy spread introduces Landau damping and stabilizes short wavelengths.

Wall Shielding



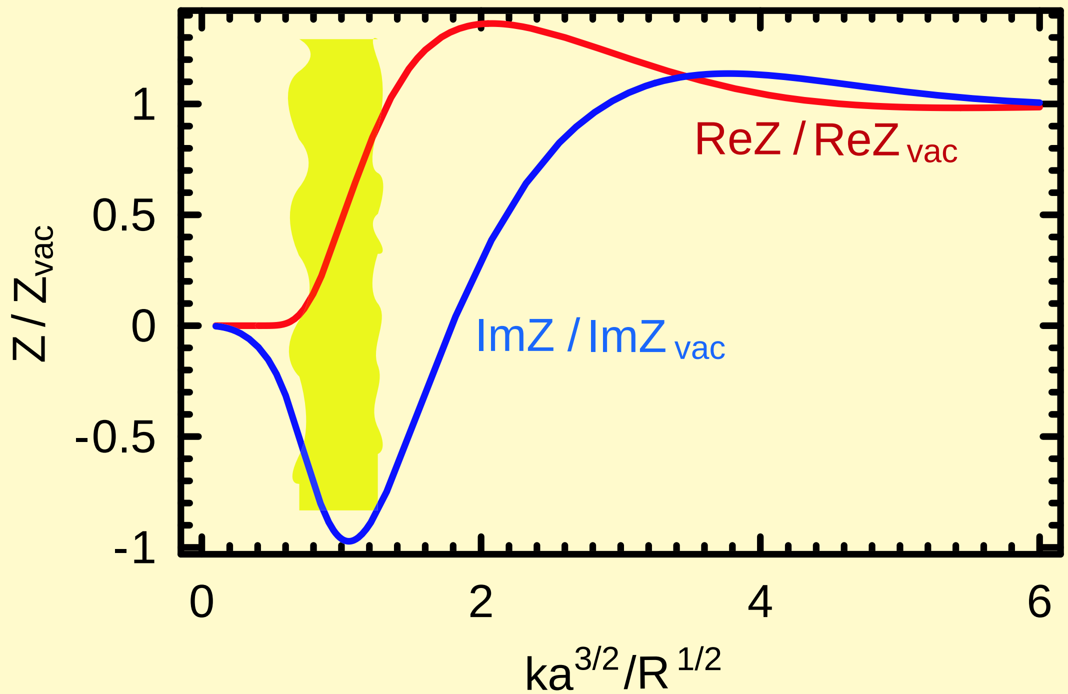
Wall shielding of CSR and finite length of the bunch suppresses the instability at large wavelengths.

Shielding occurs for $\lambda > \lambda_{\text{shield}} \sim R^{-1/2} a^{3/2}$

Unstable range of wavelenghts: $\lambda_{\text{shield}} > \lambda > \lambda_{\text{th}}$. The instability is suppressed if $\lambda_{\text{shield}} < \lambda_{\text{th}}$.

Shielding in Parallel Plates Model

Perfectly conducting parallel plates model is solvable analytically (Murphy, Krinsky and Gluckstern, 1995).



Shielding occurs for $\lambda > \lambda_{\text{shield}} \sim R^{-1/2} a^{3/2}$

CSR Instability in Existing Rings

Accelerator	ALS	LER PEP-II	LER PEP-IIU	LER KEKB*
E (GeV)	1.5	3.1	3.1	3.4
η	$1.41 \cdot 10^{-3}$	$1.31 \cdot 10^{-3}$	$1.0 \cdot 10^{-3}$	$1 \cdot 10^{-4}$
δ_0	$7.1 \cdot 10^{-4}$	$8.1 \cdot 10^{-4}$	$8.1 \cdot 10^{-4}$	$7 \cdot 10^{-4}$
$\langle R \rangle$ (m)	31.3	350	350	480
R (m)	4	13.7	13.7	16.3
a (cm)	2	2.5	2.5	2.5
I_b (mA)	30	2	2	1
σ_z (cm)	0.7	1.2	0.6	1
λ_{shield} (cm)	0.14	0.1	0.1	0.1
λ_{th} (cm)	$4.7 \cdot 10^{-3}$	1.3	0.3	0.015

*) Parameters from K. Ohmi.

Discussion

- Effect of transverse emittance was neglected:

$$\frac{\beta}{R} \ll \frac{\lambda\!\!\!^{-}}{\sigma_x}$$

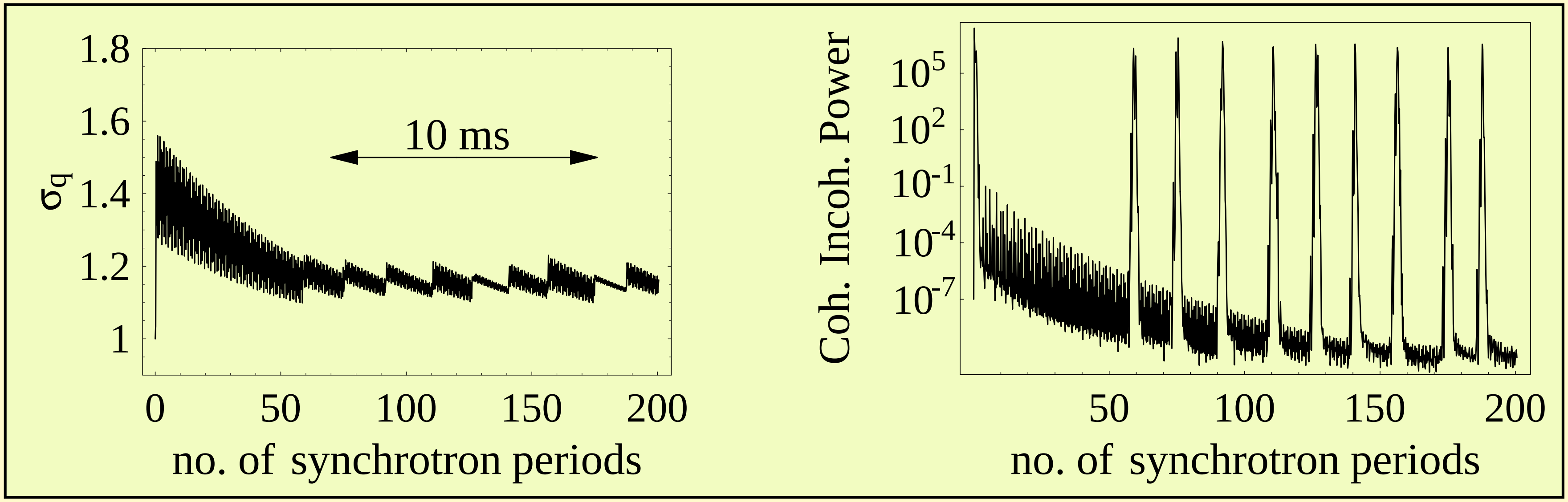
- For very short wavelengths, the CSR wake is not valid, $\sigma_\perp \lesssim l_\perp \sim (\lambda\!\!\!^{-\,2} R)^{1/3}$.

- Synchrotron damping γ_d due to incoherent radiation was neglected. $\mathrm{Im}\,\omega \to \mathrm{Im}\,\omega - \gamma_d$

- Formation time for the radiation should be smaller than the instability growth time, $I/I_A \ll \delta_0 \gamma$

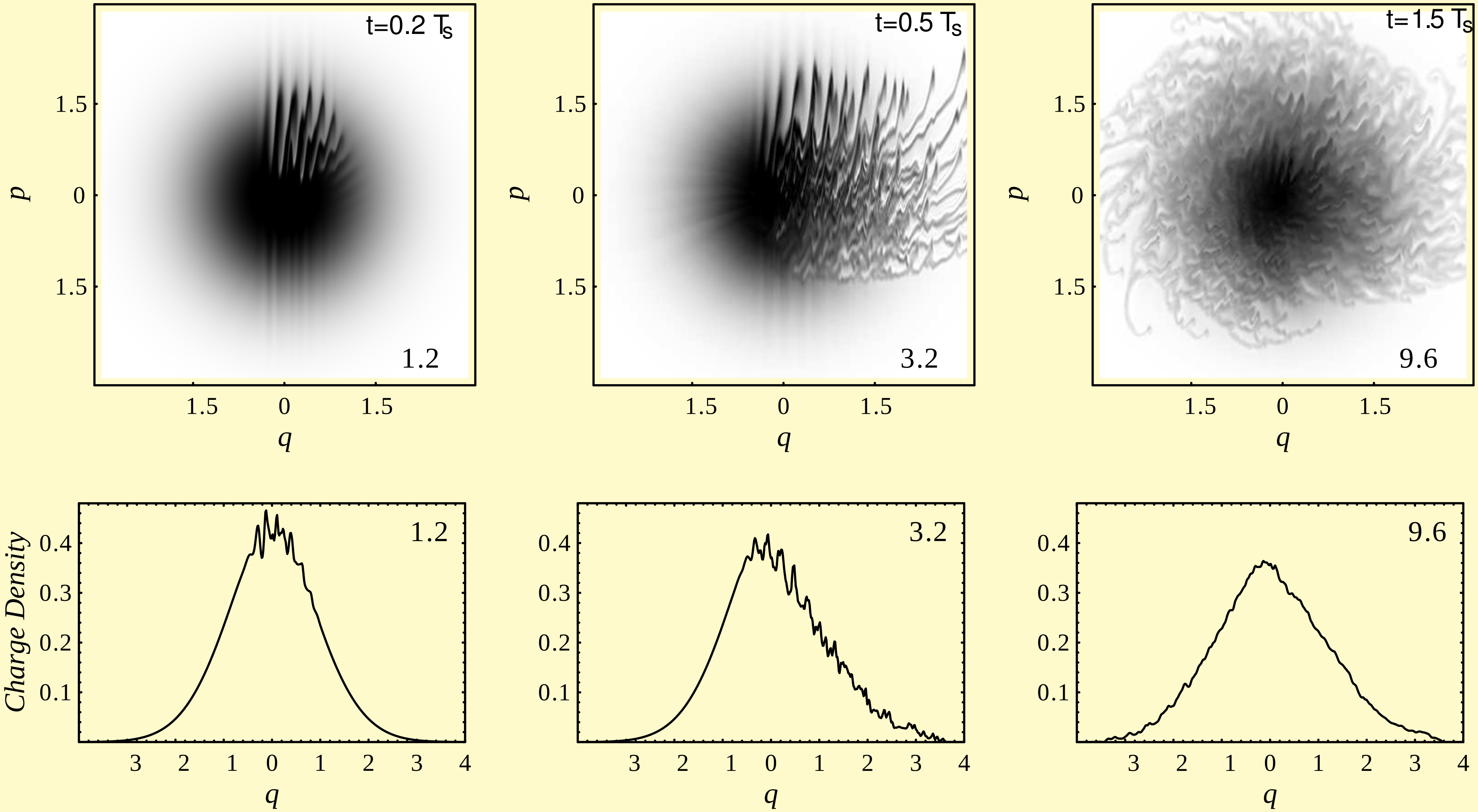
Nonlinear Regime—Computer Simulations

Not much can be done analytically. Computer simulation—Venturini and Warnock, PRL, 224802, 2002. Numerical solution of Vlasov-Fokker-Planck equation, including CSR shielding with parallel plates and damping and quantum fluctuations due incoherent radiation.



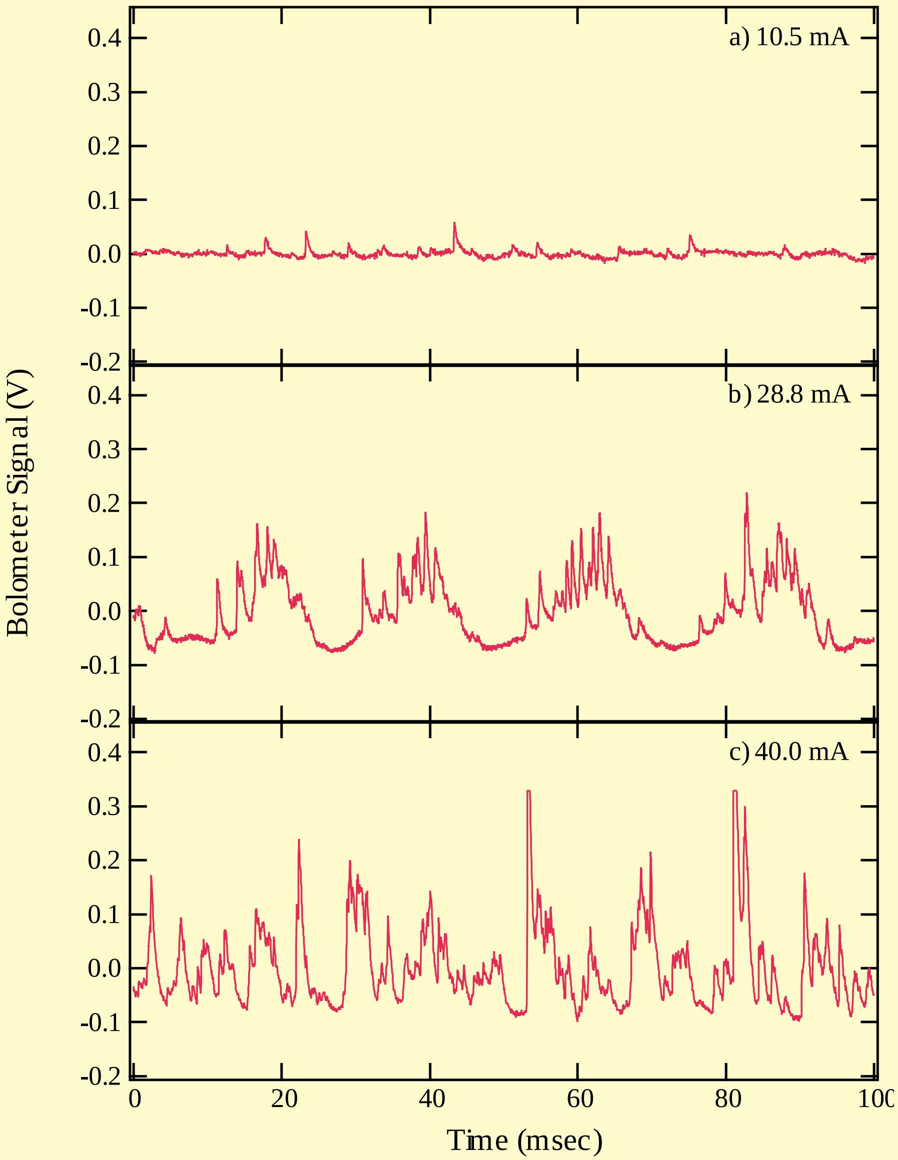
Shown are normalized bunch length and ratio of coherent to incoherent power for NSLS VUV ring.

Nonlinear Regime—Computer Simulations



Courtesy of R. Warnock

ALS Experiment

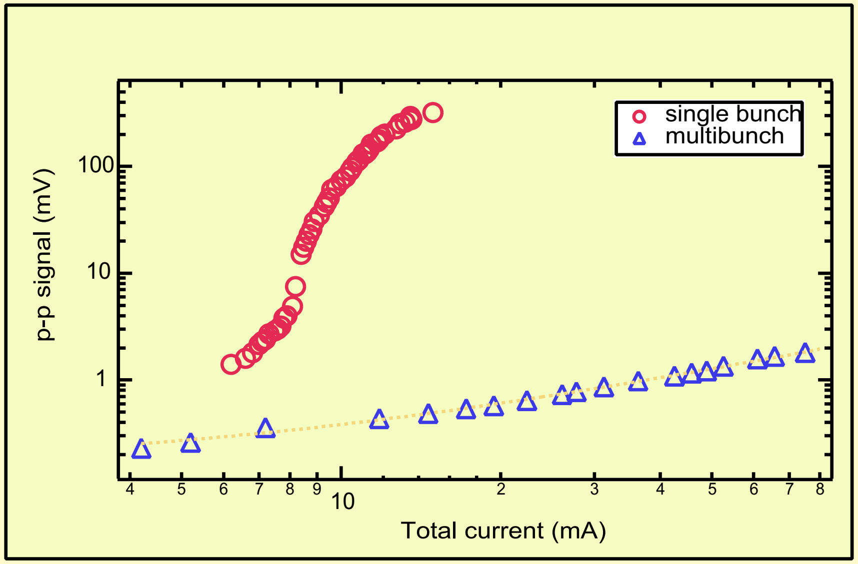


J. Byrd et al., PRL, 224801 (2002). Beam energy varied from 1.2 to 2 GeV.

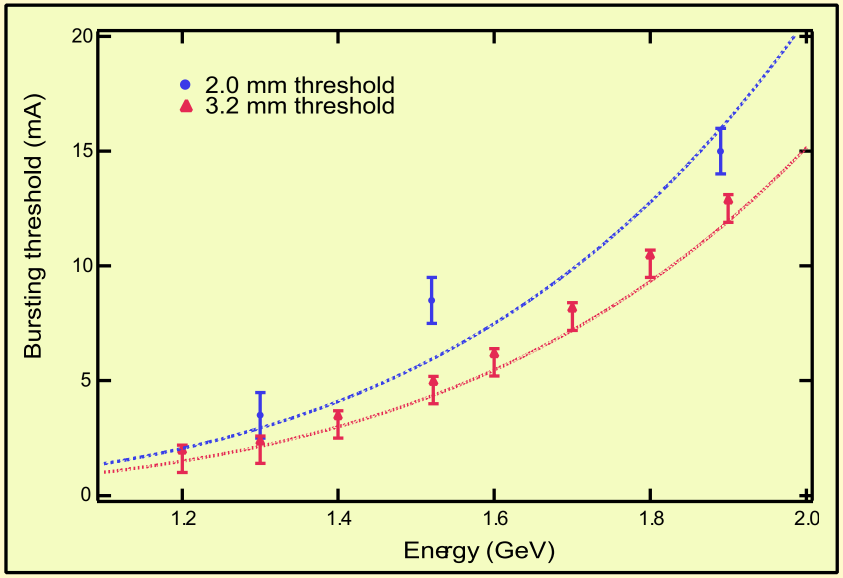
Bolometer signal as a function of total current for single and multibunch operation.

Spectrum was measured by 94 GHz microwave detector and Si bolometer (up to $\lambda = 100\ \mu$m).

ALS Experiment



Bolometer signal as a function of total current for single and multibunch operation (300 bunches).



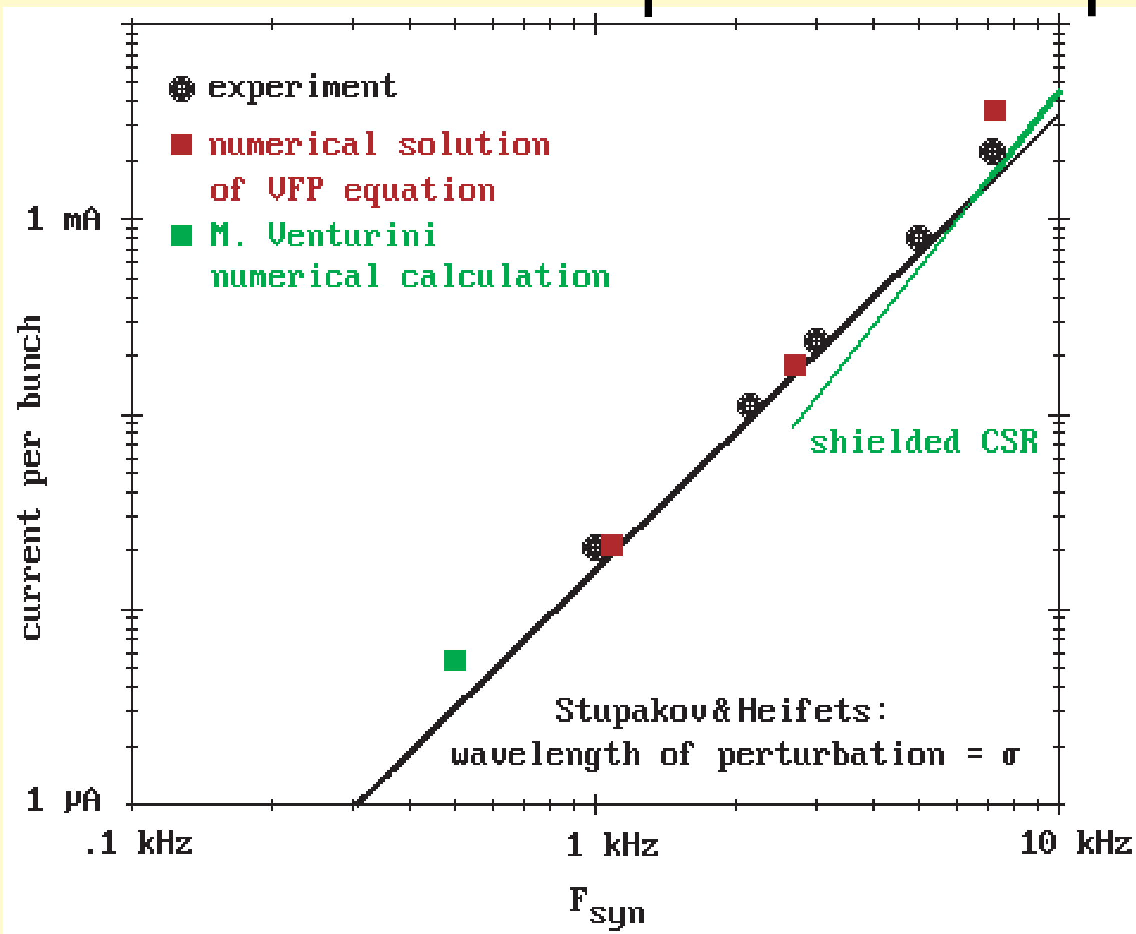
Bursting threshold as a function of electron beam energy at 3.2 and 2 mm wavelength.

Very good agreement with theory for threshold of the instability.

BESSY Experiment

Threshold of the instability vs bunch length.

1.9 ps 19 ps



Courtesy P. Kuske, PAC03.

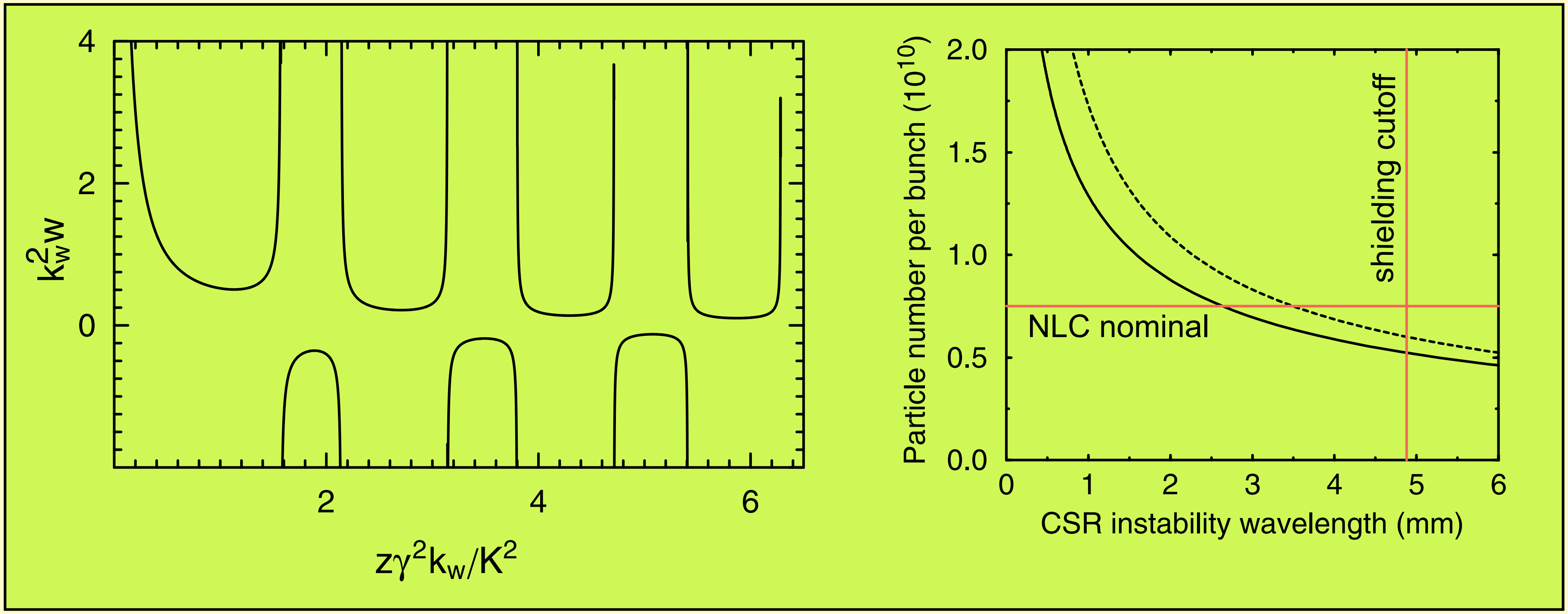
CSR Instability in Ring with Wigglers

How do wigglers affect the CSR instability?

NLC damping ring: $L_w = 46$ m, $C = 300$ m, $B = 2.15$ T, $\lambda_w = 27$ cm.

Need the CSR wake for the undulator. Saldin et. al., 1998–general treatment; J. Wu et al., 2002—$K \gg 1$. Assumptions: infinitely long wiggler, steady-state wake, and average wake over the wiggler period.

Wiggler CSR Wake and Stability in NLCDR



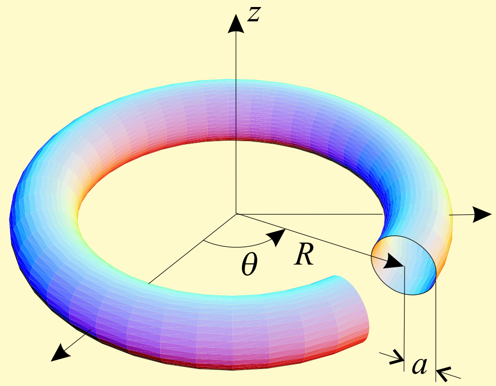
Singularity at $z = n\lambda_r$. Impedance for long wavelength

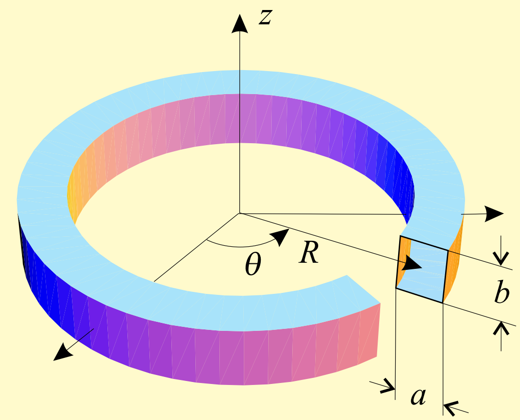
$$Z(\omega) \approx \frac{Z_0\omega}{4c}\left[1 - \frac{2i}{\pi}\log\frac{\omega}{\omega_w}\right]$$

$\omega_w = 4\gamma^2 c k_w / K^2$. Wigglers generate less CSR impedance at long wavelength than dipoles.

Shielding and Discrete Modes

Near the shielding threshold, $\lambda \sim a^{3/2}/R^{1/2}$, the vacuum CSR impedance is not valid.



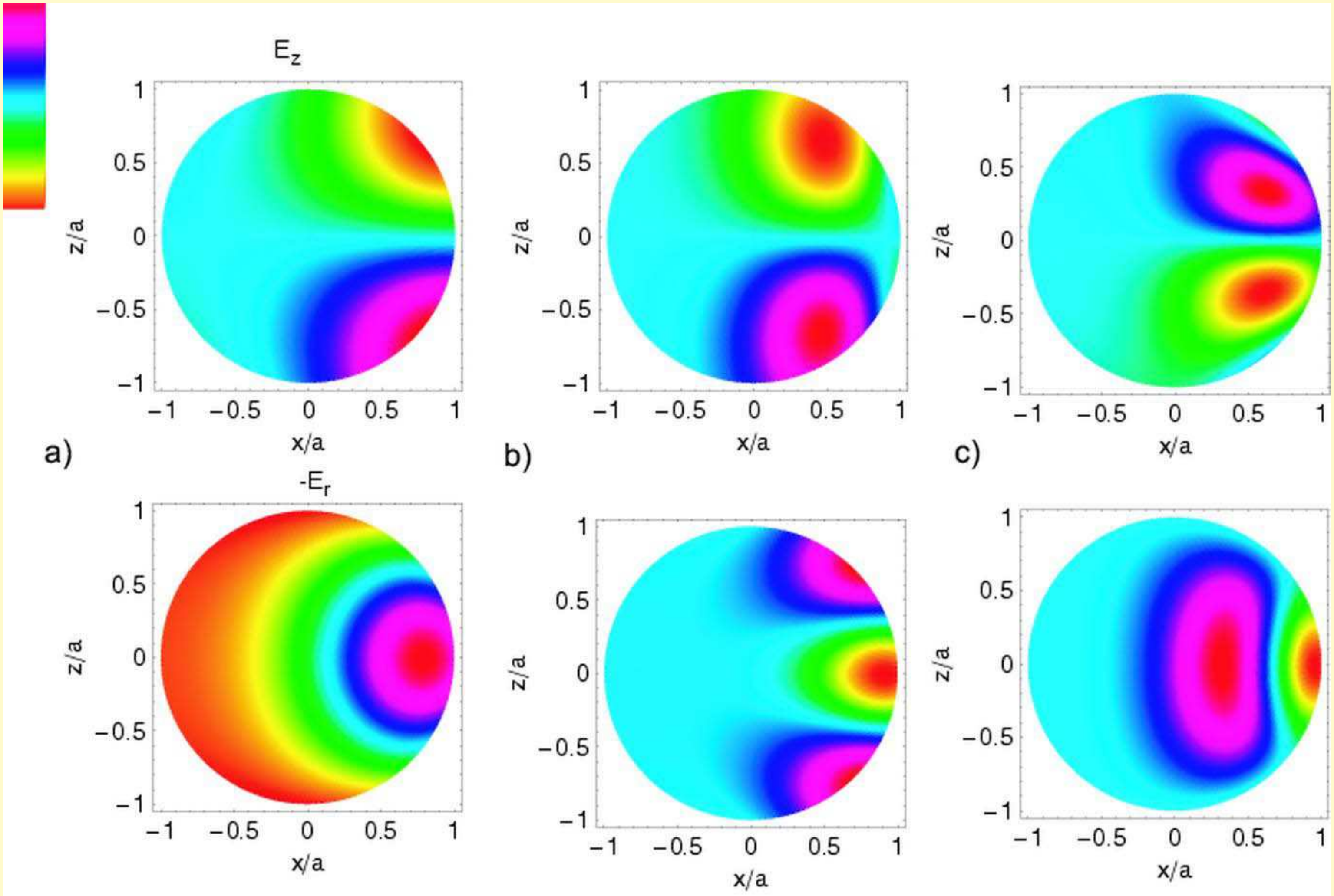


In the model of a toroidal waveguide with perfectly conduction walls and circular orbit, there are discrete synchronous modes that interact with the beam (Warnock and Morton, K.-Y. Ng, et al., late 80’s). New analysis of the shielded CSR impedance (Stupakov and Kotelnikov, 2002) deals with arbitrary shape of the toroid cross section.

Each synchronous mode in the toroid is characterized by frequency $\omega_n = ck_n$ and the loss factor κ_n (per unit length)

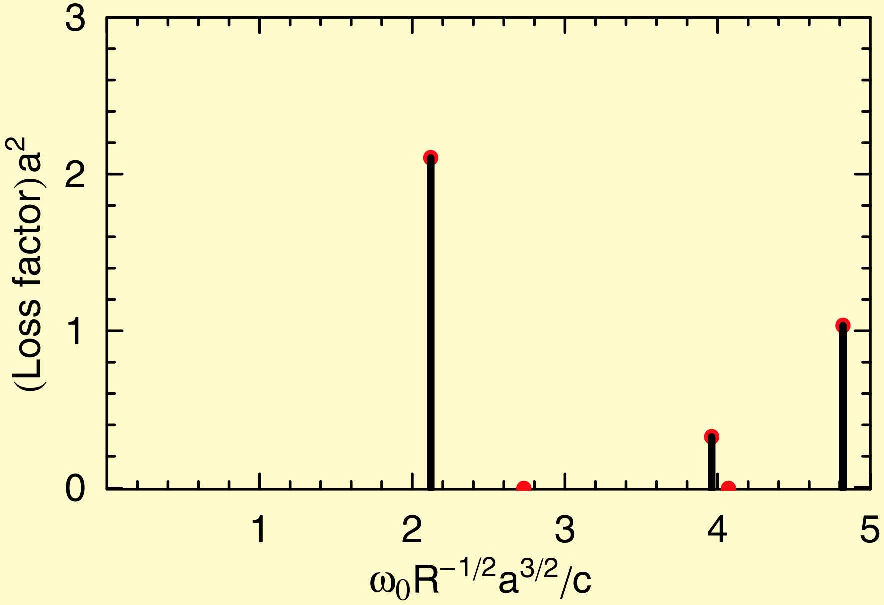
$$w_n(z) = 2\kappa_n \cos\left(\frac{\omega_n}{c} z\right)$$

Field Distributions in Toroidal Modes



Wake of a Single Toroidal Mode

Loss factors for a round cross section toroid.



Each mode has a group velocity $v_g < c$, the wake is behind the particle. For lowest modes, $\kappa_n \sim 1/a^2$, $\lambda_n \sim a^{3/2}/R^{1/2}$.

The theory (Heifets and Stupakov, 2003) parallels 1D SASE FEL, analog of the Pierce parameter

$$\rho = \left[\frac{I}{I_A}\frac{\eta\kappa}{k_0^2\gamma}(1-\beta_g)\right]^{1/3}$$

Growth rate

$$\operatorname{Im}\omega = \frac{\sqrt{3}}{2}\omega_n\rho$$

Summary

- Over the last few years, there has been a remarkable progress in understanding of microbunching and related to it coherent synchrotron radiation in electron and positron rings, both experimentally and theoretically/computationally.
- Based on a CSR wake, a linear theory was developed. It predicts thresholds of the instability and the growth rate, and is in a good agreement with recent experiments on ALS and BESSY. Simulations show qualitative agreement with observed patterns of CSR bursts. The wiggler impedance is now included into the CSR instability theory.
- More work is needed to understand the CSR instability near threshold. Experimental studies are very desirable.