• **Head-tail asymmetry** was observed at SLAC SLC damping ring, but has never been observed in proton machines.

![Figure 1: Potential-well distortion of bunch shape for various beam intensities for the SLAC SLC damping ring.](image)
• Haissinski equation

\[
\rho(\tau) = \rho_0 \left[ -\frac{\tau^2}{2\sigma^2} - \alpha_R N \int_{0}^{\tau} \rho(\tau')d\tau' \right]
\]

\[\uparrow \quad U_{\text{rf}}(\tau) \quad \uparrow \sim \rho_0 \tau\]

gives shift of peak:

\[
\frac{\tau}{\sigma_\tau} = \frac{\alpha_R N}{\sqrt{2\pi}} = \frac{e^2 \beta^2 ENR_s}{-\sqrt{2\pi}\eta T_0 \sigma_E^2}
\]

![Graph showing linear density vs \( \tau/\sigma_\tau \)]

• For proton beam, \( \alpha_R N/\sqrt{2\pi} \ll 1 \), the rf potential.

• However, at the Fermilab Recycler Ring, the rf potential \( U_{\text{rf}}(\tau) = 0 \), and head-tail asymmetry has been observed.
THE RECYCLER RF

• Barrier RF cavities are used to store $\bar{p}$.

• Merits:

  1. Beam spreads out with lower space-charge force.
  2. Can merge two batches together easily.
  3. Can compress by moving a barrier slowly.
  4. Can move batch from one location to another.
- Four 50 Ω ferrite-loaded rf cavity stations.
  - Amplifiers: 3.5 kW and broad-band (10 kHz to 100 MHz) capable of supplying 2 kV.
  - Rf waveform determined by amplitude and phase of each of the 1113 revolution harmonics
- If baseline is not zero, rf potential will be head-tail asymmetric.

![RF wave](image1)

- Nonzero baseline can come from rf error or impedance.
- Here, we are talking about $\sim 10$ V out of 2 kV (0.5%).
Figure 2: Top: Beam intensity $\sim 1 \times 10^{11}$. Bottom: Nonlinear region of Recycler high-level RF. Require flatness from 90 kHz to 1 MHz 0.26 dB in amplitude and 1.8° in phase.
Figure 3: After linearization transfer function implemented.
• With the linearization compensation properly adjusted, the slant reappears:
at higher beam intensity like \( N \gtrsim 2 \times 10^{11} \)
with barriers separation \( T_2 = 1.6 \mu s \).

\[ N = 6.4 \times 10^{11} \]
asymmetry \( \pm 14\% \)

• Now beam intensity shifts towards tail \( \rightarrow \) appears to come from resistive impedance.

• Barrier cavities have total \( \Re Z_0^\| = 200 \Omega \), visible to beam up to \( \sim 45 \) MHz (harmonics \( n = 1 \) to 500).

• \( Z_0^\| = 7.6 \Omega \) at \( n = 1 \rightarrow \) can be neglected.

• **Beam loading voltage**

\[ V_b = I_{local} \approx eN Z_0^\| \rho(\tau) \approx \frac{eN Z_0^\|}{T_2} = 12.9 \text{ V} \]
• The slant can be compensated by adding $\sim 8.82$ V to the region between the barriers.

- Assuming $\Delta E$ has Gaussian distribution, can solve Haissinski equation to obtain $\rho(\tau)$.
- Gives $\pm 17\%$ asymmetry, larger than measurement.
- To correct for asymmetry, needs 12.35 V, which is $\sim 40\%$ larger than actual compensation.
UNSOLVED PROBLEMS

Problem 1

- The voltage compensation is smaller than actually used.

- Possible reasons:

  1. Linearization has been [over compensated](#).

     It has been used to make slant zero at $1.1 \times 10^{11}$.

  2. Shunt impedance of cavities may be less than 200 $\Omega$.

     However, loaded shunt impedance may not be much less because cavities are broad-band.
• Theoretical determination of $V_{\text{comp}}$ is very general. It depends only on

$$\psi(\Delta E, \tau) = \psi(H).$$

• We tried a more realistic model with wake

$$W_0(t) = \sqrt{\frac{2}{\pi}} R_s \sigma \omega e^{-\sigma^2 t^2 / 2},$$

so that the resistive impedance

$$\Re Z_0^\parallel(\omega) = R_s e^{-\omega^2 / 2 \sigma^2}$$

rolls off around $\sigma \omega / 2\pi \approx 45$ MHz.

Negligible change in result because the beam does not have many high-frequency components.
Problem 2

- The predicted slant is not linear.

- Haissinski equation is

\[
\rho(\tau) = \rho_0 \exp \left[ \frac{\beta^2 E f_0 e V_0 T_1}{-\eta \sigma_E^2} U_{rf}(\tau) - \frac{\beta^2 e^2 N E R_s}{-\eta T_0 \sigma_E^2} \int_0^\tau \rho(\tau') d\tau' \right],
\]

In the region where potential \( U_{rf}(\tau) = 0 \),

\[
\rho' = -\alpha R N \rho^2
\]

Or

\[
\rho(\tau) = \frac{\rho_0}{1 + \alpha R N \rho_0 \tau}.
\]
• Elliptical-like distributions:

\[ \psi(\tau, \Delta E) = A \left[ \hat{\Delta E}_{0}^{2} - \Delta E^{2} \right]^{n} \longrightarrow A \left[ \hat{\Delta E}^{2}(\tau) - \Delta E^{2} \right]^{n} \]

where

\[
\hat{\Delta E}^{2}(\tau) = \Delta E_{0}^{2} \left[ 1 + bU_{rf}(\tau) - a \int_{0}^{\tau} \rho(\tau')d\tau' \right].
\]

Then

\[ \rho(\tau) = \int \psi d\Delta E = 2\gamma n A \left[ \hat{\Delta E}^{2}(\tau) \right]^{n+1/2} \]

\[ \rho' = - \left( n + \frac{1}{2} \right) a \rho_0^{2n+1} \rho^{2n+1}, \]

\[ \rho(\tau) = \rho_0 \left[ 1 + \left( n - \frac{1}{2} \right) a \rho_0 \tau \right]^{-\frac{2n+1}{2n-1}}. \]
**PROBLEM 3**

- Elliptical-like distribution with $n = \frac{1}{2}$ does not fit measured energy spread.

- 1.75 GHz Schottky signals in dBm, logarithmic scale.

- $\sigma_E = 2.3$ MeV (3.9 kHz).

- $\Delta E_{90\%} = 7.8$ MeV (13 kHz).

- Convert to linear scale.
• Elliptical distribution does not fit.

• Gaussian distribution fits better.
CONCLUSION

• We have reported head-tail asymmetry in a proton beam due to potential-well distortion of the special barrier rf.

• This slant $\rho(\tau)$ may overflow the barrier bucket when beam intensity is too high.

• When barrier bucket is too full, instabilities will occur due to resonances driven by rf jitters.

• It will also affect other application of barrier rf, like doubling proton intensity in Fermilab Main Injector.

• The required compensating beam-loading voltages are smaller than theory predicted. When the Recycler is turned on again a month later, we must make sure linearization is properly compensated but not over-compensated.

• Elliptical dist. ($n \lesssim 0.5$) fits $\rho(\tau)$ better but not $\Delta E$ dist. Gaussian dist. fits $\Delta E$ dist. better but not $\rho(\tau)$.

• We should investigate some other phase-space distributions like *cosine square*, etc.