POTENTIAL-WELL DISTORTION IN BARRIER RF

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- Head-tail asymmetry was observed at SLAC SLC damping ring, but has never been observed in proton machines.

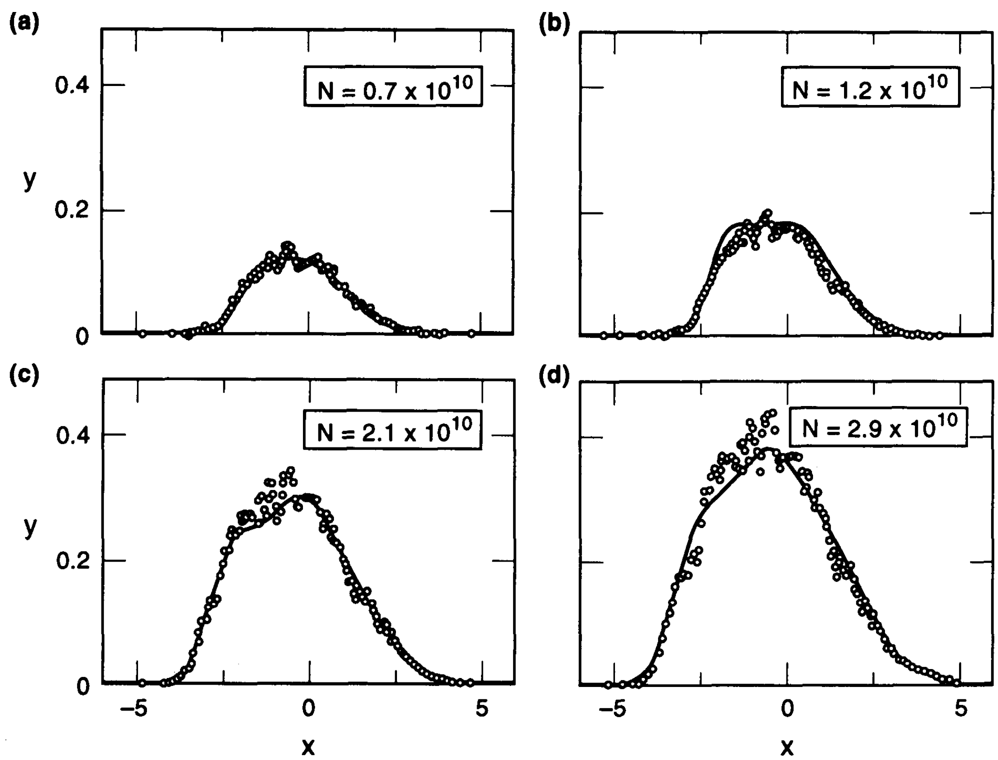


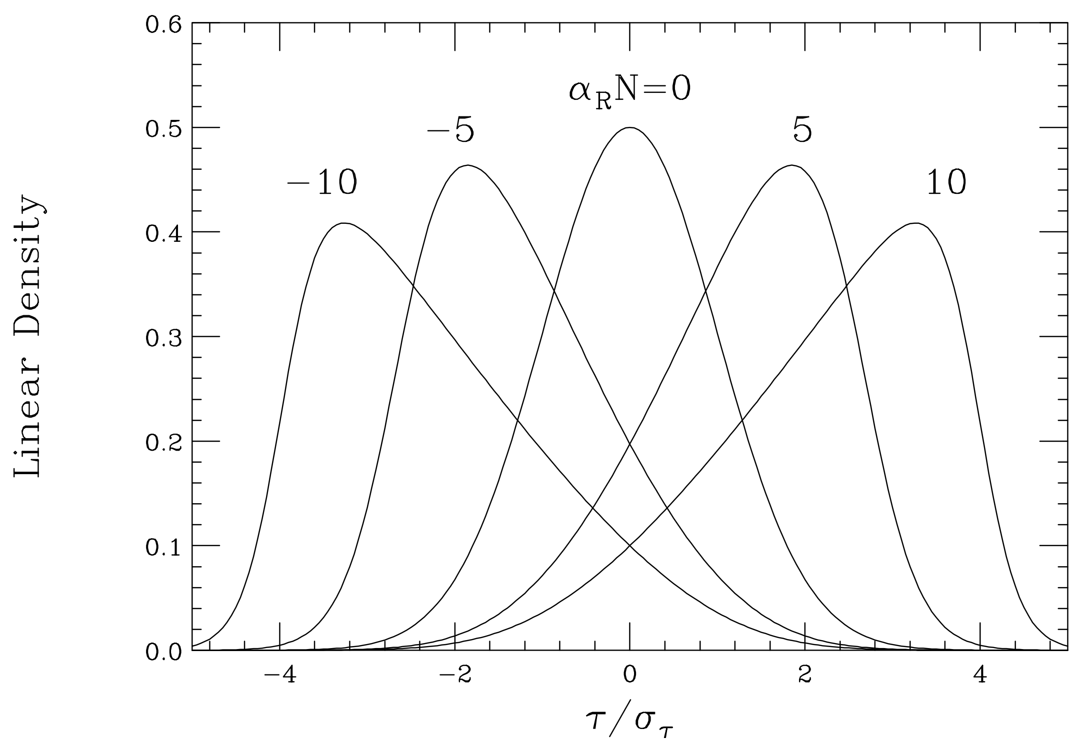
Figure 1: Potential-well distortion of bunch shape for various beam intensities for the SLAC SLC damping ring.

- Haissinski equation

$$\boxed{\rho(\tau) = \rho_0 \left[-\underbrace{\frac{\tau^2}{2\sigma_\tau^2}}_{U_{\rm rf}(\tau)} - \alpha_R N \underbrace{\int_0^\tau \rho(\tau')d\tau'}_{\sim\, \rho_0 \tau} \right]}$$

gives shift of peak:

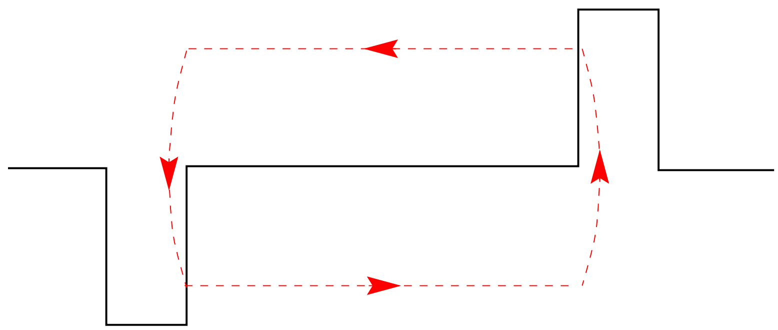
$$\frac{\tau}{\sigma_\tau} = \frac{\alpha_R N}{\sqrt{2\pi}} = \frac{e^2\beta^2 E N R_s}{-\sqrt{2\pi}\eta T_0 \sigma_E^2}$$



- For proton beam, $\boxed{\alpha_R N/\sqrt{2\pi} \ll 1,}$ the rf potential.
- However, at the Fermilab Recycler Ring, the rf potential $U_{\rm rf}(\tau) = 0,$ and head-tail asymmetry has been observed.

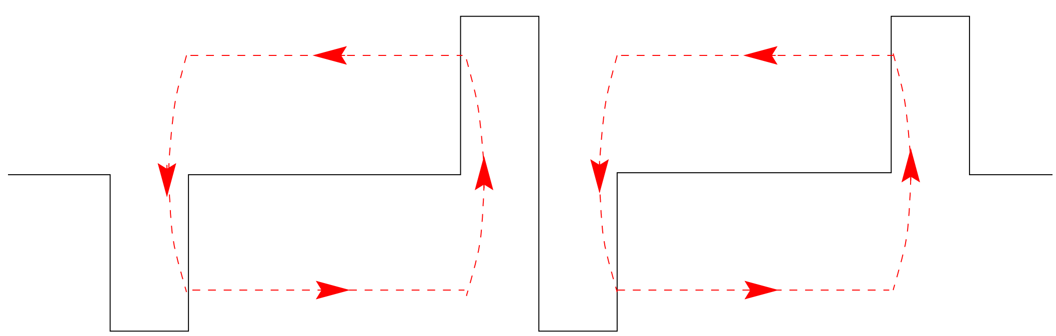
THE RECYCLER RF

- Barrier RF cavities are used to store $\bar{p}$.

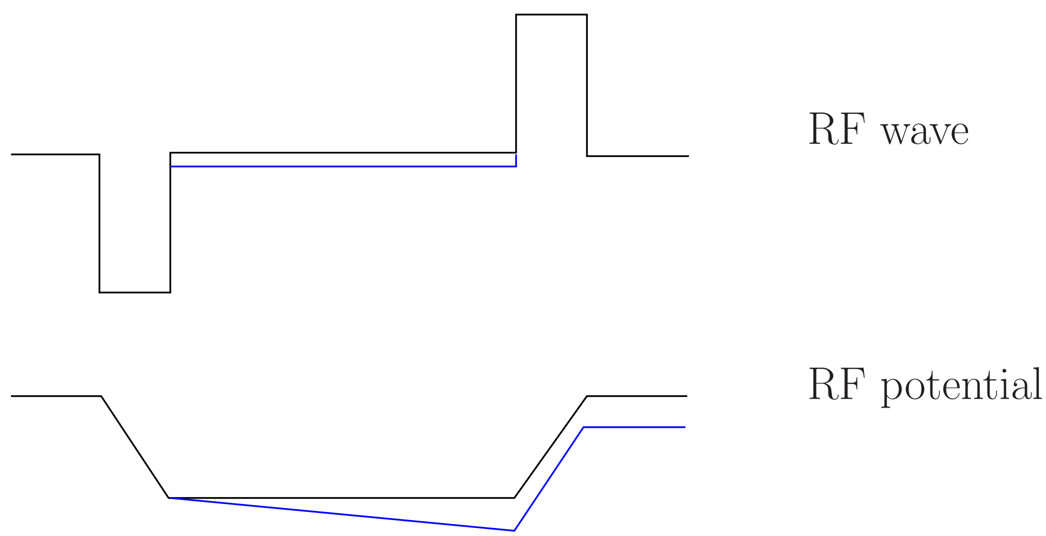


- Merits:

1. Beam spreads out with lower space-charge force.
2. Can merge two batches together easily.
3. Can compress by moving a barrier slowly.
4. Can move batch from one location to another.



- – Four 50 Ω ferrite-loaded rf cavity stations.
  - Amplifiers: 3.5 kW and broad-band (10 kHz to 100 MHz) capable of supplying 2 kV.
  - Rf waveform determined by amplitude and phase of each of the 1113 revolution harmonics
- If baseline is not zero, rf potential will be head-tail asymmetric.



- Nonzero baseline can come from rf error or impedance.
- Here, we are talking about $\sim$ 10 V out of 2 kV (0.5%).

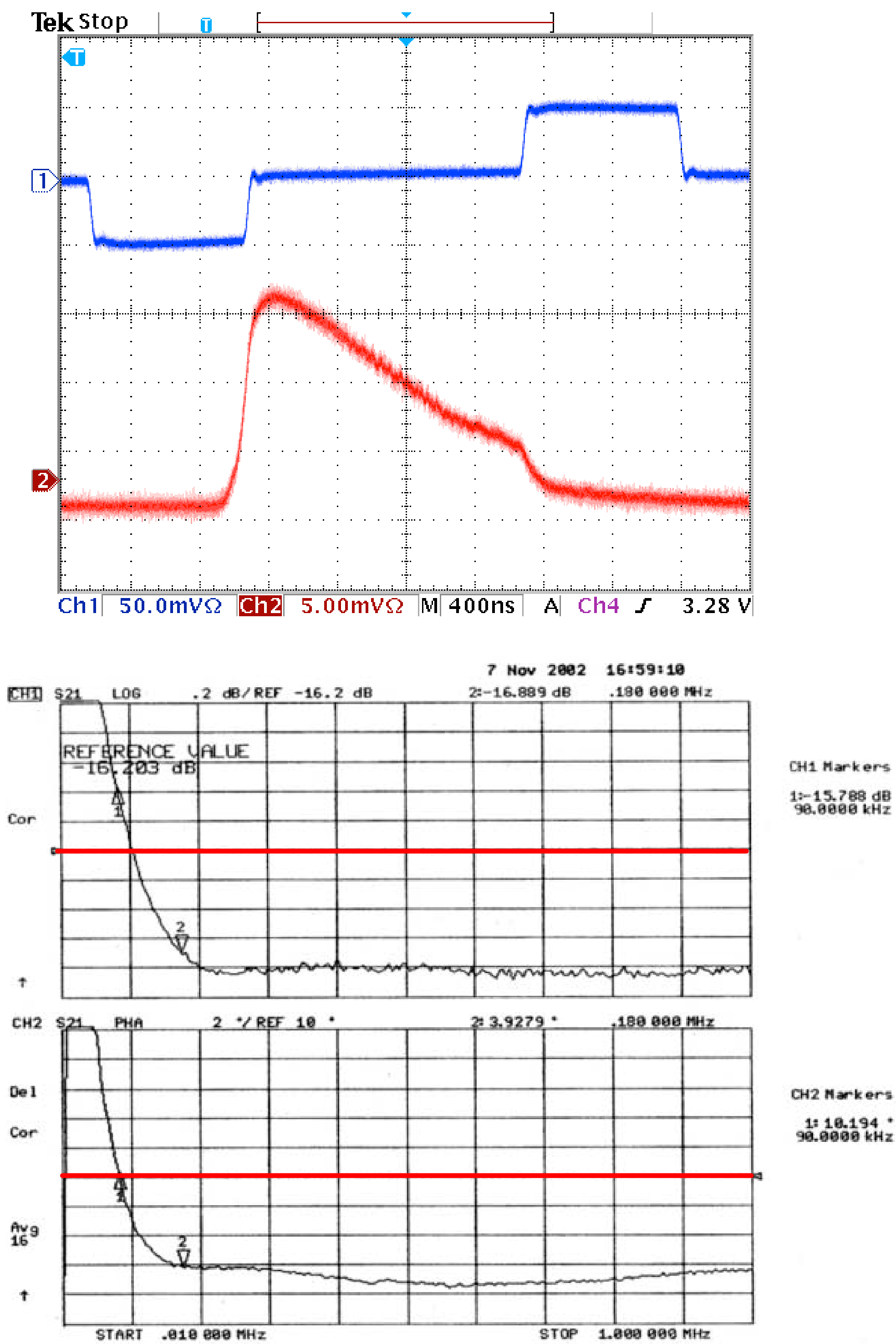


Figure 2: Top: Beam intensity $\boxed{\sim 1 \times 10^{11}.}$ Bottom: Nonlinear region of Recycler high-level RF. Require flatness from 90 kHz to 1 MHz 0.26 dB in amplitude and 1.8° in phase.

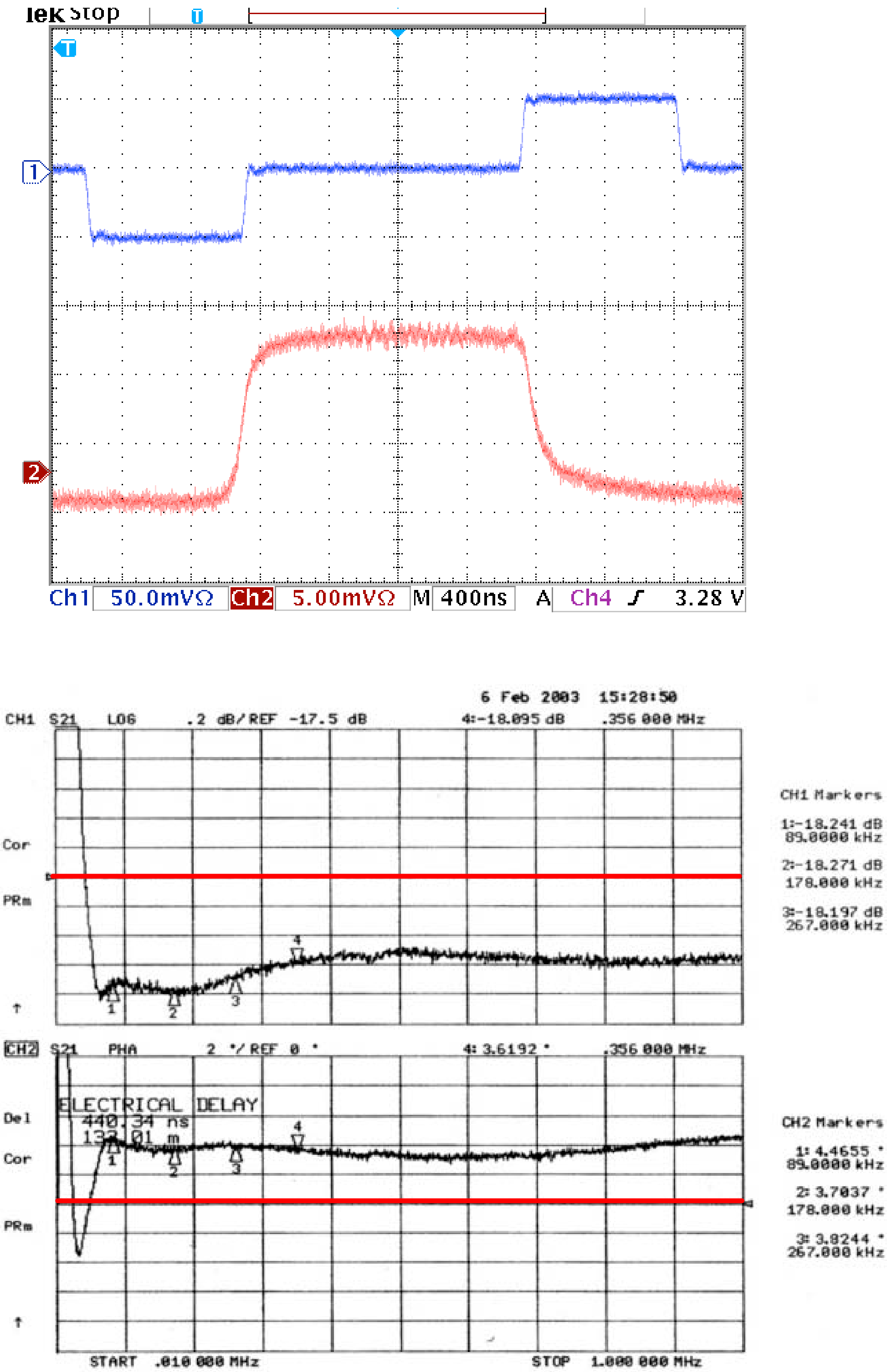
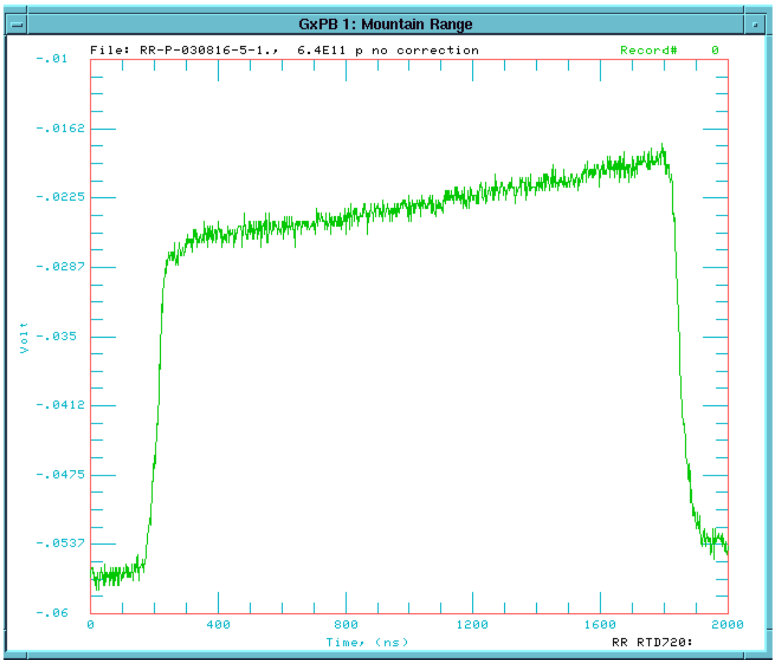


Figure 3: After linearization transfer function implemented.

- With the linearization compensation properly adjusted, the slant reappears:
  at higher beam intensity like $N \gtrsim 2 \times 10^{11}$
  with barriers separation $T_2 = 1.6\ \mu$s.



$N = 6.4 \times 10^{11}$

asymmetry $\pm 14\%$

- Now beam intensity shifts towards tail $\longrightarrow$ appears to come from resistive impedance.

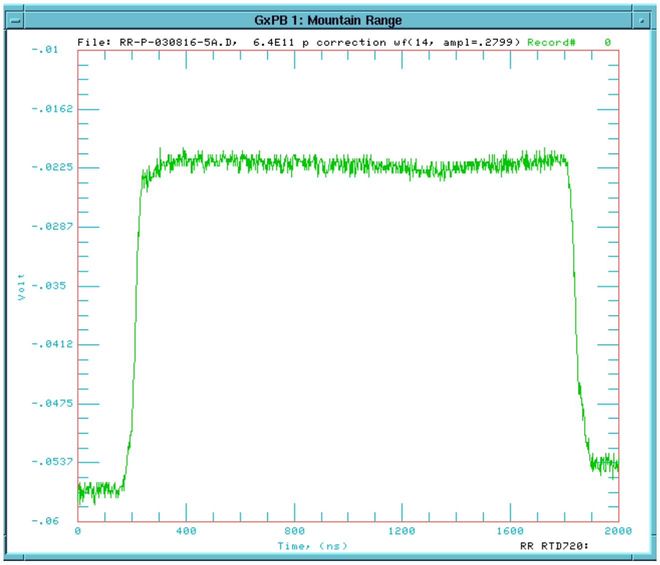
- Barrier cavities have total $\mathcal{R}e\, Z_0^{\|} = 200\ \Omega$, visible to beam up to ~ 45 MHz (harmonics $n = 1$ to 500).

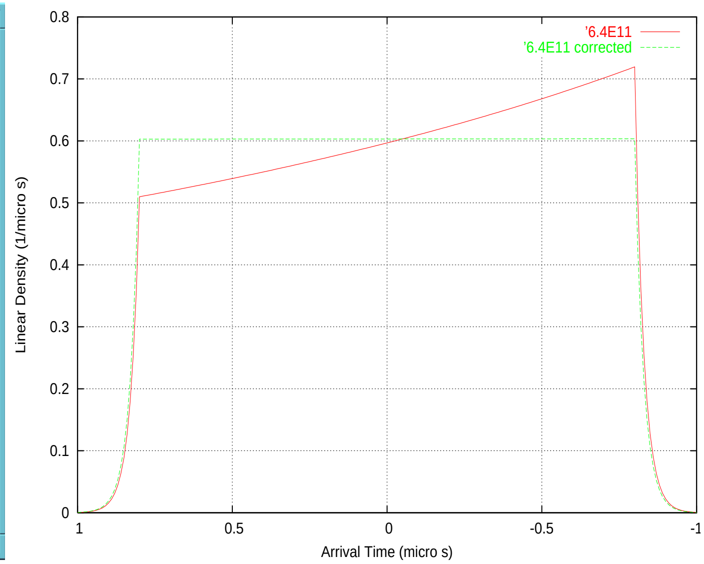
- $Z_0^{\|} = 7.6\ \Omega$ at $n = 1 \longrightarrow$ can be neglected.

- Beam loading voltage

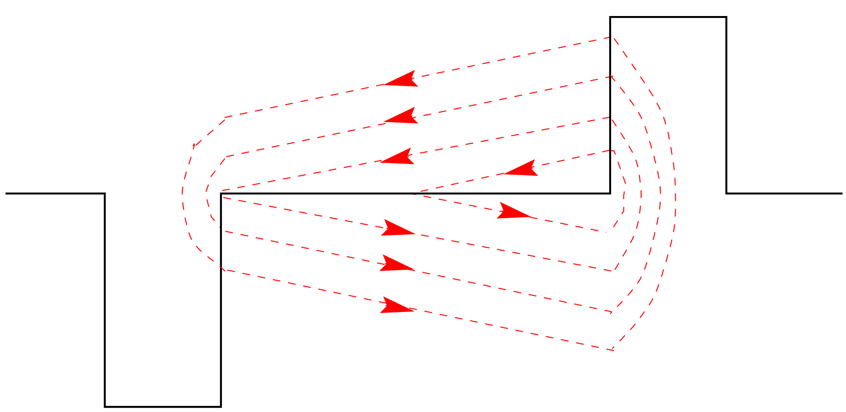
$$\boxed{V_b = I_{\rm local} \approx eNZ_0^{\|}\rho(\tau) \approx \frac{eNZ_0^{\|}}{T_2} = 12.9\ \mathrm{V}\ .}$$

- The slant can be compensated by adding $\sim$ 8.82 V to the region between the barriers.





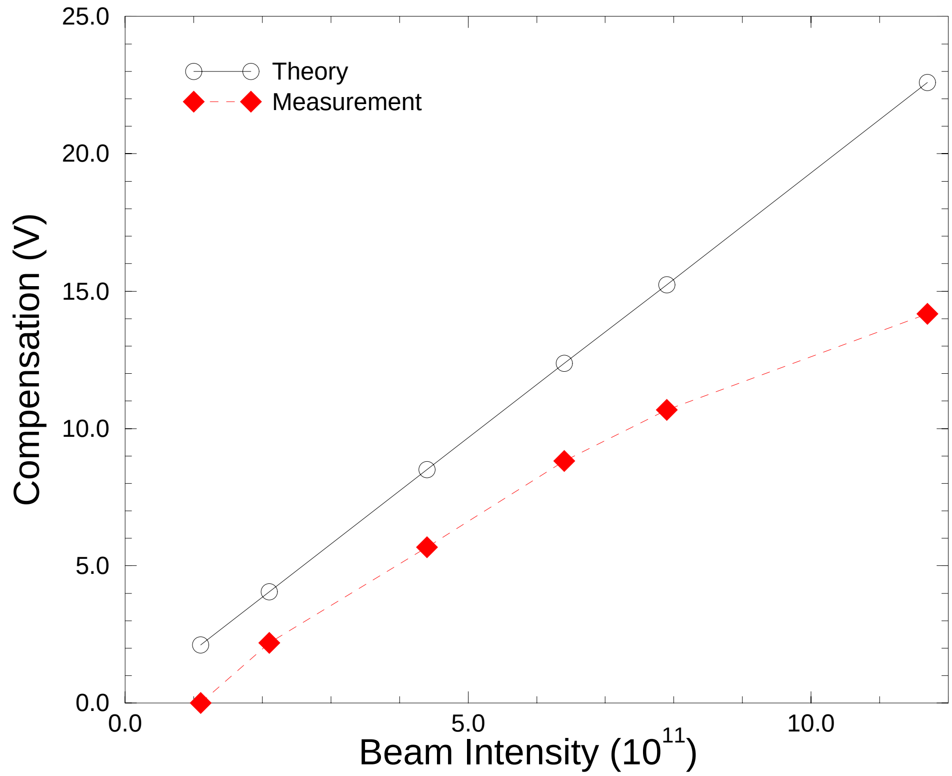
- Assuming ΔE has Gaussian distribution, can solve Haissinski equation to obtain $\rho(\tau)$.
- Gives $\pm 17\%$ asymmetry, larger than measurement.
- To correct for asymmetry, needs 12.35 V, which is $\sim 40\%$ larger than actual compensation.



UNSOLVED PROBLEMS

Problem 1

- The voltage compensation is smaller than actually used.



- Possible reasons:

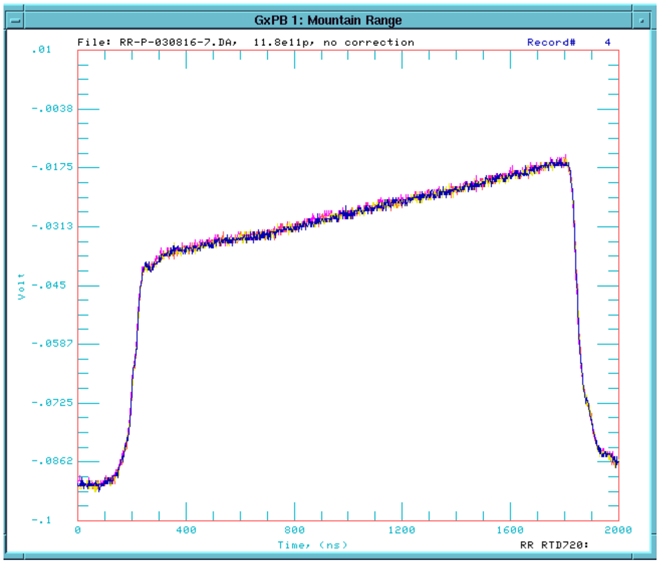
1. Linearization has been over compensated.
   It has been used to make slant zero at 1.1×10^{11}.
2. Shunt impedance of cavities may be less than 200 Ω.
   However, loaded shunt impedance may not be much less because cavities are broad-band.

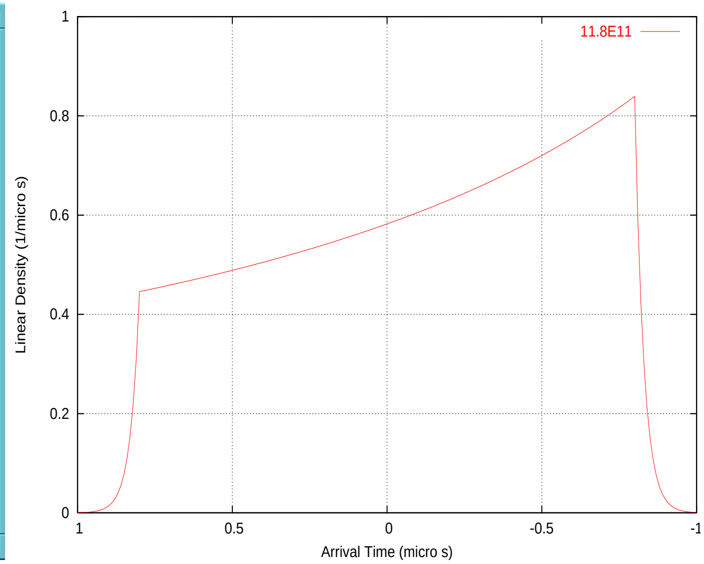
- $\boxed{\text{Theoretical determination of } V_{\text{comp}} \text{ is very general.}}$
  It depends only on
  $$\psi(\Delta E, \tau) = \psi(H) \ .$$

- We tried a more realistic model with wake
  $$W_0(t) = \sqrt{\frac{2}{\pi}} R_s \sigma_\omega e^{-\sigma_\omega^2 t^2/2} \ ,$$
  so that the resistive impedance
  $$\mathcal{R}e\, Z_0^{\|}(\omega) = R_s e^{-\omega^2/2\sigma_\omega^2}$$
  rolls off around $\sigma_\omega/2\pi \approx 45$ MHz.
  Negligible change in result because the beam does not have many high-frequency components.

Problem 2

- The predicted slant is not linear.





- Haissinski equation is

$$\rho(\tau) = \rho_0 \exp\left[\frac{\beta^2 E f_0 e V_0 T_1}{-\eta\sigma_E^2} U_{\rm rf}(\tau) - \frac{\beta^2 e^2 N E R_s}{-\eta T_0 \sigma_E^2}\int_0^\tau \rho(\tau')d\tau'\right],$$

In the region where potential $U_{\rm rf}(\tau) = 0$,

$$\rho' = -\alpha_R N \rho^2$$

Or

$$\boxed{\rho(\tau) = \frac{\rho_0}{1 + \alpha_R N \rho_0 \tau}.}$$

- Elliptical-like distributions:

$$\boxed{\psi(\tau,\Delta E) = A\left[\widehat{\Delta E}_0^2 - \Delta E^2\right]^n \longrightarrow A\left[\widehat{\Delta E}^2(\tau) - \Delta E^2\right]^n}$$

where

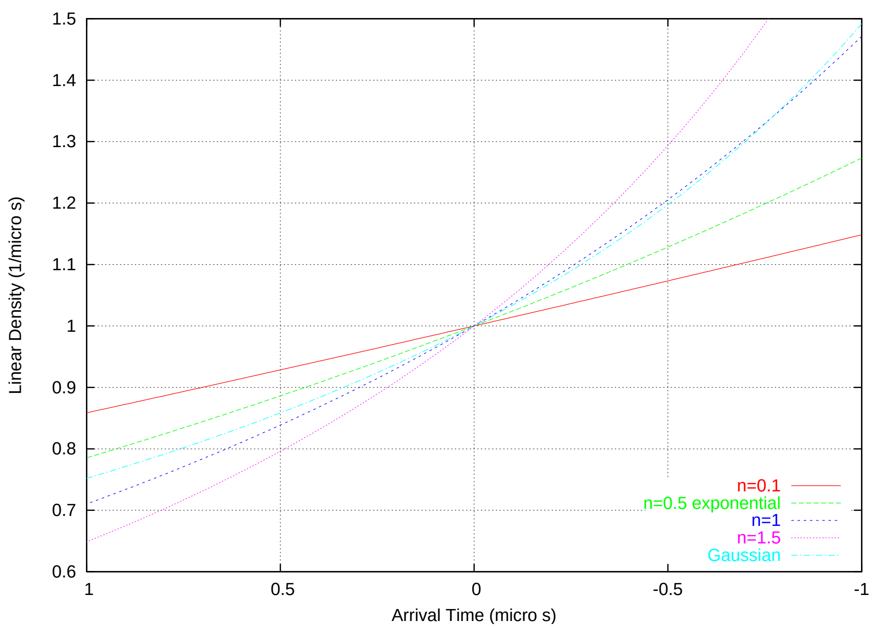
$$\widehat{\Delta E}^2(\tau) = \widehat{\Delta E}_0^2\left[1 + bU_{\rm rf}(\tau) - a\int_0^\tau \rho(\tau')d\tau'\right] .$$

Then

$$\boxed{\rho(\tau) = \int \psi d\Delta E = 2\gamma_n A\left[\widehat{\Delta E}^2(\tau)\right]^{n+1/2}}$$

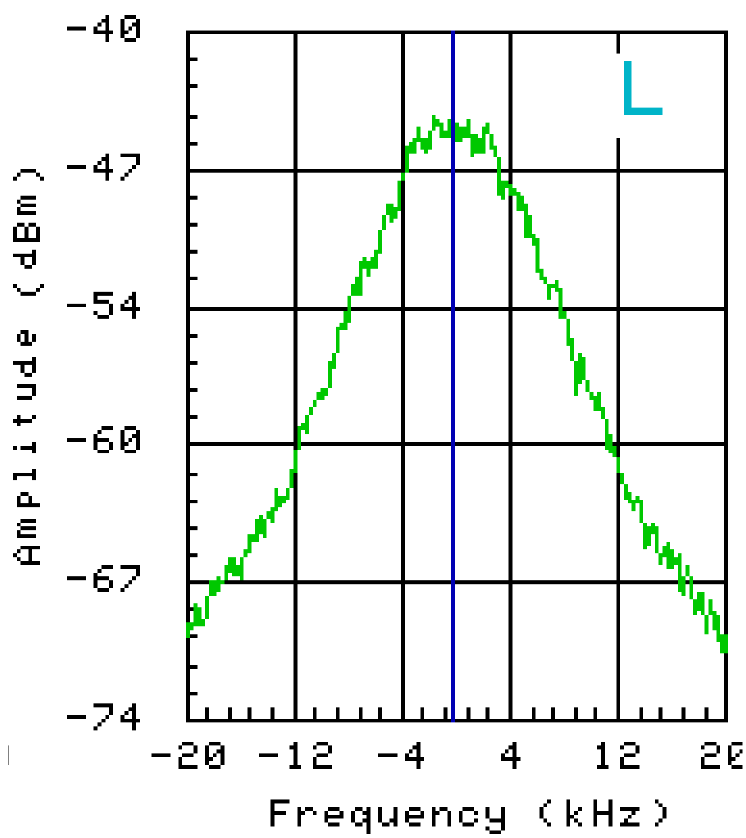
$$\rho' = -\left(n+\frac{1}{2}\right)a\rho_0^{\frac{2}{2n+1}}\rho^{\frac{4n}{2n+1}} ,$$

$$\rho(\tau) = \rho_0\left[1 + \left(n-\frac{1}{2}\right)a\rho_0\tau\right]^{-\frac{2n+1}{2n-1}} .$$

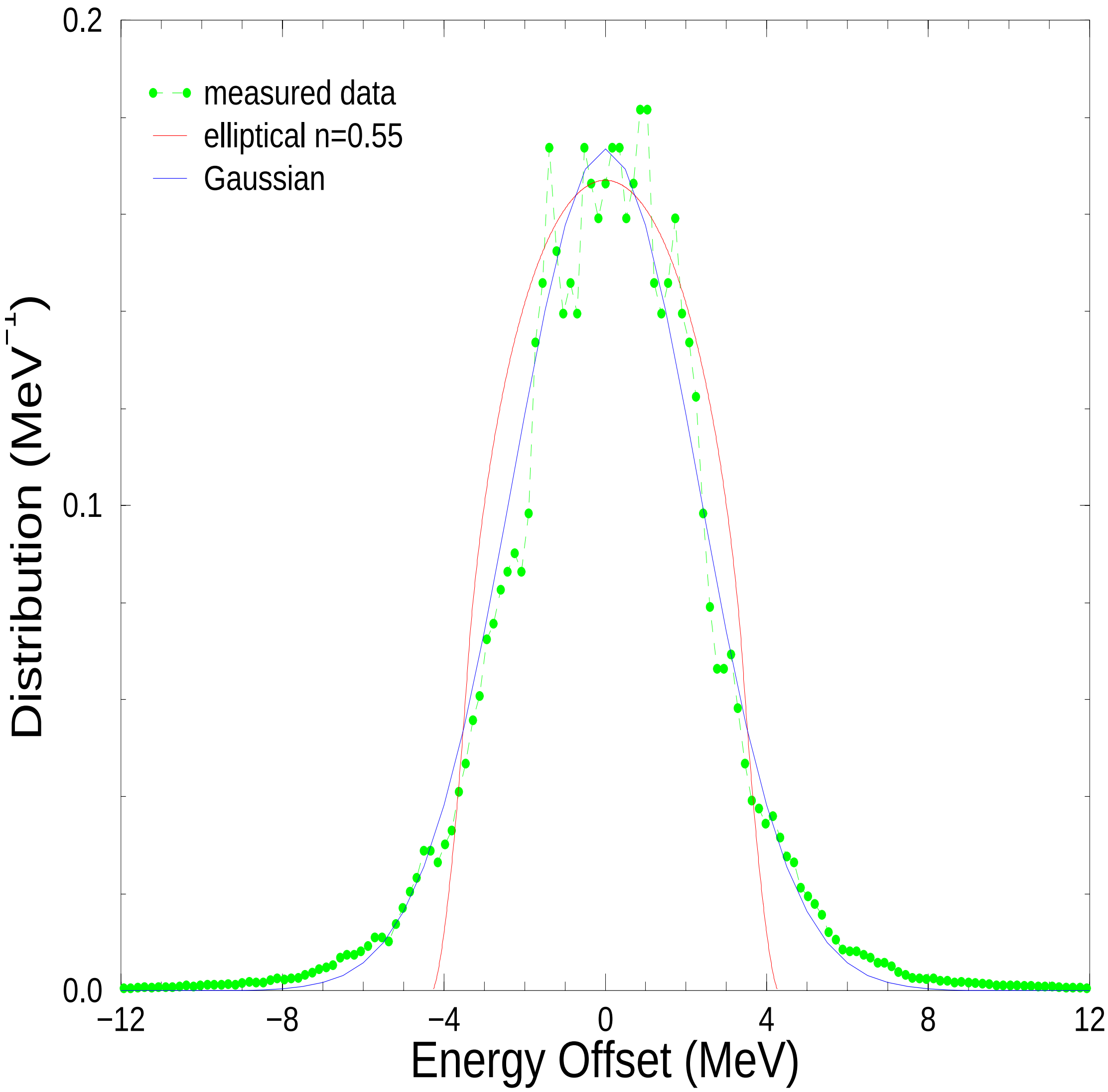


PROBLEM 3

- Elliptical-like distribution with $n = \frac{1}{2}$ does not fit measured energy spread.
- 1.75 GHz Schottky signals in dBm, logarithmic scale.
- $\sigma_E = 2.3$ MeV (3.9 kHz).
- $\Delta E_{90\%} = 7.8$ MeV (13 kHz).



- Convert to linear scale.



- Elliptical distribution does not fit.
- Gaussian distribution fits better.

CONCLUSION

- We have reported head-tail asymmetry in a proton beam due to potential-well distortion of the special barrier rf.
- This slant $\rho(\tau)$ may overflow the barrier bucket when beam intensity is too high.
- When barrier bucket is too full, instabilities will occur due to resonances driven by rf jitters.
- It will also affect other application of barrier rf, like doubling proton intensity in Fermilab Main Injector.
- The required compensating beam-loading voltages are smaller than theory predicted.
  When the Recycler is turned on again a month later, we must make sure linearization is properly compensated but not over-compensated.
- Elliptical dist. ($n \lesssim 0.5$) fits $\rho(\tau)$ better but not ΔE dist. Gaussian dist. fits ΔE dist. better but not $\rho(\tau)$.
- We should investigate some other phase-space distributions like *cosine square,* etc.