Beam-Beam Simulations for RHIC and LHC

J. Qiang, LBNL

Mini-Workshop on Beam-Beam Compensation
July 2-4, 2007, SLAC, Menlo Park, California
Outline

• Computational model
• Studies of long-range beam-beam effects at RHIC
• Tune scan a new working point at RHIC
• Simulation mismatch and offset collisions at LHC
Particle-In-Cell (PIC) Simulation

1. Initialize particles
2. Setup for solving Poisson equation
   - Advance momenta using radiation damping and quantum excitation map
   - Charge deposition on grid
   - Field solution on grid to find beam-beam forces
   - Field interpolation at particle positions
   - Advance momenta using beam-beam forces
3. Advance positions & momenta using external transfer map
4. (optional) diagnostics
Quadratic Deposition/Interpolation

\[
\begin{align*}
W_{x}^{+1} &= \frac{1}{2} \left( \frac{1}{4} - 2 - r_x + r_x^2 \right) \\
W_{x}^{0} &= \frac{3}{4} - r_x^2 \\
W_{x}^{-1} &= \frac{1}{2} \left( \frac{1}{4} + 2 - r_x + r_x^2 \right)
\end{align*}
\]
A Schematic Plot of the Geometry of Two Colliding Beams

- **Head-on collision**
- **Long-range collision**
- **Crossing angle collision**
Green Function Solution of Poisson’s Equation

\[ \phi(r) = \int G(r, r') \rho(r') dr' \ ; \ r = (x, y) \]

\[ \phi(r_i) = h \sum_{i' = 1}^{N} G(r_i - r_{i'}) \rho(r_{i'}) \]

\[ G(x, y) = -\frac{1}{2} \log(x^2 + y^2) \]

Direct summation of the convolution scales as \( N^4 \) !!!
N – grid number in each dimension
Green Function Solution of Poisson’s Equation (cont’d)

**Hockney’s Algorithm**: scales as $(2N)^2 \log(2N)$


\[
\phi_c(r_i) = h \sum_{i'=1}^{2N} G_c(r_i - r_{i'}) \rho_c(r_{i'})
\]

\[
\phi(r_i) = \phi_c(r_i) \text{ for } i = 1, N
\]

**Shifted Green function Algorithm**:

\[
\phi_F(r) = \int G_s(r, r') \rho(r') dr'
\]

\[
G_s(r, r') = G(r + r_s, r')
\]
Comparison between Numerical Solution and Analytical Solution (Shifted Green Function)

\[ E_x \]

inside the particle domain

radius
Green Function Solution of Poisson’s Equation (Integrated Green Function)

Integrated Green function Algorithm for large aspect ratio:

\[
\phi_c(r_i) = \sum_{i'=1}^{2N} G_i(r_i - r_{i'}) \rho_c(r_{i'})
\]

\[
G_i(r, r') = \oint G_s(r, r') \, dr'
\]
BeamBeam3D:
Parallel Strong-Strong / Strong-Weak Simulation Code

- Multiple physics models:
  - strong-strong (S-S); weak-strong (W-S)
- Multiple-slice model for finite bunch length effects
- New algorithm -- shifted Green function -- efficiently models long-range parasitic collisions
- Parallel particle-based decomposition to achieve perfect load balance
- Lorentz boost to handle crossing angle collisions
- Multi-IP collisions, varying phase adv,…
- Arbitrary closed-orbit separation (static or time-dep)
- Independent beam parameters for the 2 beams
Studies of Long-Range Beam-Beam Effects at RHIC
### RHIC Physical Parameters for the Long-Range Beam-Beam Simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam energy (GeV)</td>
<td>26</td>
</tr>
<tr>
<td>Protons per bunch</td>
<td>$19 \times 10^9$</td>
</tr>
<tr>
<td>$\beta^*$ (m)</td>
<td>20</td>
</tr>
<tr>
<td>Rms spot size (mm)</td>
<td>1.6/1.29</td>
</tr>
<tr>
<td>Betatron tunes</td>
<td>(0.7xx,0.7xx)</td>
</tr>
<tr>
<td>Rms bunch length (m)</td>
<td>3.6</td>
</tr>
<tr>
<td>Synchrotron tune</td>
<td>$3.7 \times 10^{-4}$</td>
</tr>
<tr>
<td>Momentum spread</td>
<td>$1.6 \times 10^{-3}$</td>
</tr>
<tr>
<td>Beam-Beam Parameter</td>
<td>0.0077/0.0095</td>
</tr>
</tbody>
</table>
RHIC Long-Range Experiment Scan 2

Scan No 2 -- colliding cogged 2 buckets from IP4, move Blue beam

Tunes (0.7351, 0.7223) (0.7282, 0.7233)

W. Fischer et al.
RHIC Long-Range Simulation Scan 2
(rms emittance vs. turn)

4.70σ separation
5.54σ
7.15σ

One million macroparticles for each beam and 128 x128 grid points

Blue Beam

Yellow Beam
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>proton energy</td>
<td>GeV</td>
<td>100</td>
</tr>
<tr>
<td>protons per bunch, $N_b$</td>
<td>$10^{11}$</td>
<td>2</td>
</tr>
<tr>
<td>emittance $\varepsilon_{N_{x,y}, 95%}$</td>
<td>mm·mrad</td>
<td>15</td>
</tr>
<tr>
<td>beta* (beam1, beam 2)</td>
<td>m</td>
<td>(0.9,1)</td>
</tr>
<tr>
<td>rms bunch length</td>
<td>m</td>
<td>0.7</td>
</tr>
<tr>
<td>rms momentum deviation</td>
<td></td>
<td>0.3e-3</td>
</tr>
<tr>
<td>synchrotron tune</td>
<td>$10^{-3}$</td>
<td>3.7</td>
</tr>
<tr>
<td>chromaticity</td>
<td></td>
<td>(2,2)</td>
</tr>
<tr>
<td>beam-beam separation</td>
<td>sigma</td>
<td>4-6</td>
</tr>
</tbody>
</table>
Computational Model

- 4x4 Linear transfer matrix between the center of sextupole magnets
- Thin lens kick from each sextupole magnet around the machine
- One turn kick to include the machine chromaticity effects with dynamic tune modulation
- Self-consistent long-range beam-beam forces
Averaged Emittance Growth with 4 and 6 Sigma Separation (with dynamic tune modulation)
Transverse Emittance Growth Evolution with/without Tune Modulation

(0.714, 0.726) 4 sigma separation
Transverse Emittance Growth Evolution with/without Static Chromaticity
(0.714, 0.726) 4 sigma separation
Transverse Emittance Growth Evolution with/without Sextupole Magnets
(0.714, 0.726) 4 sigma separation
Averaged Emittance Growth with 4 and 6 Sigma Separation
yellow beam fraction tune \((0.71, 0.69)\)
Transverse Emittance Growth Evolution at Two Resonance Islands

7th order resonance (3,4) or (4,3) 
(0.716,0.716)

7th order resonance (1,6) or (2,5) or (5, -2) 
(0.68,0.716)

7th order resonance (1,6) or (2,5) or (5, -2) 
10th order resonance (5,5)  
12th order resonance (8,-4)
Blue Gold Beam Emittance Growth vs. Wire Separation
(5 A and 50 A current)
Tune Scan Studies of New Working Points at RHIC
A schematic plot of 3 bunch collisions at RHIC
### RHIC Physical Parameters for the Tune Scan Simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam energy (GeV)</td>
<td>100</td>
</tr>
<tr>
<td>Protons per bunch</td>
<td>20e10</td>
</tr>
<tr>
<td>$\beta^*$ (m)</td>
<td>0.8</td>
</tr>
<tr>
<td>Rms spot size (mm)</td>
<td>0.14</td>
</tr>
<tr>
<td>Betatron tunes (blue)</td>
<td>(0.695, 0.685)</td>
</tr>
<tr>
<td>Rms bunch length (m)</td>
<td>0.8</td>
</tr>
<tr>
<td>Synchrotron tune</td>
<td>5.5e-4</td>
</tr>
<tr>
<td>Momentum spread</td>
<td>0.7e-3</td>
</tr>
<tr>
<td>Beam-Beam Parameter</td>
<td>0.00977</td>
</tr>
<tr>
<td>Chromaticity</td>
<td>(2.0, 2.0)</td>
</tr>
</tbody>
</table>
Intensity (experiment) and Emittance (simulation) Evolution of Blue and Yellow Proton Beams at RHIC

Intensities

Measurements (from W. Fischer, et al.)

Emittances

Simulation

Intensity (experiment) and Emittance (simulation) Evolution of Blue and Yellow Proton Beams at RHIC

(from W. Fischer, et al.)
Averaged Emittance Growth vs. Tunes (near half integer, above diagonal)
Averaged Emittance Growth vs. Tunes
(near half integer, below diagonal)
Averaged Emittance Growth vs. Tunes
(below integer, below diagonal)
Averaged Emittance Growth vs. Tunes
(above integer, below diagonal)
Mismatch and Offset Beam-Beam Collisions at LHC
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam energy (TeV)</td>
<td>7</td>
</tr>
<tr>
<td>Protons per bunch</td>
<td>10.5e10</td>
</tr>
<tr>
<td>$\beta^*$ (m)</td>
<td>0.5</td>
</tr>
<tr>
<td>Rms spot size (mm)</td>
<td>0.016</td>
</tr>
<tr>
<td>Betatron tunes</td>
<td>(0.31,0.32)</td>
</tr>
<tr>
<td>Rms bunch length (m)</td>
<td>0.077</td>
</tr>
<tr>
<td>Synchrotron tune</td>
<td>0.0021</td>
</tr>
<tr>
<td>Momentum spread</td>
<td>0.111e-3</td>
</tr>
<tr>
<td>Beam-Beam Parameter</td>
<td>0.0034</td>
</tr>
</tbody>
</table>
99.9% Emittance Growth with
Equal Beam Size and 10% Mismatched Beam Size
A Schematic Plot of LHC Collision Scheme
One Turn Transfer Map

\[ M = M_a M_1 M_b M_2 M_c M_3 M_d M_4 M_e M_5 M_f M_6 \]

\[ M = M_{6^{-1}} M_f M_6 M_a M_1 M_b M_{1^{-1}} M_1 M_2 M_3 \]

\[ M_{3^{-1}} M_c M_3 M_d M_4 M_e M_{4^{-1}} M_4 M_5 M_6 \]

Here, \( M_a \) and \( M_d \) are the transfer maps from head-on beam-beam collisions; \( M_{b,c,e,f} \) are maps from long-range beam-beam collisions; \( M_{1-6} \) are maps between collision points.

- Linear half ring transfer matrix with phase advanced:
  \[ \phi_x = 2\pi \times 31.655; \phi_y = 2\pi \times 29.66 \]

- 90 degree phase advance between long-range collision points and IPs

- 15 parasitic collisions lumped at each long-range collision point with 9.5 \( \sigma \) separation
RMS Emittance Growth vs. Horizontal Separation at LHC
(No Parasitic Collisions)
RMS Emittance Growth vs. Horizontal Separation at LHC
(With 60 lumped Parasitic Collisions)
Summary

• Strong-strong beam-beam simulations qualitatively reproduce the experimental observations at RHIC.
• Emittance growth islands are observed near the 7th and 11th order resonance for the long-range beam-beam interactions at RHIC.
• For 4 sigma separation, significant emittance growth is observed near the resonance within a short period of time. Such emittance growth decreases quickly as the increase of beam-beam separation.
• Working points above integer shows less emittance than the other working points at RHIC.
• Mismatched collisions at LHC causes extra emittance growth.
• Offset collisions including lumped parasitic collisions at LHC show strong vertical emittance growth.