NLO QCD corrections to $\mathrm{pp}/\mathrm{p\bar{p}} \to \mathrm{t\bar{t}} + \textbf{jet} + X$

Stefan Dittmaier

MPI Munich

(in collaboration with P. Uwer and S. Weinzierl)

Contents

1 Introduction

Why is $\mathrm{pp}/\mathrm{p\bar{p}} \to \mathrm{t\bar{t}} +$ jet $+ X$ interesting ?

- Top-quark properties and dynamics still not precisely known

  ↪ additional observables welcome, e.g.

  FB charge asymmetry of heavy quarks in $q\bar{q} \to \mathrm{t\bar{t}}(+$jets$)$

  origin: interference of C-odd / even parts initial and final states

  $q\bar{q} \to \mathrm{t\bar{t}}$: asymmetry appears first in NLO Kühn, Rodrigo '98

  ↪ $A_{\mathrm{FB}}(\mathrm{NLO})$ not feasible in near future (requires σ_{NNLO})

  $q\bar{q} \to \mathrm{t\bar{t}}+g$: asymmetry is LO effect Halzen, Hoyer, Kim '87

  ↪ $A_{\mathrm{FB}}(\mathrm{NLO})$ feasible and in progress

  Note: asymmetry is larger for $\mathrm{t\bar{t}}+$jet than for $\mathrm{t\bar{t}}$ inclusive process

  Situation at Tevatron: asymmetry is measurable ! Bowen, S.D.Ellis, Rainwater '05

  LHC: asymmetry requires preferred direction from partonic boost effects

- Important background process for Higgs search at the LHC via

  $\mathrm{pp}(\mathrm{WW} \to \mathrm{H}) \to \mathrm{H} + 2$jets

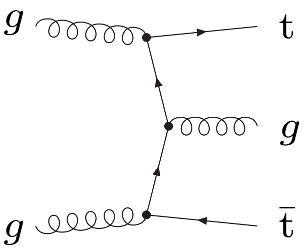
  $\mathrm{pp} \to \mathrm{t\bar{t}H} + X$

2 Calculation of NLO corrections

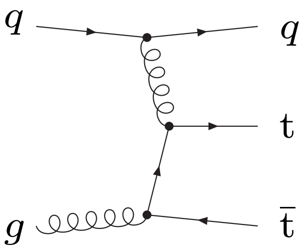
2.1 Lowest-order prediction

Some LO diagrams:

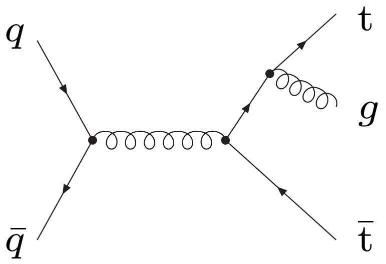
$gg \longrightarrow \mathrm{t}\bar{\mathrm{t}}g$:
(16 diagrams)



$qg \longrightarrow \mathrm{t}\bar{\mathrm{t}}q$:
(5 diagrams)



$q\bar{q} \longrightarrow \mathrm{t}\bar{\mathrm{t}}g$:
(5 diagrams)



etc.

Features of the LO cross section:

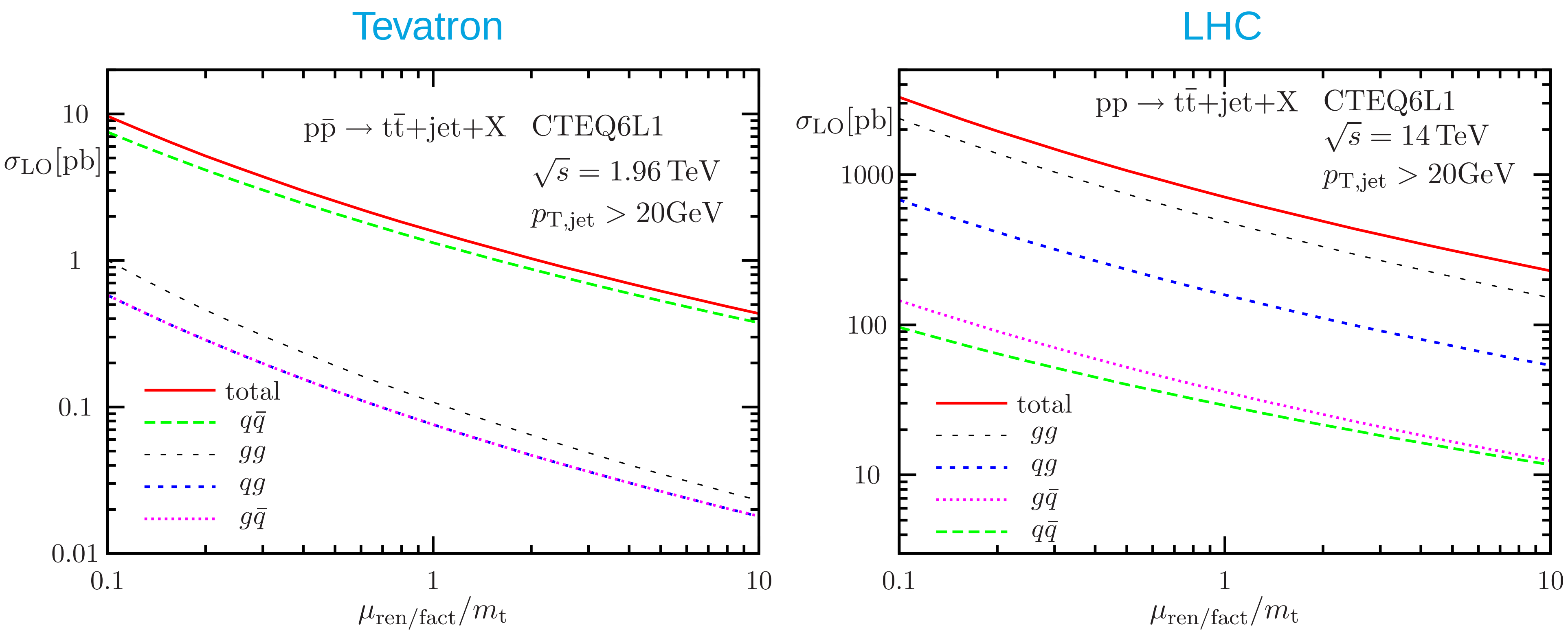
- IR safety requires at least lower cut on $p_{\mathrm{T,jet}}$

  $\hookrightarrow$ apply jet algorithm for NLO cross-section before cut on $p_{\mathrm{T,jet}}$

- LO hadron cross section $\propto \alpha_{\mathrm{s}}^3$

  $\hookrightarrow$ strong dependence on renormalization and factorization scales

Scale dependence of LO cross sections:


Tevatron

LHC

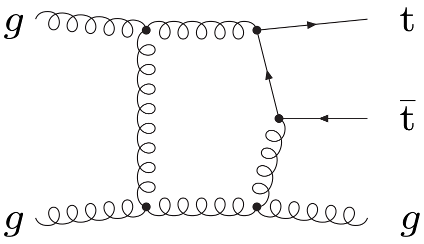
In LO: light final-state parton $\equiv$ jet

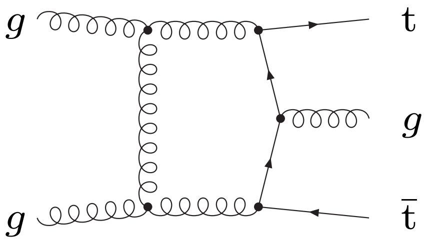
$\hookrightarrow$ no dependence on jet algorithm

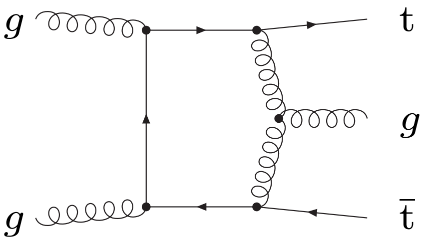
2.2 Virtual corrections

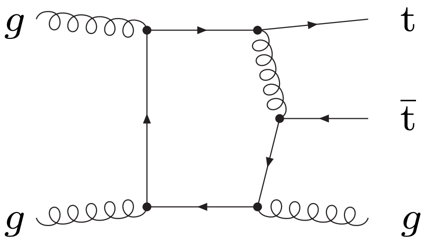
1-loop diagrams $\sim\ 350(100)$ for $gg(q\bar{q}) \to \mathrm{t}\bar{\mathrm{t}}g$

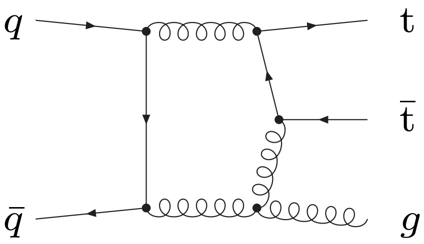
Most complicated 1-loop diagrams—pentagons of the types:

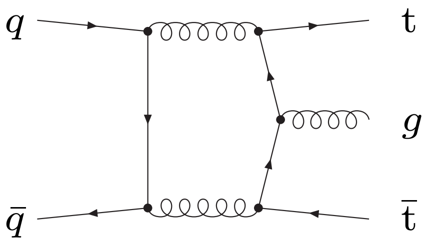












Algebraic reduction of amplitudes to standard form, e.g.

standard colour structures
$\mathcal{C}_1 = (T^{c_a}T^{c_b}T^{c_c})_{ij}$, etc.

$$\mathcal{M}_{g_a g_b \to \mathrm{t}_i \bar{\mathrm{t}}_j g_c} = \sum_{k=1}^{10} \sum_{l=1}^{144} \underbrace{F_{kl}\left(\{p_i \cdot p_j\}\right)}_{} \overbrace{\mathcal{C}_k}^{} \underbrace{\hat{\mathcal{M}}_l\left(\{p_i\}\right)}_{}$$

invariant functions
containing loop integrals

standard spinor structures
$\hat{\mathcal{M}}_1 = (\bar{v}_\mathrm{t} u_{\bar{\mathrm{t}}})(\varepsilon_a \varepsilon_b)(k_\mathrm{t} \varepsilon_c^*)$, etc.

Two independent strategies for evaluation of loop integrals

- Calculation analogous to NLO QCD calculation for $\mathrm{pp} \to \mathrm{t\bar{t}H}$ Beenakker, S.D., Krämer, Plümper, Spira, Zerwas '01,'02
  - ◇ diagrams generated with FEYNARTS 1.0 Küblbeck, Böhm, Denner '90
    and reduced with in-house MATHEMATICA routines → FORTRAN
  - ◇ analytical extraction of soft / collinear singularities Beenakker et al. '02; S.D. '03
  - ◇ reduction of 5-point to 4-point integrals according to Denner, S.D. '02
    $\hookrightarrow$ no (leading) inverse Gram det's → sufficient numerical stability
  - ◇ outlook: process will be used as further test ground for
    more sophisticated tensor reduction methods Denner, S.D. '05
    (seminumerical and/or expansion techniques)
    used at NLO EW for $\mathrm{e}^+\mathrm{e}^- \to 4f$ Denner et al. '05
- Alternative calculation
  - ◇ diagrams generated with QGRAF (Nogueira '93) and reduced with FORM → C++
  - ◇ reduction of 5-point to 4-point integrals according to Giele, Glover '04
    $\hookrightarrow$ no (leading) inverse Gram det's → still testing numerical stability
  - ◇ outlook: further numerical stabilization
    via expansion method suggested by Giele, Glover, Zanderighi '04

Strategy for extracting or translating IR (soft / collinear) singularities:

Idea: convert integrals $I^{(D)}$ in $D=4-2\epsilon$ dim.

→ 4-dim. integrals $I^{(\lambda)}$ with mass regulator λ

Procedure: consider finite and reg.-scheme-independent difference

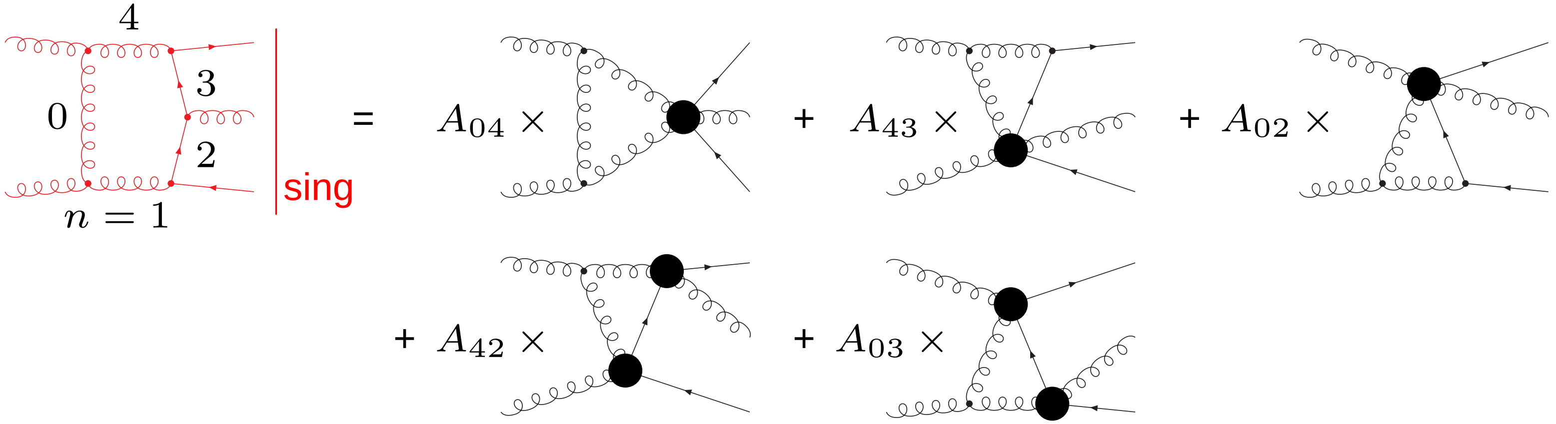
$$\left[I^{(D)} - I^{(D)}_{\text{sing}}\right]\Big|_{D\to 4} = \left[I^{(\lambda)} - I^{(\lambda)}_{\text{sing}}\right]\Big|_{\lambda\to 0}$$

$$\Rightarrow \quad I^{(D)} = I^{(D)}_{\text{sing}} + \left[I^{(\lambda)} - I^{(\lambda)}_{\text{sing}}\right]\Big|_{\lambda\to 0} + \mathcal{O}(\epsilon)$$

Note: mass-singular part can be universally constructed from 3-point integrals

↪ general result known explicitly S.D. '03 Beenakker et al. '01

An example:



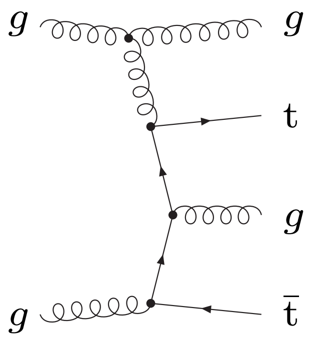
$\left.\right\}$ $\frac{1}{\epsilon^2}, \frac{1}{\epsilon}$ $\ln^2\lambda,\ \ln\lambda$

$\left.\right\}$ $\frac{1}{\epsilon}$ $\ln\lambda$

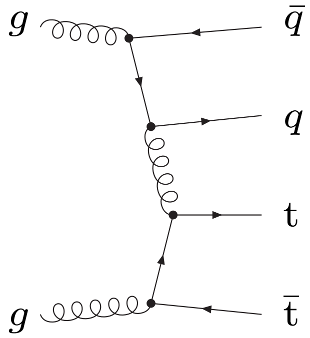
2.3 Real corrections

Some diagrams with 1-parton emission

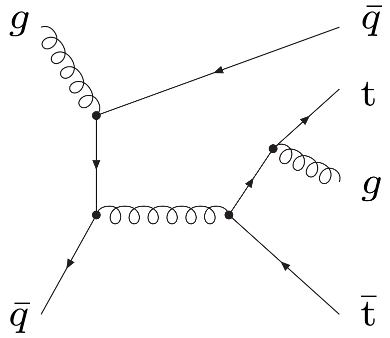
$gg \to \mathrm{t\bar{t}}gg$:
(123 diagrams)



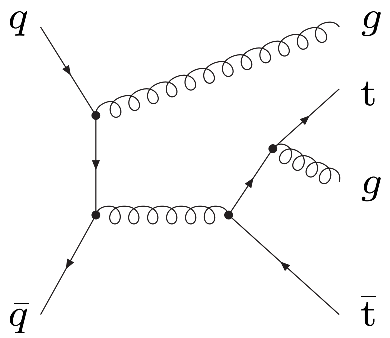
$gg \to \mathrm{t\bar{t}}q\bar{q}$:
(36 diagrams)



$g\bar{q} \to \mathrm{t\bar{t}}\bar{q}g$:
(36 diagrams)



$q\bar{q} \to \mathrm{t\bar{t}}gg$:
(36 diagrams)



etc.

Features of the calculation

- evaluation of helicity amplitudes via
  - ◇ conventional (4-dimensional) spinor techniques
    automated a la Weinzierl '05
  - ◇ Berends/Giele recurrence relations

  $\hookrightarrow$ $|\mathcal{M}|^2$ checked against MADGRAPH Stelzer, Long '94
- extraction and integration of soft / collinear singularities via
  dipole subtraction formalism Catani, Seymour '96; S.D. '99; Phaf, Weinzierl '01
  Catani, S.D., Seymour, Trocsanyi '02

Dipole subtraction formalism

$\rightarrow$ process-independent treatment of singularities in real NLO corrections

worked out for

- QCD with massless partons (Catani, Seymour ’96)
- γ radiation off massive fermions (S.D. ’99)

QCD with massive partons
Phaf, Weinzierl ’01
Catani, S.D., Seymour, Trócsányi ’02

basic idea: NLO correction to process with m partons

$$\sigma^{\mathrm{NLO}} = \underbrace{\int_{m+1} \left[\mathrm{d}\sigma^{\mathrm{real}} - \mathrm{d}\sigma^{\mathrm{sub}} \right]}_{\text{finite}} + \underbrace{\int_{m} \left[\mathrm{d}\sigma^{\mathrm{virtual}} + \mathrm{d}\bar{\sigma}^{\mathrm{sub}}_{1} \right]}_{\text{finite}} + \int_0^1 \mathrm{d}x \underbrace{\int_{m} \left[\mathrm{d}\sigma^{\mathrm{fact}}(x) + \left(\mathrm{d}\bar{\sigma}^{\mathrm{sub}}(x) \right)_{+} \right]}_{\text{finite}}$$

conditions on $\mathrm{d}\sigma^{\mathrm{sub}}$:

- sum rule: $$-\int_{m+1} \mathrm{d}\sigma^{\mathrm{sub}} + \int_{m} \mathrm{d}\bar{\sigma}^{\mathrm{sub}}_{1} + \int_0^1 \mathrm{d}x \int_{m} \left(\mathrm{d}\bar{\sigma}^{\mathrm{sub}}(x) \right)_{+} = 0$$
- asymptotics: $\sigma^{\mathrm{sub}} \sim \sigma^{\mathrm{real}}$ in all collinear/IR regions

2.4 Checks and status of the calculation

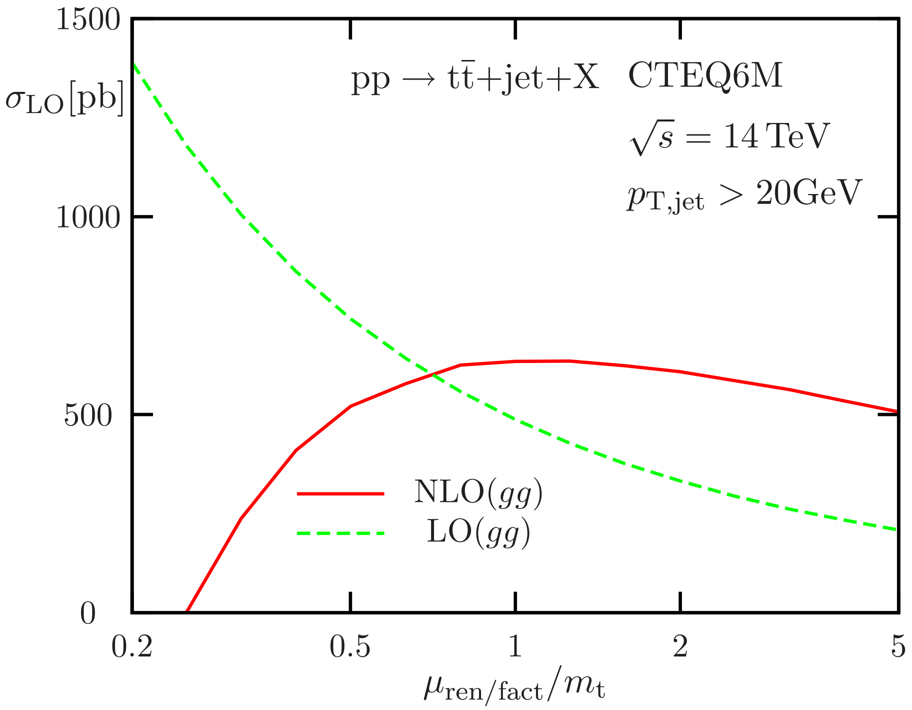
All ingredients available, but not all cross-checked:

- gg-fusion channel almost completely checked:
  - ◇ UV structure of virtual correction
  - ◇ soft and collinear structure in real and virtual corrections
  - ◇ crossing symmetries
  - ◇ all ingredients (but some parts of real corrections) confirmed in second, independent calculation
- $q\bar{q}$, gq, $g\bar{q}$ channels:
  - ◇ UV structure of virtual correction
  - ◇ soft and collinear structure in real and virtual corrections
  - ◇ crossing symmetries
  - ◇ cross-checks by second calculation still missing

3 Numerical results

LO versus NLO contributions of $gg \to \mathrm{t\bar{t}}g(g)$:

LHC



Preliminary !

Jet definition:

algorithm of S.D.Ellis, Soper ’93

with $R = 1$

applied to jets other than t and $\mathrm{\bar{t}}$

$\hookrightarrow$ Scale dependence stabilizes at NLO

4 Conclusions

The process $\mathrm{pp}/\mathrm{p\bar{p}} \to \mathrm{t\bar{t}} + \text{jet} + X$

- important background process for Higgs and other searches at the LHC
- interesting playground to investigate top-quark dynamics
  ↪ measurement of FB charge asymmetry in $\mathrm{t\bar{t}}(+\text{jets})$ already at Tevatron

↪ NLO prediction for $\mathrm{t\bar{t}}$+jet production desirable

$\mathrm{pp}/\mathrm{p\bar{p}} \to \mathrm{t\bar{t}} + \text{jet} + X$ at NLO QCD

- calculation completed, but not yet fully cross-checked
- preliminary numerical results shown
- example is important test ground for NLO methods for many-particle processes

  ↪ methods not yet exhausted,
  more complicated applications $(2 \to 4)$ feasible !

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“It’s promising for the future. The important matches are coming.” (Zinedine Zidane)