SCET for Colliders

Matthias Neubert
Cornell University

LoopFest V, SLAC – June 21, 2006

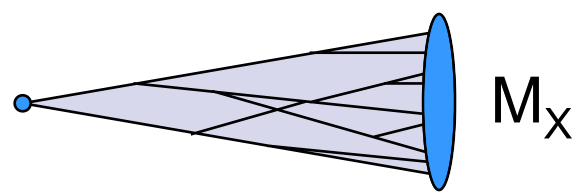
Based on work with Thomas Becher (FNAL) and Ben Pecjak (Siegen)

SCET for Colliders …

- Introduction
- Overview of SCET literature
  (hard QCD processes outside B physics)
- Parton showers [Bauer, Schwartz, hep-ph/0604065]
- Factorization in DIS ($x \to 1$) [Becher, MN, Pecjak, to appear]
- Threshold resummation in momentum space
  (DIS and Drell-Yan) [Becher, MN, hep-ph/0605050]
- Conclusions

Introduction

- Generic problem in QCD:
  - Resummation for processes with >1 scales
  - Interplay of soft and collinear emissions
    $\rightarrow$ Sudakov double logarithms
  - Jet physics: $M_X^2 \ll Q^2$
    - Soft: low momentum $p^\mu \rightarrow 0$
    - Collinear: $p \parallel p_X$ with $p^2 \rightarrow 0$



  - Examples: DIS, fragmentation, Drell-Yan, Higgs production, event shapes, inclusive B decays, …

(see talk by T. Becher)

Introduction

- Problems of scale separation often best addressed using effective field theory

Soft-Collinear Effective Theory

[Bauer, Pirjol, Stewart (2000, 2001)]

- Natural framework for studying questions of factorization, resummation, and power corrections
- Approach first developed for B physics, later applied to other hard QCD processes

Introduction

- SCET is not the invention of the wheel (given 20+ year history in this field)
- Most of what can be done with SCET can be done with conventional techniques (in fact, we never use SCET Feynman rules!)
- However, SCET may provide a novel perspective on factorization, scale separation, resummation, and power corrections in applications where interplay of soft and collinear radiation is relevant
- Existing analyses just the beginning; much room for future work

Overview of SCET literature

- Factorization for π-γ form factor, light-meson form factors, DIS, Drell-Yan, and deeply virtual Compton scattering
  [Bauer, Fleming, Pirjol, Rothstein, Stewart (2002)]
- Factorization (or "non-factorization") and threshold resummation in DIS for $x \to 1$

[Manohar (2003, 2005); Pecjak (2005); Chay, Kim (2005); Idilbi, Ji (2005); Becher, MN (2006); Becher, MN, Pecjak (in prep.)]

- p_t resummation for Drell-Yan and Higgs production
  [Gao, Li, Liu (2005); Idilbi, Ji, Yuan (2005)]
- Threshold resummation for Higgs production
  [Idilbi, Ji, Ma, Yuan (2006); Idilbi, Ji, Yuan (2006)]

Overview of SCET literature

- Nonperturbative effects on jet distributions in e^+e^- annihilation [Bauer, Manohar, Wise (2002); Bauer, Lee, Manohar, Wise (2003)]
- Universality of nonperturbative effects in event shapes [Lee, Sterman (2006)]
- Parton showers [Bauer, Schwartz (2006)]

<u>In this talk:</u>

- Factorization and threshold resummation in DIS and Drell-Yan production
- Parton showers (briefly…)

Parton Showers

[Bauer, Schwartz, hep-ph/0604065]

An interesting proposal

- Process of parton showering as a sequence of hard matchings in SCET onto operators containing increasing number of hard-collinear fields
- Sudakov logs resummed using RG equations
- Straightforward to go beyond LL approximation

(Courtesy M. Schwartz)

An interesting proposal

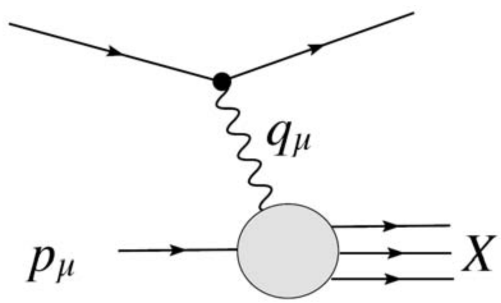
- Leading effective operator (two collinear fields) same as in Drell-Yan
  - 2-loop matching coefficient known (see below)
  - 3-loop anomalous dimension known (see below)
- Questions:
  - Is this really an advance over existing approaches (MC@NLO)?
  - How to implement in a generator?
  - Details of calculations (NLO and beyond)?
- Eagerly await long paper …!


June 19-21, 2006
LoopFest V
Radiative Corrections for the International Linear Collider:
Multi-loops and Multi-legs
Stanford Linear Accelerator Center

SCET and DIS

SCET analysis of DIS for x→1

- Simplest example of a hard QCD process
- SCET can be used to rederive elegantly all existing results
- Provides much simpler result than conventional approach for threshold resummation



$$Q^2 = -q^2$$

$$x = \frac{Q^2}{2p \cdot q}$$

$$Q^2 \gg \underbrace{Q^2(1-x)}_{\approx M_X^2} \gg \Lambda^2_{QCD}$$

- Cross section: $d^2\sigma/dx \cdot dQ^2 \sim F_2(x,Q^2)$

SCET analysis of DIS for $x \to 1$

- Will discuss:
  - Factorization for $x \to 1$
  - Threshold resummation at NNLO (N^3LL)
  - Connection with conventional approach
  - Numerical results

Factorization

Factorization for x→1

- QCD factorization formula:

$$F_2^{\text{ns}}(x,Q^2) = \sum_q e_q^2\, |C_V(Q^2,\mu)|^2\, Q^2 \int_x^1 d\xi\, J\Big(Q^2\,\frac{\xi - x}{x},\mu\Big)\, \phi_q^{\text{ns}}(\xi,\mu)$$

[Sterman (1987); Catani, Trentadue (1989); Korchemsky, Marchesini (1992)]

  - Most transparent to derive this in SCET: need hard-collinear, anti-collinear, and soft-collinear modes (called "soft" in the literature)
  - Resum threshold logarithms by solving RGEs of SCET in momentum space

Factorization for $x \to 1$

- Momentum modes in Breit frame ($\to$ fields in SCET):
  - Hard: $p_h \sim Q(1,1,1)$
  - Hard-collinear (final-state jet): $p_{hc} \sim Q(\varepsilon,1,\sqrt{\varepsilon})$
  - Anti-collinear (initial-state nucleon): $p_{\bar{c}} \sim Q(1,\lambda^2,\lambda)$
  - Soft-collinear ("soft") messengers: $p_{sc} \sim Q(\varepsilon,\lambda^2,\lambda\sqrt{\varepsilon})$

  (here ε=1-x and $\lambda \sim \Lambda/Q$)

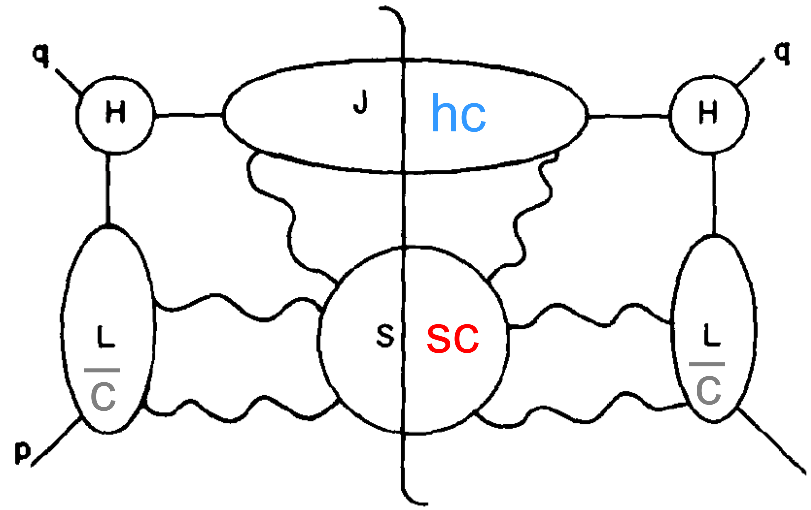


Fig. 3 1. Leading regions for DIS.

[Sterman (1987)]

SCET factorization: Outline

[Becher, MN, Pecjak, to appear]

- Step 1: At hard scale $\mu \sim Q$, match QCD vector current onto current operator in SCET
- Step 2: Hard-collinear and anti-collinear fields can interact via exchange of soft-collinear particles; at leading power, their couplings to hard-collinear fields can be removed by field redefinitions
- Step 3: After decoupling, vacuum matrix element of hard-collinear fields can be evaluated in perturbation theory (for $\mu \sim M_X = Q\sqrt{1-x}$)
- Step 4: Identify remaining nucleon matrix element over anti-collinear and soft-collinear fields with PDF in endpoint region

SCET factorization

- Step 1: current matching

$$(\bar\psi\gamma^\mu\psi)(x) \to \int dt\, \widetilde{C}_V(t, n\cdot q, \mu)\, \big(\bar\xi_{\bar c} W_{\bar c}\big)(x_-)\, \gamma_\perp^\mu \big(W_{hc}^\dagger \xi_{hc}\big)(x+t\bar n)$$

$$= C_V(\underbrace{-n\cdot q\, \bar n\cdot \boldsymbol{P}}_{Q^2}, \mu)\, \big(\bar\xi_{\bar c} W_{\bar c}\big)(x_-)\, \gamma_\perp^\mu \big(W_{hc}^\dagger \xi_{hc}\big)(x)\,.$$

  - Implication for hadronic tensor:

$$W^{\mu\nu}(p,q) = i\int d^4x\, e^{iq\cdot x}\, \langle N(p)|\, T\{J^\mu(x)\, J^{\dagger\nu}(0)\}\, |N(p)\rangle$$

$$\to |C_V(Q^2,\mu)|^2\, i\int d^4x\, e^{iq\cdot x}$$

$$\times\ \langle N(p)|\, T\big\{\big(\bar\xi_{\bar c} W_{\bar c}\big)(x_-)\, \gamma_\perp^\mu \big(W_{hc}^\dagger \xi_{hc}\big)(x)\, \big(\bar\xi_{hc} W_{hc}\big)(0)\, \gamma_\perp^\nu \big(W_{\bar c}^\dagger \xi_{\bar c}\big)(0)\big\}\, |N(p)\rangle$$

SCET factorization

- Simplest to obtain hard matching coefficient from bare on-shell QCD form factor

[Kramer, Lampe (1987, E: 1989); Matsuura, van Neerven (1988); Gehrmann, Huber, Maitre (2005); Moch, Vermaseren, Vogt (2005)]

- Matching converts IR poles into UV poles (subtraction of scaleless SCET graphs):

$$C_V(Q^2,\mu) = \lim_{\epsilon\to 0} Z_V(\epsilon,Q^2,\mu)\, F_{\text{bare}}(\epsilon,Q^2)$$

UV renormalization factor

SCET factorization

- 2-loop result (with L=ln(Q^2/μ^2)):

$$C_V(Q^2,\mu) = 1 + \frac{C_F\alpha_s}{4\pi}\left(-L^2+3L-8+\frac{\pi^2}{6}\right) + C_F\left(\frac{\alpha_s}{4\pi}\right)^2 [C_F H_F + C_A H_A + T_F n_f H_f]$$

with:

$$H_F = \frac{L^4}{2} - 3L^3 + \left(\frac{25}{2}-\frac{\pi^2}{6}\right)L^2 + \left(-\frac{45}{2}-\frac{3\pi^2}{2}+24\zeta_3\right)L + \frac{255}{8}+\frac{7\pi^2}{2}-\frac{83\pi^4}{360}-30\zeta_3$$

$$H_A = \frac{11}{9}L^3 + \left(-\frac{233}{18}+\frac{\pi^2}{3}\right)L^2 + \left(\frac{2545}{54}+\frac{11\pi^2}{9}-26\zeta_3\right)L$$
$$-\frac{51157}{648}-\frac{337\pi^2}{108}+\frac{11\pi^4}{45}+\frac{313}{9}\zeta_3$$

$$H_f = -\frac{4}{9}L^3 + \frac{38}{9}L^2 + \left(-\frac{418}{27}-\frac{4\pi^2}{9}\right)L + \frac{4085}{162}+\frac{23\pi^2}{27}+\frac{4}{9}\zeta_3$$

SCET factorization

- Step 2: decoupling transformation

$$\xi_{hc}(x) \to S_n(x_-)\,\xi_{hc}^{(0)}(x)\,, \qquad A_{hc}^{\mu}(x) \to S_n(x_-)\,A_{hc}^{\mu(0)}(x)\,S_n^{\dagger}(x_-)$$

  - Vacuum matrix element over hard-collinear fields factorizes into a jet function:

$$\langle 0|\,T\{(W_{hc}^{(0)\dagger}\xi_{hc}^{(0)})(x)\,(\bar{\xi}_{hc}^{(0)}W_{hc}^{(0)})(0)\}\,|0\rangle = \langle 0|\,T\left[\frac{\not{n}\not{\bar{n}}}{4}\,W^{\dagger}(x)\,\psi(x)\,\bar{\psi}(0)\,W(0)\,\frac{\not{\bar{n}}\not{n}}{4}\right]|0\rangle$$

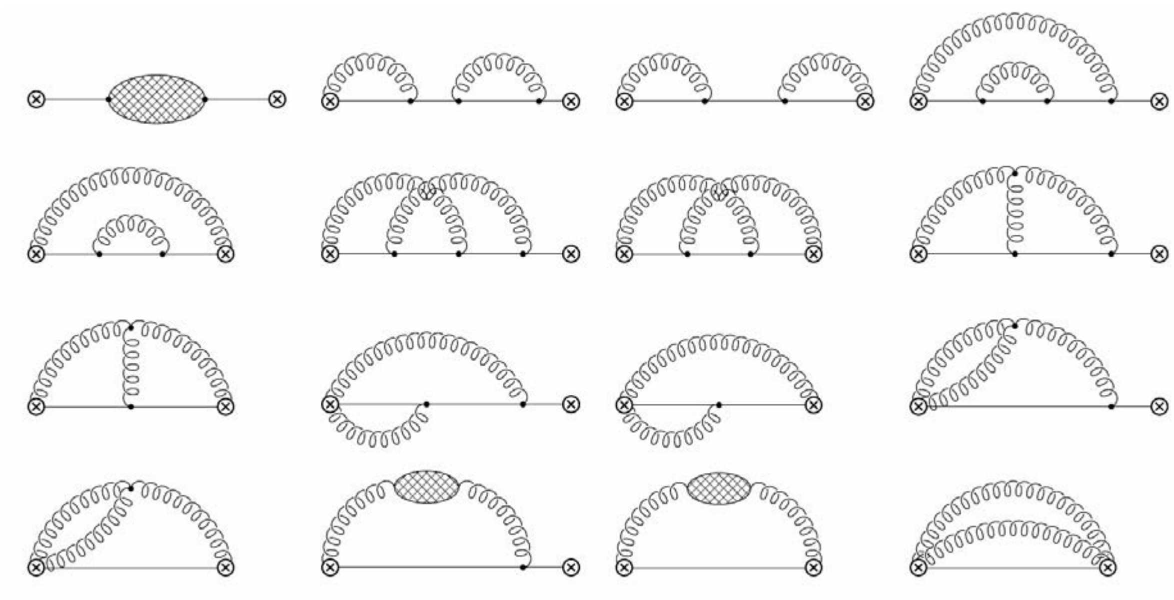
$$= \int \frac{d^4k}{(2\pi)^4}\,e^{-ik\cdot x}\,\frac{\not{n}}{2}\,\bar{n}\cdot k\,\mathcal{J}(k^2,\mu)\,.$$

SCET factorization

- Step 3: compute jet function perturbatively (known at 2-loop order)

quark propagator in light-cone gauge

$$\frac{\not n}{2}\,\bar n\cdot p\,\mathcal{J}(p^2) = \int d^4x\, e^{-ip\cdot x}\,\langle 0|\,\mathrm{T}\left\{\frac{\not n\bar{\not n}}{4}W^\dagger(0)\psi(0)\,\overline{\psi}(x)W(x)\frac{\bar{\not n}\not n}{4}\right\}|0\rangle$$



[Becher, MN, hep-ph/0603140]

SCET factorization

- Step 4: identify PDF in endpoint region

$$\phi_q^{\rm ns}(\xi,\mu)\big|_{\xi\to 1} = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, e^{-i\xi tn\cdot p}\,\langle N(p)|\,\big(\bar\xi_{\bar c}W_{\bar c}\big)(tn)\,[tn,0]_{sc}\,\frac{\not{n}}{2}\,\big(W_{\bar c}^\dagger\xi_{\bar c}\big)(0)\,|N(p)\rangle$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, e^{-i\xi tn\cdot p}\,\langle N(p)|\,\big(\bar\xi_{\bar c}^{(0)}W_{\bar c}^{(0)}\big)(tn)\,\frac{\not{n}}{2}\,W_C(t)\,\big(W_{\bar c}^{(0)\dagger}\xi_{\bar c}^{(0)}\big)(0)\,|N(p)\rangle$$

$$= Z(m,\mu)\,\frac{n\cdot p}{2\pi}\int_{-\infty}^{\infty} dt\, e^{-i\xi tn\cdot p}\,W_C(t)$$

soft-collinear Wilson loop:

$$\langle 0|\,S_{\bar n}^\dagger(tn)\,[tn,0]_{sc}\,S_{\bar n}(0)\,|0\rangle$$

[Korchemsky, Marchesini (1992)]

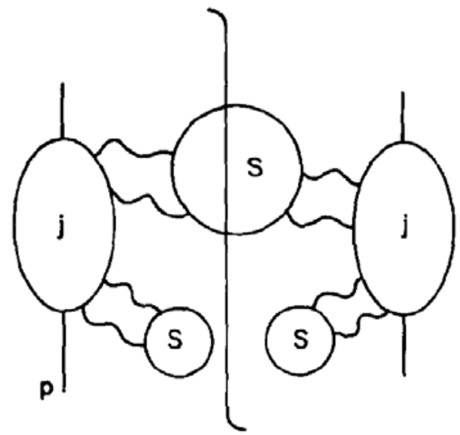


Fig. 4.2 Division of lines in LC distribution as $x \to 1$

[Sterman (1987)]

Threshold Resummation

Threshold resummation

- Traditionally, resummation is performed in Mellin moment space
  - Landau poles (in Sudakov exponent and Mellin inversion)
  - Mellin inversion only numerically
  - Non-trivial matching with fixed-order calculations in momentum space

Threshold resummation

- Define moments of structure function and PDF:

$$F_{2,N}^{\mathrm{ns}}(Q^2) = \int_0^1 dx\, x^{N-1} F_2^{\mathrm{ns}}(x, Q^2)$$
$$= C_N(Q^2, \mu_f) \sum_q e_q^2\, \phi_{q,N}^{\mathrm{ns}}(\mu_f)$$

- Short-distance coefficients C_N can be written:

$$C_N(Q^2, \mu_f) = g_0(Q^2, \mu_f)\, \exp\left[G_N(Q^2, \mu_f)\right]$$

N-independent

Threshold resummation

- Resummed exponent:

$$G_N(Q^2,\mu_f) = \int_0^1 dz\, \frac{z^{N-1}-1}{1-z} \times \left[\int_{\mu_f^2}^{(1-z)Q^2} \frac{dk^2}{k^2} A_q(\alpha_s(k)) + B_q\left(\alpha_s(Q\sqrt{1-z})\right) \right]$$

  - Integrals run over Landau pole in running couplg. (ambiguity $\sim(\Lambda/M_X)^2$ for DIS, $\sim\Lambda/M_X$ for Drell-Yan)
  - Additional singularity encountered in Mellin inversion (physical scales in moment scales are Q^2 and Q^2/N)

Threshold resummation

- Solving RG equations in SCET, we obtain all-orders resummed expressions directly in momentum space (x space)
  - Transparent physical interpretation, no Landau poles, simple analytical expressions
  - Reproduce moment-space expressions order by order in perturbation theory
- Understand IR singularities of QCD in terms of RG evolution (UV poles) in EFT

Evolution of the hard function

- RG equation:

$$\frac{dC_V(Q^2,\mu)}{d\ln\mu} = \underbrace{\left[\Gamma_{\rm cusp}(\alpha_s)\,\ln\frac{Q^2}{\mu^2} + \gamma^V(\alpha_s)\right]}_{-2\alpha_s \frac{\partial}{\partial\alpha_s} Z_V^{(1)}(Q^2,\mu)} C_V(Q^2,\mu)$$

- Exact solution:

$$C_V(Q^2,\mu) = \exp\left[2S(\mu_h,\mu) - a_{\gamma^V}(\mu_h,\mu)\right] \times \left(\frac{Q^2}{\mu_h^2}\right)^{-a_\Gamma(\mu_h,\mu)} C_V(Q^2,\mu_h)$$

- RG functions:
  - Sudakov exponent

$$S(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha\, \frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)} \int_{\alpha_s(\nu)}^{\alpha} \frac{d\alpha'}{\beta(\alpha')}$$

  - Anomalous exponent

$$a_\Gamma(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha\, \frac{\Gamma_{\rm cusp}(\alpha)}{\beta(\alpha)}$$

  - Functions of running couplings $\alpha_s(\mu)$, $\alpha_s(\nu)$

3-loop anomalous dimension γ^V

$$\gamma_0^V = -6C_F$$

$$\gamma_1^V = C_F^2\left(-3+4\pi^2-48\zeta_3\right) + C_F C_A\left(-\frac{961}{27}-\frac{11\pi^2}{3}+52\zeta_3\right) + C_F T_F n_f\left(\frac{260}{27}+\frac{4\pi^2}{3}\right)$$

$$\begin{aligned}
\gamma_2^V = {} & C_F^3\left(-29-6\pi^2-\frac{16\pi^4}{5}-136\zeta_3+\frac{32\pi^2}{3}\zeta_3+480\zeta_5\right) \\
& + C_F^2 C_A\left(-\frac{151}{2}+\frac{410\pi^2}{9}+\frac{494\pi^4}{135}-\frac{1688}{3}\zeta_3-\frac{16\pi^2}{3}\zeta_3-240\zeta_5\right) \\
& + C_F C_A^2\left(-\frac{139345}{1458}-\frac{7163\pi^2}{243}-\frac{83\pi^4}{45}+\frac{7052}{9}\zeta_3-\frac{88\pi^2}{9}\zeta_3-272\zeta_5\right) \\
& + C_F^2 T_F n_f\left(\frac{5906}{27}-\frac{52\pi^2}{9}-\frac{56\pi^4}{27}+\frac{1024}{9}\zeta_3\right) \\
& + C_F C_A T_F n_f\left(-\frac{34636}{729}+\frac{5188\pi^2}{243}+\frac{44\pi^4}{45}-\frac{3856}{27}\zeta_3\right) \\
& + C_F T_F^2 n_f^2\left(\frac{19336}{729}-\frac{80\pi^2}{27}-\frac{64}{27}\zeta_3\right)
\end{aligned}$$

Evolution of the jet function

- Integro-differential evolution equation:

$$\frac{dJ(p^2,\mu)}{d\ln\mu} = -\left[2\Gamma_{\rm cusp}(\alpha_s)\,\ln\frac{p^2}{\mu^2} + 2\gamma^J(\alpha_s)\right] J(p^2,\mu) - 2\Gamma_{\rm cusp}(\alpha_s)\int_0^{p^2} dp'^2\,\frac{J(p'^2,\mu)-J(p^2,\mu)}{p^2-p'^2}$$

- Exact solution (via Laplace transformation):

$$J(p^2,\mu) = \exp\left[-4S(\mu_i,\mu) + 2a_{\gamma^J}(\mu_i,\mu)\right] \times \widetilde{j}(\partial_\eta,\mu_i)\,\frac{e^{-\gamma_E\eta}}{\Gamma(\eta)}\,\frac{1}{p^2}\left(\frac{p^2}{\mu_i^2}\right)^\eta ,$$

with:

$$\boxed{\eta = 2\int_{\mu_0}^{\mu_i}\frac{d\mu}{\mu}\,\Gamma_c[\alpha_s(\mu)]}$$

Evolution of the jet function

- 2-loop result:

$$\widetilde{j}(L,\mu) = 1 + \frac{C_F\alpha_s}{4\pi}\left(2L^2 - 3L + 7 - \frac{2\pi^2}{3}\right) + C_F\left(\frac{\alpha_s}{4\pi}\right)^2 \left[C_F J_F + C_A J_A + T_F n_f J_f\right]$$

[Becher, MN, hep-ph/0603140]

with:

$$J_F = 2L^4 - 6L^3 + \left(\frac{37}{2} - \frac{4\pi^2}{3}\right) L^2 + \left(-\frac{45}{2} + 4\pi^2 - 24\zeta_3\right) L + \frac{205}{8} - \frac{97\pi^2}{12} + \frac{61\pi^4}{90} - 6\zeta_3$$

$$J_A = -\frac{22}{9} L^3 + \left(\frac{367}{18} - \frac{2\pi^2}{3}\right) L^2 + \left(-\frac{3155}{54} + \frac{11\pi^2}{9} + 40\zeta_3\right) L + \frac{53129}{648} - \frac{155\pi^2}{36} - \frac{37\pi^4}{180} - 18\zeta_3$$

$$J_f = \frac{8}{9} L^3 - \frac{58}{9} L^2 + \left(\frac{494}{27} - \frac{4\pi^2}{9}\right) L - \frac{4057}{162} + \frac{13\pi^2}{9}$$

3-loop anomalous dimension γ^J

$$\gamma_0^J = -3C_F$$

$$\gamma_1^J = C_F^2\left(-\frac{3}{2} + 2\pi^2 - 24\zeta_3\right) + C_F C_A\left(-\frac{1769}{54} - \frac{11\pi^2}{9} + 40\zeta_3\right) + C_F T_F n_f\left(\frac{242}{27} + \frac{4\pi^2}{9}\right)$$

$$\begin{aligned}\gamma_2^J = {} & C_F^3\left(-\frac{29}{2} - 3\pi^2 - \frac{8\pi^4}{5} - 68\zeta_3 + \frac{16\pi^2}{3}\zeta_3 + 240\zeta_5\right) \\ & + C_F^2 C_A\left(-\frac{151}{4} + \frac{205\pi^2}{9} + \frac{247\pi^4}{135} - \frac{844}{3}\zeta_3 - \frac{8\pi^2}{3}\zeta_3 - 120\zeta_5\right) \\ & + C_F C_A^2\left(-\frac{412907}{2916} - \frac{419\pi^2}{243} - \frac{19\pi^4}{10} + \frac{5500}{9}\zeta_3 - \frac{88\pi^2}{9}\zeta_3 - 232\zeta_5\right) \\ & + C_F^2 T_F n_f\left(\frac{4664}{27} - \frac{32\pi^2}{9} - \frac{164\pi^4}{135} + \frac{208}{9}\zeta_3\right) \\ & + C_F C_A T_F n_f\left(-\frac{5476}{729} + \frac{1180\pi^2}{243} + \frac{46\pi^4}{45} - \frac{2656}{27}\zeta_3\right) \\ & + C_F T_F^2 n_f^2\left(\frac{13828}{729} - \frac{80\pi^2}{81} - \frac{256}{27}\zeta_3\right)\end{aligned}$$

derived
(see below)

Evolution of the PDF

- RG invariance of DIS cross section implies evolution equation for PDF for $\xi \to 1$:

$$\frac{d}{d\ln\mu}\,\phi_q^{\rm ns}(\xi,\mu) = 2\gamma^\phi(\alpha_s)\,\phi_q^{\rm ns}(\xi,\mu) + 2\Gamma_{\rm cusp}(\alpha_s)\int_\xi^1 d\xi'\,\frac{\phi_q^{\rm ns}(\xi',\mu)}{[\xi'-\xi]_*}$$

$$= \int_\xi^1 \frac{dz}{z}\,P_{q\leftarrow q}^{\rm (endpt)}(z)\,\phi_q^{\rm ns}\!\left(\frac{\xi}{z},\mu\right)$$

with:

$$P_{q\leftarrow q}^{\rm (endpt)}(z) = \frac{2\Gamma_{\rm cusp}(\alpha_s)}{(1-z)_+} + 2\gamma^\phi(\alpha_s)\,\delta(1-z)$$

$$\boxed{\gamma^\phi = \gamma^J - \gamma^V}$$

$\rightarrow$ has been used to derive 3-loop coefficient of γ^J
[Moch, Vermaseren, Vogt (2004)]

Evolution of the PDF

- Endpoint behavior can be parameterized as

$$\phi_q^{\rm ns}(\xi,\mu_f)\Big|_{\xi\to 1} = N(\mu_f)\,(1-\xi)^{b(\mu_f)}\Big[1+\mathcal{O}(1-\xi)\Big]$$

where:

running exponent!

$$b(\mu_f) = b(\mu_0) + 2a_\Gamma(\mu_f,\mu_0)$$

$$N(\mu_f) = N(\mu_0)\,\exp\left[2a_{\gamma^\phi}(\mu_f,\mu_0)\right]\frac{e^{\gamma_E\,b(\mu_0)}\,\Gamma(1+b(\mu_0))}{e^{\gamma_E\,b(\mu_f)}\,\Gamma(1+b(\mu_f))}$$

- Will use this to perform final convolutions

Results

- Exact all-orders momentum-space formula:

$$F_2^{\rm ns}(x,Q^2) = \sum_q e_q^2 \, |C_V(Q^2,\mu_h)|^2 \left(\frac{Q^2}{\mu_h^2}\right)^{-2a_\Gamma(\mu_h,\mu_i)} \exp\left[4S(\mu_h,\mu_i) - 2a_{\gamma_V}(\mu_h,\mu_i)\right]$$

$$\times \exp\left[2a_{\gamma^\phi}(\mu_i,\mu_f)\right] \widetilde{j}\left(\ln\frac{Q^2}{\mu_i^2} + \partial_\eta, \mu_i\right) \frac{e^{-\gamma_E \eta}}{\Gamma(\eta)} \int_x^1 d\xi \, \frac{\phi_q^{\rm ns}(\xi,\mu_f)}{\left[(\xi/x - 1)^{1-\eta}\right]_*}$$

  - No integrals over Landau poles!
  - Physical scales $\mu_h \sim Q$ and $\mu_i \sim Q\sqrt{1-x}$ cleanly separated from factorization scale μ_f

Results

- Performing final convolution integral yields the K-factor:

$$K_{\mathrm{DIS}}(Q^2,x) = |C_V(Q^2,\mu_h)|^2 \left(\frac{Q^2}{\mu_h^2}\right)^{-2a_\Gamma(\mu_h,\mu_i)} \exp\left[4S(\mu_h,\mu_i) - 2a_{\gamma V}(\mu_h,\mu_i)\right]$$

$$\times \exp\left[2a_{\gamma\phi}(\mu_i,\mu_f)\right] \underbrace{\boxed{(1-x)^\eta}}_{(M_X/Q)^{2\eta}} \tilde{j}\left(\ln \frac{\underbrace{\boxed{Q^2(1-x)}}_{M_X^2}}{\mu_i^2} + \partial_\eta, \mu_i\right) \frac{e^{-\gamma_E \eta}\,\Gamma(1+b_q)}{\Gamma(1+b_q+\eta)}$$

  - Explicit dependence on physical scales Q and M_X
  - Factor $(1-x)^\eta$ is source of huge K-factor if $\mu_f > \mu_i$ (i.e., $\eta<0$)

Results

- Analogous result obtained for Drell-Yan:

$$K_{\rm DY}(s,\tau) = |C_V(-s,\mu_h)|^2 \left(\frac{s}{\mu_h^2}\right)^{-2a_\Gamma(\mu_h,\mu_i)} \exp\left[4S(\mu_h,\mu_i) - 2a_{\gamma^V}(\mu_h,\mu_i)\right]$$

$$\times \exp\left[4a_{\gamma^\phi}(\mu_i,\mu_f)\right] \underbrace{\boxed{(1-\tau)^{2\eta}}}_{(M_X/\sqrt{s})^{2\eta}} \widetilde{j}_{\rm DY}\left(\ln\frac{\boxed{s(1-\tau)^2}}{\mu_i^2} + \partial_\eta, \mu_i\right) \frac{e^{-2\gamma_E \eta}\,\Gamma(2+b_q+b_{\bar q})}{\Gamma(2+b_q+b_{\bar q}+2\eta)}$$

M_X^2

- Straightforward to expand these results order by order in RG-resummed perturbation theory (known to NNLO = N^{3}LL)

Connection with conventional approach

- Recall conventional formula (moment space):

$$C_N(Q^2,\mu_f) = g_0(Q^2,\mu_f)\,\exp\left[G_N(Q^2,\mu_f)\right]$$

$$G_N(Q^2,\mu_f) = \int_0^1 dz\,\frac{z^{N-1}-1}{1-z} \times \left[\int_{\mu_f^2}^{(1-z)Q^2} \frac{dk^2}{k^2}\,A_q(\alpha_s(k)) + B_q\big(\alpha_s(Q\sqrt{1-z})\big)\right]$$

- Work out how g_0, A_q, and B_q are related to objects in SCET (anomalous dimensions and Wilson coefficients)

Connection with conventional approach

- Find (with $\nabla = d/d\ln\mu^2$):

$$A_q(\alpha_s) = \Gamma_{\rm cusp}(\alpha_s)$$

$$e^{\gamma_E \nabla}\,\Gamma(1+\nabla)\,B_q(\alpha_s) = \gamma^J(\alpha_s) + \nabla\,\ln\widetilde{j}(0,\mu) - \left[e^{\gamma_E \nabla}\,\Gamma(\nabla) - \frac{1}{\nabla}\right]\Gamma_{\rm cusp}(\alpha_s)$$

- B_q (as well as g_0) not related to simple field-theoretic objects in EFT, but to complicated combinations of anomalous dimensions and matching coefficients

Connection with conventional approach

- It has been claimed that resummation in x-space is plagued by strong factorial growth of expansion coefficients not related to IR renormalons [Catani, Mangano, Nason, Trentadue (1996)]
- Leads to "unphysical" power corrections $\sim (\Lambda/Q)^{\gamma}$ with γ = 1.44 / 0.72 for Drell-Yan in $\overline{\text{MS}}$ / DIS scheme, and γ = 0.16 for heavy-quark production in gluon-gluon fusion

Connection with conventional approach

- In our approach this problem has been overcome!
- Indeed, perturbative convergence is better in x-space than in N-space (see below)
- Physical IR renormalon poles (unavoidable) arise in matching conditions only and are commensurate with power corrections from higher-dimensional operators in SCET:
  - $C_V(Q,\mu) \;\rightarrow\; (\Lambda/Q)^2$ at hard scale
  - $\tilde{j}(L,\mu) \;\rightarrow\; (\Lambda/M_X)^2$ at jet scale

Connection with conventional approach

- Absence of unphysical power corrections follows from very existence of effective theory
  - Difference with Catani et al. is that we fix the intermediate scale $\mu_i \sim M_X$ at the end, after all integrals are performed
  - Also, their LL approximation does not correspond to any consistent truncation in EFT approach

RG-impr. PT	Log. Approx.	Accuracy $\sim \alpha_s^n L^k$	Γ_{cusp}	γ^V, γ^J	C_V, $\widetilde{j}$
—	LL	$n+1 \le k \le 2n$ (α_s^{-1})	1-loop	tree-level	tree-level
LO	NLL	$n \le k \le 2n$ (α_s^0)	2-loop	1-loop	tree-level
NLO	NNLL	$n-1 \le k \le 2n$ (α_s)	3-loop	2-loop	1-loop
NNLO	NNNLL	$n-2 \le k \le 2n$ (α_s^2)	4-loop	3-loop	2-loop

Numerical Results

Resummed vs. fixed-order PT

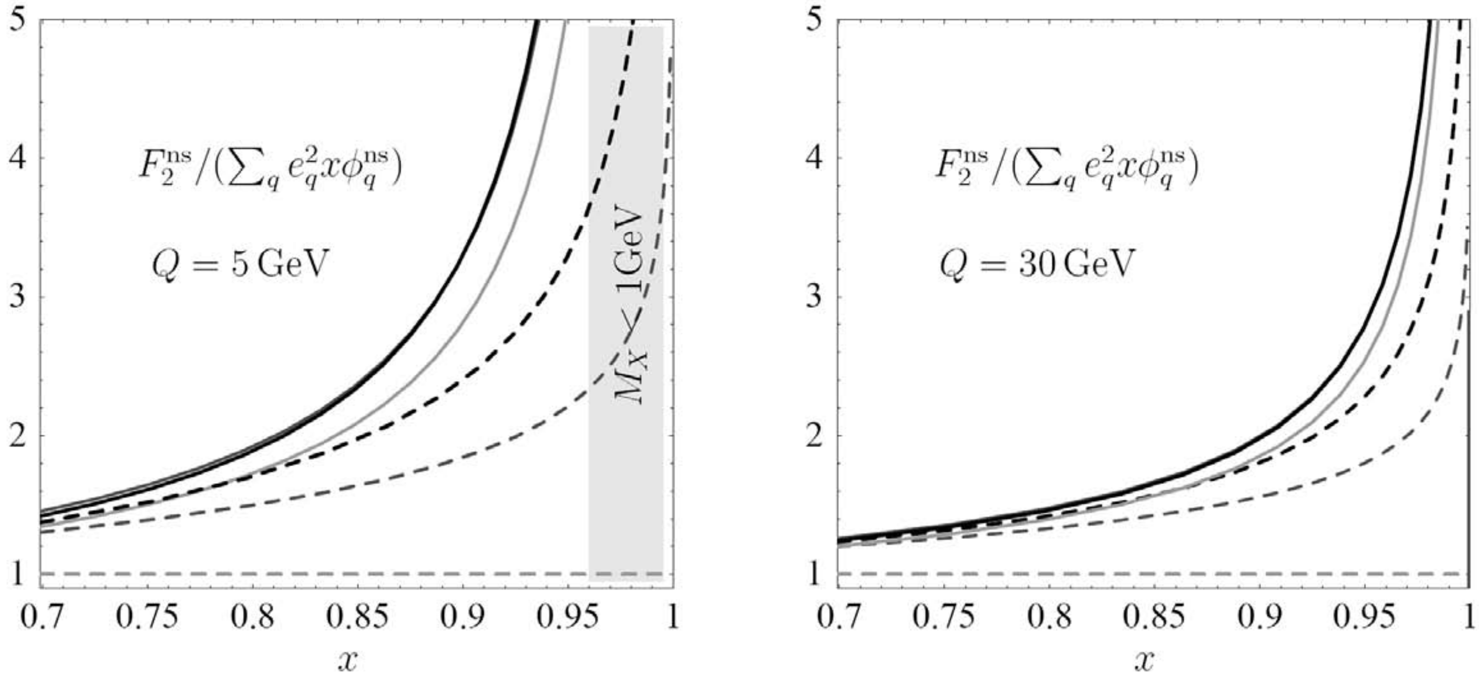


Figure 5: Comparison between fixed-order (dashed) and resummed results (solid). The light-gray curves are the LO result, dark gray NLO, black NNLO. For the resummed result, we set $\mu_h = Q$, $\mu_i = M_X$, $\mu_f = 5\mathrm{GeV}$ and $b(\mu_f) = 4$. The fixed order result with $\mu = Q$ is obtained by setting $\mu_h = \mu_i = Q$ in the resummed expression.

Perturbative uncertainties

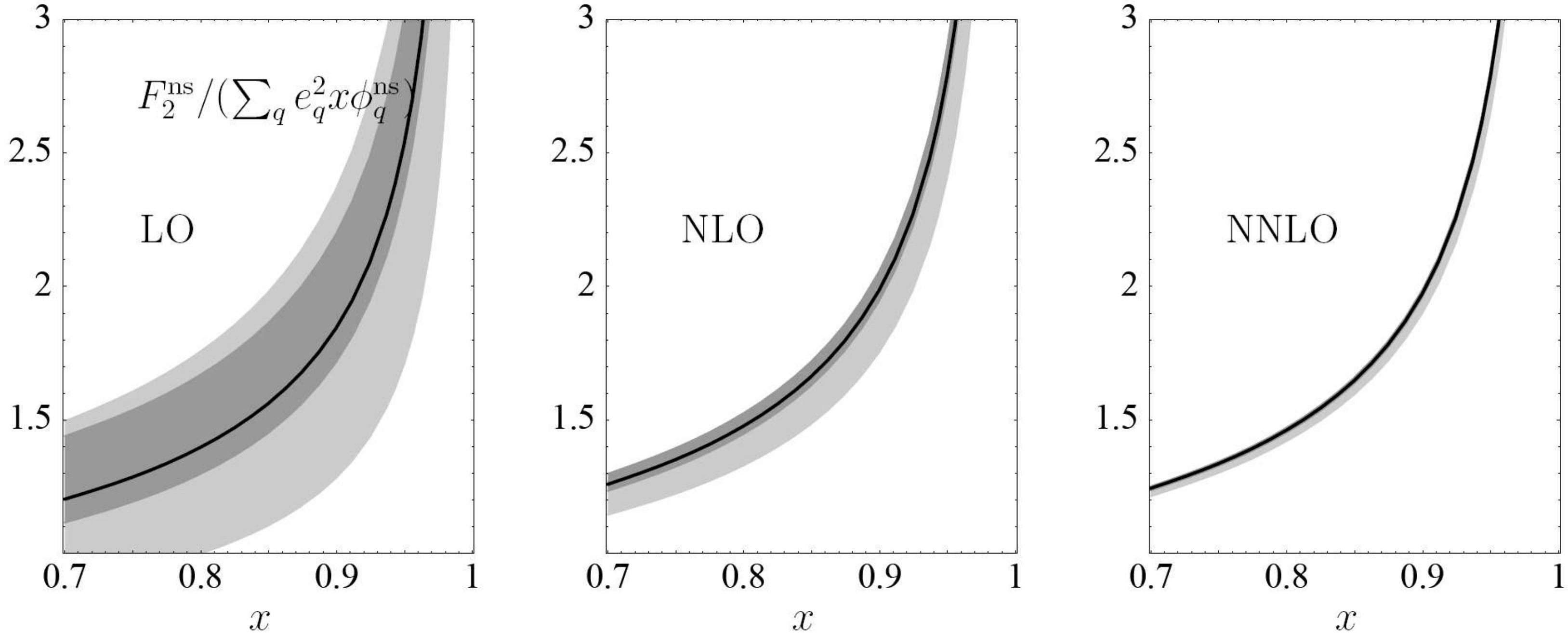


Figure 7: Scale variation of the result for $F_2^{\rm ns}$ at $Q = 30\text{GeV}$. The light-gray band is obtained by varying $M_X/2 < \mu_i < 2M_X$, while the dark-gray band arises from varying the hard scale $Q/2 < \mu_h < 2Q$. We set $\mu_f = 30\text{GeV}$ and $b(\mu_f) = 4$.

Resummation in x- vs. N-space

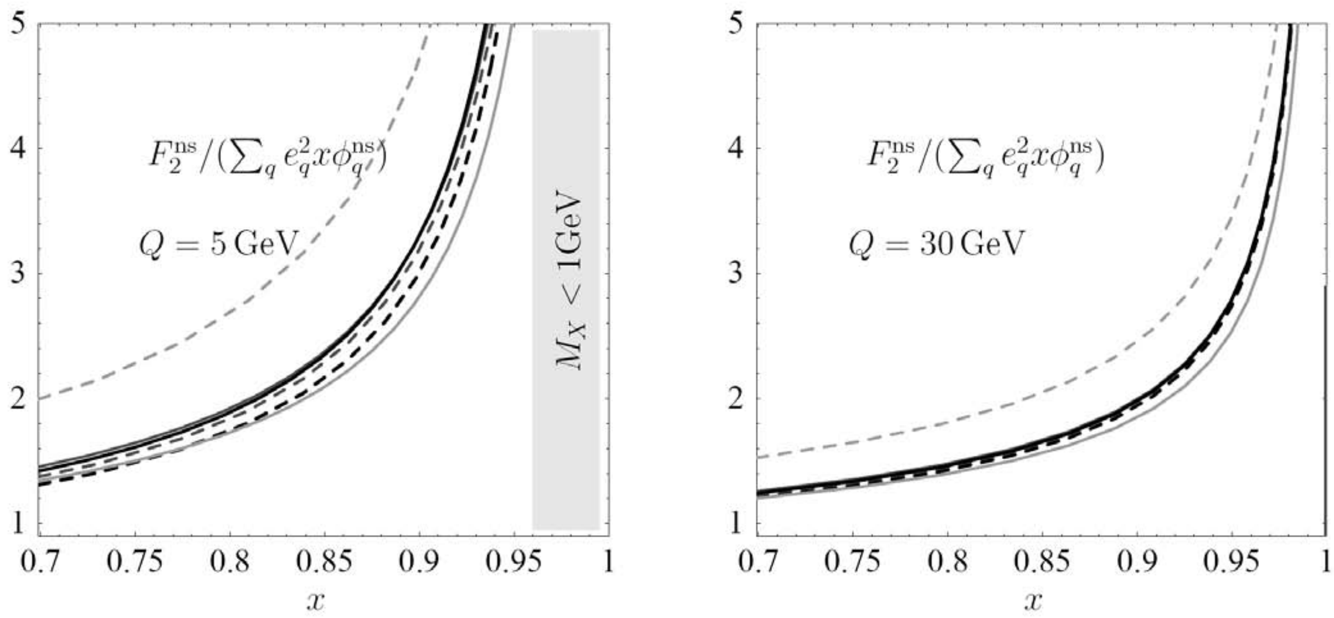


Figure 8: Comparison between Mellin-inverted moment space results (dashed) and results obtained in directly in x-space (solid). The light-gray curves are the LO result, dark gray NLO. The black lines are NNLO results and are visually indistinguishable from the NLO curves for $Q = 30\mathrm{GeV}$. We set $\mu_h = \mu_f = Q$ and $b(\mu_f) = 4$. For the intermediate scale, we choose $\mu_i = M_X$ in momentum space and $\mu_i = Q/\sqrt{N}$ in moment space.

Conclusions

- Methods from effective field theory provide powerful, efficient tools to study factorization, resummation, and power corrections in many hard QCD processes
- Have resummed Sudakov logarithms directly in momentum space by solving RGEs
- Results agree with traditional approach at every fixed order in perturbation theory, but are free of spurious Landau-pole singularities
- Easier to match with FOPT results for differential cross sections away from threshold region

Conclusions

- What else can SCET do for you?
  - Will try to get more mileage out of resummation
  - Possible to study power corrections systematically (often messy)
  - SCET approach to parton showers appears promising!
  - Understand miracles of $\mathcal{N}=4$ SUSY Yang-Mills?
  - … ?
- More at LoopFest VI