Electroweak Sudakov logarithms

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(1) **Gauge-bosons pair production with high p_T at the LHC**

– 1-loop effects [Accomando, Denner, Hollik, Meier, Kaiser]

(2) **Single gauge-boson production with high p_T at the LHC**

– 1- and 2-loop effects [Kühn, Kulesza, P., Schulze]

(3) **4-fermion neutral current processes**

– complete 2-loop logarithmic corrections ($\mathrm{N^3LL}$) [Jantzen, Kühn, Penin, Smirnov]

(4) **Progress towards 2-loop predictions for general processes**

– in NLL approximation [Denner, Jantzen, P.]

Introduction

High-energy colliders (ILC, LHC) will explore the TeV energy scale

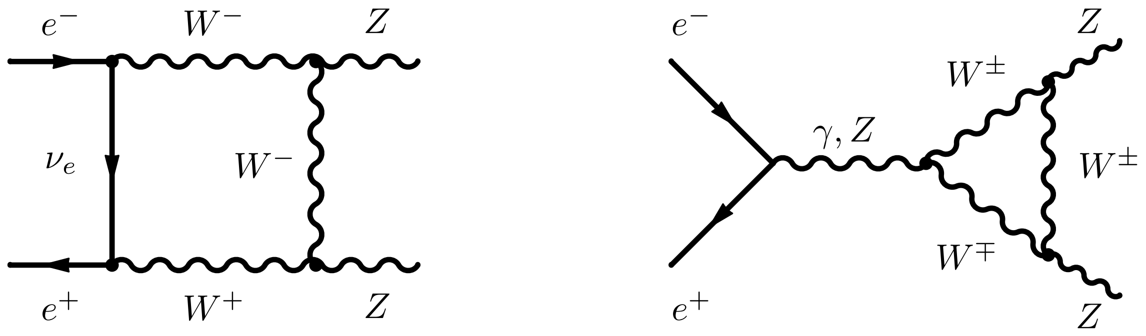
- investigate the mechanism of electroweak symmetry breaking
- search for new physics

Important implications for loop corrections within the Standard Model

- collider energy $\gg$ characteristic scale of EW corrections ($s \gg M_W^2$)
- enhancement of EW corrections due to large Sudakov logarithms

$$\ln^2\left(\frac{s}{M_{\mathrm{W}}^2}\right) \sim 25 \quad \text{at} \quad \sqrt{s} \sim 1\ \mathrm{TeV}$$

Originate from vertex and box diagrams involving virtual weak bosons



$$\Rightarrow \alpha \ln^2\left(\frac{s}{M_W^2}\right)$$

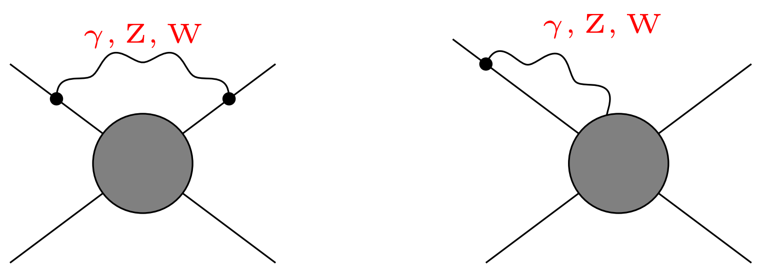
General form of 1 loop EW corrections for $s \gg M_W^2$

$$\alpha\left[C_2 \underbrace{\ln^2\left(\frac{s}{M_W^2}\right)}_{\text{LL}} + C_1 \underbrace{\ln^1\left(\frac{s}{M_W^2}\right)}_{\text{NLL}} + C_0\right] + \mathcal{O}\left(\frac{M_W^2}{s}\right)$$

Typical size of logs for $2 \to 2$ **processes at** $\sqrt{s} \simeq 1\,\text{TeV}$**: effects of** $\mathcal{O}(10\%)$

$$\left(\frac{\delta\sigma_1}{\sigma_0}\right)_{\text{LL}} \simeq -\frac{\alpha}{\pi s_{\text{W}}^2}\log^2\frac{s}{M_{\text{W}}^2} \simeq -26\% \qquad \left(\frac{\delta\sigma_1}{\sigma_0}\right)_{\text{NLL}} \simeq +\frac{3\alpha}{\pi s_{\text{W}}^2}\log\frac{s}{M_{\text{W}}^2} \simeq +16\%$$

The $\log(s/M^2)$ terms represent mass singularities and originate from



- diagrams with virtual gauge bosons ($\gamma, Z, W^\pm$) coupling to **on-shell external legs**
- **soft** and **collinear** regions

Factorization and universality at one loop

$$\mathcal{M}_1 = \Bigg(1 + \frac{\alpha}{4\pi} \underbrace{\sum_{\mathbf{legs}\, k} \delta^1_{\mathrm{EW}}(k)}_{\text{soft,coll.}}\Bigg)\, \mathcal{M}_0$$

- Born × external-leg factors (universal)
- LL and NLL for arbitrary processes ($e, \nu, u, d, t, b, \gamma, Z, W^\pm, H, g$)

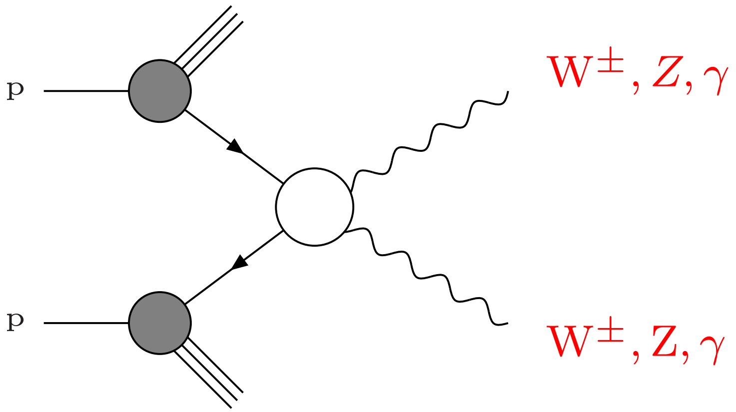
External-leg factors depend only on external-leg quantum numbers

$$\delta^1_{\mathrm{EW}}(k) \;=\; -\tfrac{1}{2} C^{\mathrm{ew}}(k) \log^2 \tfrac{s}{M^2} + \sum_{l\neq k} \sum_{a=\gamma,\mathrm{Z},\mathrm{W}^\pm} I^a(k) I^{\bar{a}}(l) \log \tfrac{r_{kl}}{s} \log \tfrac{s}{M^2} + \gamma^{\mathrm{ew}}(k) \log \tfrac{s}{M^2}$$

$$-\tfrac{1}{2} Q^2(k) \left[2 \log \tfrac{s}{m_k^2} \log \tfrac{M^2}{\lambda^2} - \log^2 \tfrac{M^2}{m_k^2} - 2 \log \tfrac{M^2}{\lambda^2} - \log \tfrac{M^2}{m_k^2} \right] + \sum_{l\neq k} Q(k) Q(l) \log \tfrac{r_{kl}}{s} \log \tfrac{M^2}{\lambda^2}$$

Denner and P. (2001)

1. One-loop EW corrections in gauge-boson pair production at the LHC



Motivation

- test YM interactions of gauge bosons, search for anomalous gauge couplings
- study region of **high invariant mass** $\hat{s} = (p_{V_1} + p_{V_2})^2$ and **large scattering angles** $\Rightarrow$ high $P_{\mathrm{T}}(V)$
- large EW logarithmic corrections in this region

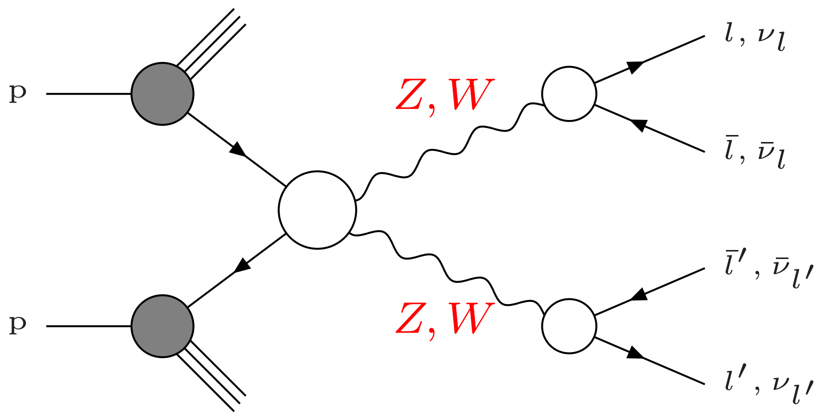
Existing calculations for pp$\to V_1 V_2$

- all processes apart from pp$\to \gamma\gamma$ computed

process	decay	corrections	
WZ, Wγ	no	NLL	Accomando, Denner, P. (2002)
Zγ	no	exact	Hollik, Meier (2004)
WW, WZ, ZZ	yes	NLL	Accomando, Denner, Kaiser (2005)
Zγ, Wγ	yes	exact+NLL	Accomando, Denner, Meier (2005)

- exact $\mathcal{O}(\alpha)$ results and/or NLL approximation depending on the process
- most recent calculations include decay of weak bosons

pp → WW, WZ, ZZ at the LHC



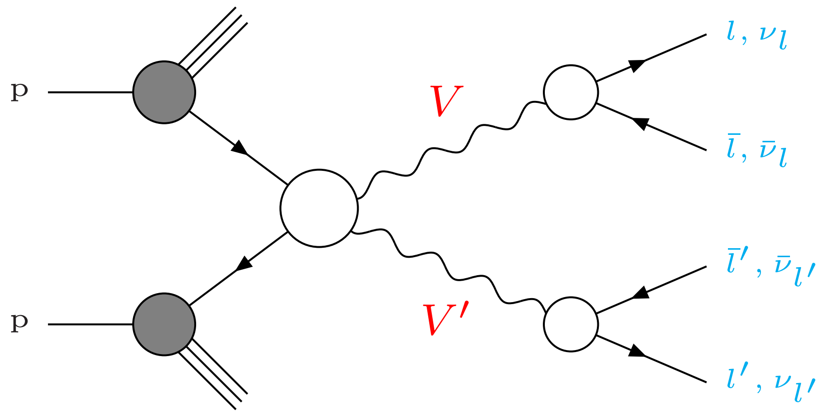
Detailed description of final state

- **leptonic decays** $l, l' = \mathrm{e}, \mu$
- **hard photon bremsstrahlung**

Electroweak corrections

- double-pole approximation (DPA)
- high-energy region $\hat{s}, |\hat{t}|, |\hat{u}| \gg M_\mathrm{W}^2$
  NLL approximation

Accomando, Denner, Kaiser (2005)



Double-pole approximation

$$\mathcal{M}_{\text{fact}}^{qq'\to VV'\to l\nu_l l'\bar{l}'} = \frac{-\sum_{\lambda,\lambda'} \mathcal{M}^{qq'\to V_\lambda V'_{\lambda'}} \mathcal{M}^{V_\lambda \to l\nu_l} \mathcal{M}^{V'_{\lambda'}\to l'\bar{l}'}}{(p_V^2 - M_V^2 + \mathrm{i}M_V\Gamma_V)\,(p_{V'}^2 - M_{V'}^2 + \mathrm{i}M_{V'}\Gamma_{V'})}$$

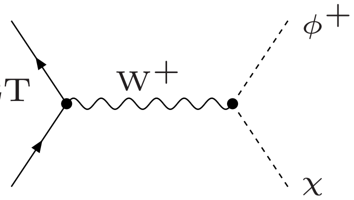
gauge-invariant factorization and separation of scales

on-shell production $\times$ **on-shell** decay

$\hat{s} \gg M^2 \Rightarrow \log(\frac{\hat{s}}{M^2})$ $\qquad$ $M^2 \gg m_l^2 \gg \lambda^2 \Rightarrow \log(\frac{\lambda^2}{M^2}),\ \log(\frac{m_l^2}{M^2})$

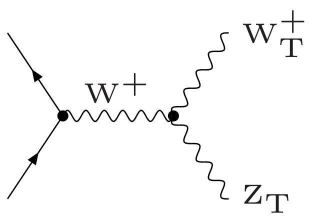
Example: electroweak NLL corrections to $q\bar{q}' \to \mathrm{WZ}$

Longitudinal polarization

$\mathcal{M}_0 \overset{\mathrm{GBET}}{=}$ 

$$\frac{\delta\mathcal{M}_1}{\mathcal{M}_0} = \frac{\alpha}{4\pi}\left\{-\log^2\frac{\hat{s}}{M_\mathrm{W}^2}\left[C^{\mathrm{ew}}_{q_\mathrm{L}} + C^{\mathrm{ew}}_{\Phi}\right] + \log\frac{\hat{s}}{M_\mathrm{W}^2}\left[-\frac{2}{s_\mathrm{W}^2}\left(\log\frac{|\hat{u}|}{\hat{s}} + \log\frac{|\hat{t}|}{\hat{s}} - \frac{s_\mathrm{W}^2}{c_\mathrm{W}^2}Y_{q_\mathrm{L}}\log\frac{\hat{t}}{\hat{u}}\right)3C^{\mathrm{ew}}_{q_\mathrm{L}} + 4C^{\mathrm{ew}}_{\Phi} - \frac{3}{2s_\mathrm{W}^2}\frac{m_t^2}{M_\mathrm{W}^2} - b_2^{(1)}\right]\right\}$$

Transverse polarization

$\mathcal{M}_0 =$ 

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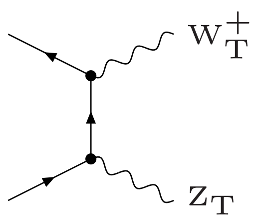

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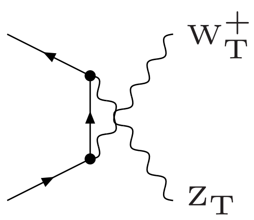
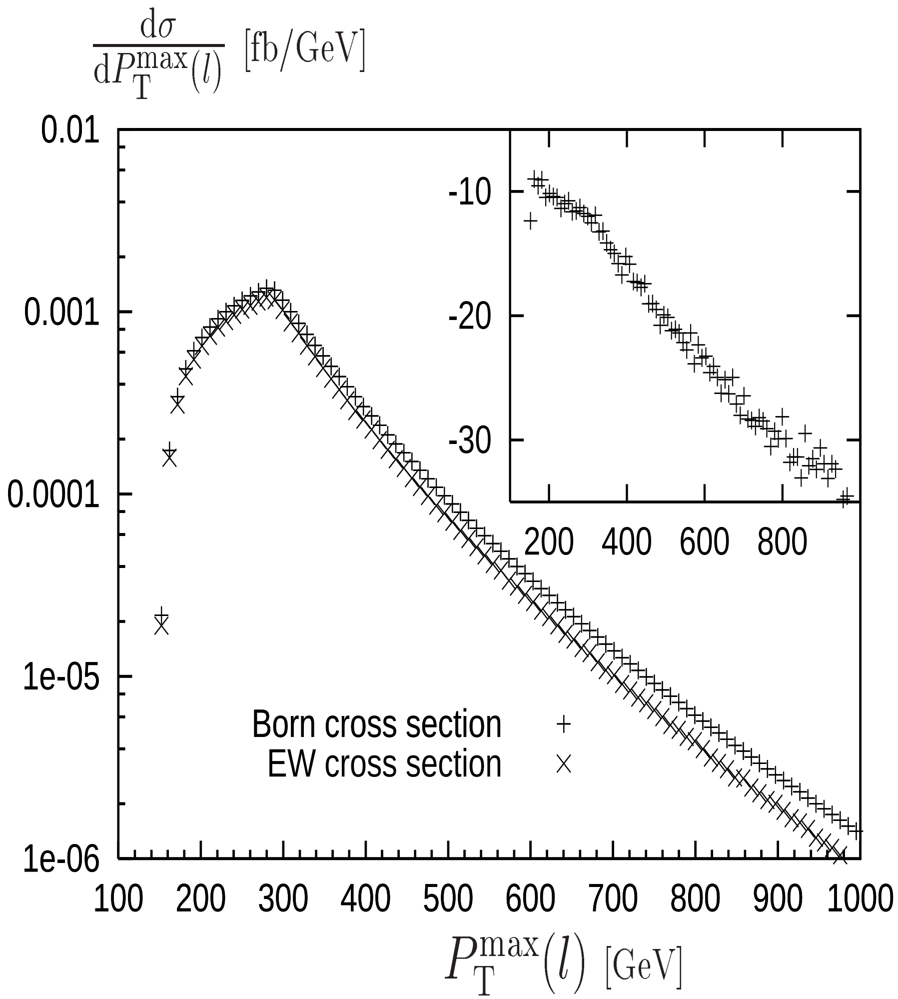


$$\frac{\delta\mathcal{M}_1}{\mathcal{M}_0} = -\frac{\alpha}{8\pi}\log^2\frac{\hat{s}}{M_\mathrm{W}^2}\left\{2C^{\mathrm{ew}}_{q_\mathrm{L}} + C^{\mathrm{ew}}_{W}\left[1 + \frac{c_\mathrm{W}^2\cos\hat{\theta}}{c_\mathrm{W}^2\cos\hat{\theta} - s_\mathrm{W}^2 Y_{q_\mathrm{L}}}\right]\right\}$$
$$+ \frac{\alpha}{4\pi}\log\frac{\hat{s}}{M_\mathrm{W}^2}\left\{-\frac{1}{s_\mathrm{W}^2}\left[\log\frac{|\hat{t}|}{\hat{s}} + \log\frac{|\hat{u}|}{\hat{s}} + \frac{c_\mathrm{W}^2}{c_\mathrm{W}^2\cos\hat{\theta} - s_\mathrm{W}^2 Y_{q_\mathrm{L}}}\log\frac{\hat{t}}{\hat{u}}\right] + 3C^{\mathrm{ew}}_{q_\mathrm{L}}\right\}$$

derived from Denner and P. (2001)

$p_{\mathrm{T}}^{\max}(l)$ distribution for $\mathrm{pp} \to \mathrm{WZ} \to e\nu_e\mu^+\mu^-$ at the LHC



Cuts for LHC detectors

- $p_{\mathrm{T}}(l)$, $p_{\mathrm{T}}^{\mathrm{miss}} > 20\,\mathrm{GeV}$, $|\eta_l| < 3$
- recombination of soft and collinear photons ($E_\gamma < 2\,\mathrm{GeV}$, $\Delta R_{l\gamma} < 0.1$)

Cuts to select gauge-bosons resonances

- $|M(l'\bar{l}') - M_Z| < 20$ GeV
- $M_{\mathrm{T}}(l\nu_l) - M_{\mathrm{W}} < 20$ GeV

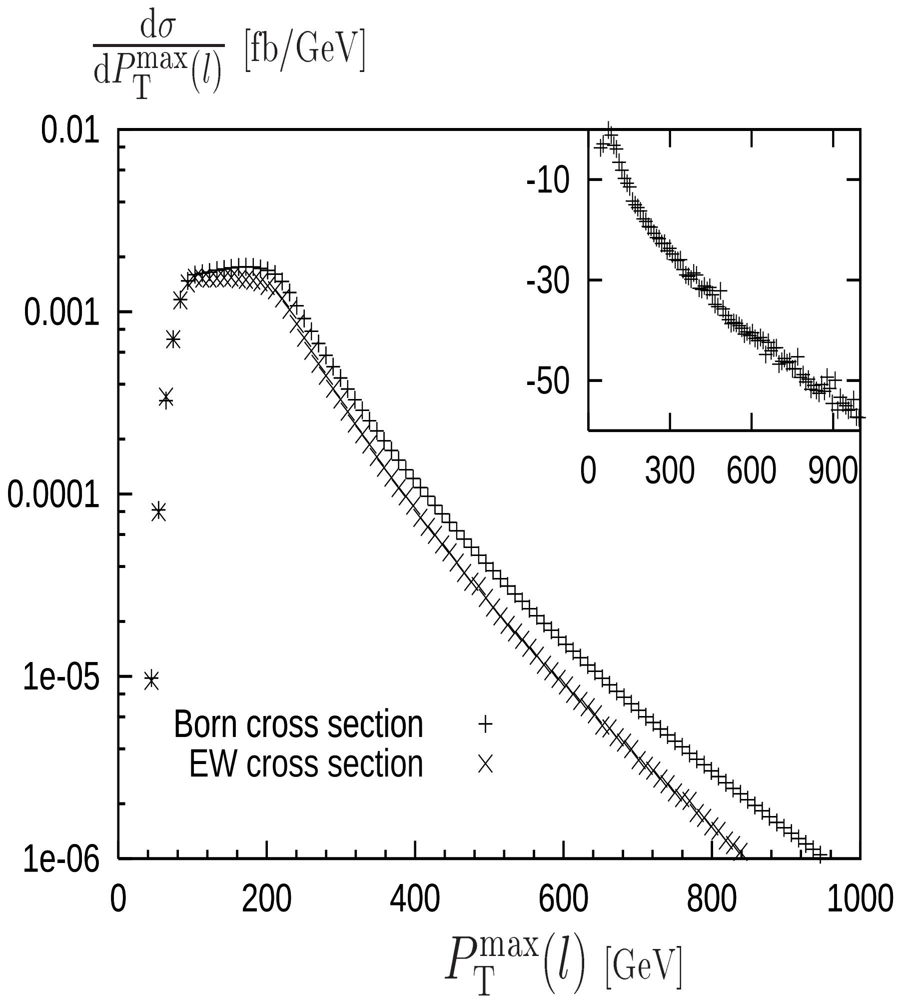
Cuts to select high-energy region

- $p_T(Z) > 300$ GeV

Large negative corrections

- increase with p_{T}
- -30% at $p_{\mathrm{T}}^{\max}(l) \sim 800$ GeV !

$p_{\mathrm{T}}^{\max}(l)$ distribution for $\mathrm{pp} \to \mathrm{ZZ} \to e^+e^-\mu^+\mu^-$ at the LHC



Cuts for LHC detectors

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Cuts to select gauge-bosons resonances

- $|M(l\bar{l}) - M_Z| < 20$ GeV

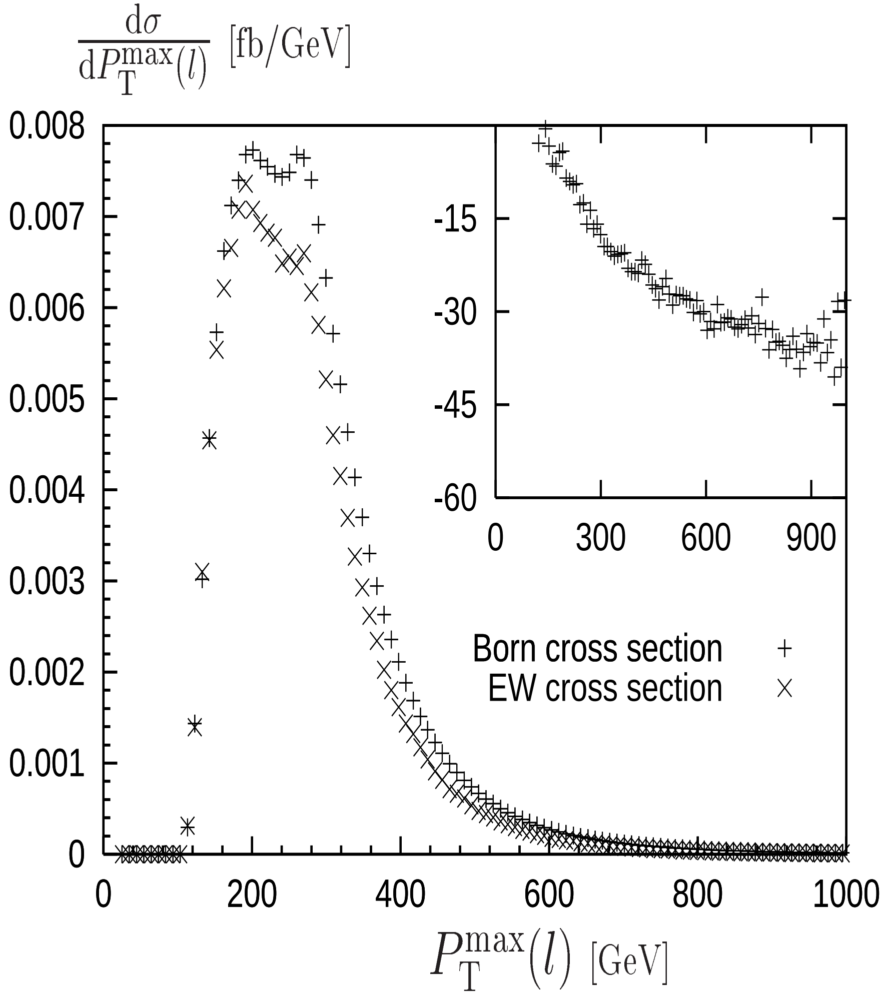
Cuts to select high-energy region

- $M_{\mathrm{inv}}(l\bar{l}l'\bar{l}') > 500\,\mathrm{GeV}$, $|\Delta y(ZZ)| < 3$

Large negative corrections

- p_{T} dependence similar as for WZ
- size larger than for WZ
- -50% at $p_{\mathrm{T}}^{\max}(l) \sim 800$ GeV !

$p_{\mathrm{T}}^{\max}(l)$ distribution for $\mathrm{pp} \to \mathrm{WW} \to e^+\nu_e\mu^-\bar{\nu}_\mu$ at the LHC



Cuts for LHC detectors

- $p_{\mathrm{T}}(l) > 20\,\mathrm{GeV}$, $p_{\mathrm{T}}^{\mathrm{miss}} > 25\,\mathrm{GeV}$, $|\eta_l| < 3$
- recombination of soft and collinear photons ($E_\gamma < 2\,\mathrm{GeV}$, $\Delta R_{l\gamma} < 0.1$)

Cuts to select gauge-bosons resonances

- W reconstruction not possible

Cuts to select high-energy region

- $M_{\mathrm{inv}}(l\bar{l}') > 500\,\mathrm{GeV}$, $|\Delta y(l\bar{l}')| < 3$

Large negative corrections

- behaviour and size similar as for WZ
- -35% at $p_{\mathrm{T}}^{\max}(l) \sim 800$ GeV !

Electroweak corrections (Δ_{EW}) vs statistical error ($\Delta_{\mathrm{stat}} = 1/\sqrt{2L\sigma_0}$)

	$\mathrm{pp} \to WZ \to l\nu_l l'\bar{l}'$		$\mathrm{pp} \to ZZ \to l\bar{l}l'\bar{l}'$		$\mathrm{pp} \to WW \to l\bar{\nu}_l \nu_{l'}\bar{l}'$	
$M_{\mathrm{inv}}^{\mathrm{cut}}$(leptons)[GeV]	Δ_{EW} [%]	Δ_{stat} [%]	Δ_{EW} [%]	Δ_{stat} [%]	Δ_{EW} [%]	Δ_{stat} [%]
500	−7.4	5.4	−15.0	8.5	−13.8	2.6
600	−9.5	7.5	−18.3	11.9	−15.9	3.7
700	−10.9	9.9	−21.0	15.7	−18.1	4.9
800	−13.3	12.8	−23.8	20.1	−20.2	6.5
900	−15.1	16.2	−26.1	25.3	−22.0	8.3
1000	−16.7	20.2	−28.1	31.2	−23.4	10.4

estimate based on $L = 100\mathrm{fb}^{-1}$ and final states with $l, l' = \mathrm{e}$ or μ

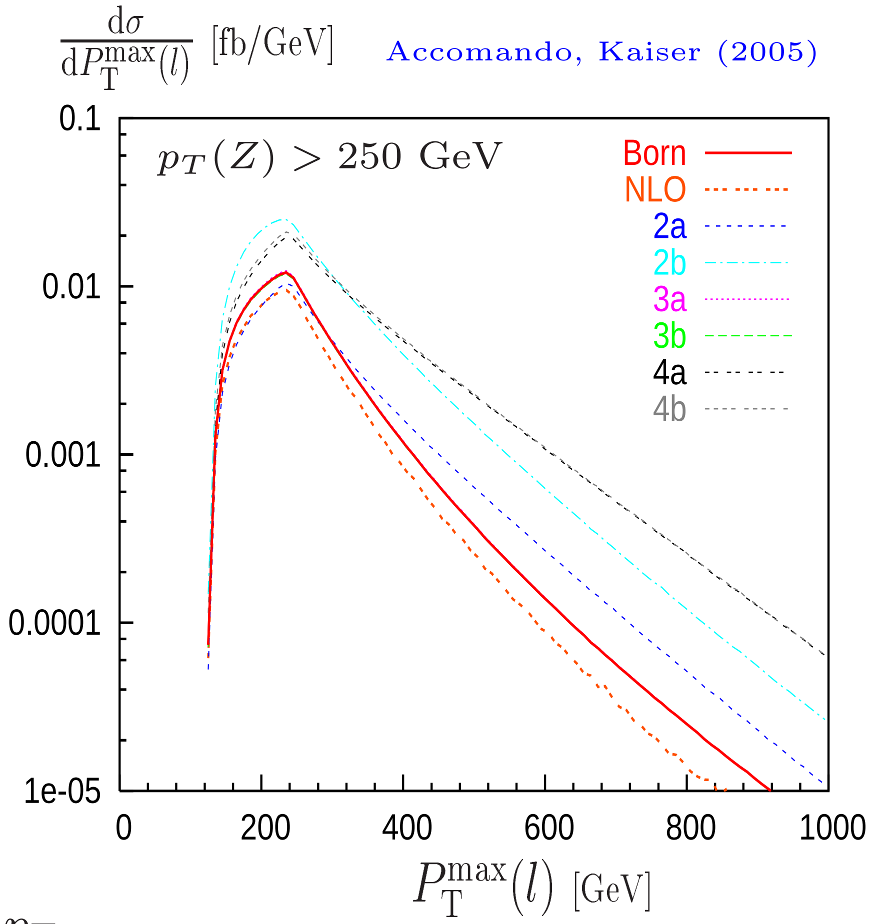
- Δ_{EW} and Δ_{stat} grow with M_{inv}(leptons)
- $\Delta_{\mathrm{EW}} \geq \Delta_{\mathrm{stat}}$ up to $M_{\mathrm{inv}} \sim 1$ TeV

Electroweak corrections vs anomalous couplings

$$\mathcal{L} = g_{WWV}\Big[g_1^V(W^\dagger_{\mu\nu}W^\mu V^\nu - W^\dagger_\mu V_\nu W^{\mu\nu}) + \kappa^V W^\dagger_\mu W_\nu V^{\mu\nu} + \frac{\lambda}{M_W^2} W^\dagger_{\rho\mu} W^\mu_\nu V^{\nu\rho}\Big]$$

Limits from LEP2 and Tevatron: $-0.054 \le \Delta g_1^Z \le 0.028$, $-0.117 \le \Delta\kappa_\gamma \le 0.061$, $-0.07 \le \Delta\lambda \le 0.012$

Scenario	λ	Δg_1^Z	$\Delta\kappa_\gamma$
$2a/2b$	0	± 0.02	0
$3a/3b$	0	0	± 0.04
$4a/4b$	± 0.02	0	0



$pp \to WZ \to e\nu_e\mu^+\mu^-$: **large effects at high** p_T

$\Rightarrow$ anomalous couplings spoil gauge cancellations and (in general) enhance σ

$\Rightarrow$ electroweak corrections reduce σ and increase the sensitivity to anomalous couplings

Electroweak corrections vs anomalous couplings

$$\mathcal{L} = g_{WWV}\Big[g_1^V (W^\dagger_{\mu\nu} W^\mu V^\nu - W^\dagger_\mu V_\nu W^{\mu\nu}) + \kappa^V W^\dagger_\mu W_\nu V^{\mu\nu} + \frac{\lambda}{M_W^2} W^\dagger_{\rho\mu} W^\mu_\nu V^{\nu\rho}\Big]$$

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Scenario	λ	Δg_1^Z	$\Delta\kappa_\gamma$
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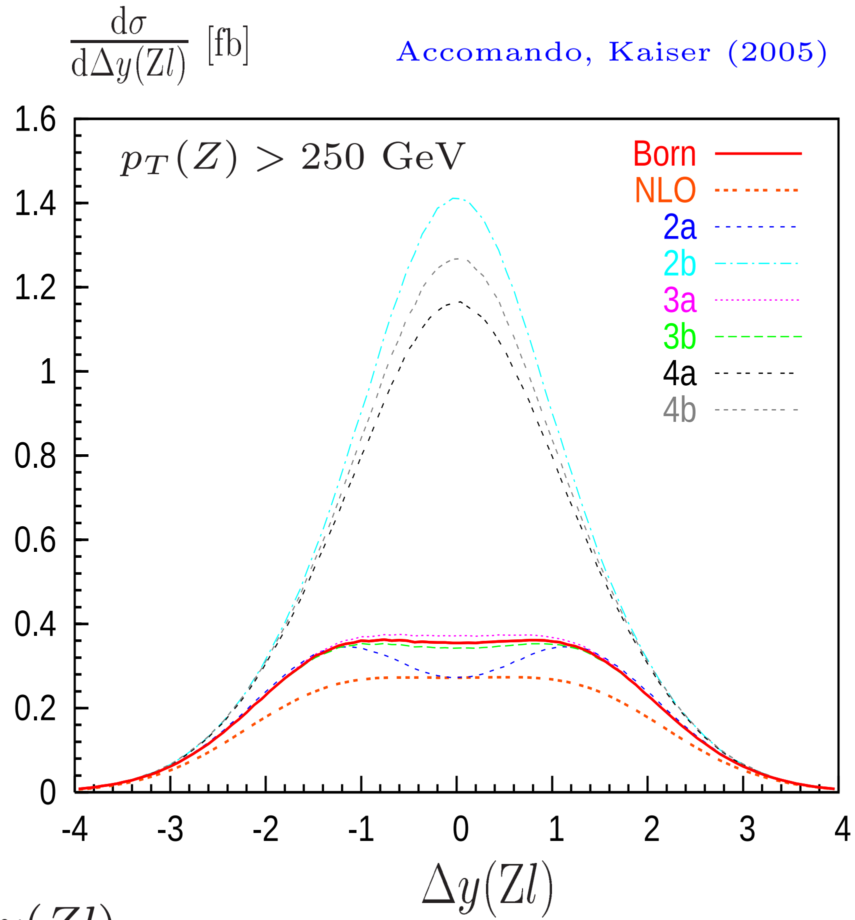
$4a/4b$	± 0.02	0	0

Accomando, Kaiser (2005)



$pp \to WZ \to l\nu_l l'\bar{l}'$: **large effects at small $\Delta y(Zl)$**

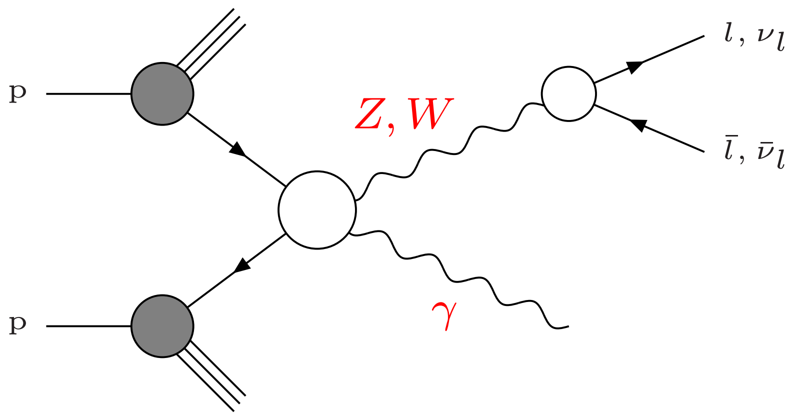
$\Rightarrow$ (in general) dip from radiation zero filled by anomalous couplings

$\Rightarrow$ dip enhanced by electroweak logarithmic corrections

One-loop predictions for pp $\rightarrow$ WW, ZZ, WZ
based on NLL approximation

how precise is the NLL approximation?

pp $\to$ Wγ, Zγ at the LHC



Detailed description of final state

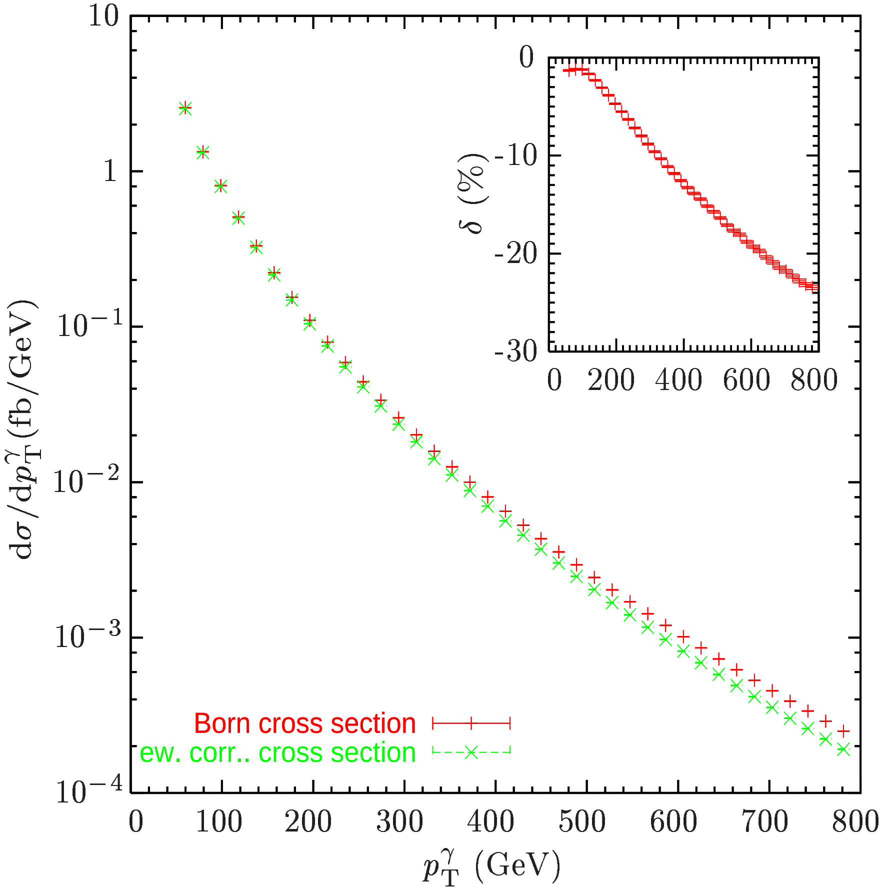
- **leptonic decays** $l = \mathrm{e}, \mu$
- **hard photon bremsstrahlung**

Electroweak corrections

- leading-pole approximation (LPA)
- exact $\mathcal{O}(\alpha)$ calculation
- high-energy region $\hat{s}, |\hat{t}|, |\hat{u}| \gg M_{\mathrm{W}}^2$
  NLL approximation

Accomando, Denner and Meier (2005)

$p_{\mathrm{T}}(\gamma)$ distribution for $\mathrm{pp} \to \mathrm{W}\gamma \to l\nu_l\gamma$ at the LHC



Cuts for LHC detectors

- $p_{\mathrm{T}}(l) > 20\,\mathrm{GeV}$, $p_{\mathrm{T}}^{\mathrm{miss}} > 50\,\mathrm{GeV}$, $|\eta_{l,\gamma}| < 2.5$, $\Delta R_{\gamma l} > 0.7$
- recombination of soft and collinear photons ($E_\gamma < 2\,\mathrm{GeV}$, $\Delta R_{l\gamma} < 0.1$)

Cut to select W resonance

- $M_{\mathrm{T}}(l\nu_l) - M_{\mathrm{W}} < 20$ GeV

Large negative corrections

- increase with p_{T}
- $\Delta_{\mathrm{EW}} \geq \Delta_{\mathrm{stat}}$ up to $p_{\mathrm{T}}(\gamma) \sim 700$ GeV
- -23% at $p_{\mathrm{T}}(\gamma) \sim 700$ GeV
- 1%-level agreement with virtual NLL approximation [Accomando, Denner, P. (2002)]

Virtual corrections to pp→ Wγ: exact corrections vs NLL approximation

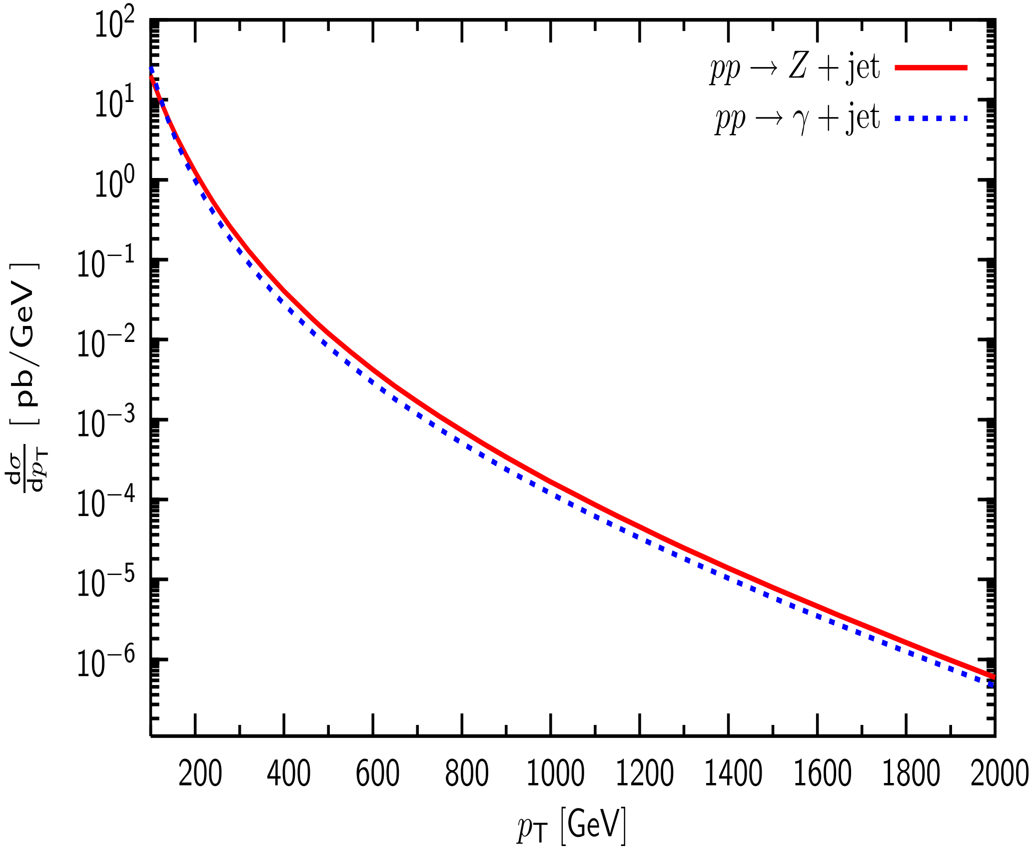
$p_T^{\gamma,c}$(GeV)	NLL	const	$\ln(\hat{t}/\hat{s})$	$\ln^2(\hat{t}/\hat{s})$
250	-6.05%	-0.26%	-1.78%	-3.95%
450	-14.4%	-0.58%	-1.97%	-3.50%
700	-22.6%	-0.74%	-1.91%	-3.05%
1000	-30.1%	-0.87%	-1.79%	-2.62%

Complete high-energy approximation

$$\underbrace{C_2 \ln^2\left(\frac{\hat{s}}{M_W^2}\right) + C_1 \ln\left(\frac{\hat{s}}{M_W^2}\right)}_{\text{NLL: correct description of large effects at high energy}} + \underbrace{B_0 + B_1 \ln\left(\frac{\hat{t}}{\hat{s}}\right) + B_2 \ln^2\left(\frac{\hat{t}}{\hat{s}}\right)}_{\text{non-enhanced terms: several percent effect (-5.5\%!)}} + \underbrace{\mathcal{O}\left(\frac{M_W^2}{\hat{s}}\right)}_{\text{suppressed } (\leq 0.5\%)}$$

Percent-level precision requires exact $\mathcal{O}(\alpha)$ corrections or at least complete high-energy approximation

2. pp $\rightarrow$ Z+jet and pp $\rightarrow \gamma$ +jet at the LHC



Precision measurement

- $\sigma(gq \rightarrow Zq) = \mathcal{O}(\alpha\alpha_S)$: large!
- clean signature (Z $\rightarrow$ leptons)

$\Rightarrow$ $\mathcal{L} \times$ gluon-PDF at 1%

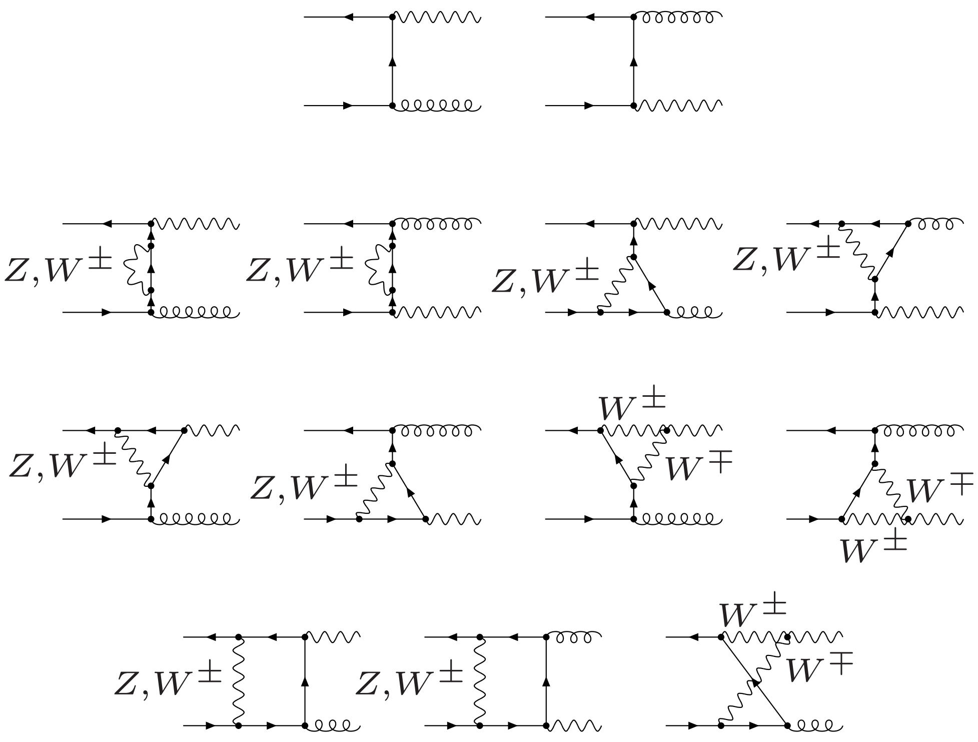
Explore TeV energy scale

- $\sigma(p_T > 1\text{TeV}) \sim 25$ fb

$\Rightarrow$ 2500 events/year

requires theoretical precision at percent level!

$pp \to Z + \text{jet}$ ($pp \to \gamma + \text{jet}$ similar)



Partonic reactions

- $q\bar{q} \to Zg, \quad gq \to Zq, \quad g\bar{q} \to Z\bar{q}$
- crossing symmetry
- dominant contribution from initial-state gluons

Weak corrections (virtual Z,W)

- exact 1-loop calculation for finite p_T
- compact 1-loop approximation for large p_T
- dominant 2-loop effects at large p_T

Kühn, Kulesza, P., Schulze (2005)

Analytic 1-loop results for $\bar{q}q \to Zg$ (compact expressions in terms of scalar integrals)

$$\overline{\sum}|\mathcal{M}_1|^2 = 8\pi^2\alpha\alpha_{\mathrm{S}}(N_{\mathrm{c}}^2-1)\sum_{\lambda=\mathrm{R,L}}\Bigg\{\left(I^Z_{q_\lambda}\right)^2\left[H_0\left(1+2\delta C^{\mathrm{A}}_{q_\lambda}\right)\right.$$

$$\left. + \frac{\alpha}{2\pi}\sum_{V=\mathrm{Z,W^\pm}}\left(I^V I^{\bar{V}}\right)_{q_\lambda} H_1^{\mathrm{A}}(M_V^2)\right] + \frac{c_{\mathrm{W}}}{s_{\mathrm{W}}^3}T^3_{q_\lambda}I^Z_{q_\lambda}\left[2H_0\delta C^{\mathrm{N}}_{q_\lambda} + \frac{\alpha}{2\pi}\frac{1}{s_{\mathrm{W}}^2}H_1^{\mathrm{N}}(M_W^2)\right]\Bigg\}$$

Asymptotic expansion: $|\hat{s}|, |\hat{t}|, |\hat{u}| \gg M_W^2$

$$H_1^{\mathrm{A/N}}(M_V^2) = \mathrm{Re}\left[g_0^{\mathrm{A/N}}(M_V^2)\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}} + g_1^{\mathrm{A/N}}(M_V^2)\frac{\hat{t}^2-\hat{u}^2}{\hat{t}\hat{u}} + g_2^{\mathrm{A/N}}(M_V^2)\right]$$

$$g_0^{\mathrm{N}}(M_W^2) = 2\left[\frac{2}{4-D} - \gamma_{\mathrm{E}} + \log\left(\frac{4\pi\mu^2}{M_Z^2}\right)\right] + \log^2\left(\frac{-\hat{s}}{M_W^2}\right) - \log^2\left(\frac{-\hat{t}}{M_W^2}\right) - \log^2\left(\frac{-\hat{u}}{M_W^2}\right)$$
$$\log^2\left(\frac{\hat{t}}{\hat{u}}\right) - \frac{3}{2}\left[\log^2\left(\frac{\hat{t}}{\hat{s}}\right) + \log^2\left(\frac{\hat{u}}{\hat{s}}\right)\right] - \frac{20\pi^2}{9} - \frac{\pi}{\sqrt{3}} + 2$$

$$g_0^{\mathrm{A}}(M_V^2) = -\log^2\left(\frac{-\hat{s}}{M_V^2}\right) + 3\log\left(\frac{-\hat{s}}{M_V^2}\right) + \frac{3}{2}\left[\log^2\left(\frac{\hat{t}}{\hat{s}}\right) + \log^2\left(\frac{\hat{u}}{\hat{s}}\right) + \log\left(\frac{\hat{t}}{\hat{s}}\right) + \log\left(\frac{\hat{u}}{\hat{s}}\right)\right] + \frac{7\pi^2}{3} - \frac{5}{2},$$

$$g_1^{\mathrm{N}}(M_V^2) = -g_1^{\mathrm{A}}(M_V^2) + \frac{3}{2}\left[\log\left(\frac{\hat{u}}{\hat{s}}\right) - \log\left(\frac{\hat{t}}{\hat{s}}\right)\right] = \frac{1}{2}\left[\log^2\left(\frac{\hat{u}}{\hat{s}}\right) - \log^2\left(\frac{\hat{t}}{\hat{s}}\right)\right]$$

$$g_2^{\mathrm{N}}(M_V^2) = -g_2^{\mathrm{A}}(M_V^2) = -2\left[\log^2\left(\frac{\hat{t}}{\hat{s}}\right) + \log^2\left(\frac{\hat{u}}{\hat{s}}\right) + \log\left(\frac{\hat{t}}{\hat{s}}\right) + \log\left(\frac{\hat{u}}{\hat{s}}\right)\right] - 4\pi^2$$

Very compact expressions

- NLL: predicted by process-independent formula [Denner and P. (2001)]
- **NNLL**: contains also $\pi^2, \log(\hat{t}/\hat{u}), \ldots$ not growing with energy

Dominant two-loop contributions for $\bar{q}q \rightarrow Zg$

$$\overline{\sum}|\mathcal{M}_2|^2 = \overline{\sum}|\mathcal{M}_1|^2 + 2\alpha^3\alpha_{\mathrm{S}}(N_c^2-1)\frac{\hat{t}^2+\hat{u}^2}{\hat{t}\hat{u}} \sum_{\lambda=\mathrm{L,R}} \left\{ \frac{1}{2}\left(I^Z_{q_\lambda} C^{\mathrm{ew}}_{q_\lambda} + \frac{c_{\mathrm{W}}}{s^3_{\mathrm{W}}} T^3_{q_\lambda}\right) \right.$$

$$\left. \times \left[I^Z_{q_\lambda} C^{\mathrm{ew}}_{q_\lambda} X_1 + \frac{c_{\mathrm{W}}}{s^3_{\mathrm{W}}} T^3_{q_\lambda} X_2 \right] - \frac{T^3_{q_\lambda} Y_{q_\lambda}}{8 s^4_{\mathrm{W}}} X_2 + \frac{1}{6} I^V_{q_\lambda} \left[I^Z_{q_\lambda}\left(\frac{b_1}{c^2_{\mathrm{W}}}\left(\frac{Y_{q_\lambda}}{2}\right)^2 + \frac{b_2}{s^2_{\mathrm{W}}} C_{q_\lambda}\right) + \frac{c_{\mathrm{W}}}{s^3_{\mathrm{W}}} T^3_{q_\lambda} b_2 \right] X_3 \right\}$$

Kühn, Kulesza, P., Schulze (2005)

includes LL and NLL terms

$$X_1 = \ln^4\left(\frac{\hat{s}}{M^2_W}\right) - 6\ln^3\left(\frac{\hat{s}}{M^2_W}\right)$$

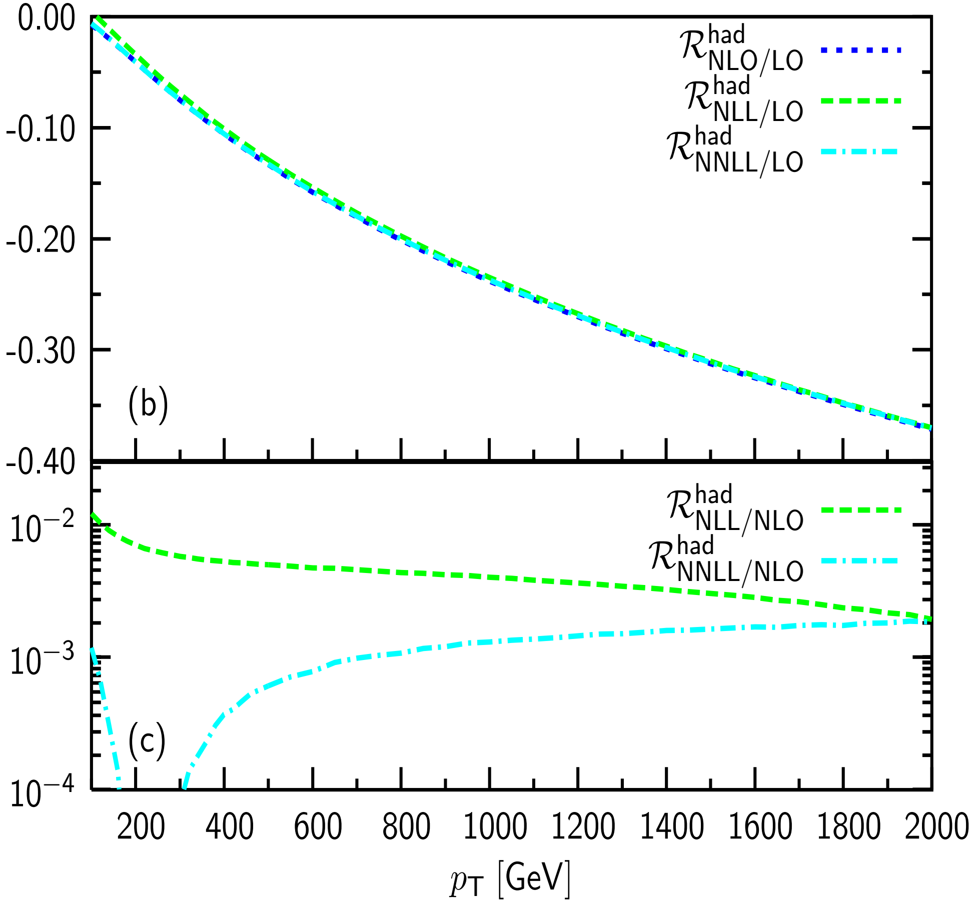
$$X_2 = \ln^4\left(\frac{\hat{t}}{M^2_W}\right) + \ln^4\left(\frac{\hat{u}}{M^2_W}\right) - \ln^4\left(\frac{\hat{s}}{M^2_W}\right)$$

$$X_3 = \ln^3\left(\frac{\hat{s}}{M^2_W}\right)$$

derived from process-independent results for two-loop Sudakov corrections

Melles (2001); Denner, Melles, P. (2003)

One-loop corrections to $\mathrm{d}\sigma/\mathrm{d}p_T$ for $pp \to Z + \mathrm{jet}$



$\overline{\mathrm{MS}}$ **input:** $\alpha_S(M_Z^2) = 0.13$, $\alpha = 1/128.1$, $s_w^2 = 0.2314$
PDFs: LO MRST2001, $\mu_{\mathrm{F}} = \mu_{\mathrm{R}} = p_{\mathrm{T}}$

Large negative corrections

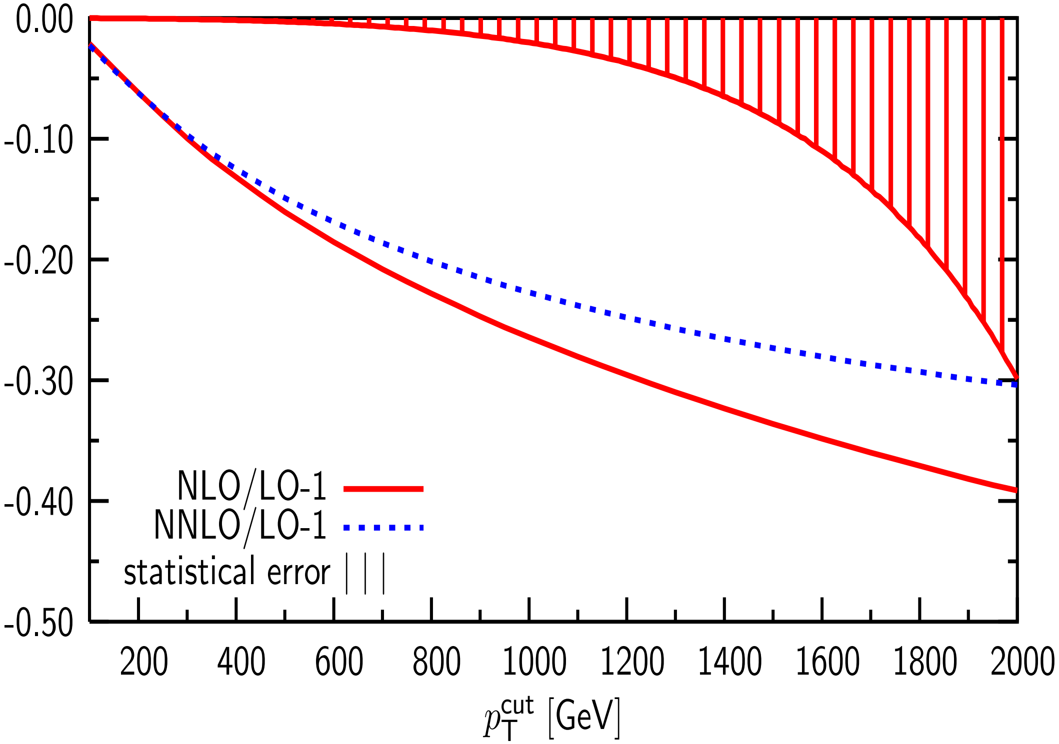
- increase with p_{T}
- -25% at $p_{\mathrm{T}} \sim 1$ TeV
- agreement with Maina, Moretti, Ross (2004)

Quality of high-energy approx.

- NLO, NLL, NNLL overlap!
- $1 \times 10^{-2} \geq \frac{\mathrm{NLL}-\mathrm{NLO}}{\mathrm{NLO}} \geq 2 \times 10^{-3}$
- $2 \times 10^{-3} \geq \frac{\mathrm{NNLL}-\mathrm{NLO}}{\mathrm{NLO}}$

$\Rightarrow$ very precise (much better than for Wγ)

1- and 2-loop corrections to $\sigma(p_T > p_T^{\mathrm{cut}})$ for pp $\rightarrow$ Z+jet



Size of corrections at $p_{\mathrm{T}} \sim 1\,\mathrm{TeV}$

- 1-loop: -26%
- 1+2-loop: -26% + 4% = -22%

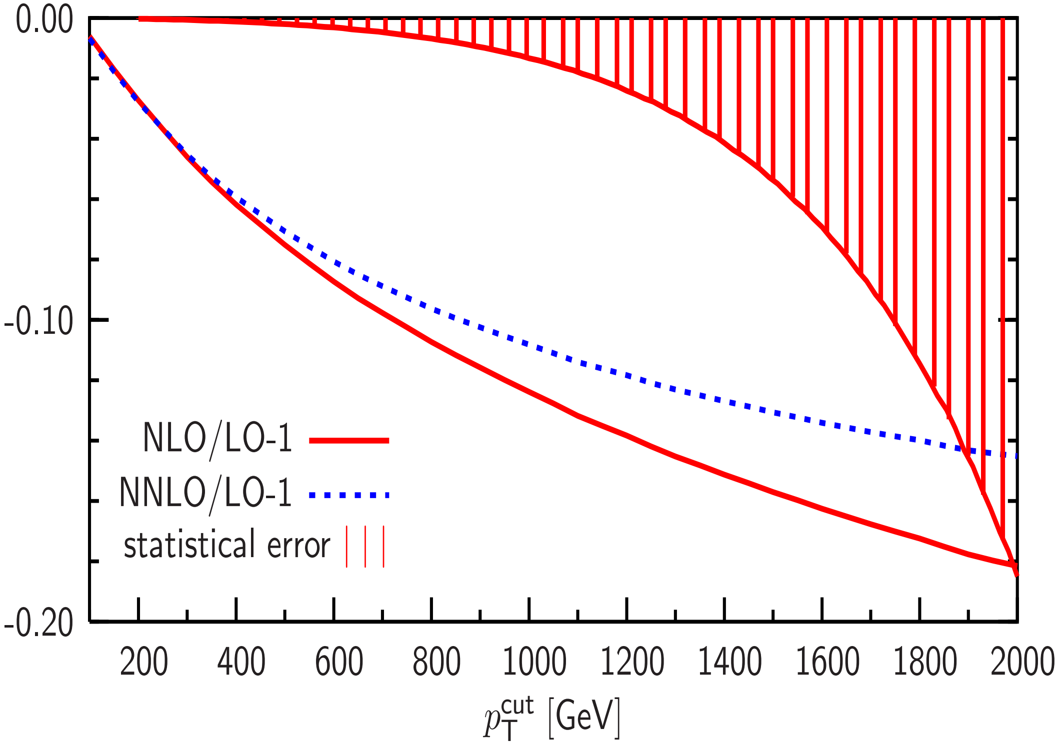
Comparison with statistical error

- $\mathcal{L} = 300\,\mathrm{fb}^{-1}$, Z $\rightarrow$ leptons
- $(\Delta\sigma/\sigma)_{\mathrm{stat}} \sim 2\%$ at 1 TeV

$\Rightarrow$ 2-loop effects not negligible!

$\overline{\mathrm{MS}}$ **input:** $\alpha_S(M_Z^2) = 0.13$, $\alpha = 1/128.1$, $s_w^2 = 0.2314$
PDFs: LO MRST2001, $\mu_{\mathrm{F}} = \mu_{\mathrm{R}} = p_{\mathrm{T}}$

1- and 2-loop corrections to $\sigma(p_T > p_T^{\mathrm{cut}})$ for pp $\rightarrow \gamma$ + jet



input: $\alpha_S(M_Z^2) = 0.13$, $\alpha = 1/137$, $s_w^2 = 1 - M_W^2/M_Z^2$
PDFs: LO MRST2001, $\mu_F = \mu_R = p_T$

Size of corrections at $p_T \sim 1$ TeV

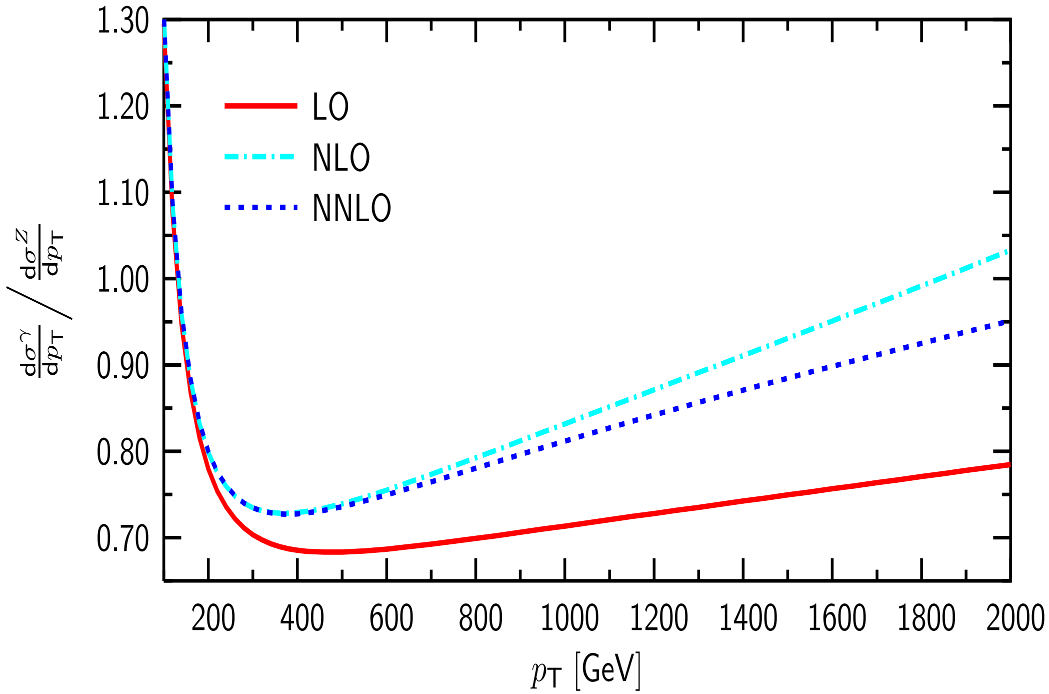
- 1-loop: -12.4%
- 1+2-loop: -12.4% + 1.6% = -10.8%

Comparison with statistical error

- $\mathcal{L} = 300\,\mathrm{fb}^{-1}$
- $(\Delta\sigma/\sigma)_{\mathrm{stat}} \sim 1.3\%$ at 1 TeV

$\Rightarrow$ 2-loop effects $\simeq$ stat. error!

Ratio of the p_T distributions for $pp \to \gamma + \text{jet}$ and $pp \to Z + \text{jet}$



$p_T \sim M_Z$: strong p_T dependence

- due to M_Z

$p_T \gg M_Z$: weak p_T dependence

- ratio determined by γ/Z couplings and up/down PDFs
- PDF-uncertainties cancel
- QCD corrections cancel

1-loop and 2-loop EW corrections

- p_T =1TeV: 0.71+0.12−0.02=0.81
- p_T =2TeV: 0.78+0.22−0.05=0.95

Electroweak Sudakov logarithms beyond one loop

Typical size of LL and NLL corrections for $2 \to 2$ processes at $\sqrt{s} \simeq 1\,\mathrm{TeV}$

1-loop effects

$$\left(\frac{\delta\sigma_1}{\sigma_0}\right)_{\mathrm{LL}} \simeq -\frac{\alpha}{\pi s_{\mathrm{W}}^2}\log^2\frac{s}{M_{\mathrm{W}}^2} \simeq -26\%$$

$$\left(\frac{\delta\sigma_1}{\sigma_0}\right)_{\mathrm{NLL}} \simeq +\frac{3\alpha}{\pi s_{\mathrm{W}}^2}\log\frac{s}{M_{\mathrm{W}}^2} \simeq +16\%$$

2-loop effects (back-of-the-envelope estimate)

$$\left(\frac{\delta\sigma_2}{\sigma_0}\right)_{\mathrm{LL}} \simeq +\frac{\alpha^2}{2\pi^2 s_{\mathrm{W}}^4}\log^4\frac{s}{M_{W}^2} \simeq 3.5\%$$

$$\left(\frac{\delta\sigma_2}{\sigma_0}\right)_{\mathrm{NLL}} \simeq -\frac{3\alpha^2}{\pi^2 s_{\mathrm{W}}^4}\log^3\frac{s}{M_{W}^2} \simeq -4.1\%$$

How to compute higher-order logarithmic corrections?

- first approach based on resummation techniques for mass singularities in QED, QCD extended to EW theory
- non-trivial due to mass gap in the gauge sector $M_A = 0 \ll M_{\mathrm{Z}} \sim M_{\mathrm{W}}$
- how to separate soft-collinear singularities due to massless photons?

InfraRed Evolution Equation (IREE)

Dependence of matrix elements on **transverse-momentum cut-off** $\mu_{\rm T}$

$$\frac{\partial \mathcal{M}}{\partial \log(\mu_{\rm T})} = K(\mu_{\rm T})\mathcal{M}$$

- factorization of mass sing. (QCD)
- kernel known for SU(N)

2 regimes with **exact gauge symmetry**

- $\mu_{\rm T} \geq M_{\rm W}$: kernel insensitive to mass gap as in symmetric SU(2) $\times$ U(1) theory with $M_A = M_{\rm Z} = M_{\rm W}$
- $\mu_{\rm T} \leq M_{\rm W}$: weak bosons frozen and kernel as in QED

Integration of IREE yields double factorization and exponentiation

$$\mathcal{M}(\mu_0) = \exp\left\{\int_{\mu_0}^{M_{\rm W}} \frac{{\rm d}\mu_{\rm T}}{\mu_{\rm T}} K_2(\mu_{\rm T})\right\} \exp\left\{\int_{M_{\rm W}}^{\sqrt{s}} \frac{{\rm d}\mu_{\rm T}}{\mu_{\rm T}} K_1(\mu_{\rm T})\right\} \mathcal{M}_{\rm Born}$$

Fadin, Lipatov, Martin, Melles (2000)

2 loop EW corrections for $s \gg M_W^2$

General form

$$\alpha^2 \left[C_4 \underbrace{\ln^4\left(\frac{s}{M_W^2}\right)}_{\text{LL}} + C_3 \underbrace{\ln^3\left(\frac{s}{M_W^2}\right)}_{\text{NLL}} + C_2 \underbrace{\ln^2\left(\frac{s}{M_W^2}\right)}_{\text{NNLL}} + C_1 \underbrace{\ln^1\left(\frac{s}{M_W^2}\right)}_{\text{N}^3\text{LL}} + C_0 \right]$$

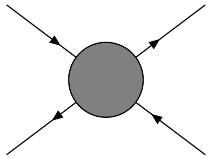
Results based on the IREE approach

- LL for **arbitrary processes** [Fadin, Lipatov, Martin, Melles (2000)]
- NLL for **arbitrary processes** [Melles (2001,2002,2003,2004)]
- NNLL for **massless** $f\bar{f} \to f'\bar{f}'$ [Kühn, Moch, Penin, Smirnov (2000,2001,2003)]
- N^3LL for **massless** $f\bar{f} \to f'\bar{f}'$ [Jantzen, Kühn, Penin, Smirnov (2004,2005)]

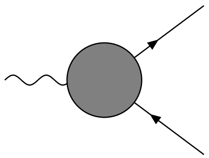
3. N^{3}LL for massless $f\bar{f} \to f'\bar{f}'$ Jantzen, Kühn, Penin, Smirnov (2004,2005)

Step A: logarithmic corrections within SU(2) Higgs model with $M_{W^\pm} = M_Z = M$ using QCD resummation techniques

- decomposition of 4-fermion amplitude

$$\mathcal{A} =$$  $$= \frac{\mathrm{i}g^2}{s}\mathcal{F}^2 \times \tilde{\mathcal{A}}$$

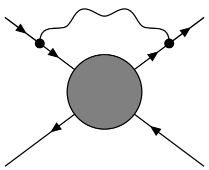
- **evolution equation** for logarithmic corrections to the **form factor** $\mathcal{F}$ [Mueller(1979), Collins(1980), Sen (1981)]

$$\mathcal{F} =$$ 

$$\frac{\partial \mathcal{F}}{\partial \ln s} = \left[\int_{M_W^2}^{s} \frac{\mathrm{d}x}{x}\gamma(\alpha(x)) + \zeta(\alpha(s)) + \xi(\alpha(M_W^2))\right]\mathcal{F}$$

  input for SU(2) Higgs model from explicit 2-loop N^{3}LL calculation using expansion by regions [Jantzen, Smirnov (2006)]

- **evolution equation** for **reduced amplitude** $\tilde{\mathcal{A}}$ [Sen(1983), Botts, Sterman (1987,1989)]

$$\tilde{\mathcal{A}} =$$ 

$$\frac{\partial \tilde{\mathcal{A}}}{\partial \ln s} = \chi\left(\alpha(s)\right)\tilde{\mathcal{A}}$$

  input for matrix of soft anomalous dimension χ derived from existing 2-loop QCD results (Higgs mechanism irrelevant)

Step B: IREE approach for extension to EW theory including photons

(i) SU(2) results translated to SU(2)×U(1) group with $M_\gamma = M$

(ii) photonic singularities for $M_\gamma = 0$ assumed to factorize according to IREE prescription

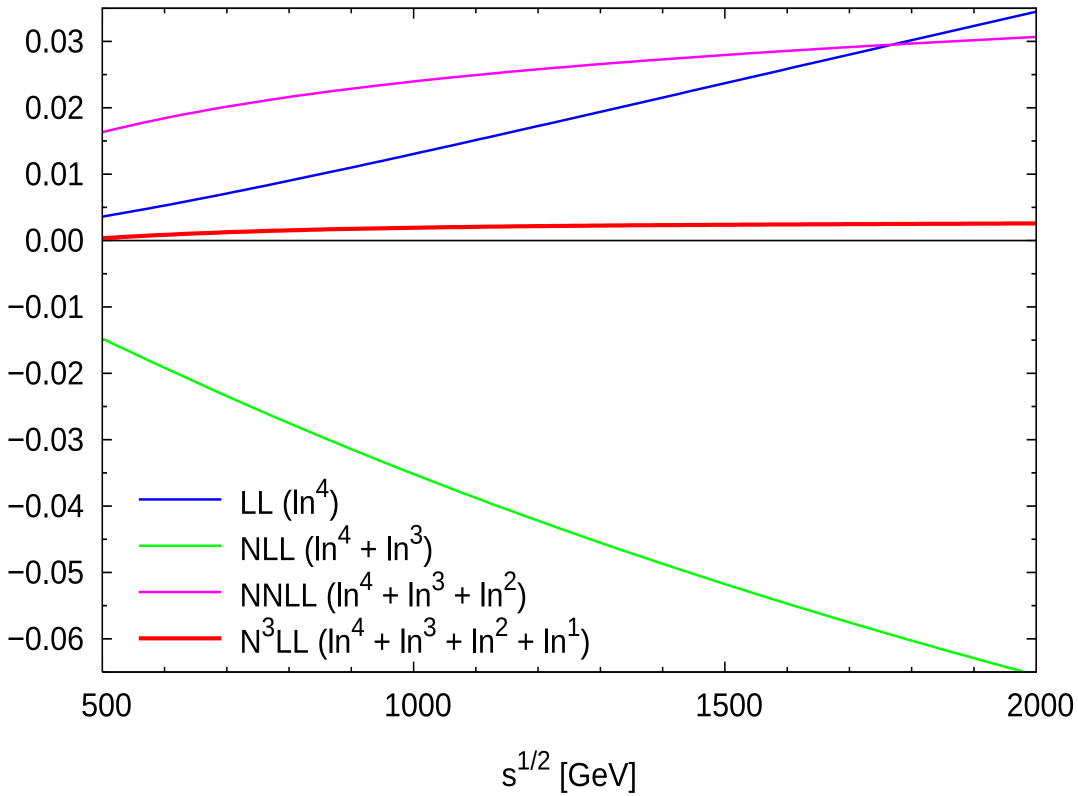
$$\mathcal{A}_{\mathrm{SU(2)}} \to \mathcal{A}_{\mathrm{SU(2)\times U(1)}}\Big|_{M_\gamma=M} \to \left(\frac{\mathcal{A}_{\mathrm{QED}}\big|_{M_\gamma=0}}{\mathcal{A}_{\mathrm{QED}}\big|_{M_\gamma=M}}\right) \times \mathcal{A}_{\mathrm{SU(2)\times U(1)}}\Big|_{M_\gamma=M}$$

confirmed by 2-loop form factor calculation for SU(2)×U(1) with $M_\gamma = 0$ and $\sin^2(\theta_W) = 0$

- mixing corrections due to nonabelian γ–W interaction expected
- but only for coefficient of $\ln^1$ term and suppressed by $\sin^2(\theta_W) \sim 0.2$

Relative 2-loop logarithmic corrections to $\sigma(\mathrm{e}^+\mathrm{e}^- \to d\bar{d})$

$$\left(\frac{\alpha}{4\pi\sin^2\theta_w}\right)^2 \left[2.79\ \ln^4\left(\frac{s}{M_\mathrm{W}^2}\right) - 51.98\ \ln^3\left(\frac{s}{M_\mathrm{W}^2}\right) + 321.34\ \ln^2\left(\frac{s}{M_\mathrm{W}^2}\right) - 606.43\ \ln^1\left(\frac{s}{M_\mathrm{W}^2}\right)\right]$$



Subleading logarithms

- increasingly large coefficients
- alternating signs

Logarithmic approximation

− total 2-loop correction very small

+ residual theoretical error $\mathcal{O}(10^{-3})$

− oscillating, bad convergence

+ better convergence expected for other processes

Open question: 2-loop EW logs for production of heavy particles (Z,W,b,t,H)?

- in contrast to massless fermions heavy particles couple directly to Higgs and Yukawa sectors and the role of symmetry breaking becomes crucial

NLL resummation prescriptions [Melles (2001,2002,2003,2004)] based on IREE

- splitting of EW theory in symmetric SU(2)×U(1) and QED regime
- effect of symmetry breaking assumed to be negligible (not proven)

Confirmed by diagrammatic two loop calculations based on EW Feynman rules

- LL for **arbitrary processes** [Beenakker, Werthenbach (200,2002); Denner, Melles, P. (2003)]
- NLL for **gluon-fermion form factor** [P. (2004)]

NLL for **general processes**: work in progress . . .

4. Two-loop electroweak NLLs for arbitrary processes: preliminary results

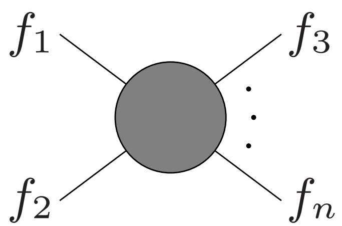
Goal

- derive 2-loop NLLs for arbitrary electroweak processes (light- and heavy fermions, gauge bosons, Higgs)
- diagrammatic 2-loop calculation based on electroweak Feynman rules

Tools for process-independent analysis already developped

- electroweak collinear Ward identities to isolate and factorize mass singularities within $\xi = 1$ gauge
- automatic algorithm for multi-scale 2-loop diagrams in NLL approximation

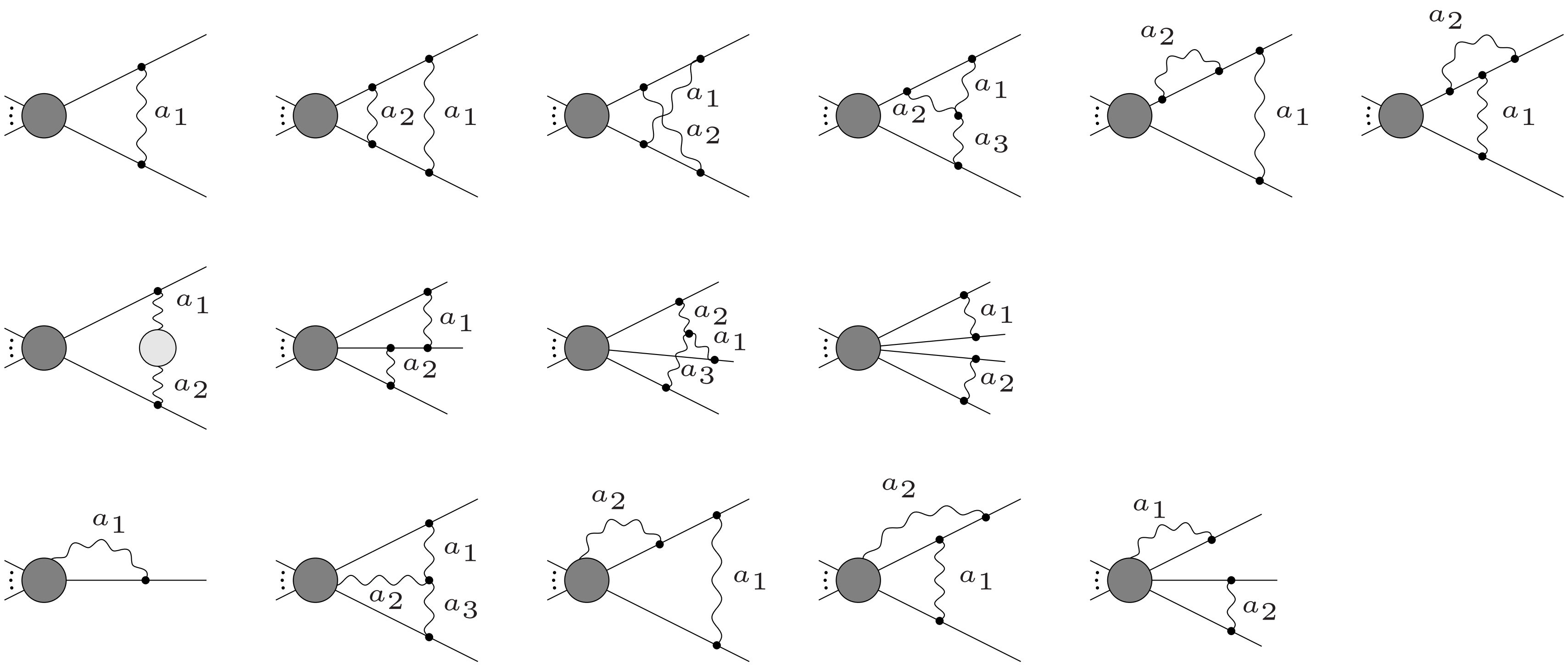
First results for general fermionic processes $f_1 f_2 \to f_3 \dots f_n$

$\mathcal{M} =$  in the limit $|(p_i + p_j)^2| \gg M_W^2$

Denner, Jantzen, P. (in preparation)

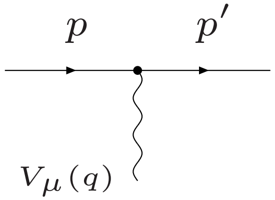
A) **1- and 2-loop diagrams that give rise to NLL mass singularities ($\xi = 1$):**

virtual gauge bosons ($a_i = \mathrm{W}^{\pm}, \mathrm{Z}, \gamma$) coupling to external fermions



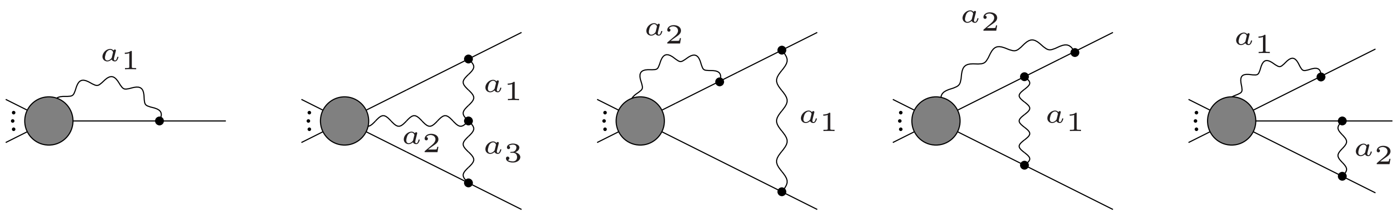
B) **Soft and collinear approximations for fermion-boson vertices**

$$\lim_{q^\mu \to x p^\mu} \left[\quad \right] = -2e p'_\mu \, I^V \qquad \text{(Dirac structure disappears)}$$

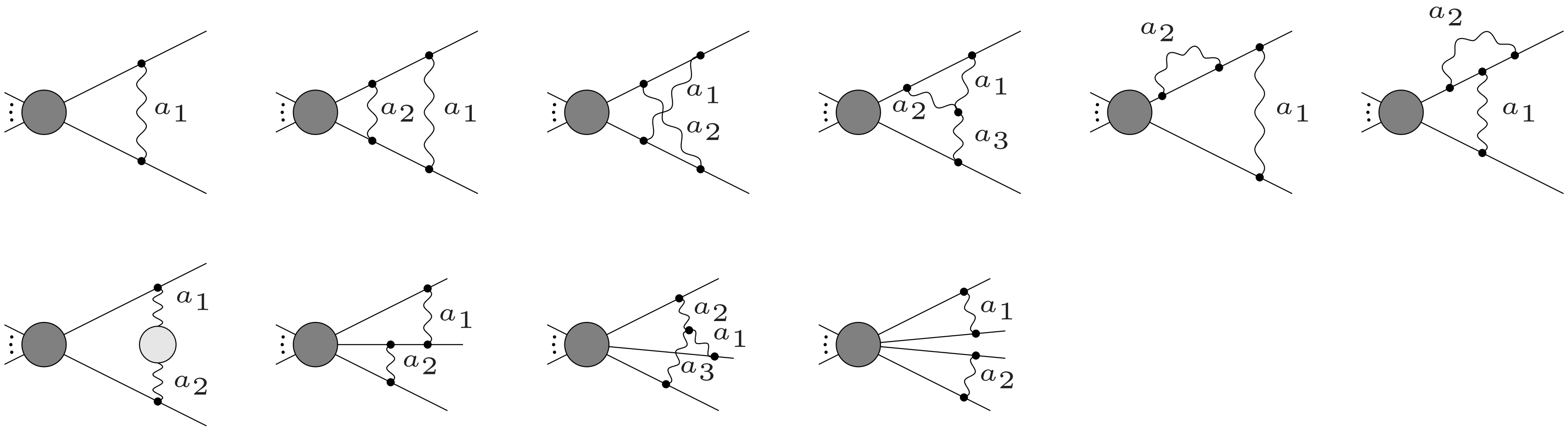


C) **Reduction to factorizable contributions**

diagrams with collinear gauge bosons that couple to external and internal lines



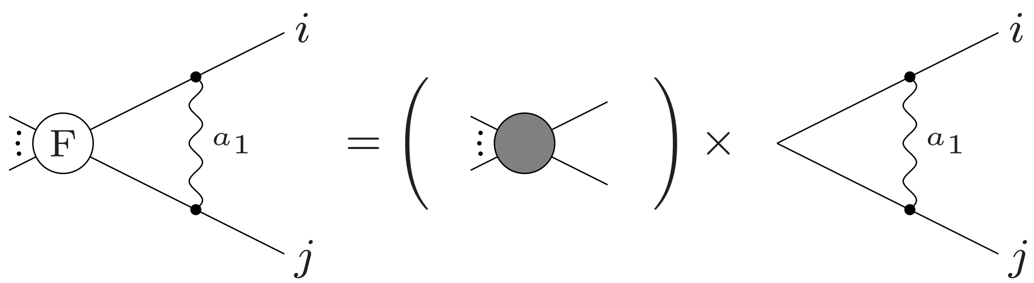
cancel against non-factorizable contributions from diagrams with gauge bosons coupling only to external lines



Reduction to factorizable contributions at one-loop level

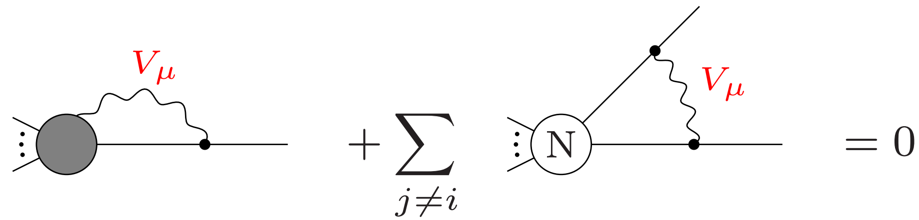
Factorizable (F):

- external-leg exchange of soft/collinear gauge bosons
- neglegt collinear momenta in hard matrix element



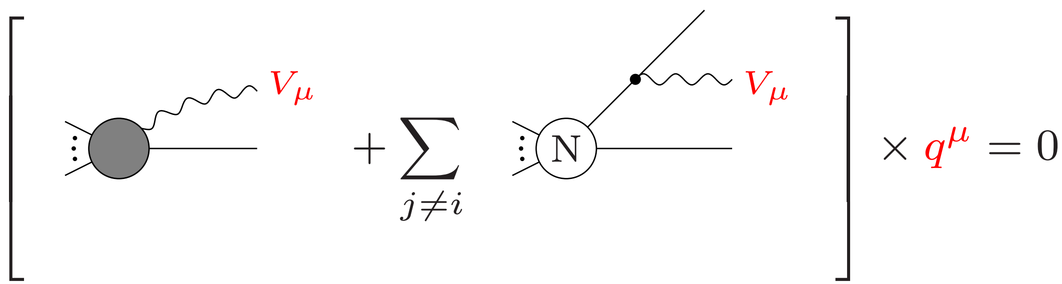
Non-factorizable (N):

- cancelled by diagrams with collinear gauge bosons coupling to internal lines



Cancellation mechanism

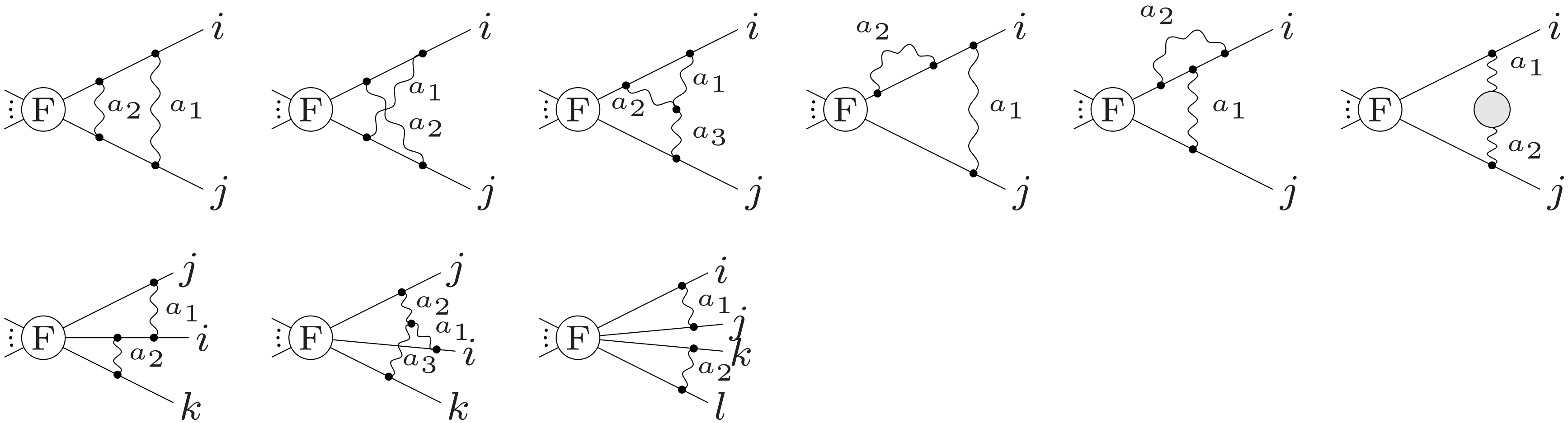
- collinear vertex prop. to q^μ
- collinear Ward identities for spontaneously broken non-abelian theories



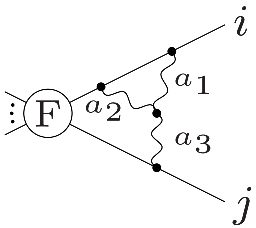
Denner, P. (2001)

Analogous collinear Ward identities derived at the two-loop level [Denner, Jantzen, P. (2006)]

$\Rightarrow$ **only factorizable contributions remain**



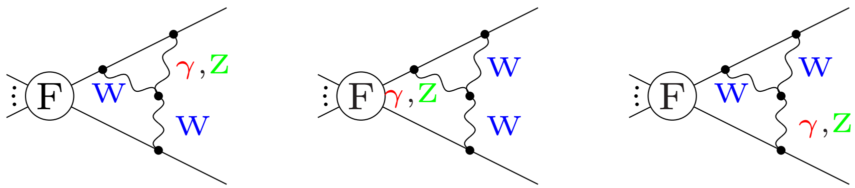
Structure of single factorizable contribution



$$= \mathcal{M}_0 \times \underbrace{\left(\mathrm{i}e^3 g_2 \epsilon^{a_1 a_2 a_3} I_i^{\bar{a}_2} I_i^{\bar{a}_1} I_j^{\bar{a}_3}\right)}_{\text{gauge couplings: SU(2) matrix}} \times \underbrace{D(M_{a_1}, M_{a_2}, M_{a_3}; r_{ij})}_{\text{2-loop integral}}$$

D) **Evaluation of 2-loop integrals**

For every topology various configurations



different scales

- 3 Mandelstam invariants: $r_{ij} = (p_i + p_j)^2$
- heavy particles: M_W, M_Z, M_H, m_t
- massless particles: γ, light fermions

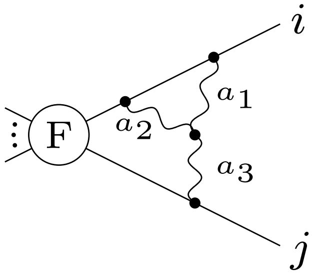
Loop integrals in the high-energy limit $L = \log\left(\frac{s}{M_\mathrm{W}^2}\right) \gg 1$ **and** $D = 4 - 2\epsilon$

$$\Rightarrow \sum_{m,n} C_{mn} \epsilon^{-m} L^n \qquad C_{mn} = C_{mn}\left(\frac{M_\mathrm{Z}}{M_\mathrm{W}}, \frac{M_\mathrm{H}}{M_\mathrm{W}}, \frac{m_\mathrm{t}}{M_\mathrm{W}}, \frac{r_{ij}}{s}\right)$$

NLL approximation with same counting for L **and** ϵ^{-1} **poles**

2-loop $\Rightarrow$ LLs $= \epsilon^{-4}, \epsilon^{-3}L^1, \epsilon^{-2}L^2, \epsilon^{-1}L^3, L^4$ NLLs $= \epsilon^{-3}, \epsilon^{-2}L^1, \epsilon^{-1}L^2, L^3$

Example of 2-loop diagram in NLL approximation



$$= \mathcal{M}_0\, \mathrm{i}e^3 g_2 \epsilon^{a_1 a_2 a_3} I_i^{\bar{a}_2} I_i^{\bar{a}_1} I_j^{\bar{a}_3} \left(\frac{s}{r_{ij}}\right)^{2\varepsilon} D(M_{a_1}, M_{a_2}, M_{a_3}; r_{ij})$$

$$D(M_\mathrm{W}, M_\mathrm{W}, M_\mathrm{W}; r_{ij}) \stackrel{\mathrm{NLL}}{=} \frac{1}{6}L^4 + \frac{3}{2}\log\left(\frac{r_{ij}}{s}\right)L^3 - 3L^2\epsilon^{-1} - 5L^3$$

$$\Delta D(M_\mathrm{W}, M_\mathrm{W}, M_\mathrm{Z}; r_{ij}) \stackrel{\mathrm{NLL}}{=} -\frac{1}{3}\log\left(\frac{M_\mathrm{Z}^2}{M_\mathrm{W}^2}\right)L^3$$

$$\Delta D(0, M_\mathrm{W}, M_\mathrm{W}; r_{ij}) \stackrel{\mathrm{NLL}}{=} -\frac{1}{3}L^3\epsilon^{-1} - \log\left(\frac{r_{ij}}{s}\right)L^2\epsilon^{-1} - \frac{7}{12}L^4 - \frac{7}{3}\log\left(\frac{r_{ij}}{s}\right)L^3 + 2L^2\epsilon^{-1} + \frac{10}{3}L^3$$

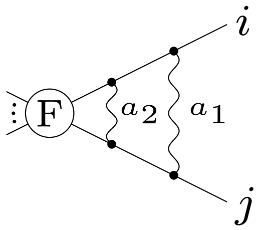
$$\Delta D(M_\mathrm{W}, M_\mathrm{W}, 0; r_{ij}) \stackrel{\mathrm{NLL}}{=} -\frac{1}{3}L^3\epsilon^{-1} - \log\left(\frac{r_{ij}}{s}\right)L^2\epsilon^{-1} - \frac{7}{12}L^4 - \frac{7}{3}\log\left(\frac{r_{ij}}{s}\right)L^3 - 6\epsilon^{-3} - 6L\epsilon^{-2} + L^2\epsilon^{-1} + \frac{17}{3}L^3$$

All loop integrals evaluated and cross checked with 2 independent methods

- automatic algorithm based on **sector decomposition** [Denner, P. (2004)]
- **expansion by regions** + Mellin barnes representation [Smirnov, Jantzen (2006)]

E) **Complete 2-loop amplitude**

$$\delta\mathcal{M}_2 = \frac{1}{2}\sum_{i\neq j}\sum_{a_1,a_2,a_3}\Bigg[$$



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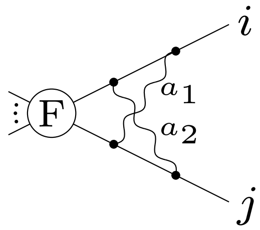

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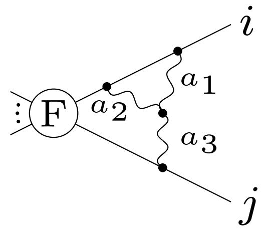

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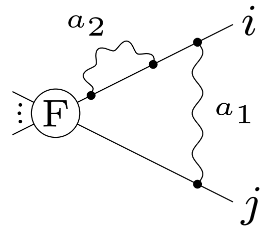

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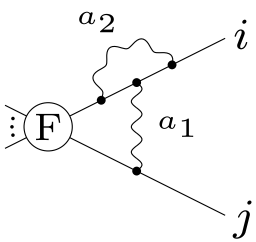

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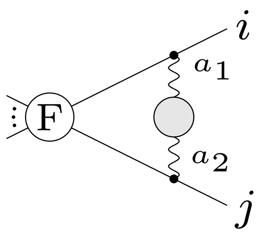
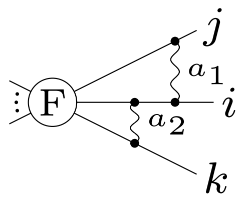


$$\Bigg] + \frac{1}{6}\sum_{i\neq j\neq k}\sum_{a_1,a_2,a_3}\Bigg[$$



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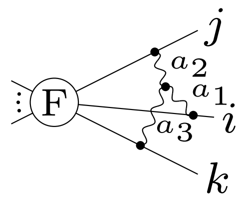
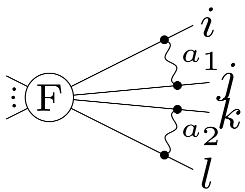


$$\Bigg] + \frac{1}{8}\sum_{i\neq j\neq k\neq l}\sum_{a_1,a_2}$$



Summations over topologies, combinations of gauge bosons $(\mathrm{W}^{\pm},\mathrm{Z},\gamma)$ and external legs $(i,j,k,l=1,\ldots,n)$ using global gauge invariance and group-theoretical identities

Result for fermionic processes $f_1 f_2 \to f_3 \dots f_n$

2-loop NLL corrections

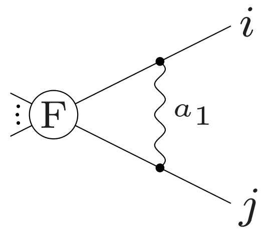
$$\mathcal{M}_{(2)} = \left(\frac{\alpha}{4\pi}\right)^2 \left[\frac{1}{2}\left(\Delta F_{\mathrm{em}}\right)^2 + \Delta F_{\mathrm{em}} F_{\mathrm{sew}} + \frac{1}{2}\left(F_{\mathrm{sew}}\right)^2 + G_{\mathrm{sew}}\right] \mathcal{M}_{(0)}$$

can be expressed in terms of 1-loop NLL corrections, $L = \log(s/M_\mathrm{W}^2)$

$$\Delta F_{\mathrm{em}} = -\frac{1}{2}\sum_{j\neq i} I_i^A I_j^A \left[-2\epsilon^{-2} - 3\epsilon^{-1} + 2\epsilon^{-1}\log\left(\frac{r_{ij}}{s}\right) - K(\epsilon, M_\mathrm{W}; r_{ij})\right]$$

$$F_{\mathrm{sew}} = -\frac{1}{2}\sum_{j\neq i}\sum_{a=A,Z,\pm} I_i^{\bar{a}} I_j^{a} \times$$

$$\underbrace{\left\{-L^2 - \frac{2}{3}L^3\epsilon - \frac{1}{4}L^4\epsilon^2 + \left[\frac{3}{2} - \log\left(\frac{r_{ij}}{s}\right)\right]\left(2L + L^2\epsilon + \frac{1}{3}L^3\epsilon^2\right)\right\}}_{K(\epsilon, M_\mathrm{W}; r_{ij})}$$



and 1-loop β-function coeff. $\times$ 1-loop functions

$$G_{\mathrm{sew}} = -\frac{1}{2}\sum_{i=1}^{n}\left[b_1^{(1)} g_1^2 \left(\frac{Y_i}{2}\right)^2 + b_2^{(1)} g_2^2 C_{F,i}\right]\frac{1}{\epsilon}\left[K(2\epsilon, M_\mathrm{W}, s) - \left(\frac{-s}{\mu^2}\right)^{\epsilon} K(\epsilon, M_\mathrm{W}, s)\right]$$

Result for fermionic processes $f_1 f_2 \to f_3 \dots f_n$

Two-loop results consistent with double factorization and exponentiation

$$\mathcal{M}_{(0)} + \mathcal{M}_{(1)} + \mathcal{M}_{(2)} \equiv \exp\left[\left(\frac{\alpha}{4\pi}\right) \Delta F_{\mathrm{em}}\right] \exp\left[\left(\frac{\alpha}{4\pi}\right) F_{\mathrm{sew}} + \left(\frac{\alpha}{4\pi}\right)^2 G_{\mathrm{sew}}\right] \mathcal{M}_{(0)}$$

- electromagnetic singularities factorize and exponentiate separately
- remaining part as within a symmetric SU(2)×U(1) theory with $M_\gamma = M_\mathrm{W} = M_\mathrm{Z}$
- in agreement with IREE prescription [Kühn et. al. (2000), Melles (2003)]

Summary

Large EW logs for gauge boson production with high p_T at LHC

(1) WW, WZ, ZZ, Wγ, Zγ ($p_T \sim 500\,\text{GeV}$)

- 1-loop logs: -15% to -30%
- non-log terms sizable for Wγ, Zγ (-6%) unknown for WW, WZ, ZZ

(2) Z + jet and γ + jet ($p_T \sim 1\text{TeV}$)

- 1-loop logs: -26% (Z) and -12.5% (γ)
- non-log terms small (1%)
- 2-loop NLLs significant: $+4\%$ (Z) and $+1.5\%$ (γ)

Recent progress at 2-loop level

(3) $f\bar{f} \to f'\bar{f}'$

- all 2-loop logs from $\ln^4$ to $\ln^1$ included
- large cancellations between leading and subleading terms
- complete two-loop smaller than 1%

(4) towards NLLs for arbitrary processes

- diagrammatic with EW Feynman rules
- all aspects of symmetry breaking
- collinear Ward identities, automatic algorithm for loop calculations
- first results for fermionic processes