Two-loop contributions to the Higgs sector in the MSSM with complex parameters

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Outline

- Higgs bosons in the complex MSSM
- Mass of the lightest Higgs boson
- Higher order contributions to this mass
The Higgs potential in the MSSM

Higgs potential:

\[
V_{\text{Higgs}} = \frac{g^2 + g_1^2}{8} (H_1^+ H_1 - H_2^+ H_2)^2 + \frac{g^2}{2} |H_1^+ H_2|^2 \\
+ |\mu|^2 (H_1^+ H_1 + H_2^+ H_2) \quad \mu: \text{coupl. betw. Higgs superfields} \\
+ (m_1 H_1^+ H_1 + m_2 H_2^+ H_2) \quad \text{soft breaking terms} \\
+ (\epsilon_{ij} |m_3|^2 e^{i\varphi m_3^2} H_1^i H_2^j + h.c.)
\]

Two Higgs doublets (\(v_i: \text{Higgs vac. exp. value}\)):

\[
H_1 = \begin{pmatrix}
    v_1 + \frac{1}{\sqrt{2}} (\phi_1^0 - i \zeta_1^0) \\
    -\phi_1^-
\end{pmatrix}, \\
H_2 = e^{i \xi} \begin{pmatrix}
    v_2 + \frac{1}{\sqrt{2}} (\phi_2^0 + i \zeta_2^0)
\end{pmatrix}
\]

2 phases
Higgs bosons

At Born level: no CP-violation:

- one phase in the Higgs potential: \( V_{\text{Higgs}} = \cdots + \epsilon_{ij} |m_3^2| e^{i \phi m_3^2 H_{1}^i H_{2}^j} + \cdots \)
  elimination via Peccei-Quinn transformation

- phase difference \( \xi \) of Higgs doublets: vanishes because of minimum condition

Physical mass eigenstates (at Born level):

- 5 Higgs bosons: 3 neutral \( H^0, h^0, A^0 \); 2 charged \( H^\pm \)

Masses of the Higgs bosons:

- not all independent: here: \( H^\pm \)-mass \( M_{H^\pm} \) (and \( \tan \beta \)) as free parameter
  \[ \tan \beta = \frac{v_2}{v_1} \]: ratio of the Higgs vac. expect. values

- lightest Higgs boson: \( h^0 \)
Mass of the lightest Higgs boson

Upper theoretical Born mass bound: \( M_{h^0} \leq M_Z = 91 \text{ GeV} \)

with quantum corrections of higher orders: \( M_{h^0} \lesssim 135 \text{ GeV} \)

dependent on the MSSM parameters: particularly on parameter phases

- Discovery of the Higgs boson:
  - accurate measurement
  - precise prediction
  \( \Rightarrow \) strong bounds on the MSSM parameter, e.g. on \( \tan \beta = \frac{v_2}{v_1} \)
  - i.e. inclusion of higher order corrections with full phase dependence

- Before the discovery: **Exclusion** of parts of the parameter space
Determination of the Higgs masses

Two-point-function:

\[
\Gamma(k^2) = k^2 - M_{\text{Born}}^2 + \begin{pmatrix}
\hat{\Sigma}_{H^0 H^0}(k^2) \\
\hat{\Sigma}_{H^0 h^0}(k^2) \\
\hat{\Sigma}_{H^0 A^0}(k^2) \\
\hat{\Sigma}_{h^0 h^0}(k^2) \\
\hat{\Sigma}_{h^0 A^0}(k^2) \\
\hat{\Sigma}_{A^0 A^0}(k^2)
\end{pmatrix}
\]

diagonal matrix with squared Born masses

diag\left(M_{H^0_{\text{Born}}}^2, M_{h^0_{\text{Born}}}^2, M_{A^0_{\text{Born}}}^2\right)

determining the zero of \(\text{det}(\Gamma(k^2)) \Rightarrow M_{h_1}, M_{h_2}, M_{h_3}\)

Real parameters:

\[
\hat{\Sigma}_{H^0 A^0}(k^2) = \hat{\Sigma}_{h^0 A^0}(k^2) = 0 \Rightarrow M_{h^0} = M_{h_1}
\]

no mixing between CP-even and CP-odd states
Renormalized two-loop self energies $\hat{\Sigma}$

Calculation of the dominant two-loop contributions ($\alpha_t = \lambda_t^2/(4\pi)$):

- Terms of order $O(\alpha_t \alpha_s)$ for complex parameters

Known: two-loop leading-log contributions

- [Pilaftsis, Wagner]
- [Carena, Ellis, Pilaftsis, Wagner]
Renormalized two-loop self energies $\hat{\Sigma}$

Calculation of the dominant two-loop contributions:

- Terms of order $\mathcal{O}(\alpha_t\alpha_s)$ for complex parameters

contributing self energy diagrams ($\phi = h^0, H^0, A^0$):

![Diagram of self-energy contributions](image-url)
Renormalized two-loop self energies $\hat{\Sigma}$

Calculation of the dominant two-loop contributions:

- Terms of order $\mathcal{O}(\alpha_t\alpha_s)$ for complex parameters

Contribution self energy diagrams:

- generation of diagrams with FeynArts

  [Küblbeck, Böhm, Denner],[Hahn]

- tensor reduction with TwoCalc

  [Weiglein, Scharf, Böhm]

- extraction of relevant terms:
  
  - use vanishing external momenta
  
  - use vanishing electroweak gauge couplings $g$, $g'$
Renormalized two-loop self energies $\hat{\Sigma}$

Calculation of the dominant two-loop contributions:

- within an **on-shell** scheme in the Higgs sector:
  - no shift of the minimum of the Higgs potential: $\delta t_\phi = -T_\phi$

  Calculation of tadpole diagrams like:
  $$(\phi = h^0, H^0, A^0)$$

  - define the $H^\pm$-mass $M_{H^\pm}$ as the **pole mass**: $\delta M_{H^\pm} = \Sigma_{H^+H^-}$

  \[\Rightarrow\] directly related to a **physical observable**

  Calculation of selfenergy diagrams like:
Renormalized two-loop self energies $\hat{\Sigma}$

Calculation of the dominant two-loop contributions:

- within an **on-shell** scheme in the Higgs sector:

  renormalized self energies (examples):

  $\hat{\Sigma}_{h^0 h^0}^{(2)} = \Sigma_{h^0 h^0}^{(2)} - c_{\alpha-\beta}^2 \delta M_{H^\pm}^{(2)}$

  $$- \frac{e s_{\alpha-\beta}}{4 M_Z c_W s_W} \left[ s_2 (\alpha-\beta) \delta t_{h^0}^{(2)} + 2 (1 + c_{\alpha-\beta}) \delta t_{h^0}^{(2)} \right],$$

  $\hat{\Sigma}_{h^0 A^0}^{(2)} = \Sigma_{h^0 A^0}^{(2)} - \frac{e s_{\alpha-\beta}}{2 M_Z c_W s_W} \delta t_{A^0}^{(2)}.$
Renormalized two-loop self energies \( \hat{\Sigma} \)

Calculation of the dominant two-loop contributions:

- with parameters of the top (and bottom) sector defined at one-loop:
  
  - top quark mass and top squark masses on-shell
  
  - \( \tilde{b}_1 \)-mass (\( = \tilde{b}_L \)-mass) determined by \( SU(2) \)-relation (\( m_b = 0 \)):
    \[
    m_{\tilde{b}_1} = m_{\tilde{b}_1} (m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_t, \theta_{\tilde{t}}, \varphi_{\tilde{t}})
    \]
  
  - generalization of the mixing angle condition:
    \[
    \widetilde{\text{Re}} \hat{\Sigma}_{\tilde{t}_{12}} (m_{\tilde{t}_1}^2) + \widetilde{\text{Re}} \hat{\Sigma}_{\tilde{t}_{12}} (m_{\tilde{t}_2}^2) = 0
    \]
    
    \[
    \Rightarrow \quad (\delta \theta_{\tilde{t}} + i \sin \theta_{\tilde{t}} \cos \theta_{\tilde{t}} \delta \varphi_{\tilde{t}}) e^{i \varphi_{A_t}} = \frac{\widetilde{\text{Re}} \hat{\Sigma}_{\tilde{t}_{12}} (m_{\tilde{t}_1}^2) + \widetilde{\text{Re}} \hat{\Sigma}_{\tilde{t}_{12}} (m_{\tilde{t}_2}^2)}{2(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)}
    \]
  
  - \( A_t = |A_t| e^{i \varphi_{A_t}} \) is then determined \((A_t: \text{trilinear coupling})\):
    \[
    A_t = A_t (m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_t, \theta_{\tilde{t}}, \varphi_{\tilde{t}})
    \]

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Phases in couplings

Phases relevant at two-loop level:

▶ squark sector:

▶ phase $\varphi_{A_t}$ of the trilinear coupling $A_t$

▶ phase of $\mu$ (small), $\mu$: Higgsino mass parameter

constraints from measurements of electr. dipole moments

▶ gluino sector:

▶ phase $\varphi_{\tilde{g}}$ of the gluino mass parameter

in the gluino-squark-quark-vertex

soft breaking parameter
Results: $\varphi_{A_t}$-dependence (small $M_{H^\pm}$)

- Quantum corrections of $\mathcal{O}(\alpha_t \alpha_s)$ change the qualitative behaviour of $M_{h_1}$. 

$M_{h_1}$ [GeV]

<table>
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<th>$\varphi_{A_t} / \pi$</th>
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<tr>
<td>0</td>
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<tr>
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<tr>
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<tr>
<td>2</td>
</tr>
</tbody>
</table>

$O(\alpha)$

$O(\alpha + \alpha_t \alpha_s)$

$M_{H^\pm} = 150$ GeV

$|A_t| = 2.0$ TeV
Results: $\varphi_{A_t}$-dependence (large $M_{H^\pm}$)

- Quantum corrections of $O(\alpha_t \alpha_s)$ change the qualitative behaviour of $M_{h_1}$.
- Qualitative behaviour of $M_{h_1}$ depends strongly on $|A_t|$. 
Results: $|A_t|$-dependence (large $M_{H^\pm}$)

- One-loop: $\varphi_{A_t} = \pi$ “shifts” towards lower values of $|A_t|$ with respect to $\varphi_{A_t} = 0$.
- Two-loop: also the value of the maximum of $M_{h_1}$ depends on the phases.
Results: $\varphi_{A_t}$ - versus $\varphi_{X_t}$-dependence (large $M_{H^\pm}$)

size of the squark mixing: $X_t := A_t - \mu^* \cot \beta$

- Quantum corrections are smaller for constant absolute value of the squark mixing, $|X_t| = \text{const.}$
Results: dependence on the gluino mass $m_{\tilde{g}}$

- Large effects in the threshold region: $m_{\tilde{t}_2} = m_{\tilde{g}} + m_t$
Conclusions

- Quantum corrections are important for a precise prediction of the mass of the lightest Higgs boson $M_{h_1}$:
  - They can induce CP-violation.
  - Dominant corrections: from the top sector

- Contributions of $\mathcal{O}(\alpha_t \alpha_s)$ for complex parameters:
  - Necessary: Renormalization of the sfermion sector for complex parameters
  - Phases are relevant at the two-loop level.
  - Two-loop contributions can change qualitative behaviour of $M_{h_1}$.
  - are currently included into FeynHiggs.