Two-loop contributions to the Higgs sector in the MSSM with complex parameters

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Outline

- Higgs bosons in the complex MSSM
- Mass of the lightest Higgs boson
- Higher order contributions to this mass

The Higgs potential in the MSSM

Higgs potential:

gauge couplings

$$V_{\text{Higgs}} = \frac{g^2 + g'^2}{8}(H_1^+ H_1 - H_2^+ H_2)^2 + \frac{g^2}{2}|H_1^+ H_2|^2$$

$$+ |\mu|^2 (H_1^+ H_1 + H_2^+ H_2) \qquad \mu\text{: coupl. betw. Higgs superfields}$$

$$+ (m_1^2 H_1^+ H_1 + m_2^2 H_2^+ H_2) \qquad \text{soft breaking terms}$$

$$+ (\epsilon_{ij} |m_3^2| e^{i\varphi_{m_3^2}} H_1^i H_2^j + h.c.)$$

Two Higgs doublets (v_i: Higgs vac. exp. value):

$$H_1 = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(\phi_1^0 - i\zeta_1^0) \\ -\phi_1^- \end{pmatrix}, \qquad H_2 = e^{i\xi} \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}(\phi_2^0 + i\zeta_2^0) \end{pmatrix}$$

$\Longrightarrow$ 2 phases

Higgs bosons

At Born level: no CP-violation:

- one phase in the Higgs potential: $V_{\text{Higgs}} = \cdots + \epsilon_{ij} |m_3^2| e^{i\varphi_{m_3^2}} H_1^i H_2^j + \ldots$
  elimination via Peccei-Quinn transformation
- phase difference ξ of Higgs doublets:
  vanishes because of minimum condition

Physical mass eigenstates (at Born level):

- 5 Higgs bosons: 3 neutral H^0, h^0, A^0; 2 charged $H^\pm$

Masses of the Higgs bosons:

- not all independent: here: $H^\pm$-mass $M_{H^\pm}$ (and $\tan\beta$) as free parameter
  $\tan\beta = \frac{v_2}{v_1}$: ratio of the Higgs vac. expect. values
- lightest Higgs boson: h^0

Mass of the lightest Higgs boson

Upper theoretical Born mass bound: $M_{h^0} \leq M_Z = 91$ GeV

with quantum corrections of higher orders: $M_{h^0} \lesssim 135$ GeV

dependent on the MSSM parameters:
particularly on parameter phases

- Discovery of the Higgs boson:

accurate measurement

precise prediction

mass of the Higgs boson

$\Rightarrow$ strong bounds on the MSSM parameter, e.g. on $\tan\beta = \frac{v_2}{v_1}$

i.e. inclusion of higher order corrections with full phase dependence

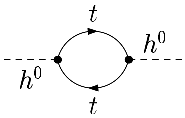
- Before the discovery: Exclusion of parts of the parameter space

Determination of the Higgs masses

Two-point-function:

$$\Gamma(k^2) = k^2 - \mathrm{M}^2_{\mathrm{Born}} + \begin{pmatrix} \hat{\Sigma}_{H^0 H^0}(k^2) & \hat{\Sigma}_{H^0 h^0}(k^2) & \hat{\Sigma}_{H^0 A^0}(k^2) \\ \hat{\Sigma}_{H^0 h^0}(k^2) & \hat{\Sigma}_{h^0 h^0}(k^2) & \hat{\Sigma}_{h^0 A^0}(k^2) \\ \hat{\Sigma}_{H^0 A^0}(k^2) & \hat{\Sigma}_{h^0 A^0}(k^2) & \hat{\Sigma}_{A^0 A^0}(k^2) \end{pmatrix}$$

diagonal matrix with
squared Born masses
$\mathrm{diag}(M^2_{H^0_{\mathrm{Born}}}, M^2_{h^0_{\mathrm{Born}}}, M^2_{A^0_{\mathrm{Born}}})$



determining the zero of $\det\big(\Gamma(k^2)\big) \Rightarrow M_{h_1},\ M_{h_2},\ M_{h_3}$

Real parameters:

$$\hat{\Sigma}_{H^0 A^0}(k^2) = \hat{\Sigma}_{h^0 A^0}(k^2) = 0 \Rightarrow M_{h^0} = M_{h_1}$$

no mixing between CP-even and CP-odd states

Renormalized two-loop self energies $\hat{\Sigma}$

Calculation of the dominant two-loop contributions ($\alpha_t = \lambda_t^2/(4\pi)$):

Yukawa coupling

new

- Terms of order $\mathcal{O}(\alpha_t \alpha_s)$ for complex parameters

  known: two-loop leading-log contributions [Pilaftsis, Wagner]

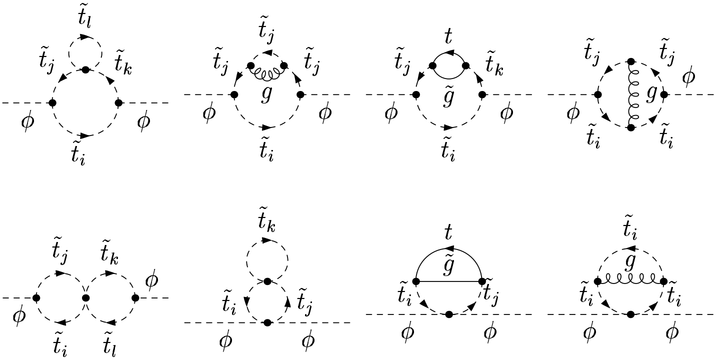
  [Carena, Ellis, Pilaftsis, Wagner]

Renormalized two-loop self energies $\hat{\Sigma}$

Calculation of the dominant two-loop contributions :

- Terms of order $\mathcal{O}(\alpha_t \alpha_s)$ for complex parameters

  contributing self energy diagrams ($\phi = h^0, H^0, A^0$):



Renormalized two-loop self energies $\hat{\Sigma}$

Calculation of the dominant two-loop contributions :

- Terms of order $\mathcal{O}(\alpha_t \alpha_s)$ for complex parameters

  contributing self energy diagrams:

  - generation of diagrams with `FeynArts`

    [Küblbeck, Böhm, Denner],[Hahn]

  - tensor reduction with `TwoCalc` [Weiglein, Scharf, Böhm]

  - extraction of relevant terms:

    - use vanishing external momenta

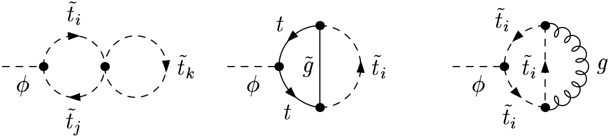
    - use vanishing electroweak gauge couplings g, g'

Renormalized two-loop self energies $\hat{\Sigma}$

Calculation of the dominant two-loop contributions :

- within an **on-shell** scheme in the Higgs sector:
  - no shift of the minimum of the Higgs potential: $\delta t_\phi = -T_\phi$

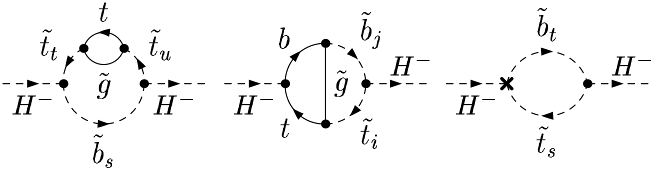
    calculation of tadpole diagrams like: $(\phi = h^0, H^0, A^0)$



  - define the $H^\pm$-mass $M_{H^\pm}$ as the **pole mass**: $\delta M_{H^\pm} = \Sigma_{H^+H^-}$

    $\Rightarrow$ directly related to a **physical observable**

    calculation of selfenergy diagrams like:



Renormalized two-loop self energies $\hat{\Sigma}$

Calculation of the dominant two-loop contributions :

- within an **on-shell** scheme in the Higgs sector:

  renormalized self energies (examples):

$$\hat{\Sigma}^{(2)}_{h^0 h^0} = \Sigma^{(2)}_{h^0 h^0} - c^2_{\alpha-\beta}\,\delta M^2_{H^\pm}{}^{(2)} - \frac{e\ s_{\alpha-\beta}}{4 M_Z c_W s_W}\left[s_{2(\alpha-\beta)}\,\delta t^{(2)}_{H^0} + 2\left(1 + c^2_{\alpha-\beta}\right)\delta t^{(2)}_{h^0}\right],$$

$$\hat{\Sigma}^{(2)}_{h^0 A^0} = \Sigma^{(2)}_{h^0 A^0} - \frac{e\ s_{\alpha-\beta}}{2 M_Z c_W s_W}\,\delta t^{(2)}_{A^0}\ .$$

Renormalized two-loop self energies $\hat{\Sigma}$

Calculation of the dominant two-loop contributions :

- with parameters of the top (and bottom) sector defined at one-loop:
  - top quark mass and top squark masses on-shell
  - $\tilde{b}_1$-mass ($= \tilde{b}_L$-mass) determined by $SU(2)$-relation ($m_b = 0$):
    $$m_{\tilde{b}_1} = m_{\tilde{b}_1}(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_t, \theta_{\tilde{t}}, \varphi_{\tilde{t}})$$
  - generalization of the mixing angle condition:
    $$\widetilde{\mathrm{Re}}\hat{\Sigma}_{\tilde{t}_{12}}(m^2_{\tilde{t}_1}) + \widetilde{\mathrm{Re}}\hat{\Sigma}_{\tilde{t}_{12}}(m^2_{\tilde{t}_2}) = 0$$
    $$\Rightarrow \qquad (\delta\theta_{\tilde{t}} + i \sin\theta_{\tilde{t}} \cos\theta_{\tilde{t}} \delta\varphi_{\tilde{t}}) e^{i\varphi_{\tilde{t}}} = \frac{\widetilde{\mathrm{Re}}\Sigma_{\tilde{t}_{12}}(m^2_{\tilde{t}_1}) + \widetilde{\mathrm{Re}}\Sigma_{\tilde{t}_{12}}(m^2_{\tilde{t}_2})}{2(m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2})}$$
  - $A_t = |A_t| e^{i\varphi_{A_t}}$ is then determined (A_t: trilinear coupling):
    $$A_t = A_t(m_{\tilde{t}_1}, m_{\tilde{t}_2}, m_t, \theta_{\tilde{t}}, \varphi_{\tilde{t}})$$

Phases in couplings

Phases relevant at two-loop level:

- squark sector:
  - phase φ_{A_t} of the trilinear coupling A_t
  - phase of μ (small), μ: Higgsino mass parameter

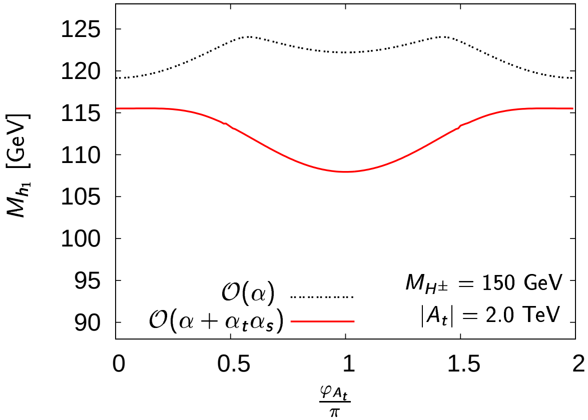
    constraints from measurements of electr. dipole moments

soft breaking parameter

- gluino sector:
  - phase $\varphi_{\tilde{g}}$ of the gluino mass parameter

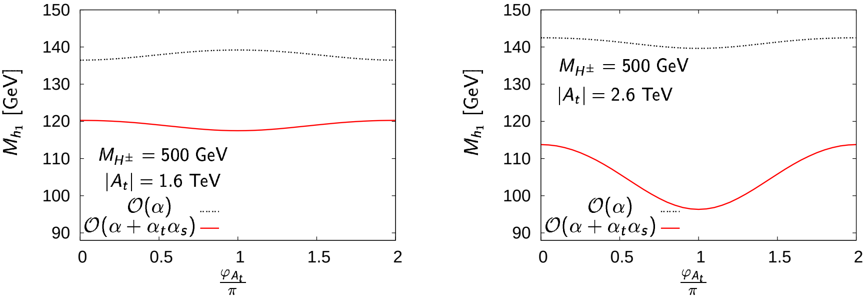
    in the gluino-squark-quark-vertex

Results: φ_{A_t}-dependence (small $M_{H^\pm}$)



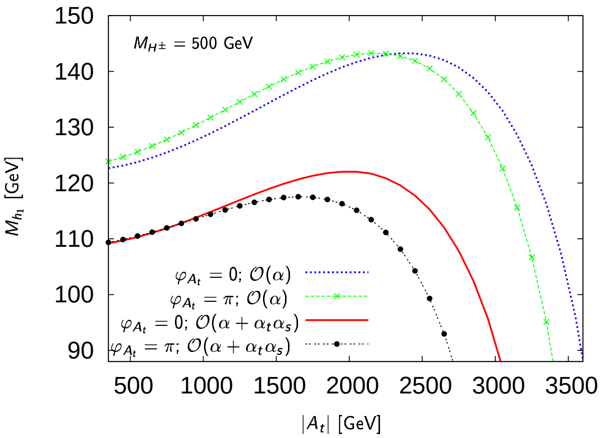
- Quantum corrections of $\mathcal{O}(\alpha_t\alpha_s)$ change the qualitative behaviour of M_{h_1}.

Results: φ_{A_t}-dependence (large $M_{H^\pm}$)



- Quantum corrections of $\mathcal{O}(\alpha_t\alpha_s)$ change the qualitative behaviour of M_{h_1}.
- Qualitative behaviour of M_{h_1} depends strongly on $|A_t|$.

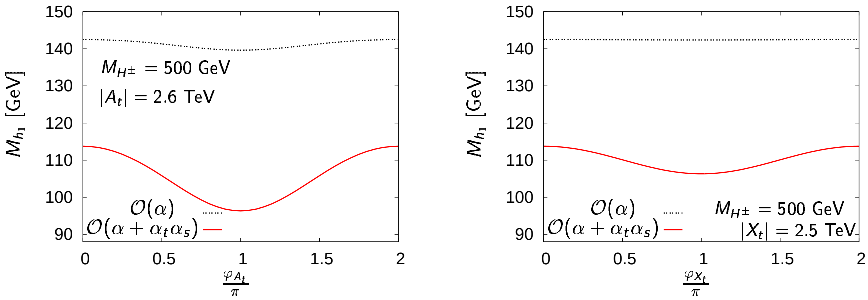
Results: $|A_t|$-dependence (large $M_{H^\pm}$)



- One-loop: $\varphi_{A_t} = \pi$ "shifts" towards lower values of $|A_t|$ with respect to $\varphi_{A_t} = 0$.
- Two-loop: also the value of the maximum of M_{h_1} depends on the phases.

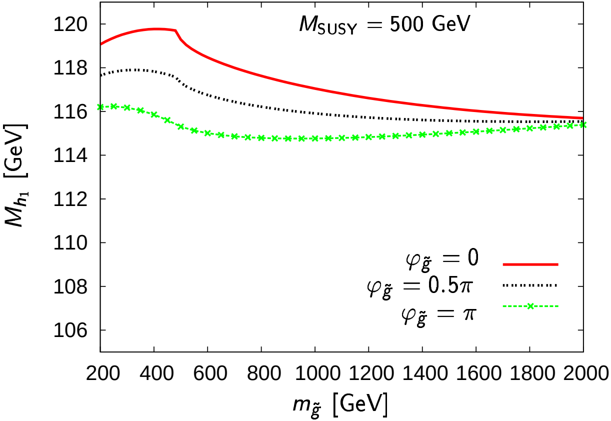
Results: φ_{A_t}- versus φ_{X_t}-dependence (large $M_{H^\pm}$)

size of the squark mixing: $X_t := A_t - \mu^* \cot\beta$



- Quantum corrections are smaller for constant absolute value of the squark mixing, $|X_t| = \text{const.}$

Results: dependence on the gluino mass $m_{\tilde{g}}$



- Large effects in the threshold region: $m_{\tilde{t}_2} = m_{\tilde{g}} + m_t$

Conclusions

- Quantum corrections are important for a precise prediction of the mass of the lightest Higgs boson M_{h_1}:
  - They can induce CP-violation.
  - Dominant corrections: from the top sector
- Contributions of $\mathcal{O}(\alpha_t \alpha_s)$ for complex parameters:
  - Necessary: Renormalization of the sfermion sector for complex parameters
  - Phases are relevant at the two-loop level.
  - Two-loop contributions can change qualitative behaviour of M_{h_1}.
  - are currently included into FeynHiggs.