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A pNRQCD approach to $t\bar{t}$ near threshold

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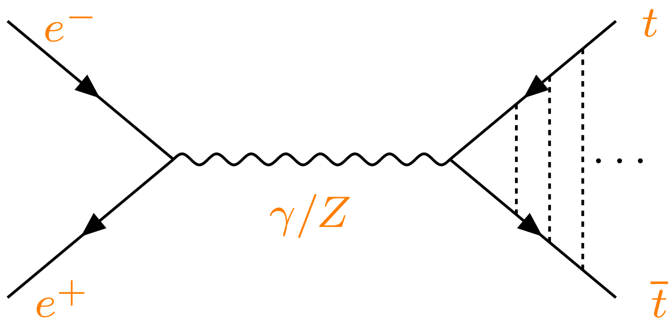
BASED ON WORK DONE IN COLLABORATION WITH

A. PINEDA AND M. BENEKE, V. SMIRNOV

outline

A "NNLL" computation	• introduction • outline of calculation using pNRQCD • results and theoretical error
Electroweak effects	• QED effects • from $e^+e^- \to t\bar{t}$ to $e^+e^- \to W^+bW^-\bar{b}$ • unstable particle effective theory
Outlook	• further improvements • towards NNNLO



$$R \equiv \frac{\sigma(e^+e^- \to Q\overline{Q})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

problem with three scales:

- hard: m
- soft: $\vec{p} \sim mv \sim m\alpha_s$
- ultrasoft: $E = \sqrt{s} - 2m \sim mv^2 \sim m\alpha_s^2$

hierarchy of scales: $m \gg mv \gg mv^2 \gg \Lambda_{\rm QCD}$

fixed order:

$$\sigma(= R) = v \sum_n \left(\frac{\alpha_s}{v}\right)^n \times \left\{1 \;({\rm LO});\, \alpha_s, v \;({\rm NLO});\, \alpha_s^2, v^2, \alpha_s v \;({\rm NNLO})\right\}$$

resummed:

$$\sigma = v \sum_n \left(\frac{\alpha_s}{v}\right)^n \sum_l (\alpha_s \log v)^l \times \left\{1 \;({\rm LL});\, \alpha_s, v \;({\rm NLL});\, \alpha_s^2, v^2, \alpha_s v \;({\rm NNLL})\right\}$$

introduction

- exploit $\alpha_s \ll 1$ and $v \ll 1$ $\longrightarrow$ double expansion
- identify modes [Beneke, Smirnov] $\Rightarrow$ asymptotic expansion (method of regions)

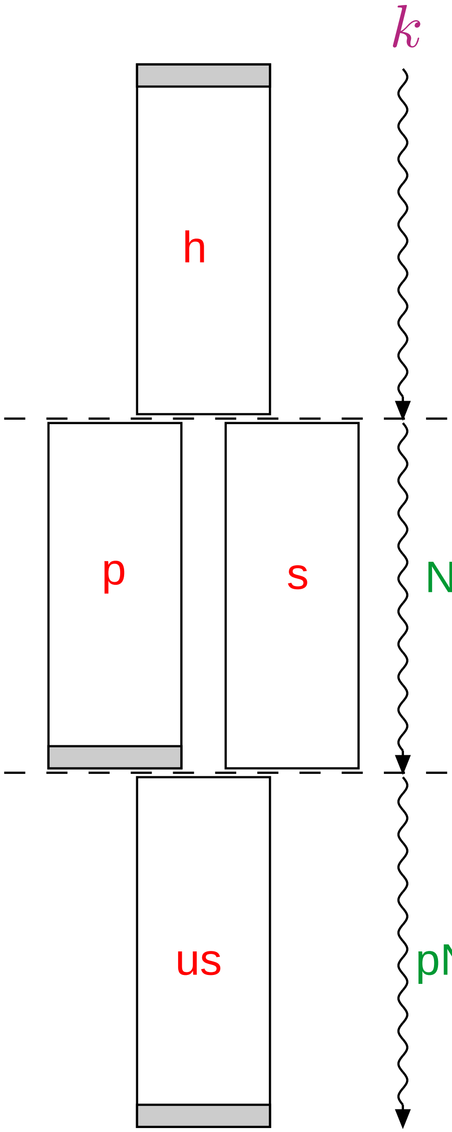
hard	$k^\mu \sim m$
soft	$k^\mu \sim mv$
potential	$k^0 \sim mv^2;\ \vec{k} \sim mv$
ultrasoft	$k^\mu \sim mv^2$

- integrate out 'unwanted' modes (final state described by potential quarks and ultrasoft gluons):
  QCD (h,s,p,u) $\longrightarrow$ NRQCD (s,p,u)$\longrightarrow$ pNRQCD (p$|_q$,u)
- matching of currents
- done to NNLO [Beneke et.al; Hoang et.al; Melnikov et.al; Yakovlev; . . .]
- use threshold mass, not pole mass [Bigi et.al; Beneke; Hoang et.al; Pineda]

introduction

underlying theory

$\mu_h \sim m$ QCD $\mathcal{L}_{\rm QCD}(\psi_h, \psi_s, \psi_p, g_h, g_s, g_p, g_{us})$



effective theory I

$\mu_s \sim mv$ NRQCD $\mathcal{L}_{\rm NRQCD}(\psi_s, \psi_p, g_s, g_p, g_{us})$

effective theory II

$\mu_{us} \sim mv^2$ pNRQCD $\mathcal{L}_{\rm pNRQCD}(\psi_p, g_{us})$

calculation

In full QCD: $q^2 = s = (E + 2m)^2$

$$R(s) = \frac{4\pi\, e_q^2}{s}\,\mathrm{Im}\left[-i\int d^4x\, e^{iqx}\langle 0|T\{j^\mu(x) j_\mu(0)\}|0\rangle\right]$$

current: (Z exchange not included)

$$j^\mu \equiv \bar{Q}\gamma^\mu Q \to c_1\,\chi^\dagger\sigma^i\psi - \frac{c_2}{6m^2}\,\chi^\dagger\sigma^i\,(i\mathbf{D})^2\,\psi + \ldots$$

in pNRQCD :

$$R(E) = \frac{24\pi e_q^2 N_c}{s}\left(c_1^2 - c_1 c_2\frac{E}{3\,m}\right)\mathrm{Im}\,G(0,0,E)$$

calculation

NRQCD Lagrangian [Caswell, Bodwin, Braaten, Lepage]

$$\begin{aligned}
\mathcal{L}_{\rm NRQCD} &= \psi^\dagger \left(iD^0 + c_k \frac{\vec{D}^2}{2m} \right) \psi + \frac{c_4}{8m^3}\, \psi^\dagger \vec{D}^4 \psi - \frac{g\, c_F}{2m}\, \psi^\dagger \sigma^i B^i\, \psi \\
&+ \frac{g\, c_D}{8m^2}\, \psi^\dagger \left[D^i, E^i \right] \psi + \frac{i g\, c_S}{8m^2}\, \psi^\dagger \sigma^{ij} \left[D^i, E^j \right] \psi + (\psi \leftrightarrow \chi) \\
&+ \frac{d_{ss}}{m^2}\, \psi^\dagger \psi\, \chi^\dagger \chi + \frac{d_{sv}}{m^2}\, \psi^\dagger \sigma^i \psi\, \chi^\dagger \sigma^i \chi \\
&+ \frac{d_{vs}}{m^2}\, \psi^\dagger T^a \psi\, \chi^\dagger T^a \chi + \frac{d_{vv}}{m^2}\, \psi^\dagger \sigma^i T^a \psi\, \chi^\dagger \sigma^i T^a \chi + \mathcal{L}_{\rm light}
\end{aligned}$$

- all calculations in momentum space, using dimensional regularization in $D = 4 - 2\epsilon$ dimensions thus e.g: $\sigma^i B^i = (i/4)[\sigma^i, \sigma^j]\, G^{ij}$
- resum $\log(\mu_h/\mu_s)$ in c_i and d_{ij} using renormalization group
- RGI: single heavy quark sector as in HQET [Bauer, Manohar, . . .]
  RGI: four heavy quark operators [Pineda]

pNRQCD Lagrangian [Pineda, Soto]

$$
\begin{aligned}
\mathcal{L}_{\rm pNRQCD} &= \psi^\dagger \left(iD^0 + \frac{\partial^2}{2m} \right) \psi + \chi^\dagger \left(iD^0 - \frac{\partial^2}{2m} \right) \chi \\
&+ \int d^3r \left(\psi^\dagger T^a \psi \right) \frac{-4\pi C_F \alpha_s}{q^2} \left(\chi^\dagger T^a \chi \right) \\
&+ \int d^3r \left(\psi^\dagger T^a \psi \right) \delta V \left(\chi^\dagger T^a \chi \right) \\
&+ \psi^\dagger \left(\frac{\partial^4}{8m^3} - g_s\, \vec{x} \cdot \vec{E} \right) \psi + \chi^\dagger \left(-\frac{\partial^4}{8m^3} - g_s\, \vec{x} \cdot \vec{E} \right) \chi
\end{aligned}
$$

- leading order Coulomb potential is LO effect
- remaining terms in potential, δV (Breit-Fermi potential, static potential [Schröder, Peter], non-analytic potential ...) included perturbatively
- (some) matching coefficients in δV have to be known in D dimensions
- ultrasoft effects enter at NNNLO

The renormalization group improved pNRQCD potential: [Pineda, AS]

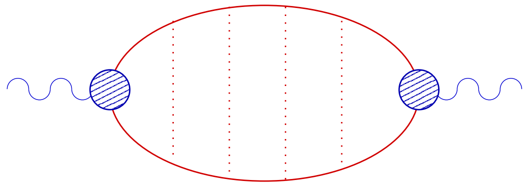
$$
\begin{aligned}
V_{NNLL} &= -4\pi C_F \frac{\alpha_{\tilde{V}_s}}{\vec{q}^{\,2}} \\
& -C_F C_A D_s^{(1)} \frac{\pi^2}{m\, q^{1+2\epsilon}} (1-\epsilon) \frac{(4\pi)^\epsilon \Gamma^2(\frac{1}{2}-\epsilon)\Gamma(\frac{1}{2}+\epsilon)}{\pi^{3/2}\Gamma^2(1-2\epsilon)} \\
& -\frac{2\pi C_F D_{1,s}^{(2)}}{m^2} \frac{\vec{p}^{\,2}+\vec{p}^{\,\prime\,2}}{\vec{q}^{\,2}} + \frac{\pi C_F D_{2,s}^{(2)}}{m^2} \left(\left(\frac{\vec{p}^{\,2}-\vec{p}^{\,\prime\,2}}{\vec{q}^{\,2}} \right)^2 - 1 \right) \\
& +\frac{3\pi C_F D_{d,s}^{(2)}}{m^2} - \frac{4\pi C_F D_{S^2,s}^{(2)}}{d\, m^2} [\mathbf{S}_1^i, \mathbf{S}_1^j][\mathbf{S}_2^i, \mathbf{S}_2^j] + \ldots
\end{aligned}
$$

- use renormalization-group equations to evolve potentials D_X from μ_s to μ_{us}, resumming $\log \mu_s/\mu_{us}$. [Pineda]
- LL running of D_X known $\longrightarrow$ potential known at NNLL

calculation

We use dimensional regularization throughout, perform all calculations in momentum space and always use $\overline{\mathrm{MS}}$-subtraction [Beneke, AS, Smirnov]

$$G_c(\vec{r},\vec{r}\,',E)\Big|_{\vec{r}=\vec{r}\,'=0} \equiv \int \frac{d^d\vec{p}}{(2\pi)^d}\,\frac{d^d\vec{p}\,'}{(2\pi)^d}\,\tilde{G}_c(\vec{p},\vec{p}\,',E)$$



$$G_c(0,0,E) = -\frac{\alpha_s\,C_F\,m^2}{4\pi}\left(\frac{1}{2\lambda}+\frac{1}{2}\ln\frac{-4mE}{\mu^2}-\frac{1}{2}+\gamma_E+\psi(1-\lambda)\right)$$

where $\lambda \equiv C_F\,\alpha_s/(2\sqrt{-E/m})$; This sums all potential gluon (ladder) diagrams

for higher-order corrections evaluate single and double insertions

$$\delta G_c(0,0,E) = \int \prod \frac{d^d\vec{p}_i}{(2\pi)^d}\,\tilde{G}_c(\vec{p}_1,\vec{p}_2,E)\,\delta V(\vec{p}_2,\vec{p}_3)\,\tilde{G}_c(\vec{p}_3,\vec{p}_4,E)$$

calculation

current

- c_1 needed at two loop [Czarnecki, Melnikov; Beneke, AS, Smirnov]
- higher dimensional operators of single heavy quark sector mix into lower dimensional operators in heavy quark-antiquark sector through potential loops
- need NLL matching coefficients of NRQCD to obtain NLL current $\Rightarrow$ done [Pineda; Hoang, Manohar, Stewart]

$$\mu_s \frac{d}{d\mu_s} c_1 = -\frac{C_F^2}{4}\alpha_s \left(\alpha_s - \frac{2}{3} D^{(2)}_{S^2,s} - 3D^{(2)}_{d,s} + 4D^{(2)}_{1,s}\right) - \frac{C_A C_F}{2} D^{(1)}_s$$

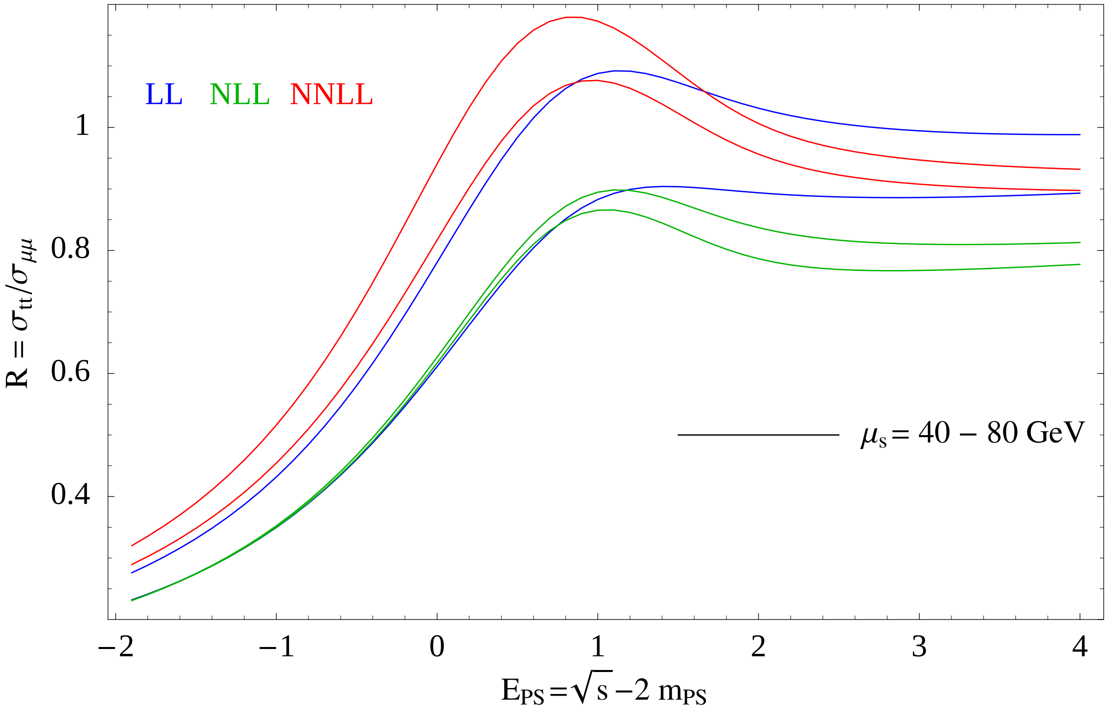
- however NNLL current not complete, only partial results available [Kniehl et.al; Hoang; Penin et.al.] $\Rightarrow$ NNLL $\longrightarrow$ 'NNLL'
- these are the only missing NNLL terms

vNRQCD vs. pNRQCD

- resummation of $\log v$ done before at “NNLL” using vNRQCD [Hoang, Manohar, Stewart, Teubner]
- in vNRQCD there is only one step in the matching procedure and the correlation between the scales is fixed from the start $\mu_{us} = \mu_s^2/m$
- in the pNRQCD approach the correlation between the scales is taken into account in the RG solutions
- done (so far) only for the spin dependent term [Penin, Pineda, Steinhauser, Smirnov], thus NNLL $\Rightarrow$ “NNLL”
- independent variation of μ_s and μ_h $\rightarrow$ more conservative error estimate, μ_h dependence is now larger than μ_s dependence.
- ideally, we also would like to independently vary the ultrasoft scale μ_{us}, i.e. $\mu_{us} = \mu_s^2/\mu_h \rightarrow \mu_{us} \sim \mu_s^2/\mu_h$; has not been done so far

results

μ_s dependence of fixed-order results



$$\mu_s^2 \sim 4m_t\sqrt{E^2+\Gamma_t^2}$$

$$m_{PS} = 175 \text{ GeV}$$

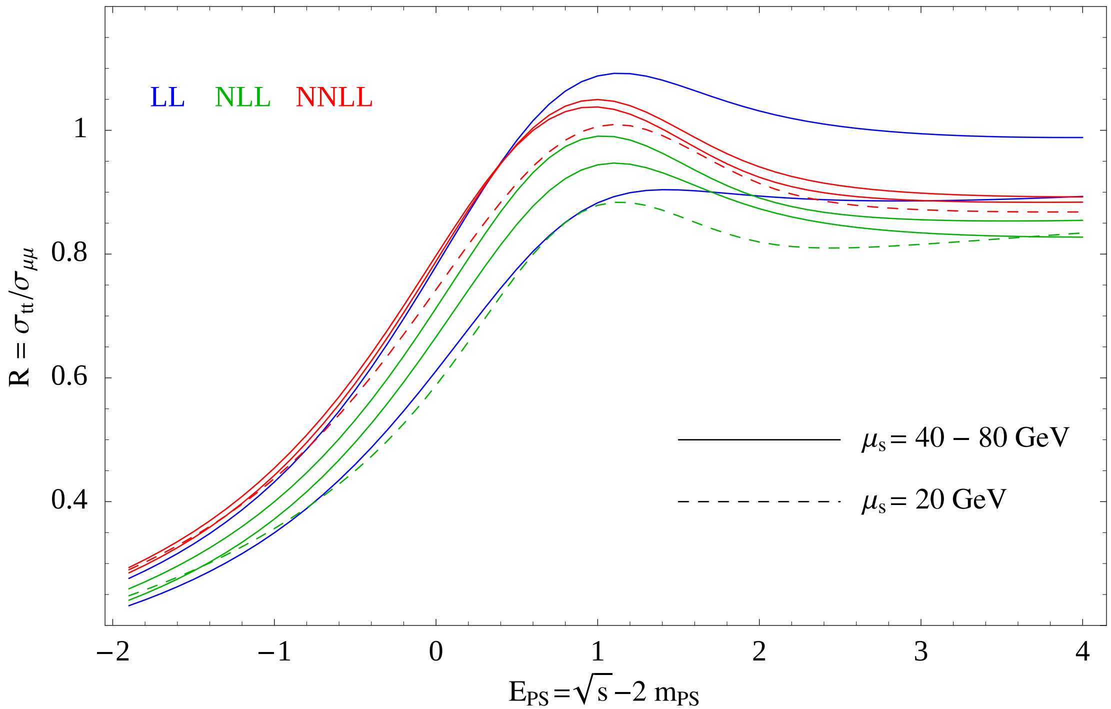
$$\Gamma_t = 1.4 \text{ GeV}$$

$$\mu_F = 20 \text{ GeV}$$

- normalization of cross section has a large theoretical error, scale dependence increases from NLO to NNLO and NNLO corrections as large as NLO corrections!
- no top width / Yukawa coupling measurement

results

μ_s dependence of renormalization-group improved results



[Pineda, AS]

$$\mu_s^2 \sim 4 m_t \sqrt{E^2 + \Gamma_t^2}$$

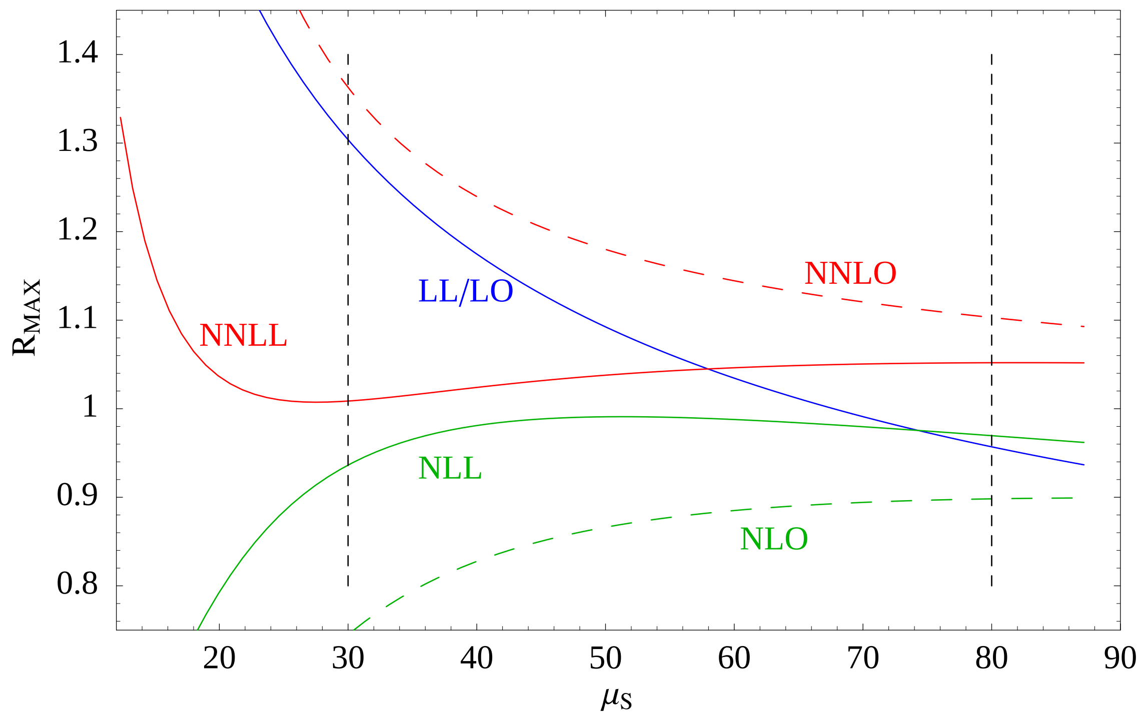
$$m_{PS} = 175 \text{ GeV}$$

$$\Gamma_t = 1.4 \text{ GeV}$$

$$\mu_F = 20 \text{ GeV}$$

- normalization of cross section much more stable, confirms previous results by [Hoang, Manohar, Stewart, Teubner]
- μ_s scale-dependence bands do not overlap $\longrightarrow$ estimate of theoretical error ??

μ_s dependence of renormalization-group improved results



[Pineda, AS]

$$\mu_s^2 \sim 4m_t\sqrt{E^2+\Gamma_t^2}$$

$$m_{PS} = 175 \text{ GeV}$$

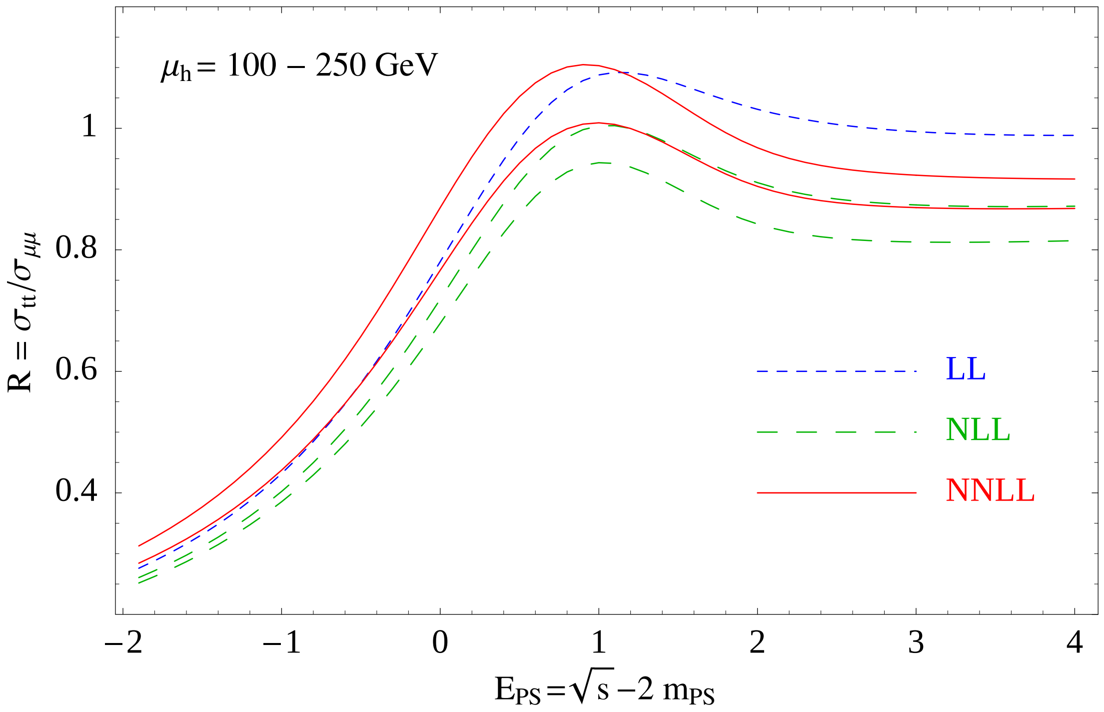
$$\Gamma_t = 1.4 \text{ GeV}$$

$$\mu_F = 20 \text{ GeV}$$

- ’problem’ with small scales solved by including multiple insertions of Coulomb potentials [Beneke, Kiyo, Schuller]
- reliable region for soft scale: $30 \text{ GeV} \leq \mu_s \leq 80 \text{ GeV}$

μ_h dependence of renormalization-group improved results

[Pineda, AS]



$\mu_s^2 \sim 4m_t\sqrt{E^2 + \Gamma_t^2}$

$m_{PS} = 175$ GeV

$\Gamma_t = 1.4$ GeV

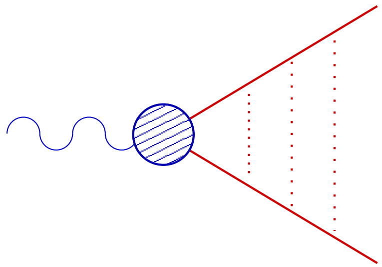
$\mu_F = 20$ GeV

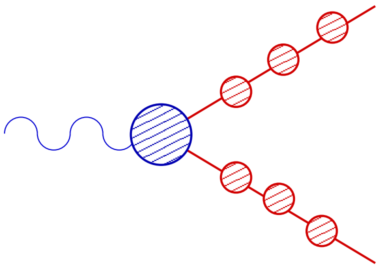
- μ_h scale dependence larger than μ_s scale dependence
- μ_h scale-dependence bands do overlap $\longrightarrow$ more reliable estimate of theoretical error for normalization: $\sim 10\%$

leading order

- top quark propagator $(E - \frac{\vec{p}^2}{2m_t})^{-1}$ scales as $\frac{1}{mv^2} \sim \frac{1}{m\alpha_s^2} \sim \frac{1}{m\alpha_{ew}}$
- the width $\Gamma_t \sim m\alpha_{ew}$ is a LO effect, $E \to E + i\Gamma$ [Fadin, Khoze]

$$\frac{1}{E - \frac{\vec{p}^2}{2m_t}} \to \frac{1}{E - \frac{\vec{p}^2}{2m_t} + i\Gamma_t}$$





Coulomb singularity $v \to 0$

resum $(\alpha_s/v)^n$ (potential gluon exchange)

systematic expansion in α and v

propagator pole $\Gamma \to 0$

resum $(\Gamma/m)^n$ (self-energy insertions)

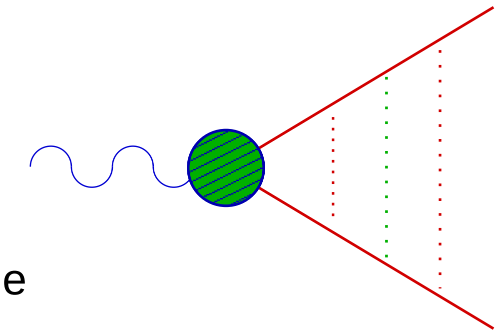
systematic expansion in α and Γ

electroweak

higher-order electroweak corrections for stable top

- NLO QED corrections: single potential photon exchange suppressed by $\alpha/v \sim \alpha_s^2/v \sim v$

$$V \to V - \frac{4\pi\alpha\, e_q^2}{q^2}$$

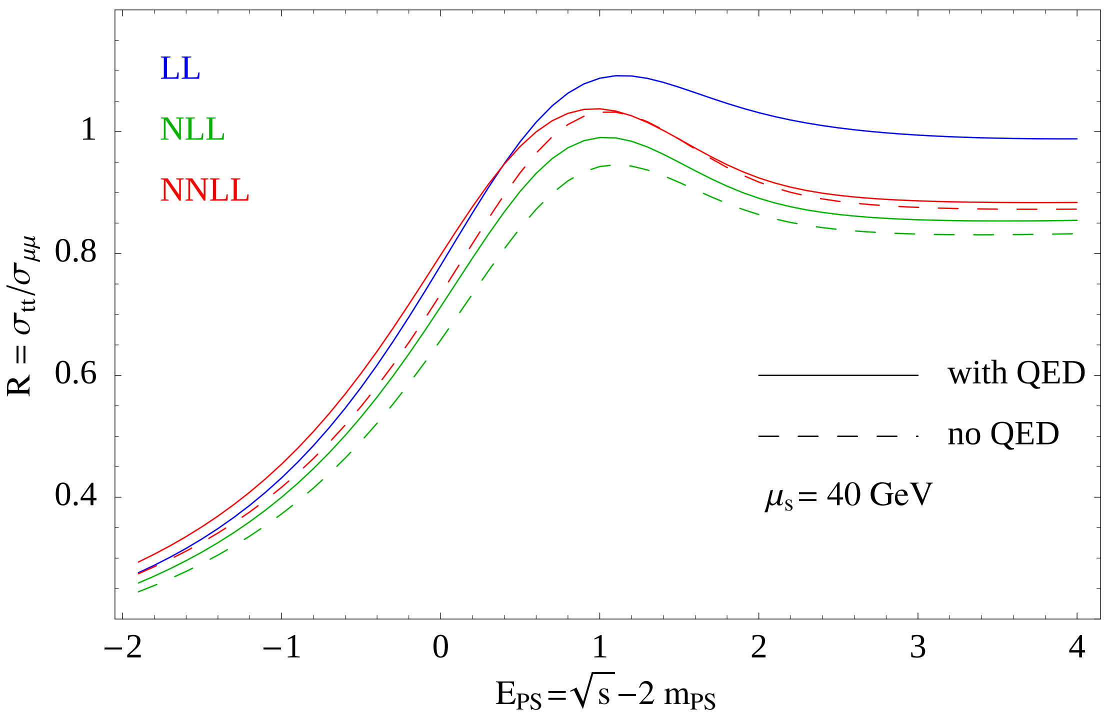


- NNLO QED corrections: double potential photon exchange $(\alpha/v)^2 \sim v^2$ and hard corrections

$$c_1 \to c_1 - \frac{2e_q^2\alpha}{\pi}$$

- beyond NNLO: many corrections of order $\alpha\,\alpha_s$ e.g. Higgs mass dependence δm_t up to $20 - 40\ \mathrm{MeV}$ [Eiras, Steinhauser]

NLO and NNLO QED corrections for stable top



[Pineda, AS]

$\mu_s^2 \sim 4m_t\sqrt{E^2 + \Gamma_t^2}$

$m_{PS} = 175\ \mathrm{GeV}$

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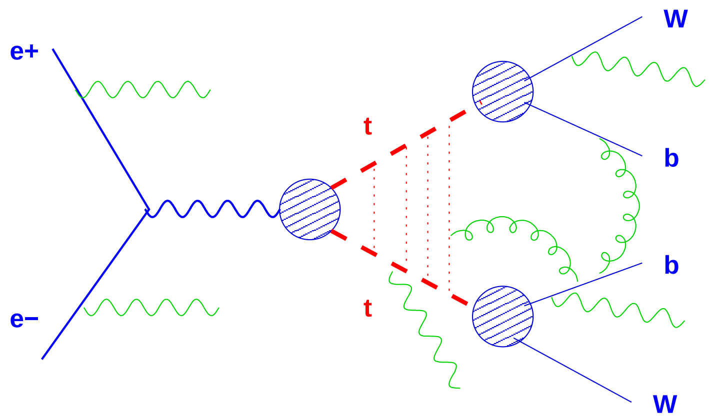
$\mu_F = 20\ \mathrm{GeV}$

- shift in position of peak, i.e. $\delta m_t \sim 100\ \mathrm{MeV}$, about the same size as NNLL corrections

electroweak

electroweak corrections beyond “stable” top

- Strictly speaking, it does not make sense to talk about $\sigma(e^+e^- \to t\bar{t})$ (or any cross section with an unstable particle in the final state).
- for threshold scan, $\delta m_t \ll \Gamma_t$, thus $\sigma(e^+e^- \to t\bar{t}) \to \sigma(e^+e^- \to W^+W^-b\bar{b})$
- QCD and electroweak effects

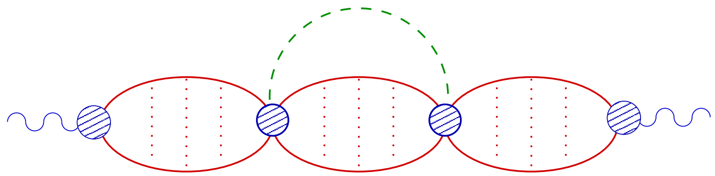


- electroweak effects are important! partially computed $\delta m_t = 30 - 50$ MeV [Hoang, Reisser] effective theory approach to unstable particles

- use effective theory methods (again!) to systematically expand in small parameter $\delta \equiv (p^2 - m^2)/m^2 \sim \Gamma/m$ [Chapovsky, Khoze, AS, Stirling]
- identify relevant modes (soft/resonant modes from HQET and NRQCD, collinear modes from SCET) $\longrightarrow$ asymptotic expansion [Beneke, Chapovsky, AS, Zanderighi]
- integrate out 'unwanted' modes $\longrightarrow$ tower of effective theories (Unstable Particle Effective Theory)
- hard effects correspond to factorizable corrections
- non-factorizable corrections due to still dynamical modes
- this is neither a "quick-fix" nor a "free lunch", it is a method to identify the minimal amount of calculation to be done for a systematic expansion in the small parameters (as for NRQCD)
- gauge invariance is automatic since the split into the various contributions respects gauge invariance

outlook

- needed: c_1 fully NNLL and all electroweak effects
- ultrasoft effects (retardation effects)
  - due to chromoelectric dipole operator $\vec{x} \cdot \vec{E}$
  - NNNLO effects α_s^3 (NNLL part $\alpha_s^3 \ln \alpha_s$ already included)
  - potentially particularly important: $\alpha_s^3 \sim \alpha_s^2(\mu_s)\,\alpha_s(\mu_{us})$



- full NNNLO.....
  - compute all insertions (up to triple insertions), some results available: [Beneke, Kiyo, Schuller]
  - compute all potentials, some results available: [Kniehl, Penin, Steinhauser, Smirnov]
  - bottleneck: three-loop static potential and current matching coefficient
- more exclusive quantities

conclusions

- the theory for $t\bar{t}$ production near threshold is in good shape and further progress is on its way
- achieving $\delta m_t \sim 100\ \mathrm{MeV}$ and $\delta R_{\mathrm{max}} \sim 3\%$ relies on further theoretical progress (and the patience to actually do a threshold scan!!)
  - full NNLL !!
  - at least ultrasoft (if not full) NNNLO
  - fully take into account instability of top quark
- more exclusive final states ?
- tools are set up, but a lot of (tedious) additional work required
- this is one of the rare problems that is very fascinating from a theoretical point of view and extremely relevant from an experimental point of view