Regularization of supersymmetric theories

New results on dimensional reduction

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Outline

Properties of DREG/DRED (status Jan. 2005)

DREG:

Dim. Regularization (DREG)

D dimensions
D Gluon/photon-components
4 Gluino/photino-components

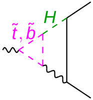
DRED:

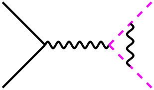
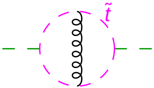
Dim. Reduction (DRED)

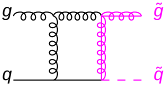
D dimensions
4 Gluon/photon-components
4 Gluino/photino-components

Regularization of SUSY

Motivation: some important observables/calculations...

- $(g-2)_\mu$ 

$\rightarrow$ no problem with regularization
- 1-Loop processes  $\rightarrow$ DRED preserves SUSY!!
- M_h 

$\rightarrow$ DRED SUSY-preserving??
- LHC 

$\rightarrow$ DRED violates factorization!?

Summary: Properties of DREG and DRED

DREG:	consistent	SUSY-violation	factorization
	$+$	$-$	$+$
DRED:	consistent	SUSY	factorization
	$+$	$(+)$	$(+)$

There is no consistent SUSY regularization

theoretical question:
SUSY renormalizable? Anomalies?

practical question:
Which scheme is best in practical computations?

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Renormalizability, no anomalies: proven indep. of reg.
SUSY [Piguet, Sibold 1985] [Piguet et al],
MSSM [Hollik, Kraus, Roth, Rupp, Sibold, DS 2002]

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Which scheme is best in practical computations? This talk

Outline

1. Introduction
2. Consistency of DRED
3. Factorization in DRED
4. Supersymmetry and M_h-calculations
5. Conclusions

Outline

Where does the inconsistency come from?

DREG: "D-dimensional space" can be consistently defined
$\Rightarrow$ no inconsistency like 1=0: [Wilson'73],[Collins]

DRED: in original form: problem

1. algebraic id.: $g^{(4)}{}_{\mu\nu} g^{(D)}{}_{\rho}{}^{\nu} = g^{(D)}{}_{\mu}{}^{\rho}$ etc
2. 4-dim id.: $\det \begin{pmatrix} g^{\mu_1\nu_1} & \dots & g^{\mu_1\nu_5} \\ \vdots & & \vdots \\ g^{\mu_5\nu_1} & \dots & g^{\mu_5\nu_5} \end{pmatrix} = 0,$ Fierz, ...

(1)+(2) $\Rightarrow$ inconsistent, 1=0

Consistent DRED

Idea:

Use only algebraic id. (1) but no 4-dim id. (2)

- should be consistent [Avdeev, Chochia, Vladimirov 1981]
- mathematical construction of quantities satisfying (1) possible
  $\Rightarrow$ proof: DRED is mathematically consistent if only (1) is used [DS 2005]

Consequences in practice:

- algebraic id. of DRED as usual
- one cannot rely on index counting or Fierz identities
- for many SUSY loop calculations, this doesn't make a difference

Quantum Action Principle in DRED

Using the consistent formulation of DRED, one can prove the quantum action principle in DRED

$$i\,\delta_{\mathrm{SUSY}}\langle T\,\phi_1\ldots\phi_n\rangle = \langle T\,\phi_1\ldots\phi_n\Delta\rangle$$

Useful to study symmetry-properties of regularizations

Proof has to be carried out for each regularization,

BPHZ [Lowenstein et al '71]
DREG [Breitenlohner, Maison '77]
DRED [DS 2005]

Outline

Factorization-problem

Problem: DRED, $m \neq 0$

$$\sigma^{\mathrm{DRED}}(GG \to t\bar{t}G) \xrightarrow{2\|3} \sim \frac{1}{k_2 k_3} P_{g\to gg}\ \sigma^{\mathrm{DRED}}(GG \to t\bar{t})$$
$$+ \frac{1}{k_2 k_3} K_g\ \sigma^{\mathrm{puzzle}}$$

[Beenakker, Kuijf, van Neerven, Smith ’88] [van Neerven, Smith ’04] [Beenakker, Höpker, Spira, Zerwas ’96]

- One “solution” in practice (unsatisfactory complication): resort to DREG $\Rightarrow$ SUSY-restoring cts necessary
- Fundamental question: where does the seemingly non-factorizing term σ^{puzzle} come from?

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clue: mismatch!

Dim. Reduction (DRED)

D dimensions

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DRED and the gluon

4-component Gluon in DRED G	=	D-component gauge field g	+	ϵ-scalars ϕ

For polarization sums: $g_{\mu\nu}^{(4)} = g_{\mu\nu}^{(D)} + g_{\mu\nu}^{(\epsilon)}$

g and ϕ have to be treated seperately!

DRED and the gluon

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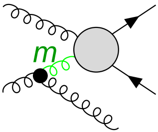
- Simple kinematics: e.g. $GG \to q\bar{q}$ (massless) $\quad \sigma_{GG\to q\bar{q}} = \sigma_{Gg\to q\bar{q}} = \sigma_{G\phi\to q\bar{q}}$
- in general / here: $GG \to t\bar{t}$ (massive) $\quad \sigma_{GG\to q\bar{q}} \neq \sigma_{Gg\to q\bar{q}} \neq \sigma_{G\phi\to q\bar{q}}$

Factorization — result

Main result:



- reconciled DRED and factorization [Signer, DS '05]

$$\sigma^{\mathrm{DRED}}(GG \to t\bar{t}G) \to P_{G\to gG}\ \sigma_{Gg} + P_{G\to\phi G}\ \sigma_{G\phi}$$

- understood origin of non-factorizing term

$$K_g\ \sigma^{\mathrm{puzzle}} \to P_{\phi\to g\phi}\ [\sigma_{Gg} - \sigma_{G\phi}]$$

Factorization — result

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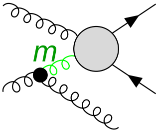

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Practical consequences

- hadron processes can be computed using DRED

Outline

Symmetries of regularizations

In principle, we don't have to bother whether a regularization preserves symmetries

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In practice, life is easier with a symmetry-preserving regularization!

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- counterterms $\Gamma^{\rm ct}$ also preserve symmetries:

  $g \to g + \delta g$, $m \to m + \delta m$ —"multiplicative renormalization"
- most common situation, often assumed without proof

Problem: SUSY of DRED

- DRED preserves SUSY in simple cases
- Does DRED preserve SUSY in general?
- Or at least in cases that are relevant in practice?

DRED preserves SUSY — What does it mean?

SUSY $\Leftrightarrow$ ST-identities $0 = \delta_{\mathrm{SUSY}} \langle T \phi_1 \ldots \phi_n \rangle$

- ST-identities must be satisfied after renormalization

> DRED preserves SUSY if the ST-identities are already satisfied on the regularized level

Quantum action principle as a tool

Quantum action principle:

STI $$\delta_{\mathrm{SUSY}}\langle T\phi_1\ldots\phi_n\rangle = 0$$

$$\updownarrow$$

valid in DRED $\Leftrightarrow$ $$\langle T\phi_1\ldots\phi_n\Delta\rangle = 0 \qquad \Delta = \delta_{\mathrm{SUSY}}\mathcal{L}$$

Sample application: QCD-gauge invariance in DREG

$$\delta_{\mathrm{gauge}}\mathcal{L}_{\mathrm{QCD}}^{\mathrm{DREG}} = \Delta = 0 \quad \Rightarrow \quad \delta_{\mathrm{gauge}}\langle T\phi_1\ldots\phi_n\rangle = 0$$

- $\Rightarrow$ DREG preserves all QCD Slavnov-Taylor identities at all orders

Quantum action principle as a tool

Quantum action principle:

STI $\qquad \delta_{\mathrm{SUSY}}\langle T\phi_1\ldots\phi_n\rangle = 0$

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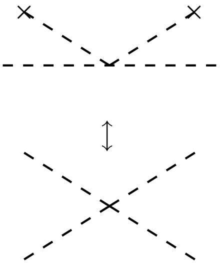
valid in DRED $\Leftrightarrow$ $\qquad \langle T\phi_1\ldots\phi_n\Delta\rangle = 0 \qquad \Delta = \delta_{\mathrm{SUSY}}\mathcal{L}$

application here: SUSY of DRED:

$\delta_{\mathrm{SUSY}}\mathcal{L}^{\mathrm{DRED}} = \Delta \neq 0$ gives rise to Feynman rules [DS '05]

- DRED probably does not preserve all SUSY-identities
- but checking particular ST-identities is simplified using the Q.A.P.

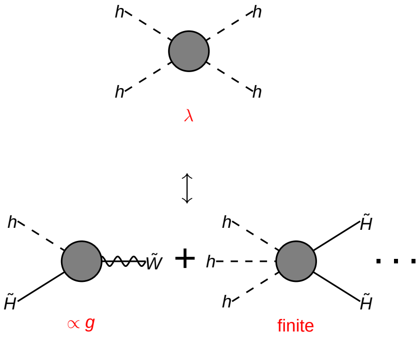
Higgs boson mass and quartic coupling



Higgs mass

- M_h governed by quartic Higgs self coupling λ
- $\lambda \propto g^2$ in SUSY

Quartic coupling and SUSY

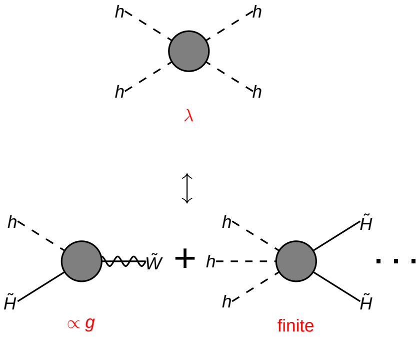


$$0 \overset{?}{=} \delta_{\mathrm{SUSY}} \langle h h h \tilde{H} \rangle$$

Slavnov-Taylor identity

- expresses $\lambda \propto g^2$
- If it is satisfied by DRED $\Leftrightarrow$ multiplicative renormalization o.k.
- Needs to be verified at 2-loop level

Quartic coupling and SUSY

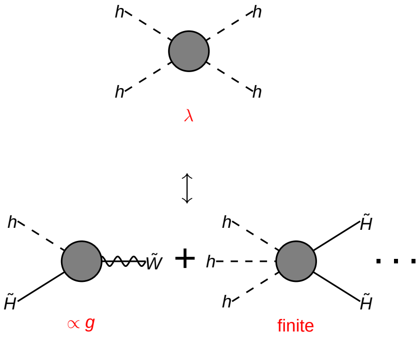


$$0 \stackrel{?}{=} \delta_{\rm SUSY}\langle hhh\tilde{H}\rangle \equiv \langle \Delta hhh\tilde{H}\rangle$$

Method:

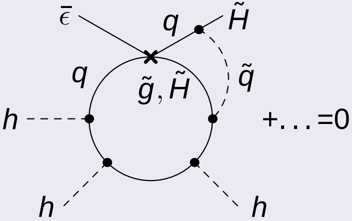
- Use quantum action principle
- replace ST-identity by $\langle \Delta hhh\tilde{H}\rangle = 0 \Leftrightarrow$

Quartic coupling and SUSY



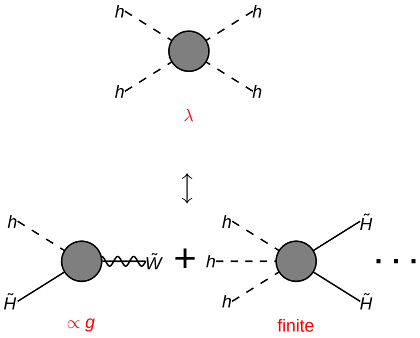
STI valid if

$$\langle \Delta h h h \tilde{H} \rangle = 0 \Leftrightarrow$$



Explicit computation $\Rightarrow$ STI valid in DRED at two-loop level [Hollik, DS '05]

Quartic coupling and SUSY



Results:

- Two-loop STI valid in DRED (in Yukawa-approximation, $\mathcal{O}(\alpha_{t,b}^2, \alpha_{t,b}\alpha_s)$)
- for M_h-calculation at this order, multiplicative renormalization correct
- Previous calculations sufficient

Explicit computation $\Rightarrow$ STI valid in DRED at two-loop level [Hollik, DS '05]

How much do we know now?

old: many SUSY identities checked in DRED:

1-Loop Ward identities [Capper,Jones,van Nieuvenhuizen'80]
β-functions [Martin, Vaughn '93] [Jack, Jones, North '96]
1-Loop S-matrix relation [Beenakker,Höpker,Zerwas'96]
1-Loop Slavnov-Taylor identities [Hollik,Kraus,DS'99] [Hollik,DS'01] [Fischer,Hollik,Roth,DS'03]

new: further 2-loop ST-identities [DS'05] [Hollik, DS'05]

Status:

- sufficient for one-loop SUSY processes
- sufficient for two-loop Higgs masses and further mass relations
  $\Rightarrow$ multiplicative renormalization o.k.
  $\Rightarrow$ no SUSY-restoring counterterms

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DREG:	consistent	SUSY-violation	factorization
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Outline

Summary & Outlook

Comparison of DREG and DRED:

- Factorization: holds in DREG and DRED, slightly more complicated in DRED due to different partons g, ϕ

  $\longrightarrow$ streamlined prescription for hadron processes in DRED?

- Consistency, quantum action principle: ok in DREG and DRED
- SUSY: DREG breaks SUSY already in simplest cases, DRED preserves SUSY in many cases up to 2-Loop, but not at all orders

  $\longrightarrow$ further checks of e.g. RG-running at 3-Loops?