Spin Transport and Depolarization

Kaoru Yokoya, KEK

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Beam-beam Depolarization

• Precession
• Spin-flip radiation

Thanks to K. Moening
Depolarization by Precession

Orbit deflection $\theta \Rightarrow$ Spin precession $\gamma a \theta$

$$\Delta P = 1 - \langle \cos(\gamma a \theta) \rangle = \frac{1}{2} \langle (\gamma a \theta)^2 \rangle$$

Typical deflection angle

$$\theta_0 \equiv \frac{D_x \sigma_z}{\sigma_x} = \frac{D_y \sigma_z}{\sigma_y} = \frac{2N r_e}{\gamma (\sigma_x + \sigma_y)}$$

$D_{x,y}$ Disruption parameter

$\sigma_z$ R.m.s. bunch length

$N$ Number of particle in a bunch

$\sigma_{x,y}$ Transverse beam size

$r_e$ Classical electron radius

R.m.s deflection angle

$$\theta_{x,rms} \approx 0.55 \theta_0, \quad (D_x \ll 1)$$

$$\theta_{y,rms} \approx \frac{0.55 \theta_0}{[1 + (0.5D_y)^5]^{1/6}}$$
\[ \Delta P_{prec} = \frac{1}{2} (\gamma a)^2 \left( \frac{2 \times 0.55 N r_e}{\gamma (\sigma_x + \sigma_y)} \right)^2 \left[ 1 + \frac{1}{[1 + (0.5 D_y)^5]^{1/3}} \right] \]

Or, by using a formula for \( n_\gamma \) (number of photons)

\[ \Delta P_{prec} = 0.00324 \left\{ 1 + \frac{1}{[1 + (0.5 D_y)^5]^{1/3}} \right\} F_p(\gamma) n_\gamma^2 \]

- Effect of vertical deflection is much smaller when \( D_x \ll 1 \ll D_y \)
The coefficient $a$ is actually a function of $\gamma$ given by (V. N. Baier)

$$a = \frac{\alpha}{2\pi} f_a(\gamma), \quad f_a(\gamma) = \frac{2}{\gamma} \int_0^{\infty} \frac{x \, dx}{(1 + x)^3} \int_0^{\infty} \sin \left[ \frac{x}{\gamma} \left( t + \frac{t^3}{3} \right) \right] \, dt$$
Depolarization by Spin-Flip Radiation

In a similar form as precession,

\[ \Delta P_{flip} = 2n_{\gamma} F_f(\gamma) \]

- \( \Delta P_{prec} \) and \( \Delta P_{flip} \) are comparable at 500GeV NLC/GLC
- But \( \Delta P_{prec} \) dominates at 500GeV TESLA
Luminosity-Weighted Polarization

- $\Delta P_{lum}$ is 1/4 to 1/3 of $\Delta P_{tot}$ in the absence of errors.
- To estimate $\Delta P_{lum}$, the theoretical relation between $\Delta P_{lum}$ and $\Delta P_{tot}$ may be used, if polarization accuracy 0.1% is enough.
- However, ...
Offset and Polarization Tilt

Depolarization vs horizontal offset

No initial tilt

![Graph showing depolarization vs horizontal offset for no initial tilt.](image)

Initial hor.tilt 50mrad

![Graph showing depolarization vs horizontal offset for initial horizontal tilt 50mrad.](image)
My simulation (not a cross check — same code is used)

- \( \Delta P \) is sensitive to \( x \)-offset in the presence of initial hor.tilt

- Cannot rely on the simple theoretical relation between \( \Delta P_{\text{lum}} \) and \( \Delta P_{\text{tot}} \) (though better to exclude initial \( \Delta P \))
Depolarization vs vertical offset

- Less sensitive to vertical offset/tilt, as expected
• Possible cause of polarization tilt
  
  o Mis-orientation at the gun only causes depolarization but not tilt at IP (because of smearing in DR)
  o Tilt of precession axis in the DR in the presence of errors can cause a tilt at IP
    (depolarization in DR is not an issue ... $\cos\phi$ only)

• Polarization tilt at IP should be corrected
  
  o Transverse polarization (before collision) should be measured (1-2% accuracy is enough)
  o Should be corrected by the spin rotation after DR
  o This is particularly true in the case of large crossing angle (solenoid effect)
Finally a Good News

- CAIN has been the only beam-beam simulation code with spin
- But D. Schulte is going to include spin tracking in Guinea-Pig