Acceleration and Focusing by superstrong laser fields and their applications to a laser micro-collider

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Outline

- Which is better for high energy acceleration, plasma wakefields or vacuum laser fields?
- Laser acceleration and focusing mechanisms
- Electron-positron micro-collider concept

Necessity for high energy acceleration mechanism

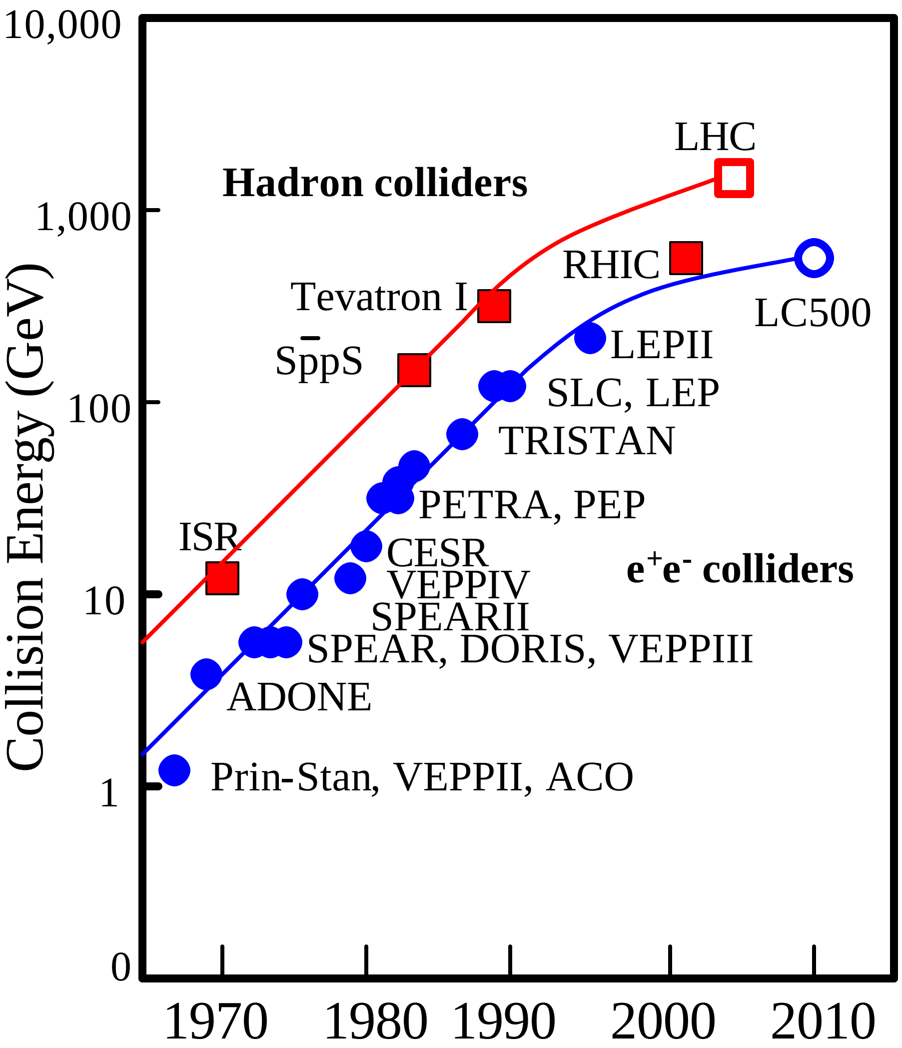
The Laser Acceleration of Particles Workshop, 1982, Los Alamos

"The realization that we seem to be **near the end of the road for conventional accelerators** has generated renewed interest in the possibility of **accelerators with super-high accelerating fields**."

(AIP Conference Proceedings No. 91, 1982)

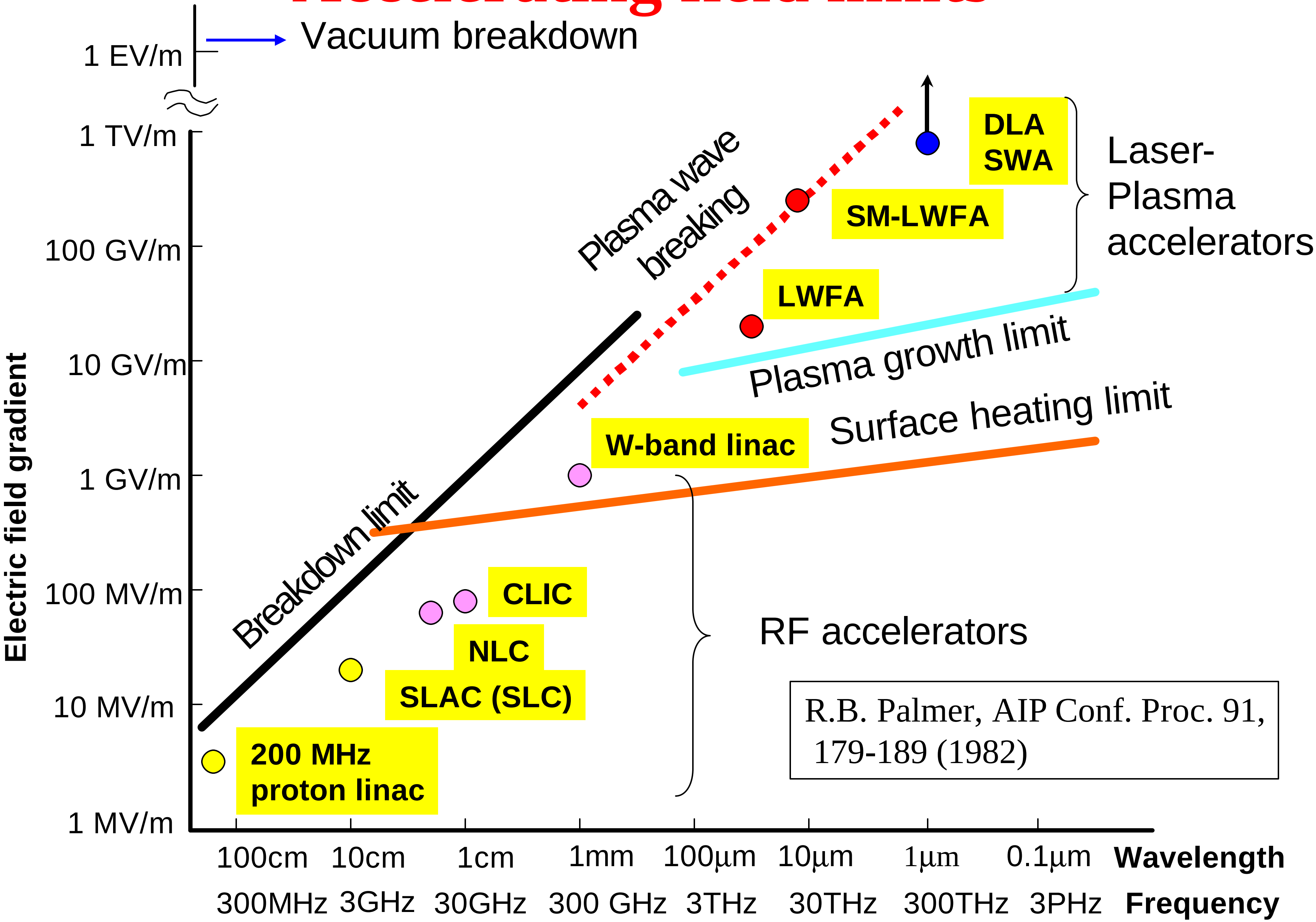
To keep enregy frontier trend

Energy frontier Accelerators



Effective constituent collision energy of Colliders (Physics Today, Jan. 2001)

Accelerating field limits



R.B. Palmer, AIP Conf. Proc. 91, 179-189 (1982)

Is plasma wakefield a high energy acceleration mechanism?

• The plasma wave-breaking causes **background electron trapping**.
• The accelerating field should be less than the plasma **wave-breaking limit**.
• The energy gain is limited by **dephasing** even if the optical guiding is accomplished.
• **Beam-plasma instabilities** breaks high quality beam acceleration.
• **Field control** is difficult.
• High energy **background** particle and photon radiations

Many **disadvantages** kill a role of high energy accelerators.

The maximum accelerating energy

The maximum energy gain of particles trapped in the electric field E_{max} of which phase velocity v_p is given by

$$\Delta\gamma_{max} \approx \frac{1}{\pi}\frac{eE_{max}}{mc^2}L_{acc}$$ for the acceleration length $$L_{acc} \approx \frac{c\lambda_w}{c - v_p}$$

$$\Delta\gamma_{max} \cong \frac{\lambda_w}{1-\beta_p}\frac{eE_{max}}{\pi mc^2} = 4\gamma_p^2\frac{eE_{max}}{mc^2 k_w}$$

where λ_w is the wavelength of the electric field,

$$k_w = 2\pi/\lambda_w \qquad \beta_p = v_p/c \qquad \gamma_p = \frac{1}{\sqrt{1-\beta_p^2}}$$

Assuming the maximum electrostatic potential $$\frac{eE_{max}}{mc^2 k_w} \approx 1$$

$$\boxed{\Delta\gamma_{max} \approx 4\gamma_p^2}$$

Super-high energy particle acceleration in plasma

The phase velocity of the electron plasma wave is given by

$$\beta_p = \frac{v_p}{c} = \sqrt{1 - \frac{n_e}{n_c}} \qquad \gamma_p = \left(\frac{n_c}{n_e}\right)^{1/2}$$

where n_c is the critical plasma density,

$$n_c\left[\mathrm{cm}^{-3}\right] = \frac{\pi}{r_e \lambda_0^2} = 1.11 \times 10^{21} \lambda_0^{-2} [\mu m]$$

with the classical electron radius $\quad r_e = \frac{e^2}{mc^2} = 2.818 \times 10^{-13} [\mathrm{cm}]$

The maximum energy gain is approximately given by

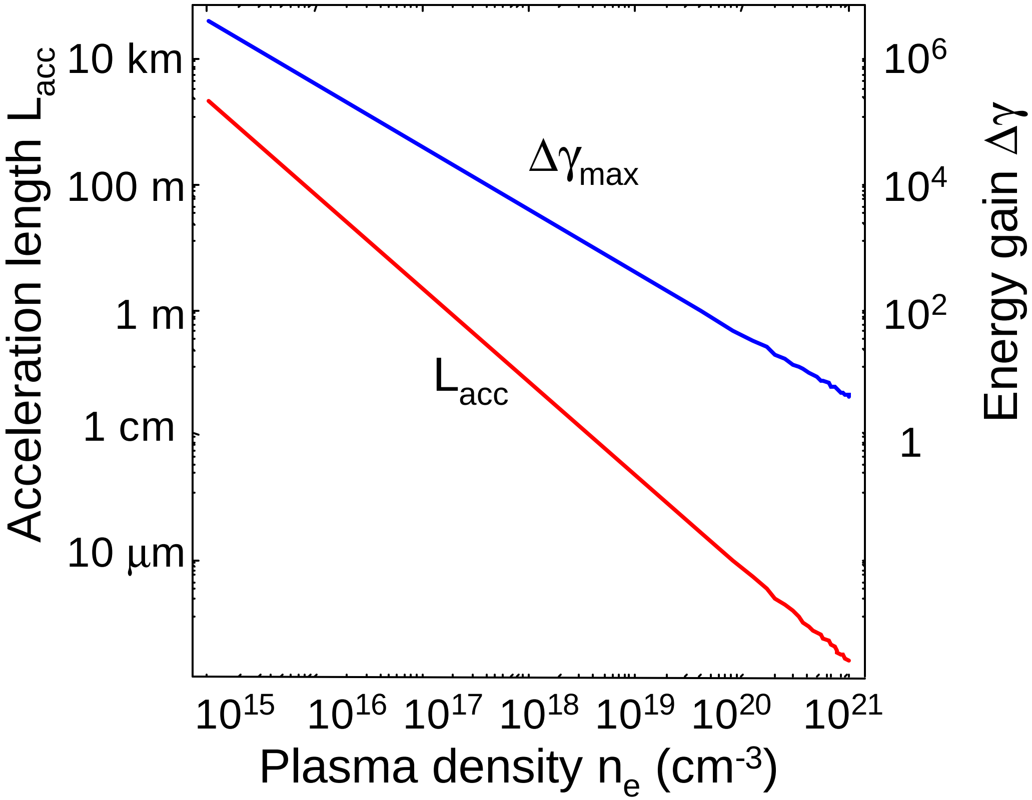
$$\Delta\gamma_{\max} \approx 4\gamma_p^2 = 4\frac{n_c}{n_e} = \frac{4 \times 10^{21}}{\lambda_0^2 [\mu\mathrm{m}] n_e \left[\mathrm{cm}^{-3}\right]}$$

The acceleration length is $$L_{acc} \approx \frac{\lambda_p}{1-\beta_p} = 2\gamma_p^2 \lambda_p$$

With the plasma wavelength $$\lambda_p = \frac{2\pi}{k_p} = \sqrt{\frac{\pi}{r_e n_e}}$$

$$L_{acc}[\mathrm{cm}] \approx 2\frac{n_c}{n_e}\sqrt{\frac{\pi}{r_e n_e}} = \frac{2}{\lambda_0^2}\left(\frac{\pi}{r_e n_e}\right)^{3/2} \approx \frac{7.4\times 10^{27}}{\lambda_0^2[\mu\mathrm{m}]\, n_e^{3/2}\left[\mathrm{cm}^{-3}\right]}$$

A single stage energy gain



A naive estimate of 2.5 TeV electron linac

Consider a laser driver of $\lambda_0 = 1\mu\text{m}$

For 2.5 TeV electron acceleration,

the required energy gain is $\Delta\gamma \approx 5\times 10^6$

Necessary plasma density is given by $n_e \approx 8\times 10^{14}\,\text{cm}^{-3}$

Necessary acceleration length becomes $L_{acc} \sim 3.3$ km

The required accelerating gradient is 0.76 GeV/m

The accelerator length will be the order of 1 km
even if the multi-staging can be accomplished.

It is not so attractive compared to the conventional scheme.

Laser acceleration field

The laser fields are written as $\mathbf{E} = -\frac{\partial \mathbf{A}}{c\partial t}, \quad \mathbf{B} = \nabla \times \mathbf{A}$

in terms of the vector potential **A** of the laser field.

For a linearly polarized laser field, $\mathbf{A} = A_0 \cos(kz - \omega t)\mathbf{e}_\perp$

Defining the normalized vector potential as $\mathbf{a} = \frac{e\mathbf{A}}{m_e c^2}$

the **laser strength parameter** is given by

$$a_0 = \left(\frac{2e^2\lambda_0^2 I}{\pi m_e^2 c^5}\right)^{1/2} \cong 0.855 \times 10^{-9} I^{1/2}\left[\mathrm{W/cm^2}\right]\lambda_0[\mu\mathrm{m}]$$

where I is the laser peak intensity, and

$\lambda_0 = 2\pi c/\omega_0$ is the laser wavelength with frequency ω_0

The amplitude of the transverse electric field of linealy polarized laser is

$$E_L[\mathrm{TV/m}] = \frac{m_e c^2 k}{e} a_0 \cong 3.21 \frac{a_0}{\lambda[\mu\mathrm{m}]} \cong 2.7 \times 10^{-9} I^{1/2}\left[\mathrm{W/cm^2}\right]$$

e.g. $I = 1\times10^{18}$ W/cm^2 gives $E_L = 2.7$ TV/m

Lawson-Woodward Theorem

The net energy gain of a relativistic electron interacting with an electromagnetic field in vacuum is zero.

The theorem assumes that
(i) the laser field is in vacuum with no walls or boundaries present,
(ii) the electron is highly relativistic ($v \sim c$) along the acceleration path,
(iii) no static electric or magnetic fields are present,
(iv) the region of interaction is infinite,
(v) ponderomotive effects (nonlinear forces, e.g. $\mathbf{v} \times \mathbf{B}$ force) are neglected.

(J.D. Lawson, IEEE Trans. Nucl. Sci. NS-26, 4217, 1979)

Acceleration mechanism must violate the Lawson-Woodward theorem.
Usually it is not difficult!

Acceleration by Higher-Order Gaussian Modes

The x-polarized Hermite-Gaussian TEM_{10} mode is given by

$$E_x = \frac{4E_0 w_0 x}{\sqrt{2}w^2(z)} \exp\left(-\frac{r^2}{w^2(z)}\right)\sin\psi$$

$$E_z = \frac{4E_0 w_0}{\sqrt{2}kw^2(z)} \exp\left(-\frac{r^2}{w^2(z)}\right)\left[\left(1-\frac{2x^2}{w^2(z)}\right)\cos\psi - \frac{2zx^2}{Z_R w^2(z)}\sin\psi\right]$$

where $\psi = kz - \omega t + \dfrac{zr^2}{Z_R w^2(z)} - 2\tan^{-1}\left(\dfrac{z}{Z_R}\right) + \phi_0$ $\quad w(z) = w_0\left(1+\dfrac{z^2}{Z_R^2}\right)^{1/2}$

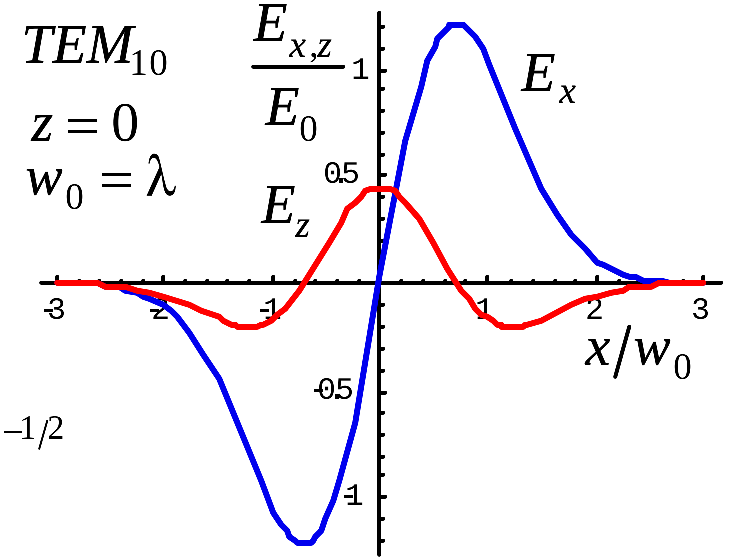
$$P = \frac{c}{4}E_0^2 w_0^2 \qquad E_0 = \frac{2}{w_0}\sqrt{\frac{P}{c}}$$

On the z-axis E_x=0 and the axial field accelerating an electron with v_z ~ c

$$E_z = \frac{4E_0 w_0}{\sqrt{2}kw^2(z)}\cos\psi_z$$

where

$$\psi_z = -k\int\frac{dz}{2\gamma_z^2} - 2\tan^{-1}\left(\frac{z}{Z_R}\right) + \phi_0 \qquad \gamma_z = \left(1-\frac{v_z^2}{c^2}\right)^{-1/2}$$



The phase velocity of the accelerating field is

$$v_{ph} = c\left[1-\theta_d^2/(1+\hat{z})^2\right]^{-1}$$

where $\theta_d = (2/kZ_R)^{1/2} = \lambda/\pi w_0$ is the divergence angle.

Near the focal point $|z| \le Z_R$ $v_{ph} \cong 1+\theta_d^2 = 1+1/(2\gamma_c^2)$ $\gamma_c = (2\theta_d^2)^{-1/2}$

The slippage distance is given by $Z_s \cong \dfrac{\pi Z_R/2}{1+\gamma_c^2/\gamma_z^2}$

For $\gamma_z >> \gamma_c$ $Z_s = \dfrac{\pi Z_R}{2}$

The particle injection energy for a finite interaction length is given by

$$E_{inj} = m_0 c^2\left(\frac{m_0 c^2}{4eE_{z0}/k}\right) + \frac{eE_{z0}}{k} \quad T_{inj} = m_0 c^2\left(\frac{m_0 c^2}{4\theta_d^2\sqrt{Z_0 P/2\pi}} - 1\right) + \theta_d^2\sqrt{\frac{Z_0 P}{2\pi}}$$

where $Z_0 = 377\Omega$

The **maximum energy gain** is given by

$$\Delta W[\mathrm{MeV}] \cong eE_{z0}\cdot Z_R = 2e\sqrt{2P/c} = \sqrt{(2/\pi)Z_0 P[\mathrm{TW}]} \cong 15.5\sqrt{P[\mathrm{TW}]}$$

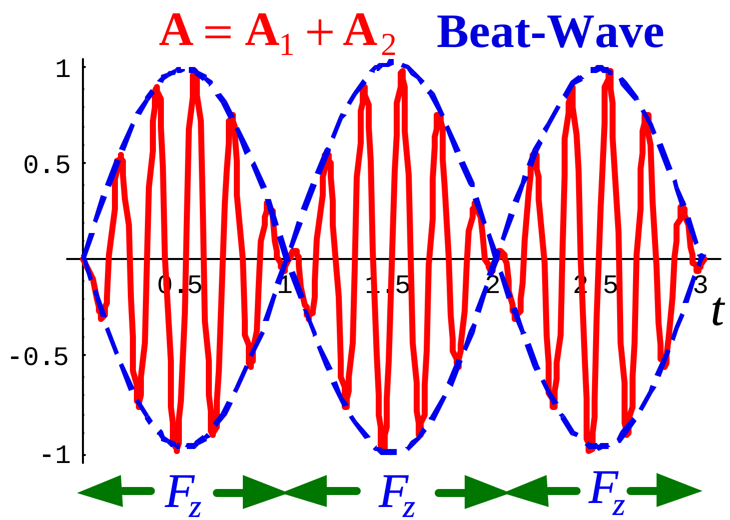
e.g. For $\lambda = 1\mu\mathrm{m}, w_0 = 10\mu\mathrm{m}, P = 10\mathrm{TW}$

$$T_{inj} = 2.1\mathrm{MeV} \qquad \Delta W = 49\,\mathrm{MeV}$$

Vacuum Beat Wave Accelerator

Two laser beams of different frequencies cause an axial acceleration from the beat term in the **v×B** force.

(E. Esarey, PRE 52, 5443,1995)



$$\frac{d\mathbf{p}}{dt} = -e[\mathbf{E} + (\mathbf{v}\times\mathbf{B})/c] \quad \mathbf{E} = -\frac{\partial \mathbf{A}}{c\partial t} \quad \mathbf{B} = \nabla\times\mathbf{A}$$

Using $\mathbf{u} = \mathbf{p}/mc = \gamma\beta \qquad \mathbf{a} = e\mathbf{A}/mc^2$

$$\gamma = (1+u^2)^{1/2} = (1-\beta^2)^{1/2}$$

$$\frac{d\mathbf{u}}{dt} = \frac{\partial \mathbf{a}}{\partial t} - \frac{c\mathbf{u}}{\gamma}\times(\nabla\times\mathbf{a}) \qquad \frac{d\gamma}{dt} = \frac{\mathbf{u}}{\gamma}\cdot\frac{\partial \mathbf{a}}{\partial t} \qquad \rightarrow \mathbf{u}_\perp = \mathbf{a}_\perp$$

The axial component of the **nonlinear ponderomotive force** is

$$F_z = -\frac{e}{c}(\mathbf{v}\times\mathbf{B})_z = -\frac{mc^2}{2\gamma}\frac{\partial}{\partial z}(\mathbf{a}\cdot\mathbf{a})$$

The total laser field is $\mathbf{A}(z,r,t) = \mathbf{A}_1 + \mathbf{A}_2$ $\quad \mathbf{A}_1, \mathbf{A}_2$; the vector potential of laser 1 and 2.

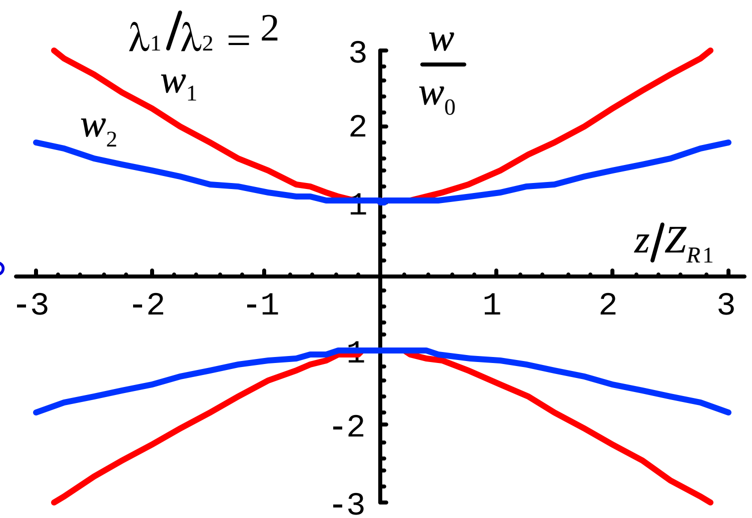
$$a^2 = \mathbf{a}\cdot\mathbf{a} = \hat{a}_1^2 + \hat{a}_2^2 + 2\hat{a}_1\hat{a}_2\cos(\psi_2 - \psi_1) + a_z^2 \qquad \hat{a}_i = (a_{0i}w_{0i}/w_i)\exp(-r^2/w_i^2)$$

The energy equation along the axis is $\displaystyle \frac{d\gamma}{dz} = \frac{a_{01}a_{02}}{\gamma\beta_z}\Delta k \sin(\phi_{02} - \phi_{01})$

Along the axis r=0 and near the focus of the two lasers

$$v_{ph}/c \cong 1-(1-Z_{R1}/Z_{R2})/(\Delta k Z_{R1})$$

$$v_{ph} < c \quad \text{for} \quad Z_{R2} > Z_{R1}$$



In an infinite interaction region $z_I = -\infty, z_F = \infty$

$$\gamma_F^{\,2} - \gamma_I^{\,2} = 2\pi a_{01} a_{02} \Delta k Z_R$$

Energy gain

The accelerated electron energy for the injection energy W_I and $P_1 = P_2$

$$W_F(\text{MeV}) = \left\{ [W_I(\text{MeV})]^2 + 750(\lambda_1/\lambda_2 - 1) P_1(\text{TW}) \right\}^{1/2}$$

e.g.

$$\left.\begin{array}{l} W_I = 20\text{MeV} \\ \lambda_1 = 2\lambda_2 = 1\mu\text{ m} \\ w_{01} \approx \lambda_1 \\ P = 10\text{TW} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} W_F = 89\text{MeV} \\ \Delta W \approx 70\text{MeV} \end{array}\right.$$

Effect of Radiative Losses

An attainable maximum energy is limited by radiative losses due to transverse quiver oscillations.

$$P_R \cong \frac{2}{3} r_e mc^3 \gamma^2 a_0^2 k^2 \left[\frac{1}{2\gamma_z^2} + \frac{1}{kZ_R}\right]^2$$

$r_e = e^2 / mc^2$; the classical electron radius

The electron energy equation with radiative loss is

$$\frac{d\gamma}{dz} \cong a_{01} a_{02} \frac{\Delta k}{\gamma} - \frac{2}{3} r_e \gamma^2 \left(\frac{a_{01}^2}{Z_{R1}^2} + \frac{a_{02}^2}{Z_{R2}^2}\right)$$

Assuming $\Delta k = k_1, a_{01} = a_{02}, Z_{R1} = Z_{R2}$

the maximum electron energy is given by

$$W_{max} \cong \pi mc^2 (w_{01}/\lambda_1)(3w_{01}/r_e)^{1/3}$$

$$W_{max}[\mathrm{GeV}] \cong 1.3 \left(w_{01}^{4/3}[\mu\mathrm{m}]/\lambda_1[\mu\mathrm{m}]\right)$$

e.g.

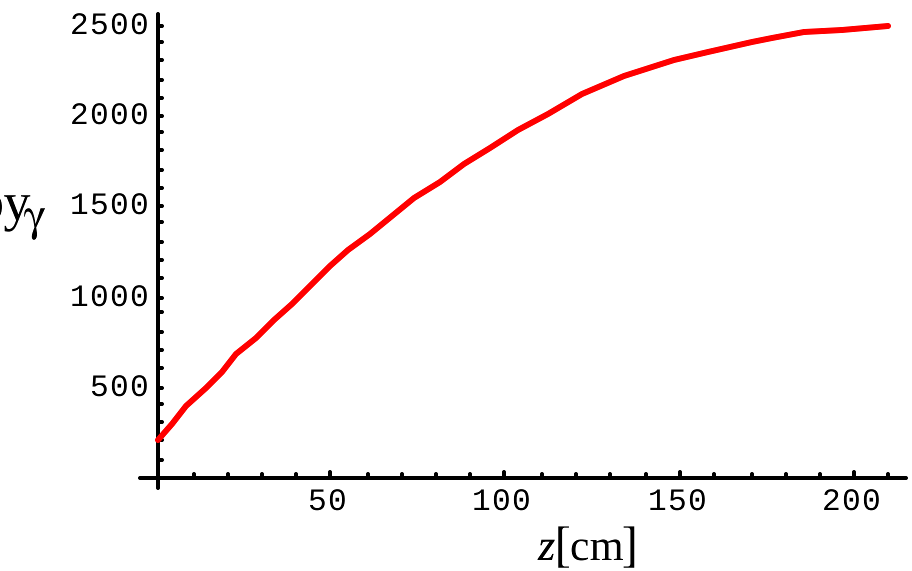
$W_{max} \cong 1.3\mathrm{GeV}$ for $w_{01} \approx \lambda_1 = 2\lambda_2 = 1\mu\mathrm{m}$

Electron Energy vs Distance

$W_I = 100\mathrm{MeV}$

$a_0 = 1$

$w_{01} \approx \lambda_1 = 2\lambda_2 = 1\mu\mathrm{m}$



Ponderomotive acceleration

Energy-momentum equations of electrons in a linearly polarized plane wave

$$\frac{d\mathbf{p}}{dt} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \qquad \frac{d\gamma}{dt} = -\frac{e}{mc^2}\mathbf{v}\cdot\mathbf{E}$$

where

$$\beta = \frac{\mathbf{v}}{c}, \gamma = \frac{1}{\sqrt{1-\beta^2}}, \mathbf{p} = \gamma mc\beta$$

$$\mathbf{E} = \hat{\mathbf{x}}E_0(\phi)\sin\phi, \quad \mathbf{B} = \hat{\mathbf{y}}\frac{E_0(\phi)}{c}\sin\phi \quad \phi(t) = \omega\left[t - \frac{z(t)}{c}\right]$$

$$\frac{d}{dt}(\gamma\beta_x) = -\frac{eE_0}{mc}(1-\beta_z)\sin\phi$$

$$\frac{d}{dt}(\gamma\beta_y) = 0$$

$$\frac{d}{dt}(\gamma\beta_z) = -\frac{eE_0}{mc}\beta_x\sin\phi$$

$$\frac{d}{dt}\gamma = -\frac{eE_0}{mc}\beta_x\sin\phi$$

$$\beta_y = const = 0 \qquad \gamma(1-\beta_z) = const = \gamma_0(1-\beta_0)$$

$$a_0 = \frac{eE_0}{mc\omega} = \frac{eE_0\lambda}{2\pi mc^2}$$

$$\gamma(\phi) = \gamma_0\left[1 + a_0^2(\cos\phi - 1)^2\left(\frac{1+\beta_0}{2}\right)\right]$$

$$\beta_x(\phi) = \frac{2a_0(\cos\phi - 1)}{\gamma_0\left[2 + a_0^2(1+\beta_0)(\cos\phi - 1)^2\right]}$$

$$\beta_z(\phi) = 1 - \frac{2(1-\beta_0)}{\gamma_0\left[2 + a_0^2(1+\beta_0)(\cos\phi - 1)^2\right]}$$

(F.V. Hartemann et al.,PR E,51,4833,1995)

Energy and trajectory of an electron

The maximum energy at $\phi = (2n+1)\pi$

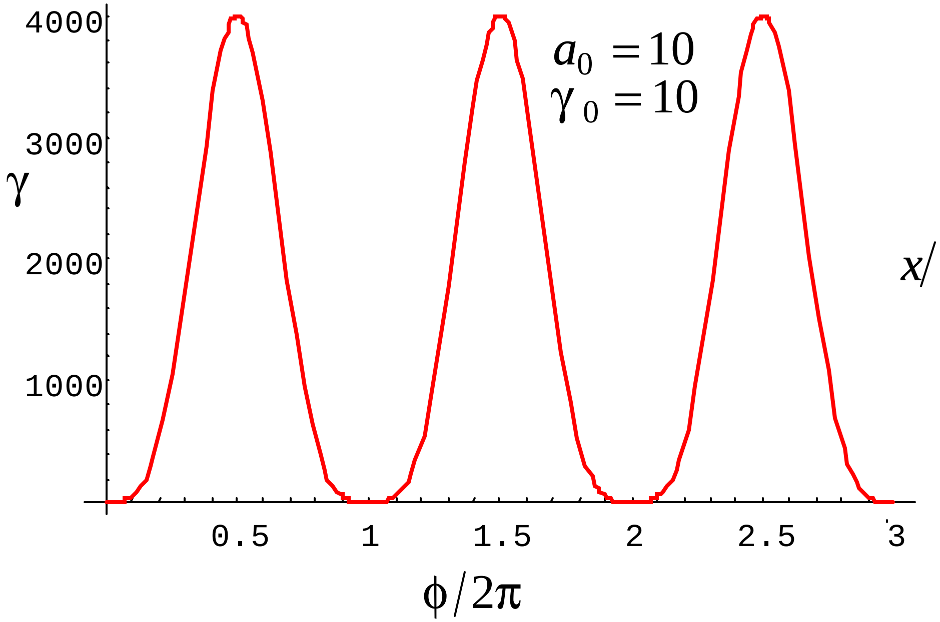
$$\gamma_{max} = \gamma_0\left[1 + 2a_0^2(1+\beta_0)\right] = \gamma_0\left[1 + \frac{e^2 I\lambda^2}{\pi^2\varepsilon_0 m^2 c^5}(1+\beta_0)\right] \quad \text{with} \quad E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}}$$

The transverse and longitudinal positions are

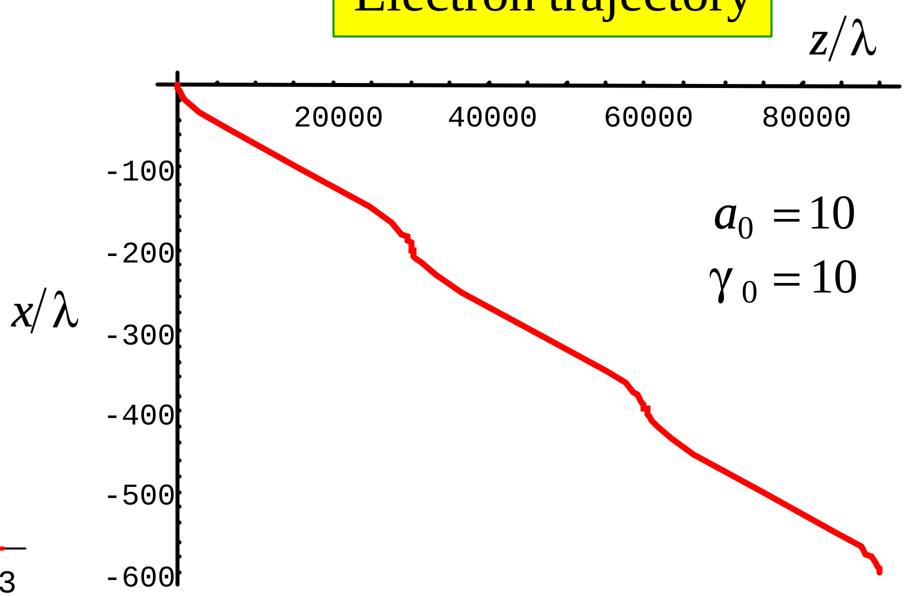
$$x(\phi) = \frac{ca_0(\sin\phi - \phi)}{\omega\gamma_0(1-\beta_0)}$$

$$z(\phi) = \frac{ca_0}{\omega(1-\beta_0)}\left\{\phi\left[\beta_0 + \frac{3}{4}a_0^2(1+\beta_0)\right] + a_0^2(1+\beta_0)\left[\frac{\sin 2\phi}{8} - \sin\phi\right]\right\}$$

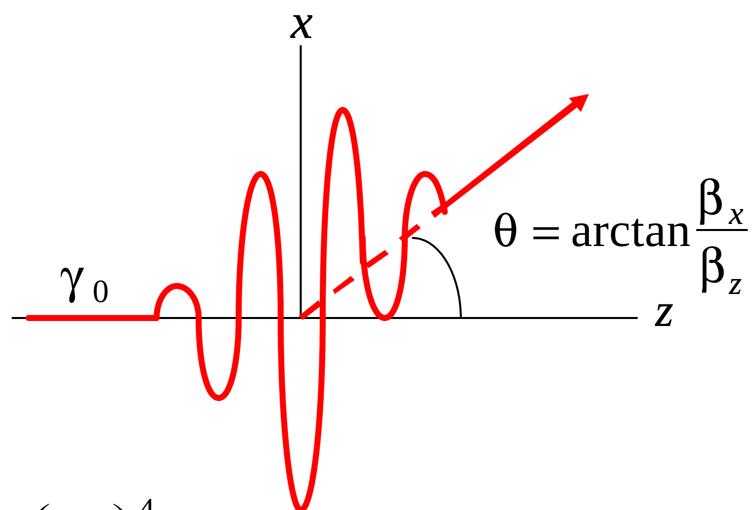
Electron energy



Electron trajectory



Ponderomotive scattering



- The quiver amplitude becomes comparable to the beam waist in the high field.
- The electron can be scattered away with a high energy as the interaction is terminated within a wavelength.

Threshold power given by $x \approx \frac{\lambda}{\pi}\gamma_0 a_0 = w_0$

$$a_{0th} = \frac{\pi w_0}{\gamma_0 \lambda} \quad \Rightarrow \quad P_{max} \approx \frac{\pi^5}{\gamma_0^2}\left(\frac{\varepsilon_0 m^2 c^5}{e^2}\right)\left(\frac{w_0}{\lambda}\right)^4 = \frac{0.21}{\gamma_0^2}\left(\frac{w_0}{\lambda}\right)^4 [\text{TW}]$$

Scattering angle θ

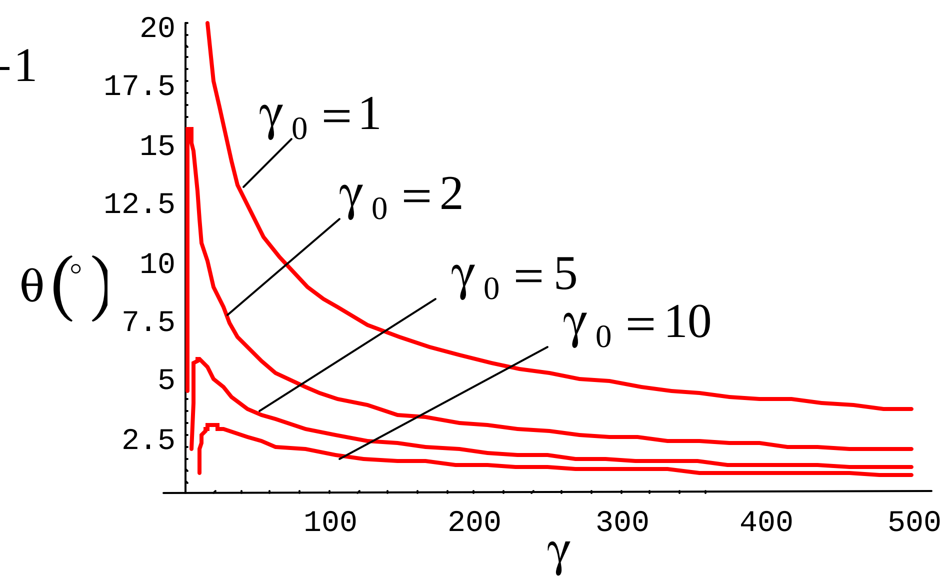
$$\begin{aligned} \gamma(1-\beta_z) &= \gamma_0(1-\beta_0) \\ 1/\gamma^2 &= 1-\beta_x^2-\beta_z^2 \end{aligned} \quad \Rightarrow \quad \left(\frac{\beta_x}{\beta_z}\right)^2 = \frac{\gamma^2-1}{(\gamma\beta_z)^2} - 1$$

$$\theta(\gamma) = \arctan\frac{\left[\left(\frac{2}{1+\beta_0}\right)\left(\frac{\gamma}{\gamma_0}-1\right)\right]^{1/2}}{\gamma - \gamma_0(1-\beta_0)}$$

$\beta_0 = 0 \quad \gamma_0 = 1 \quad \Rightarrow \quad \theta = \arctan\sqrt{\frac{2}{\gamma - 1}}$

At $\theta = 0$ $\gamma = \gamma_0$ $\Rightarrow$ No acceleration

Scattering angle



Is vacuum laser acceleration possible to accelerate particles up to super high energy?

Consider energy gain of laser acceleration in vacuum.

Higher Order Gaussian Mode Acceleration

$$\Delta\gamma \approx 2\sqrt{2}a_0 \frac{w_0}{\lambda_0} \qquad \text{(HGMA)}$$

Laser Beat Wave Acceleration

$$\Delta\gamma \approx 2\pi^{3/2} a_0 \frac{w_0}{\lambda_0} \qquad \text{(LBWA)}$$

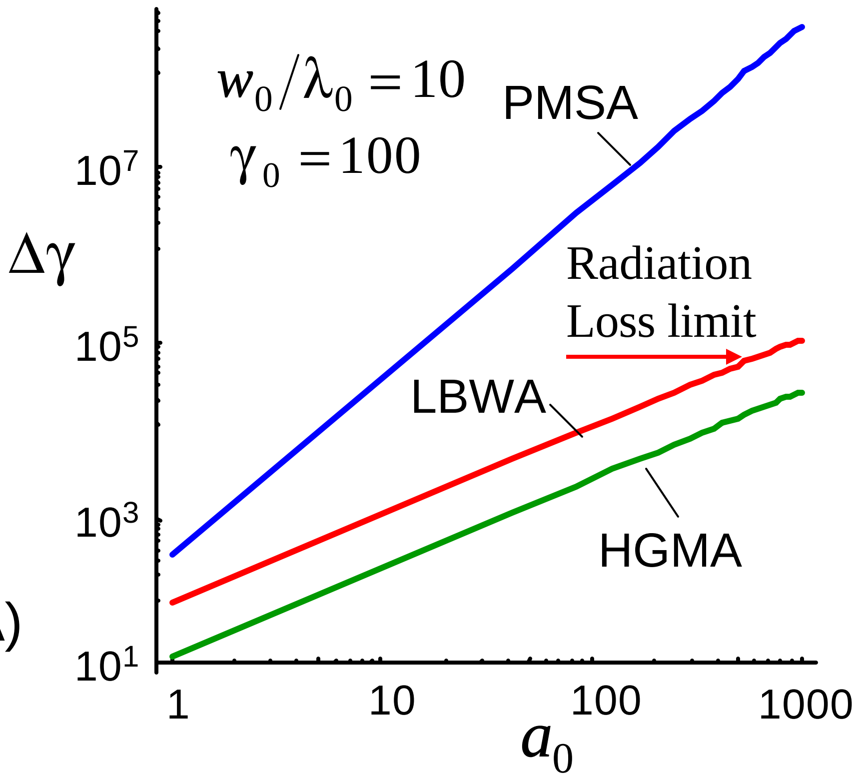
assuming $\lambda_0 = \lambda_1 = 2\lambda_2$

Radiation loss limit $\gamma_{max} \cong \pi \frac{w_0}{\lambda_0}\left(\frac{3w_0}{r_e}\right)^{1/3}$

Poderomotive scattering for $\beta_0 \sim 1$

$$\Delta\gamma \approx 4\gamma_0 a_0^2 \quad \text{with} \quad a_0 > \frac{\pi w_0}{\gamma_0 \lambda_0} \qquad \text{(PMSA)}$$

Energy gain as a function of laser intensity

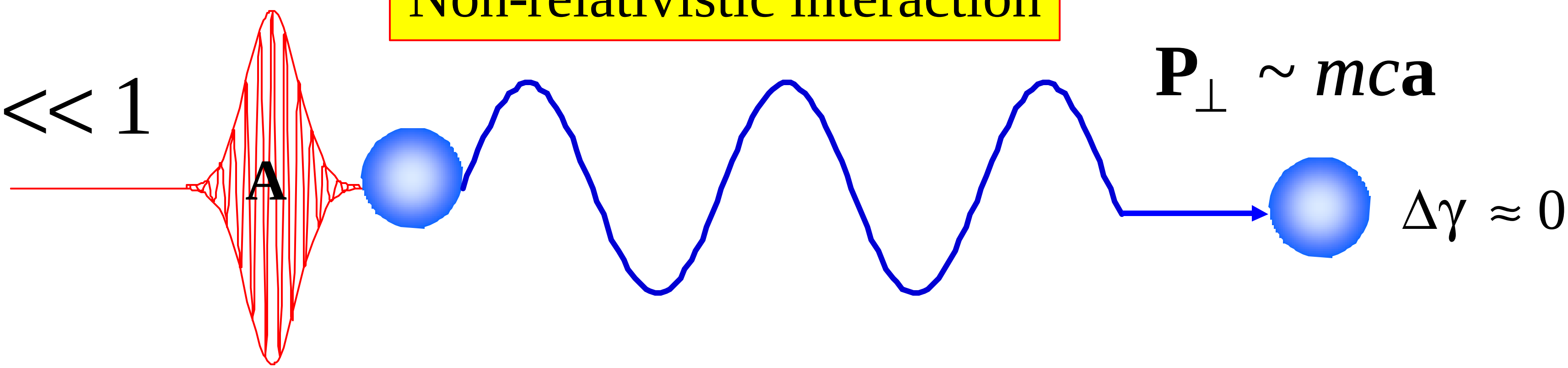


Direct laser-particle interaction

$\mathbf{p}_\perp = \frac{e}{c}\mathbf{A}_\perp \qquad \mathbf{a} \equiv \frac{e\mathbf{A}}{mc^2}$ is the normalized vector potential.

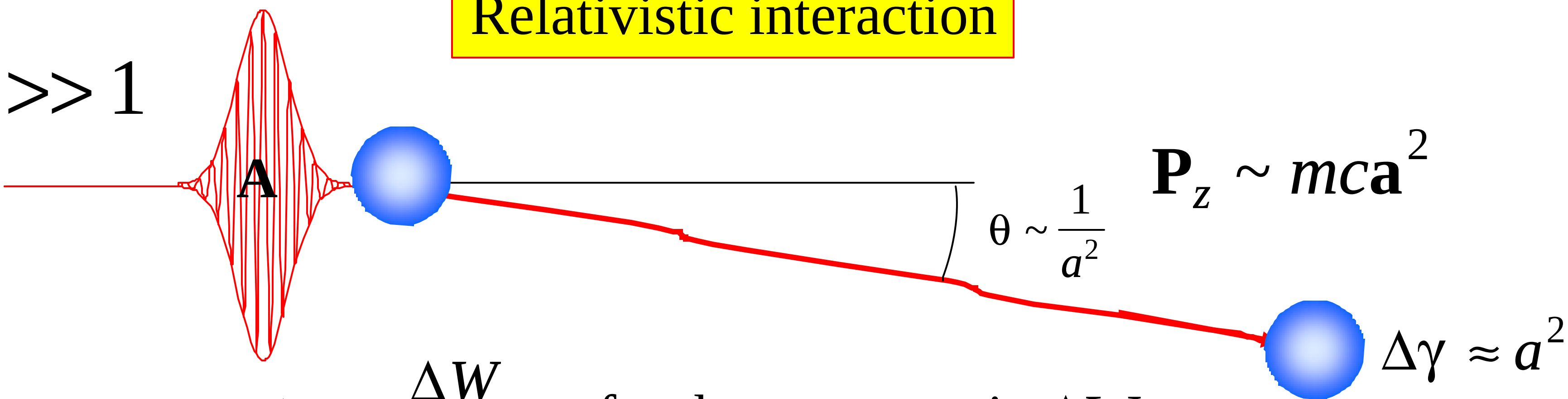
Non-relativistic interaction

$\mathbf{a} << 1$



Relativistic interaction

$\mathbf{a} >> 1$



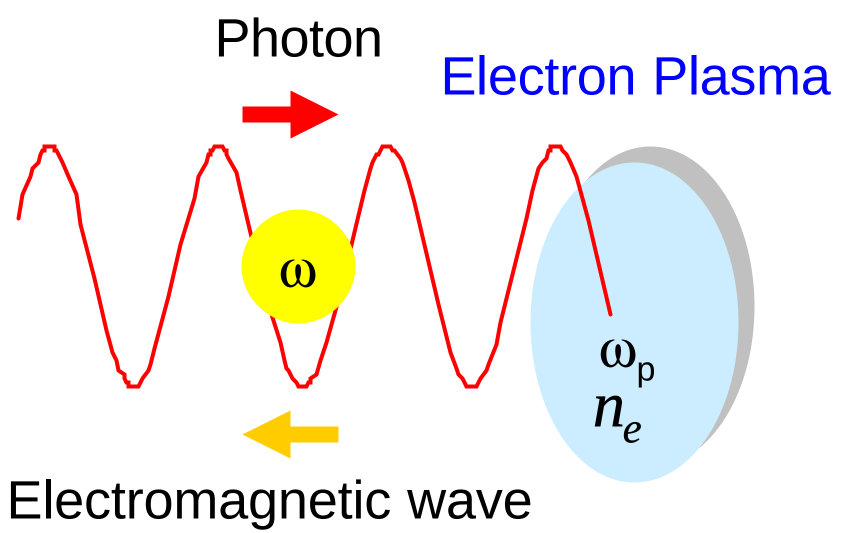
where $\Delta\gamma \equiv \frac{\Delta W}{mc^2}$ for the energy gain ΔW.

Reflection of electromagnetic wave on an electron plasma

The dispersion relation of the electromagnetic wave in plasma with electron density n_e is

$$\omega^2 = k^2 c^2 + \omega_p^2$$

where $$\omega_p = \sqrt{\frac{4\pi n_e e^2}{m}}$$



This means that a photon inside plasma has an effective mass:

$$m_{ph} = \frac{\hbar \omega_p}{c^2}$$

Photon-'Electron Plasma' Interaction

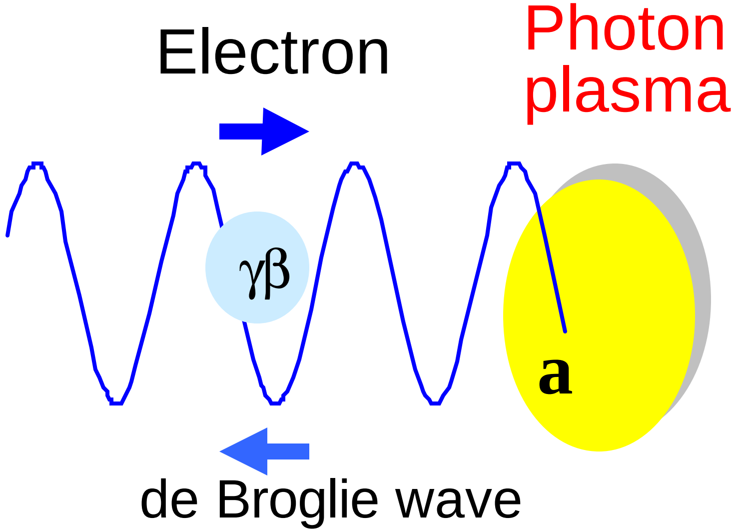
The reflection condition of the electromagnetic wave (photon) is given by

$$\omega < \omega_p$$

Reflection of an electron on an intense photon field

In an analogy to reflection of electromagnetic wave from the electron plasma, an electron in an intense laser field has a mass shift

$$\Delta m_e^2 = \bar{\mathbf{a}}^2 m_e^2 = \frac{1}{2} a_0^2 m_e^2$$



Considering a laser pulse as a 'plasma' of photons, the photon plasma frequency is

$$\omega_{ph,p} = \frac{\bar{\mathbf{a}} m_e c^2}{\hbar}$$

Electron-'Photon Plasma' Interaction

An electron de Broglie wave will be reflected for its frequency

$$\omega_e = \frac{m_e c^2 \gamma\beta}{\hbar} < \omega_{ph,p} = \frac{\bar{\mathbf{a}} m_e c^2}{\hbar} \qquad \text{i.e.} \qquad \gamma\beta < \bar{\mathbf{a}}$$

An electrom with momentum $\gamma\beta$ is expelled in the condition:

$$\gamma < \sqrt{1 + \bar{\mathbf{a}}^2} = \left(1 + \frac{a_0^2}{2}\right)^{1/2}$$

Particle acceleration in super-strong laser fields in plasma

Basic equations

Laser field and electron space charge field can be given by the vector potential **A** and the scaler potential Φ

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - \nabla\Phi \qquad \mathbf{B} = \nabla \times \mathbf{A}$$

The equation of a particle motion is written as

$$\frac{d\mathbf{p}}{dt} = -\left(e\mathbf{E} + \frac{\mathbf{v}}{c}\times\mathbf{B}\right) = e\left(\nabla\Phi + \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}\right) - \frac{\mathbf{v}}{c}\times(\nabla\times\mathbf{A}) \qquad \text{with} \qquad \frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla$$

The equation of energy is $\frac{d}{dt}\gamma mc^2 = e\mathbf{v}\cdot\left(\nabla\Phi + \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}\right)$

The Maxwell's equations are

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\mathbf{A} = \frac{4\pi}{c}\mathbf{j} - \nabla\left(\frac{1}{c}\frac{\partial\Phi}{\partial t} + \nabla\cdot\mathbf{A}\right) \qquad \nabla\cdot\left(\nabla\Phi + \frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}\right) = -4\pi\rho$$

where **j** is the current density, $\mathbf{j} = \langle -en\mathbf{v}\rangle$

ρ is the charge density, $\rho = \langle -en\rangle$

The continuity equation of plasma density n is

$$\frac{\partial n}{\partial t} + \nabla \cdot n\mathbf{v} = 0$$

assuming the plasma is an ensemble of cold fluids.

1D field-electron interaction in plasma

In a 1D laser field according to the Coulomb gauge $\nabla \cdot \mathbf{A} = 0$
letting $A_z = 0$

the vector potential is expressed as

$$\mathbf{A} = A_x \mathbf{e}_x + A_y \mathbf{e}_y \quad \text{for circular polarization,}$$

$$\mathbf{A} = A_x \mathbf{e}_x \quad \text{for linear polarization.}$$

Consider the laser pulse frame propagating at a group velocity in plasma

$$v_g \equiv \frac{\partial \omega}{\partial k} = c\sqrt{1 - \frac{\omega_p^2}{\omega_0^2}} \qquad \text{where} \qquad \omega_p = \sqrt{\frac{4\pi n_e e^2}{m}}$$

is the plasma frequency for the ambient electron density n_e.

Transforming the momentum equation to a new variable, $\zeta = z - v_g t$

From the transverse component of the equation, $\frac{d}{d\zeta}\left(\mathbf{p}_\perp - \frac{e}{c}\mathbf{A}_\perp\right) = 0$

This gives conservation of canonical transverse momentum.

$$\mathbf{p}_\perp = \frac{e}{c}\mathbf{A}_\perp$$

The electron quiver velocity is given by $\mathbf{v}_\perp = \frac{\mathbf{p}_\perp}{\gamma m} = \frac{e}{\gamma mc}\mathbf{A}_\perp$

with the Lorentz factor $\gamma = \left(1-\beta^2\right)^{-1/2}$

Defining the normalized vector potential $\mathbf{a} \equiv \frac{e\mathbf{A}}{mc^2}$ ➡ $\beta_\perp = \frac{\mathbf{a}_\perp}{\gamma}$

The longitudinal component of the energy equation becomes

$$\frac{d}{d\zeta}\left(p_z - \frac{\gamma mc^2}{v_g} + \frac{e}{v_g}\Phi\right) = 0$$

This gives $\gamma - \gamma\beta_g\beta_z - \phi = \text{const.} = \gamma_0\left(1-\beta_g\beta_0\right)$

where $\beta_g = \frac{v_g}{c}$ $\phi \equiv \frac{e\Phi}{mc^2}$

assuming that prior to laser interaction $\begin{cases} \mathbf{a}_\perp = \phi = 0 \\ \gamma = \gamma_0,\ \beta_z = \beta_0 \end{cases}$

Using $\gamma = \sqrt{1 + \mathbf{u}_\perp^2 + u_z^2} = \gamma_\perp \gamma_z$ where $\gamma_\perp = \sqrt{1 + \mathbf{a}_\perp^2}$ $\gamma_z = \frac{1}{\sqrt{1 - \beta_z^2}}$

the solutions on β and γ are given by

$$\beta_\perp = \frac{\mathbf{a}}{\gamma} \qquad \beta_z = \frac{\beta_g \gamma_\perp^2 \pm H_0 \left[H_0^2 - \left(1 - \beta_g^2\right)\gamma_\perp^2 \right]^{1/2}}{H_0^2 + \beta_g^2 \gamma_\perp^2}$$

$$\gamma = \gamma_g^2 H_0 \pm \gamma_g \beta_g \left[\gamma_g^2 H_0^2 - \gamma_\perp^2 \right]^{1/2} = \frac{H_0^2 + \beta_g^2 \gamma_\perp^2}{H_0 \mp \beta_g \left[H_0^2 - \left(1 - \beta_g^2\right)\gamma_\perp^2 \right]^{1/2}}$$

with $H_0 = \gamma_0 \left(1 - \beta_g \beta_0\right) + \phi$

Space-charge potential

The space-charge potential ϕ is obtained from

$$\frac{1}{c}\frac{\partial n}{\partial t} + \nabla \cdot (n\beta) = 0 \qquad \text{(Continuity equation)}$$

$$\nabla^2 \phi = k_p^2 \left(n/n_e - 1 \right) \qquad \text{(Poisson's equation)}$$

where n is the electron density, $k_p = \omega_p / c$

In an initial equilibrium (prior to the laser pulse), the space-charge potential is negligible, i.e. ϕ=0

Assuming $n = n(\zeta)$

$$\frac{d}{d\zeta}\left[n\left(\beta_g - \beta_z\right)\right] = 0 \quad \Rightarrow \quad n = n_e \frac{\beta_g - \beta_0}{\beta_g - \beta_z}$$

For $\beta_g \approx 1$ letting $\psi = \dfrac{\phi}{\gamma_0 (1-\beta_0)}$ $\hat{k}_p = \dfrac{k_p}{\gamma_0^{3/2}(1-\beta_0)}$

$$\frac{d^2\psi}{d\varsigma^2} = \frac{\hat{k}_p^2}{2}\left[\frac{1+a^2}{(1+\psi)^2} - 1\right]$$

(Nonlinear wake equation)

In the short-pulse limit $\tau_L << (ck_p)^{-1}$ $L_0 = c\tau_L << \dfrac{\lambda_p}{2\pi}$

where τ_L is a laser pulse duration. $|\psi| << 1$

In the long-pulse limit $\tau_L >> (ck_p)^{-1}$ $L_0 = c\tau_L >> \dfrac{\lambda_p}{2\pi}$

$$\psi \approx \left(1 + a^2\right)_s^{1/2} - 1$$

The averaging over a laser wavelength,

$$\left(1 + a^2\right)_s^{1/2} \cong \left(1 + a_0^2/2\right)^{1/2}$$

is nearly constant.

In the short-pulse limit $\phi \approx 0$ let $\beta_g = 1$ for vacuum

$$\gamma = \gamma_0\left(1+\frac{1+\beta_0}{2}a^2\right)_s = \gamma_0\left(1+\frac{1+\beta_0}{4}a_0^2\right) \longrightarrow \frac{1}{2}\gamma_0 a_0^2$$

$$\beta_z = \frac{\beta_0 + \frac{1+\beta_0}{2}a^2}{1+\frac{1+\beta_0}{2}a^2} = \frac{\beta_0 + \frac{1+\beta_0}{4}a_0^2}{1+\frac{1+\beta_0}{4}a_0^2} \longrightarrow 1$$

for $\begin{cases} a_0 >> 1 \\ \beta_0 \approx 1 \end{cases}$

In the short-pulse limit $\phi \sim 0$

$$\gamma = \gamma_g^2\gamma_0(1-\beta_g\beta_0) \pm \gamma_g\beta_g\left[\gamma_g^2\gamma_0^2(1-\beta_g\beta_0)^2 - \gamma_\perp^2\right]^{1/2}$$

$$= \gamma_g^2\gamma_0(1-\beta_g\beta_0) \pm \gamma_g\beta_g\left[\gamma_g^2\gamma_0^2(\beta_g-\beta_0)^2 - \mathbf{a}^2\right]^{1/2}$$

For $\gamma_g\gamma_0(1-\beta_g\beta_0) > \gamma_\perp$

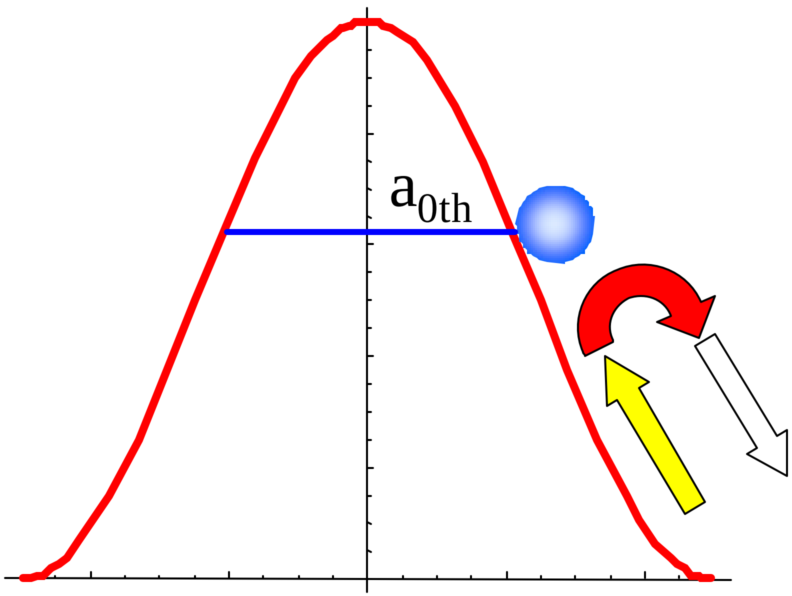
the maximum and minimum energies are

$$\begin{cases} \gamma_{\max} = \gamma_g^2\gamma_0(1-\beta_g\beta_0) - \gamma_g\beta_g\left[\gamma_g^2\gamma_0^2(\beta_g-\beta_0)^2 - a_0^2\right]^{1/2} \\ \gamma_{\min} = \gamma_0 \end{cases}$$

For $\gamma_g \gamma_0 (1 - \beta_g \beta_0) \le \gamma_\perp$

$$\begin{cases} \gamma_{\max} = \gamma_g^2 \gamma_0 \left[(\beta_g - \beta_0)^2 + \gamma_0^{-2} \right] \\ \gamma_{\min} = \gamma_0 \end{cases}$$

Scattering by ponderomotive potential



Scattering condition

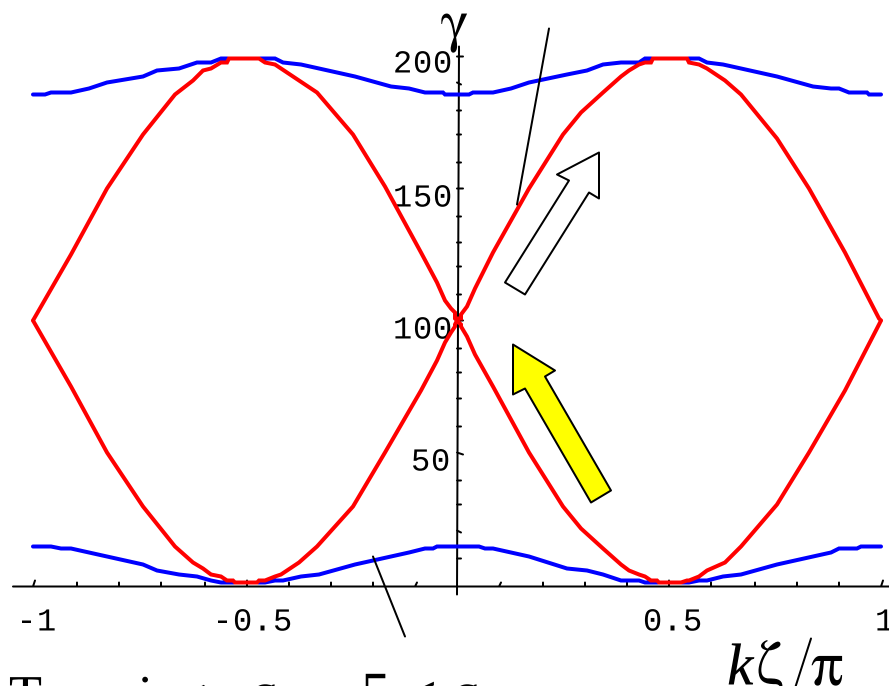
$$\gamma_g \gamma_0 (1 - \beta_g \beta_0) \le \gamma_{Lth}$$

$$\gamma_{Lth} = \left(1 + a_{0th}^2 \cos^2 k\zeta\right)^{1/2}$$

$$a_{0th} = \gamma_g \gamma_0 (\beta_g - \beta_0)$$

Electron trajectory in phase space

Scattering $a_0 \ge a_{0th} = \sqrt{\gamma_g^2 - 1}$



Trapping $a_0 = 5 < a_{0th}$

$\gamma_g = 10, \gamma_0 = 1, \beta_0 = 0$

Simulations of ponderomotive acceleration

Below threshold $a_0 = 5$

QuickTimeý Ç²
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Group velocity

$$\beta_g = \frac{v_g}{c} = 0.8$$

$$\gamma_g = 1.67$$

Above threshold $a_0 = 10$

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Ponderomotive acceleration and focusing

High energy booster acceleration of a pair-beam can be accomplished by the relativistic ponderomotive acceleration with focusing in **vacuum** or **tenuous plasma**.

Acceleration

The final energy is obtained approximately as

$$\gamma_f \approx a_0^2$$

for a particle initially at rest.

$$E_f\,[\mathrm{GeV}] \approx 0.37\times 10^{-21}\, I\left[\mathrm{W/cm^2}\right]\lambda_0^2\,[\mu\mathrm{m}]$$

E.g. At $\lambda_0 = 0.8\mu\mathrm{m}$

$I = 10^{22}\,\mathrm{W/cm^2}$ ➡ $E_f \approx 2.4\mathrm{GeV}$

$I = 10^{23}\,\mathrm{W/cm^2}$ ➡ $E_f \approx 24\mathrm{GeV}$

$I = 10^{24}\,\mathrm{W/cm^2}$ ➡ $E_f \approx 240\mathrm{GeV}$

$I = 10^{25}\,\mathrm{W/cm^2}$ ➡ $E_f \approx 2.4\mathrm{TeV}$

Focusing

The electric field of Hermite-Gaussian modes is given by

$$\mathbf{E}^{l,m} = \mathrm{Re}\, E_0^{l,m} \mathbf{e}^{l,m} e^{-i\omega t + i\psi_{l,m}(r,z)} H_l\left(\frac{x}{\sqrt{2}\sigma_\perp}\right) H_m\left(\frac{y}{\sqrt{2}\sigma_\perp}\right) \frac{\sigma_{\perp 0}}{\sigma\perp} \exp\left[-\frac{r^2}{4\sigma_\perp^2} - \frac{(z-ct)^2}{4\sigma_z^2}\right]$$

$\sigma_{\perp 0}$ is the rms beam size.

$\sigma_\perp = \sigma_{\perp 0}\sqrt{1 + z^2/Z_R^2}$

Z_R is the Rayleigh length.

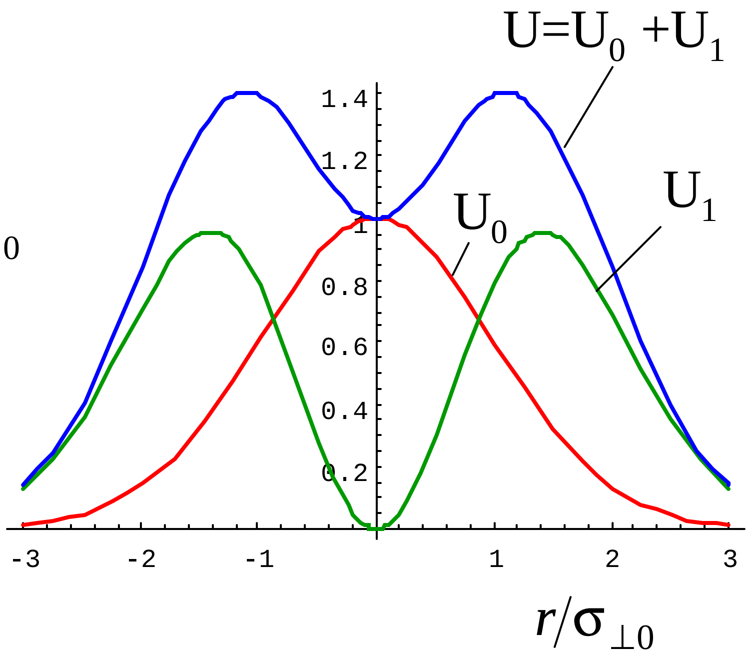
σ_z is the beam pulse length.

Consider the lowest mode of the laser beam with $l=m=0$, and the amplitude electric field E_0

$$a_0 = \frac{e\lambda\!\!\!^{-}}{\sqrt{2}mc^2} E_0 \qquad \lambda\!\!\!^{-} = c/\omega$$

The ponderomotive potential is

$$U_0(r,z,t) = a_0^2 \frac{\sigma_{\perp 0}^2}{\sigma_\perp^2} \exp\left[-\frac{r^2}{2\sigma_\perp^2} - \frac{(z-ct)^2}{2\sigma_z^2}\right]$$



The amplitude of the quiver oscillations is estimated to be

$$\frac{\Delta x}{\sigma_{\perp 0}} \approx \frac{a_0 \lambda\!\!\!^{-}}{\sigma_{\perp 0}} \approx \frac{2\lambda\!\!\!^{-}(Z_R/\sigma_z)^{1/2}}{(Z_R\lambda\!\!\!^{-}/2)^{1/2}} \approx \sqrt{8\frac{\lambda\!\!\!^{-}}{\sigma_z}}$$

Off axis particles in a Gaussian laser beam are accelerated by the radial ponderomotive force and quickly expelled from the beam in the radial direction.

To avoid transverse instability of motion, superposition of a Gaussian mode with l=m=0 and higher order modes, e.g. a linear combination of l=1, m=0 and l=0, m=1 with the same polarization vectors $\mathbf{e}^{1,0} = \mathbf{e}^{0,1}$, and $E^{1,0} = iE^{0,1}$ creates a potential well in the radial direction:

$$U_1(r,z,t) = a_1^2 \frac{r^2\sigma_{\perp 0}^2}{\sigma_\perp^4} \exp\left[-\frac{r^2\sigma_{\perp 0}^2}{2\sigma_\perp^4} - \frac{(z-ct)^2}{2\sigma_z^2}\right]$$

where a_1 is the dimensionless vector potential.

The total ponderomotive potential is given by $U = U_0 + U_1$

The focusing force is given by

$$\frac{F_r}{mc^2} = \frac{\partial U}{\partial r} = \left[\left(2a_1^2 - a_0^2\right)\frac{r\sigma_{\perp 0}^2}{\sigma_\perp^4} - a_1^2\frac{r^3\sigma_{\perp 0}^2}{\sigma_\perp^6}\right]\exp\left[-\frac{r^2}{2\sigma_\perp^2} - \frac{(z-ct)^2}{2\sigma_\perp^2}\right]$$

Focusing strength at r=0, and z-ct=0

$$K_F = \frac{F_t}{\gamma m c^2 r} = \frac{2a_1^2 - a_0^2}{\gamma \sigma_{\perp 0}^2}$$

The beam envelope equation on the rms beam radius σ_{rb} is

$$\frac{d^2\sigma_{rb}}{dz^2} + K_F \sigma_{rb} - \frac{r_e N}{\sqrt{2\pi}\beta^2\gamma^3\sigma_{zb}\sigma_{rb}} - \frac{\varepsilon_b^2}{\sigma_{rb}^3} = 0$$

Space charge force

Thermal emittance

where N is the number of electrons in the bunch,

σ_{zb} is the rms bunch length,

ε_b is the geometric emittance, $\varepsilon_b = \varepsilon_n / \gamma\beta$

ε_n the normalized emittance

r_e is the classical electron radius.

Estimate of a focused beam size

1) The space-charge-force dependent beam size

The equilibrium radius is obtained from $\frac{d^2\sigma_{rb}}{dz^2} = 0$

$$\sigma_{rb} \cong \frac{\sqrt{r_e N}}{(2\pi)^{1/4} K_F^{1/2} \beta \gamma^{3/2} \sigma_{zb}^{1/2}} \cong \frac{\sqrt{r_e N}}{(2\pi)^{1/4} \left(2a_1^2 - a_0^2\right)^{1/4} \sigma_{zb}^{1/2}} \frac{\sigma_{\perp 0}}{\gamma}$$

Assuming $a_1 = a_0 \quad \sigma_{\perp 0} = r_0/2 \quad \sigma_{zb} \approx \lambda\!\!\!^{-}_0$

$$\sigma_{rb} \approx \frac{(2\pi)^{1/4}}{2} \frac{r_0}{a_0^{5/2}} \sqrt{\frac{r_e N}{\lambda_0}}$$

$$\sigma_{rb}[\mathrm{pm}] \approx 2 \times 10^{24} \frac{\sqrt{N}}{I^{5/4}\left[\mathrm{W/cm^2}\right]} \frac{r_0[\mu\mathrm{m}]}{\lambda_0^3[\mu\mathrm{m}]}$$

e.g. For $\lambda_0 = 0.8\mu\mathrm{m}$

$r_0 = 10\mu\mathrm{m}$

$N_p = 1 \times 10^{10}$

$I = 1.06 \times 10^{25}\,\mathrm{W/cm^2}$

$\Rightarrow$ $\boxed{\sigma_{rb} \approx 0.2\ \mathrm{pm}}$

2) The emittance dependent beam size

If an electron-positron pair beam is focused, the space chrage force will be neglected.

$$\sigma_{rb} \cong \frac{\sqrt{\varepsilon_b}}{K_F^{1/4}} = \frac{\gamma^{1/4}\sqrt{\varepsilon_b \sigma_{\perp 0}}}{\left(2a_1^2 - a_0^2\right)^{1/4}}$$

Assuming $a_1 = a_0$ $\sigma_{\perp 0} = r_0/2$ $\varepsilon_b \cong \varepsilon_n/\gamma \approx \varepsilon_n/a_0^2 \approx ƛ/a_0^4$

$$\sigma_{rb} \approx \frac{\sqrt{\varepsilon_n \sigma_{\perp 0}}}{a_0} = \frac{1}{2a_0^2}\sqrt{\frac{\lambda_0 r_0}{\pi}}$$

$$\sigma_{rb}[\mathrm{pm}] \approx \frac{4\times 10^{23}}{I\left[\mathrm{W/cm^2}\right]}\sqrt{\frac{r_0}{\lambda_0}}$$

e.g. For $\lambda_0 = 0.8\mu\mathrm{m}$

$r_0 = 10\mu\mathrm{m}$

$N_p = 1\times 10^{10}$

$I = 1.06\times 10^{25}\,\mathrm{W/cm^2}$

$\sigma_{rb} \approx 0.13$ pm

Laser micro collider

Two counter propagating laser-accelerated beams make a micro collider.

The space charge limited luminosity

$$L = \frac{N_p^2 f_{rep}}{4\pi\sigma_{rb}^2} \cong \frac{a_0^5 \lambda_0 N_p f_{rep}}{\sqrt{2}\pi^{3/2} r_e r_0^2}$$

$$L\left[\mathrm{cm}^{-2}\mathrm{s}^{-1}\right] \approx 2\times 10^{-30} I^{5/2}\left[\mathrm{W/cm}^2\right]\lambda_0^6[\mu\mathrm{m}] r_0^{-2}[\mu\mathrm{m}] N_p f_{rep}$$

e.g. $I = 1.06\times 10^{25}\,\mathrm{W/cm}^2$ ➡ $E_{\mathrm{C.M.}} = 5\mathrm{TeV}$

$$\left.\begin{aligned} \lambda_0 &= 0.8\mu\mathrm{m} \\ r_0 &= 10\mu\mathrm{m} \\ N_p &= 1\times 10^{10} \end{aligned}\right\} \quad L \approx 2\times 10^{40} f_{rep}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}$$

Required peak power and pulse energy

P = 17 EW $E_L > 2\times 4$ kJ

e^+e^- pair-beam micro-collider

Two counter-propagating laser-accelerated pair beams will create a new e^+e^-, e^-e^-, e^+e^+ micr-size collider without beam disruption at collision.

The emittance-limited luminosity is

$$L = \frac{N_p^2 f_{rep}}{4\pi\sigma_{rb}^2} \cong \frac{a_0^4 N_p^2 f_{rep}}{\lambda_0 r_0}$$

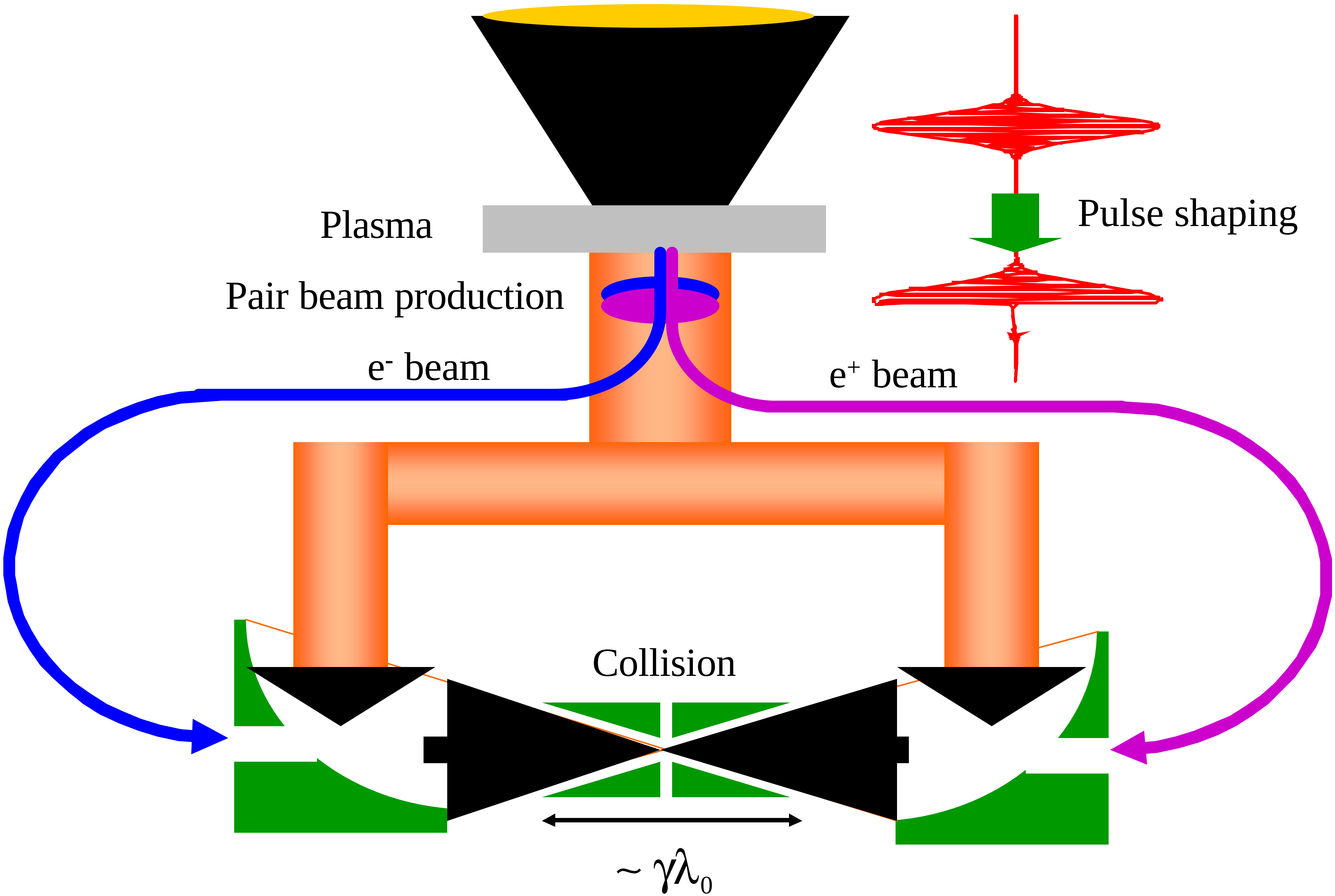
where N_p is the number of accelerated e^+e^- pairs and f_{rep} is the repetition rate of laser pulses.

$$L\left[\mathrm{cm}^{-2}\mathrm{s}^{-1}\right] \approx 5.3\times10^{-27} I^2\left[\mathrm{W/cm}^2\right]\lambda_0^3[\mu\mathrm{m}]r_0^{-1}[\mu\mathrm{m}]N_p^2 f_{rep}[\mathrm{Hz}]$$

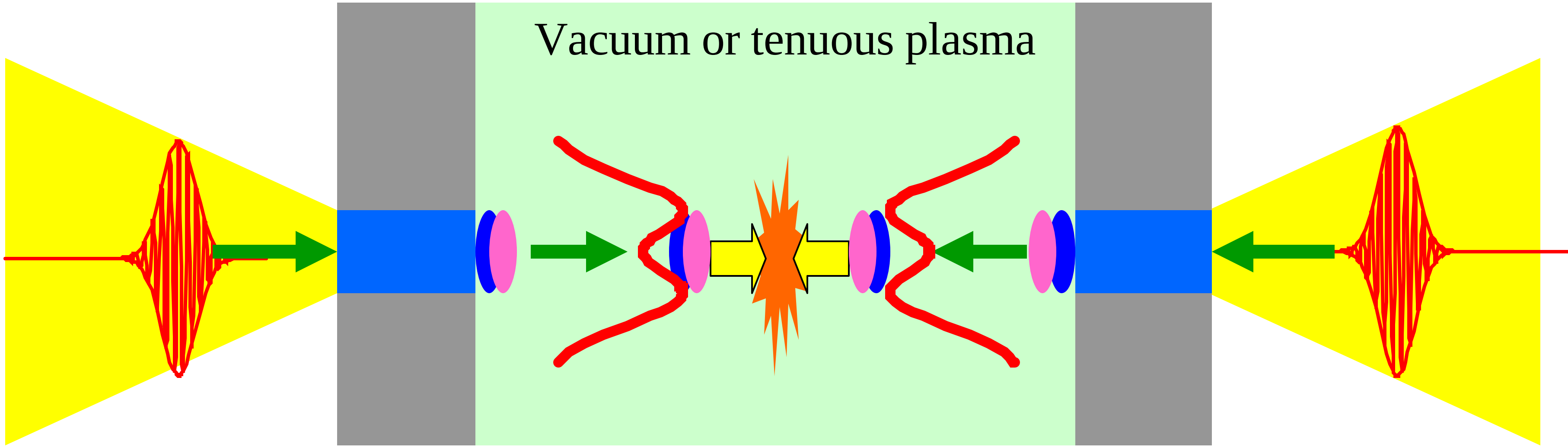
E.g. For $I = 1.06\times10^{25}\,\mathrm{W/cm}^2$

$$\left.\begin{aligned} N_p &= 1\times10^{10} \\ \lambda_0 &= 0.8\mu\mathrm{m} \\ r_0 &= 10\mu\mathrm{m} \end{aligned}\right\} \quad \boxed{L \approx 3\times10^{42} f_{rep}\ \mathrm{cm}^{-2}\mathrm{s}^{-1}}$$

Laser Micro Collider Concept



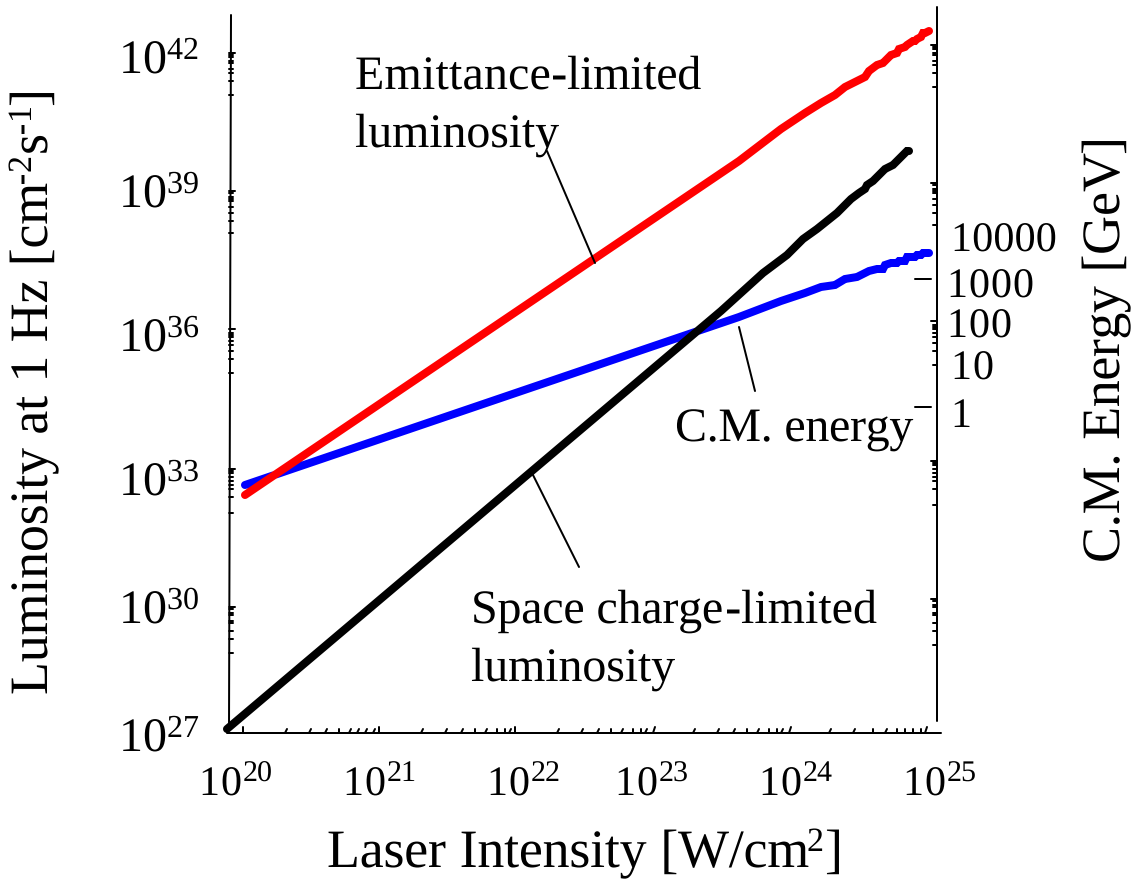
$E_{C.M.}$ [GeV]	I [W/cm^2]	P [PW/pulse]	E_L [J]	L [cm^{-2}s^{-1}] at 1 Hz	
1	2.1×10^{21}	3.3	1.6	1.2×10^{35}	Φ factory
4	8.4×10^{21}	13	6.4	2×10^{36}	J/Ψ factory
10	2.1×10^{22}	33	16	1.2×10^{37}	B factory
500	1.1×10^{24}	1700	800	3×10^{40}	
1000	2.1×10^{24}	3300	1600	1.2×10^{41}	
5000	1.1×10^{25}	17000	8000	3×10^{42}	



Pair-beam micro-collider concept

Luminosity of laser micro-colliders

$\lambda_0 = 0.8\mu m \quad r_0 = 10\mu m \quad N_p = 1\times 10^{10}$



Summary

- The plasma acceleration concept has difficulties in high energy gain and high beam quality. ➡ { The beam injector, The e^+e^- pair beam generator

  A single stage energy gain < 1 GeV

- The proper shaped superstrong laser fields are useful for accelerating and focusing electron and/or positron beams.

$$\left.\begin{array}{l} I = 10^{25}\,\mathrm{W/cm^2} \\ N_p = 1\times 10^{10} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} E_f \approx 2.4\,\mathrm{TeV} \\ \sigma_{rb} \approx 0.13\ \mathrm{pm} \end{array}\right.$$

- Two counter-propagating electron and positron beams result in a micro-collider with extremely high luminosity.

$$\left.\begin{array}{l} I = 10^{25}\,\mathrm{W/cm^2} \\ N_p = 1\times 10^{10} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} L \approx 2\times 10^{40} f_{rep}\ \mathrm{cm^{-2}s^{-1}} \\ L \approx 3\times 10^{42} f_{rep}\ \mathrm{cm^{-2}s^{-1}} \end{array}\right.$$

Advanced Accelerator Research & Development at KEK

- There are great interests in advanced accelerator researches, growing in the national and industry institutes, and the universities around Japan.
- The C.O.E. on A.A. R&D is conceived in KEK to organize developments of the future high energy accelerator technologies, holding long term visions of the post-LC, or the post-B factory.
- We are eager to make collaboration and a word-wide network for A.A. R&D.

The 1st Workshop on Advanced Accelerator R&D
April 24, 25, 2003 at KEK, Tsukuba, Japan

Advanced Accelerator R&D NETWORK