

# Photon-Photon Interactions

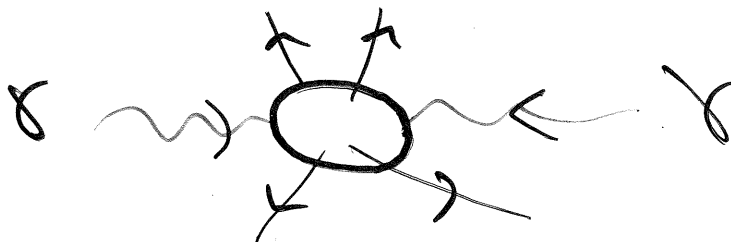
Stan Brodsky

SLAC

$e^+e^-$  Physics at Intermediate Energies

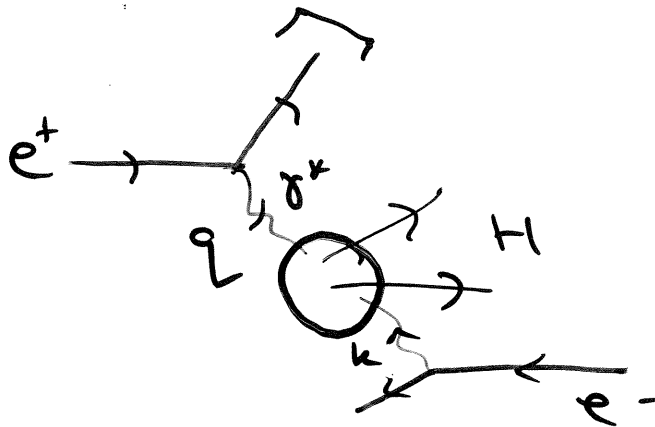
PSP-N Workshop

May 2, 2001



$$\gamma^* \gamma \rightarrow H \quad \text{at PEP-N}$$

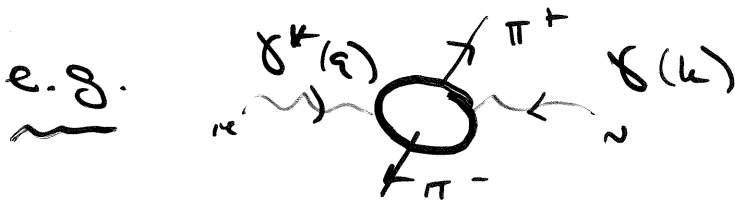

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tag scattered  $e^-$

$$\gamma^* \gamma \rightarrow H$$

$$\rightarrow \pi^0, \eta^0, \pi^+ \pi^-, \pi^0 \pi^0, k^+ k^- \dots$$



$$T^{\mu\nu} = i \int d^4x e^{-iq \cdot x} \langle \pi^+(p^+) \pi^-(p^-) | T J^{\mu}(x) J^{\nu}(0) | 0 \rangle$$

\* Study real and virtual Compton scattering on  $\pi$  target via crossing

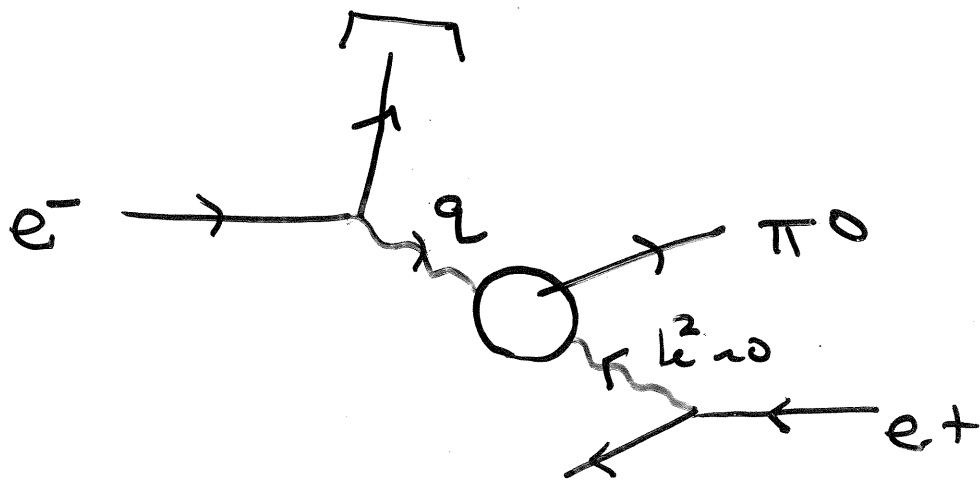
- L, T polarization
- planar correlations

# Photon - to - Pion

## Transition Form Factor



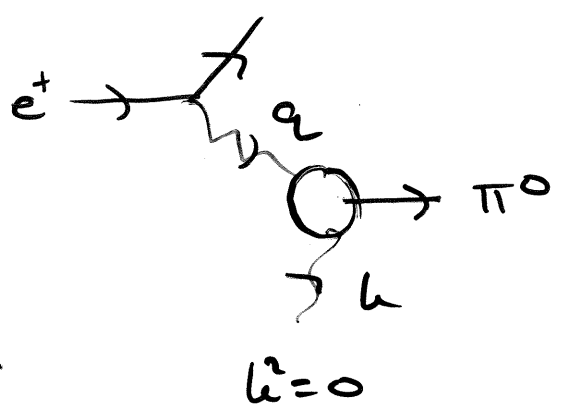
\* Measure in  $e^+e^-$  colliders



CLEO

$\gamma^* \gamma \rightarrow \pi^0, \eta, \eta', \eta_c \dots$

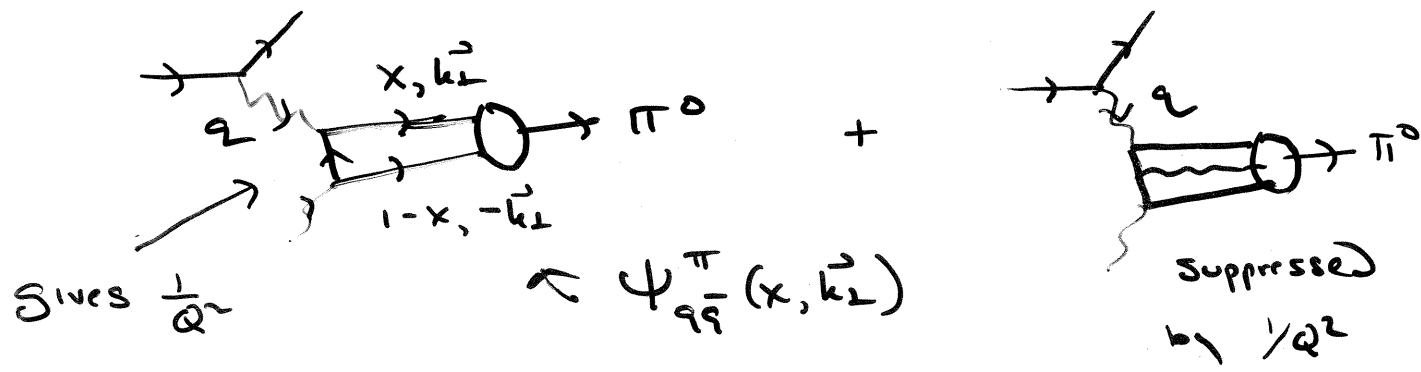
Simplest example of exclusive process



$F_{\gamma\pi^0}(Q^2)$   
 $\pi^0 \rightarrow \gamma\gamma$  at  $Q^2 = 0$ .

For  $Q^2 \gg \Lambda_{QCD}^2$  analyze in PQCD

LePage  
SID

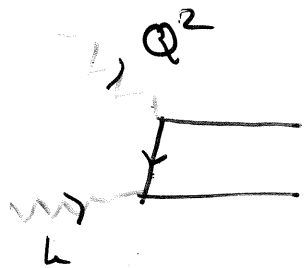


\*  $F_{\gamma\pi^0}(Q^2) = \frac{1}{Q^2} 2\sqrt{n_c} (e_u^2 - e_d^2) \int_0^1 dx \left( \frac{1}{x} + \frac{1}{1-x} \right) \Phi_\pi(x, Q)$

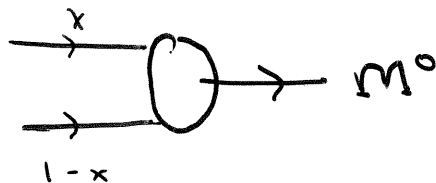
\*  $\Phi_\pi(x, Q) = \int \frac{d^2k_\perp}{16\pi^3} \Psi_{q\bar{q}}(x, \vec{k}_\perp)$  pion distribution amplitude

PQCD:

$$F_{\gamma \rightarrow M^0}(Q^2) \sim \frac{1}{Q^2} \int_0^1 \frac{dx}{1-x} \phi_M(x, \bar{Q})$$



$T_H$



$$\phi_M(x, Q) = \int_{k_\perp^2 < Q^2} d^2 k_\perp \psi_{g\bar{q}}(x, \vec{k}_\perp)$$

\*  $T_H (\gamma^* \gamma \rightarrow \underbrace{q\bar{q}}_{\text{collinear}}) \sim \frac{1}{Q^2(1-x)}$   
 $\mathcal{O}(Q)^{\rightarrow}$

\* Higher Fock states:  $\frac{1}{Q^4}$

Other diagrams  $\mathcal{O}(Q^2(Q^2))!$

\*  $\phi_M(x, Q) = \sum_{n=0}^{\infty} a_n P_n(x) \left( \ln \frac{Q^2}{\Lambda^2} \right)^{-\delta_n}$    
 log evolution

\*  $\lambda_n = \lambda_q + \lambda_{\bar{q}} = 0$ . HHC test  
 " f<sup>0</sup>

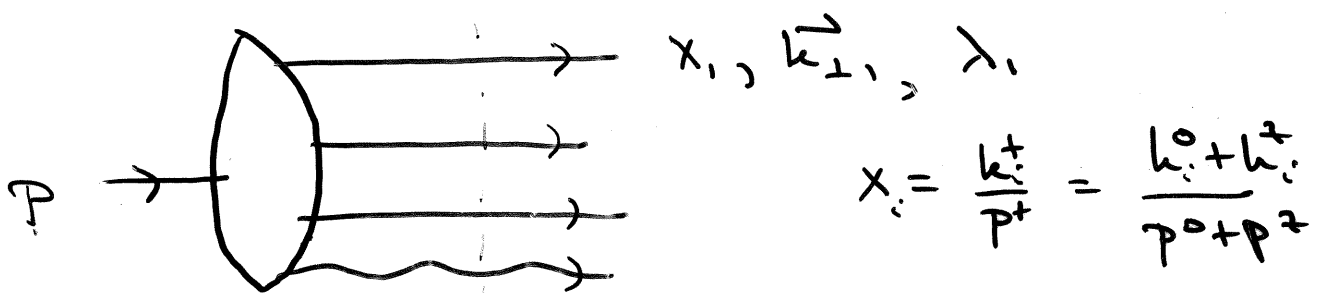
\*\* Small part of Fock state dominates

$$\phi_M \sim \psi(x, b_\perp \sim \frac{1}{Q})$$

# Light-Cone Wavefunctions and QCD Phenomena

Non-Perturbative  
QCD

$\{\Psi_n\}$ : translation: hadrons  $\Rightarrow$  q, g



fixed  $\tau = t + z/c$

Dirac

$$|\Psi\rangle = \sum_n |n\rangle \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\leftarrow$  free q, g basis

$$\sum_{i=1}^n x_i = 1, \quad \sum_{i=1}^n \vec{k}_{\perp i} = 0$$

“Light-cone Fock expansion”

boost invariant

Frame-indep.

$$\tau = t + z/c$$

Dirac  
Bjorken, Haag, Sjoen  
Lepage + SSB  
Pauli + SSB

Equation of motion

$$i \frac{\partial}{\partial \tau} |\Psi_H\rangle = P^- |\Psi_H\rangle = \frac{M_H^2 + P_\perp^2}{P^+} |\Psi_H\rangle$$

$$H_{LC} = P^- P^+ - P_\perp^2$$

$P^+, P_\perp$   
kinematical

$$H_{LC} |\Psi_H\rangle = M_H^2 |\Psi_H\rangle$$

⇒ eigenvalue problem for LC Hamiltonian

Insert complete set of  $H_{LC}^0$  eigenstates  
 ed or singlet  
 eigenstate

$$\sum_n |n\rangle \langle n| = \mathbb{I}$$

$$\sum_n \langle m | H_{LC} | n \rangle \langle n | \Psi_H \rangle = M_H^2 \langle m | \Psi_H \rangle$$

DLCQ

⇒ Heisenberg matrix form of eigenvalue problem

$$|\Psi_H\rangle = \sum_n |n\rangle \langle n | \Psi_H \rangle = \sum_n |n\rangle \psi_{n/H}(x_i, \vec{k}_{\perp i}, \dots)$$

⇒ LC Fock expansion of eigenstate  $|\Psi_H\rangle$

\* Given  $\{\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)\}$

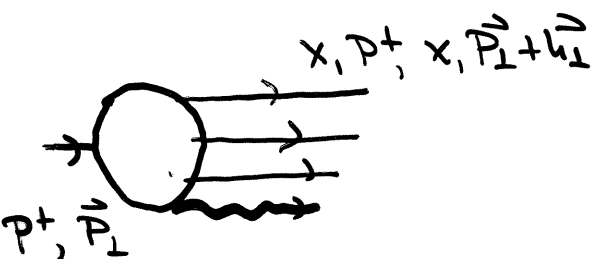
wavefunction known for all  $P^\mu$ !

relative coordinates

$$|P^+, \vec{P}_\perp\rangle = \sum_n \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \prod_j \frac{1}{\sqrt{x_j}}$$

$$|x_i, P^+, x_i, \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i\rangle$$

absolute coordinates



In equal-time theory (instant form)

boosts mix with interactions

changing  $\vec{P} \rightarrow \vec{P}'$  as complicated

as solving  $H|\Psi\rangle = E|\Psi\rangle$



# Factorization of Exclusive Amplitudes at Large Momentum Transfer

$$* M = \int [dx] \pi \phi_i(x, Q) \overline{T_H}$$

Lepage 823  
Efremov Radjov  
Chernyak et al

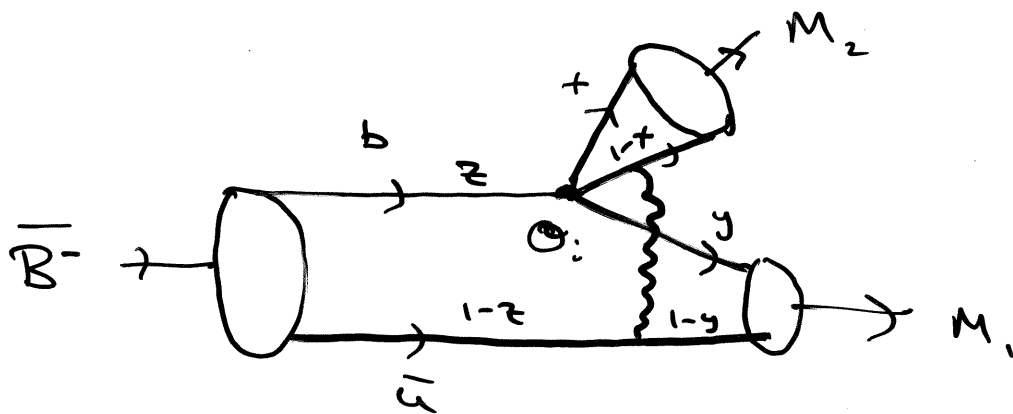
- Accurate to leading twist  $+ O(\frac{1}{Q})^2$
- $\phi_i$ : universal distribution amplitude
- $T_H$ : hard scattering amplitude
  - perturbative calculable in QCD
- Hadron helicity conservation  $\sum_{unif} \lambda_H = \sum_{final} \lambda_H$
- Quark Counting Rules
- Color Transparency
- End-point suppression by Sudakov effect

Many applications: form factors, higher twist  $\Delta$   
 fixed QCD scattering, heavy hadron decay

# New Analyses of $B \rightarrow MM$ in PQCD

BBNS : Beneke, Buchalla, Neubert, Seidenfeld

kLS : Keum, Li, Sunde



"octet"  
non-factorizable

$\alpha_s(k^2)$  : BLM scale

$$* M_{B \rightarrow M_1 M_2} = \int_0^1 dz \int_0^1 dx \int_0^1 dy$$

$$\phi_B(z, Q) T_H(z, x, y; Q) \phi_{M_2}(x, Q) \phi_{M_1}(y, Q)$$

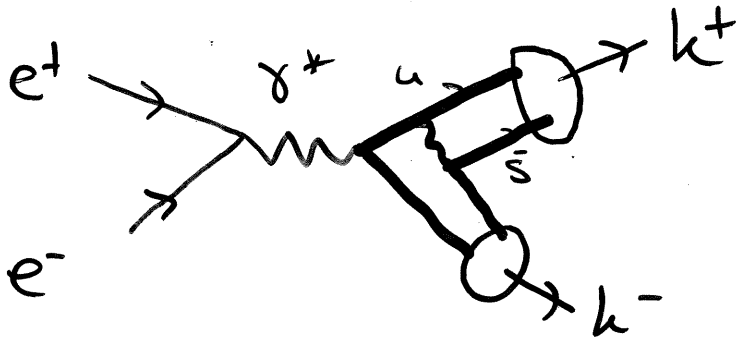
$$\uparrow \quad \bar{b} \bar{u} \rightarrow q \bar{q} \quad q \bar{q}$$

\* No end-point singularities

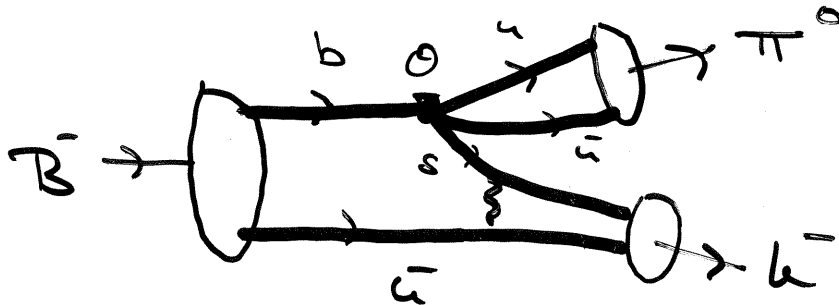
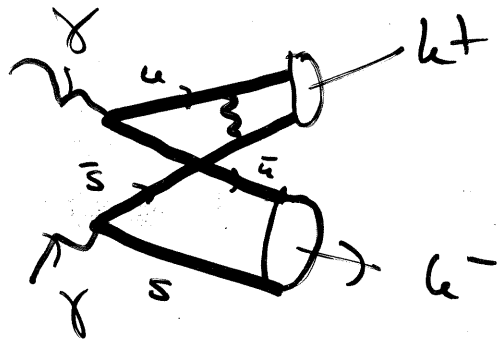
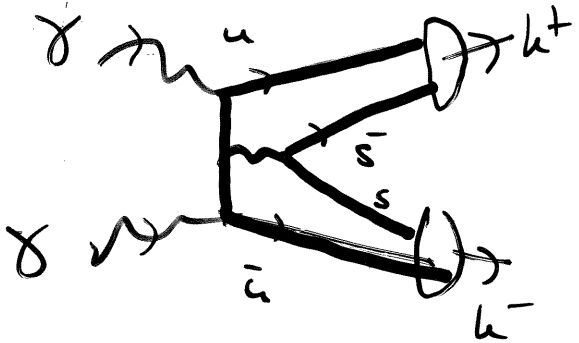
\* Color Transparency

\*  $Q^2 = O(M_B^2)$  : Factorization scale

# Universal Light-Front Wavefunctions



G.P. Lepage  
SJB



Henry  
Szepevari  
SJB

## Common Ingredients:

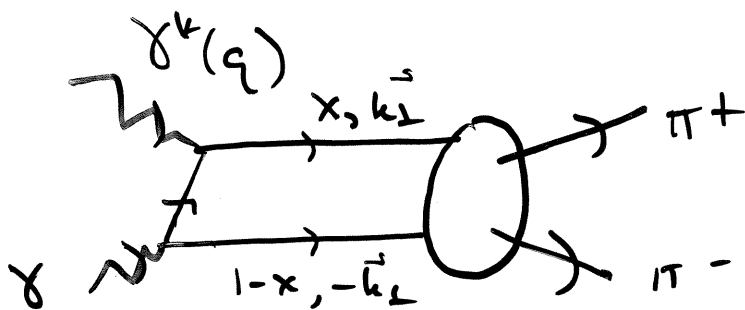
\*  $\Phi_\pi(x, Q)$ ,  $\Phi_h(x, Q)$

Distribution  
Amplitudes

\*  $\alpha_s(Q)$  at low scales

Use  $\gamma^* \gamma \rightarrow M \bar{M}$

to predict B-decays



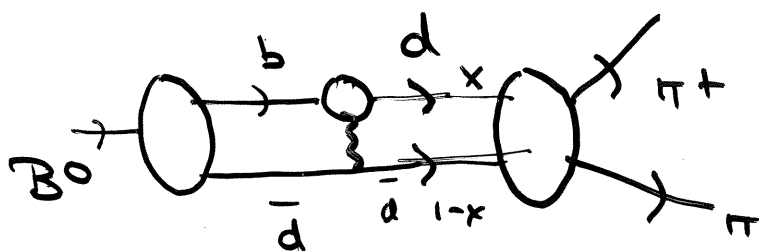
$$k_\perp^2 < (1-x) Q^2$$

Measures

$$\frac{1}{Q^2} \int_0^1 \frac{dx}{1-x}$$

$$\Phi_{\pi^+\pi^-}(x, k_{\perp \max}^2, M_{\pi\pi}^2)$$

M. Diehl et al



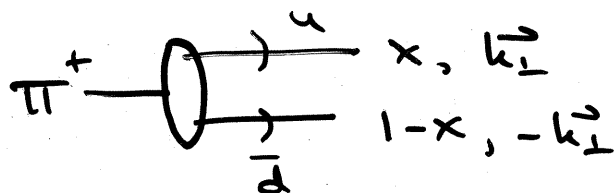
Measures

$$\Phi_{\pi^+\pi^-}(x, k_{\perp \max}^2 = M_B^2, M_B^2)$$

weighted by  $\Phi_B^B(x)$

# Pion Distribution Amplitude

$$\Phi_{\pi}(x, Q^2) = \int \frac{d^2 k_{\perp}}{16\pi^2} \Psi_{q\bar{q}/\pi}^{(\pi)}(x, \vec{k}_{\perp})$$



$$\sim \Psi_{q\bar{q}/\pi}(x, b_{\perp} \sim O(1/Q))$$

$$\Phi_{\pi}(x, Q) = \int \frac{dz^- P_{\pi}^+}{4\pi} e^{ix P_{\pi}^+ z^-/2}$$

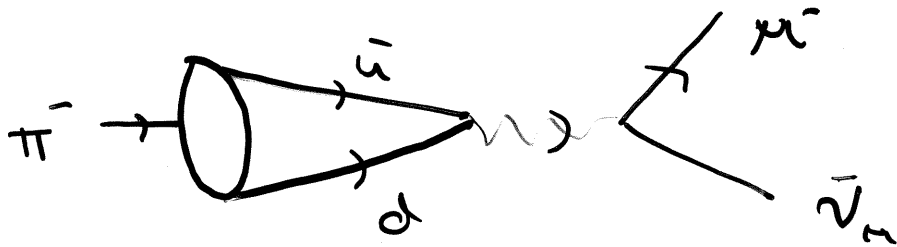
$$\langle 0 | \bar{\Psi}(0) \frac{\gamma^+ \gamma^5}{2\sqrt{2}n_c} \Psi(z) | \pi \rangle^{(\pi)} \Big|_{z^+ = z_{\perp}^2 = 0}$$

$$P \exp \int_0^1 ds i g A(sz) \cdot z = 1 \quad \text{in } A^+ = 0 \text{ gauge}$$

$$= \int \frac{dk^-}{2\pi} \Psi_{BS}(k, p)$$

obeys: OPE, RGE, Evolution Eqn.

Valence wavefunction normalized  
to pion decay constant



$$\int_0^1 dx \phi_\pi(x, Q) = \frac{F_\pi}{2\sqrt{3}}$$

# Evolution Eqn. for Distribution Amplitudes

$$\begin{aligned}\phi_M(x, Q) &= \int \frac{d^2 k_\perp}{16\pi^3} \psi_{q\bar{q}}^{(\phi)}(x, k_\perp) \\ &= x_1 x_2 \tilde{\phi} \quad \Theta(Q^2 - \frac{k_\perp^2}{x(1-x)})\end{aligned}$$

$$x_1 x_2 \frac{\partial}{\partial \ln Q^2} \tilde{\phi}_M(x, Q) = \frac{\alpha_s(Q^2)}{4\pi} \int_0^1 dy V(x, y) \tilde{\phi}_M(y, Q)$$

$$* \quad V(x, y) = 2 C_F x_1 y_2 \Theta(y_1 - x_1) \left( \delta_{h_1 \bar{h}_2} + \frac{\Delta}{y_1 - x_1} \right) + (1 \leftrightarrow 2)$$

$$\Delta \tilde{\phi} = \tilde{\phi}(y_1, Q) - \tilde{\phi}(x_1, Q)$$

$$x_1 = x, \quad x_2 = 1-x$$

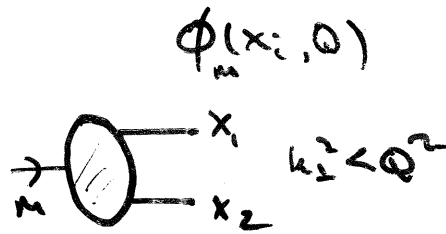
$\delta_{h_1 \bar{h}_2} = 1$  opp hel  
Favors  
opp hel.

For baryons:

$$* \quad V(x_i, y_j) = 2 x_1 x_2 x_3$$

$$\cdot \sum_{i \neq j} \Theta(y_i - x_i) \delta(x_k - y_k) \frac{y_j}{x_j} \left( \frac{\delta_{h_i \bar{h}_j}}{x_i + x_j} + \frac{\Delta}{y_i - x_i} \right)$$

Hadron Distribution Amplitude



- key non-perturbative input  
 to hadronic exclusive processes  
 at large momentum transfer

Rigorous results

$$\phi(x_i, Q) \equiv \left( \ln \frac{Q^2}{\Lambda^2} \right)^{-\delta_n / 2\beta_0}$$

$$\int \prod_{i=1}^n \{ d^2 k_{\perp}^{(i)} \Theta(Q^2 - m_n^2) \} \delta^{(2)}(\sum k_{\perp}^{(i)}) \Psi^{val}(x_i, k_{\perp}^{(i)})$$

\*  $\phi_m(x_i, Q) = x_1 x_2 \sum_n Q_n C_n^{3/2}(x_1, x_2) \left( \ln \frac{Q^2}{\Lambda^2} \right)^{-\delta_n / 2\beta_0}$

non-perturbative input  $\nearrow$   $\uparrow$  from conformal symmetry

Labels:  $\delta_n$ , Efremov Radyushin

$$Z(Q^2) = \frac{1}{2\pi} \int_0^{Q^2} \frac{d\ell^2}{\ell^2} \alpha_S(\ell^2)$$

deviations from conformal symmetry

$$e^{-\delta_n Z(Q^2)} \Rightarrow \left( \ln \frac{Q^2}{\Lambda^2} \right)^{-\delta_n / 2\beta_0}$$



## "Asymptotic" Distribution Amplitude

$$\phi_{\text{asympt}}(x) = \sqrt{3} x(1-x) F_{\pi}$$

- Leading anomalous dimension in OPE
- Solution to QCD evolution eqn  
at  $Q^2 \rightarrow \infty$

SJB + GPL

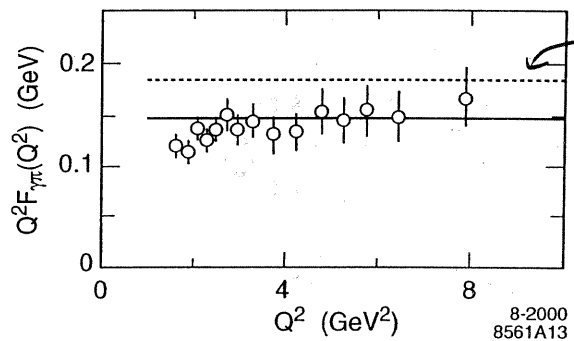
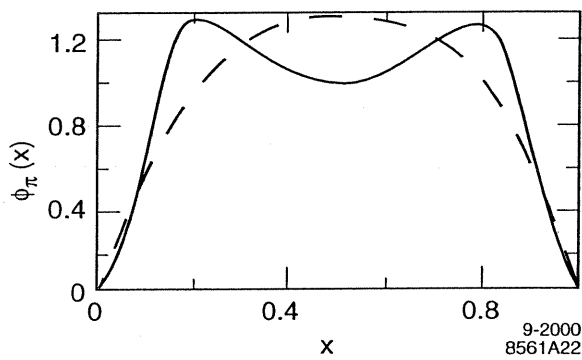
- $\frac{\partial}{\partial \ln Q^2} \phi(x, Q^2) = \int dy V(x, y) \phi(y, Q^2)$

- Non-perturbative methods

DLCQ + Transverse lattice

Dalkey  
Burkhardt

- Normalized by  $\pi \rightarrow e\nu$



— transverse lattice/DLCQ

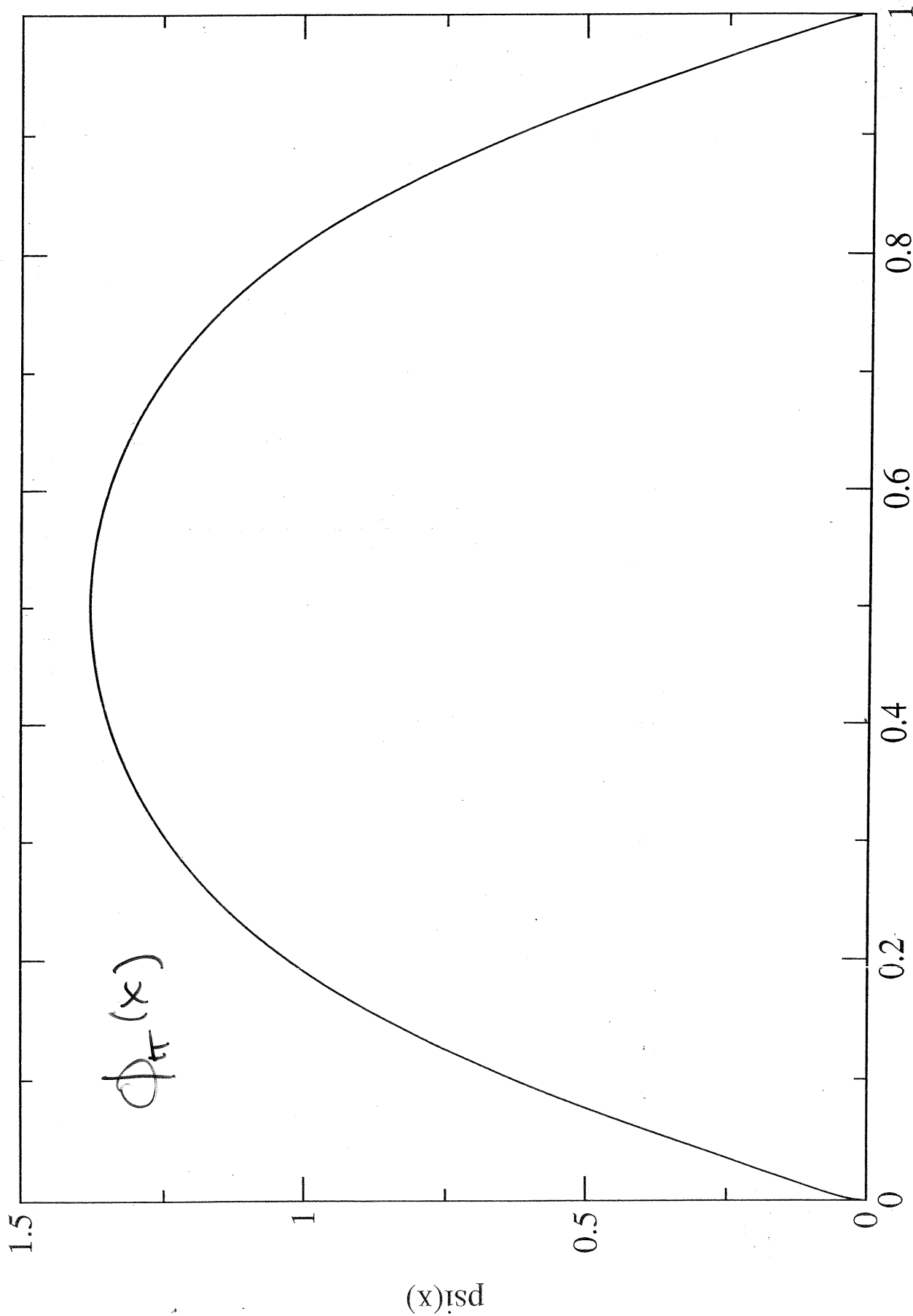
Delley

--- Asymptotic dist. Ampl.

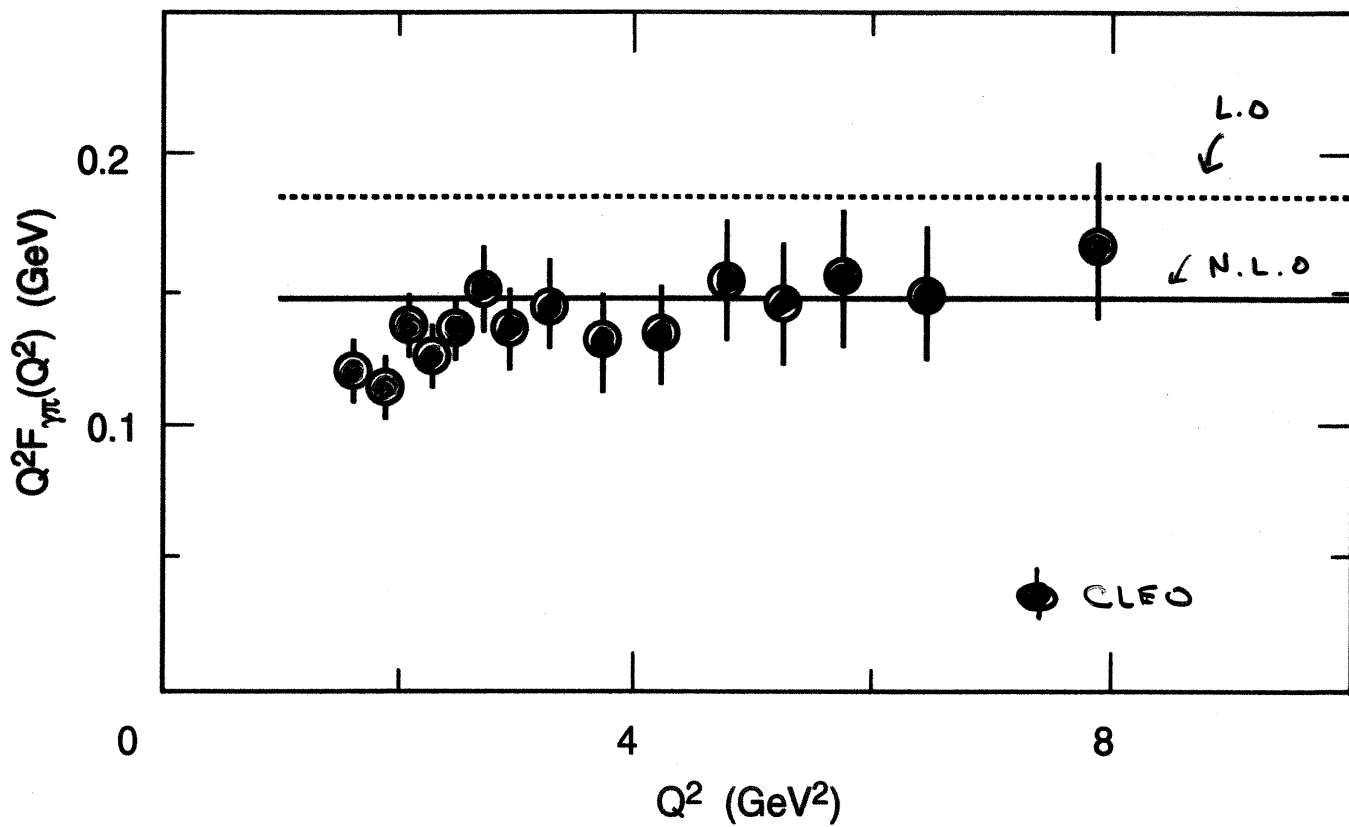
Burkert

Burkhardt  
+ Seel

Transverse Lattice + DLCQ



x



8-2000  
8561A13

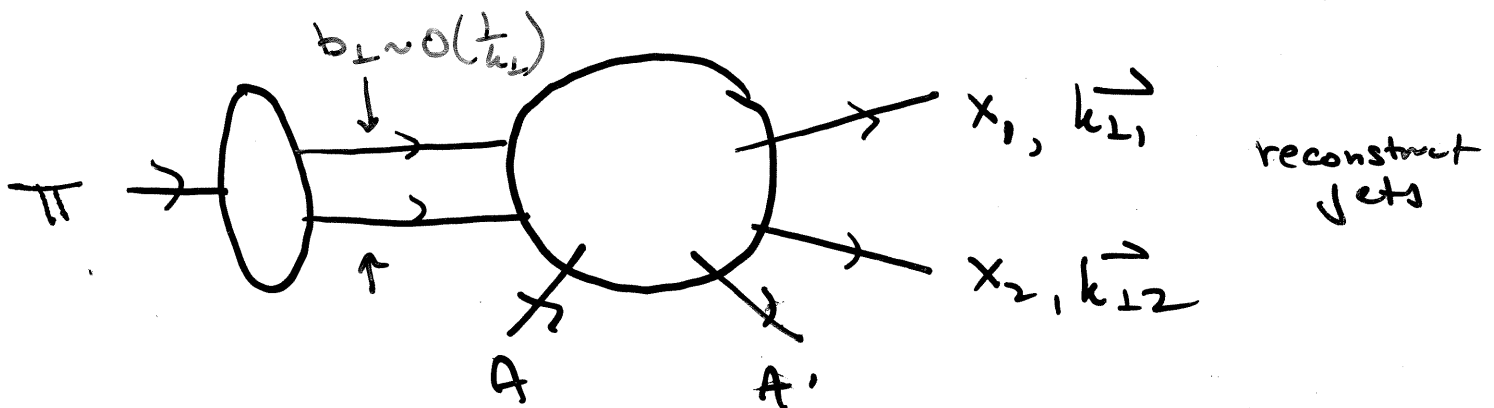
assumes  $\phi_{\pi}(x) = \phi_{\text{asymp}}(x)$   
 $= \sqrt{3} F_{\pi} x(1-x)$

Measure  $\Psi_H^{\text{val}}(x, k_\perp)$  via Diffractive Dissociation

$$\pi A \rightarrow \rho \bar{\rho} A'$$

Bertsch, Goldhaber  
Gunion, 213

Frankfurt, Miller, Strikman



$$x_1 + x_2 \approx 1$$

$$q_\perp = -k_\perp - k_\perp' < R_A^{-1}$$

$k_\perp$  large: color transparency  $M_A \sim A' m_A$

$$q_\perp \text{ small } \frac{d\sigma}{dq_\perp^2} \propto e^{-q_\perp^2 R_A^2/3}$$

$$* \int \frac{d\sigma}{dq_\perp^2} dq_\perp^2 \sim \frac{A^2}{R_A^2} \sim A^{4/3}$$

$$* x, k_\perp \text{ det} \Rightarrow \left| \frac{\partial^2}{\partial k_\perp^2} \Psi(x, k_\perp^2) \right|^2$$

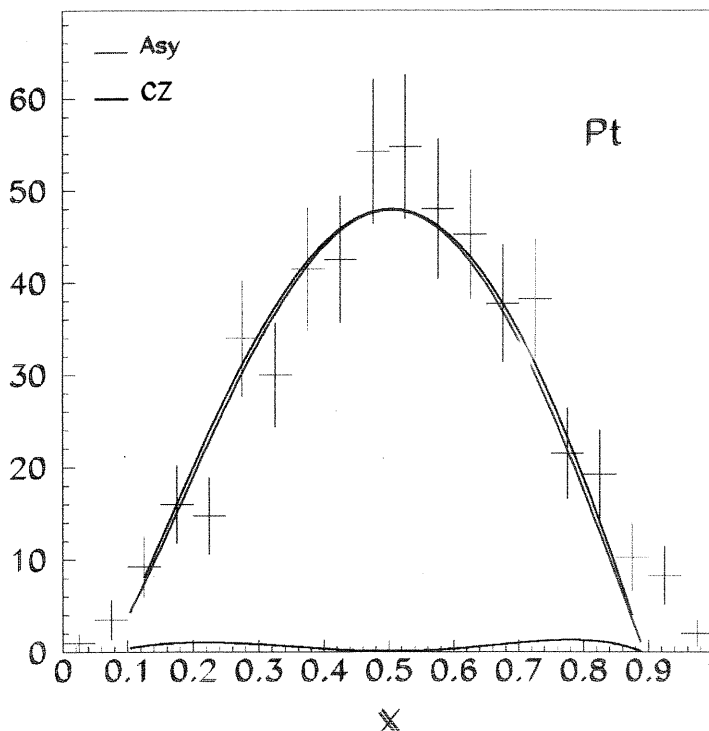
# E791 DATA - THE $q\bar{q}$ MOMENTUM WAVE FUNCTION

## AS MEASURED BY THE DI-JETS

- Use the diffractive di-jets to extract the momentum  $x$  distribution.
- Fit to a combination of the two wave function simulations.

$$\phi^2 = \alpha \phi_{Asy}^2 + \beta \phi_{CZ}^2$$

$$k_t > 1.5 \text{ GeV}/c ; Q^2 \sim 10 \text{ GeV}^2$$



$$\Phi_{Asy} (x, Q^2)$$

$$\Rightarrow \text{for } \chi(1-x) \sqrt{s}$$

Soln to  
DGLAP  
evol. eqn.

$$\Phi_{\pi}^{Asy} \propto x(1-x)$$

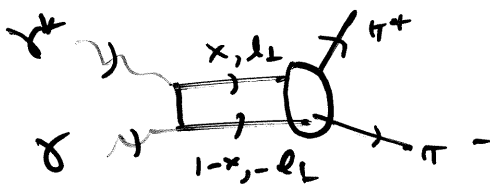
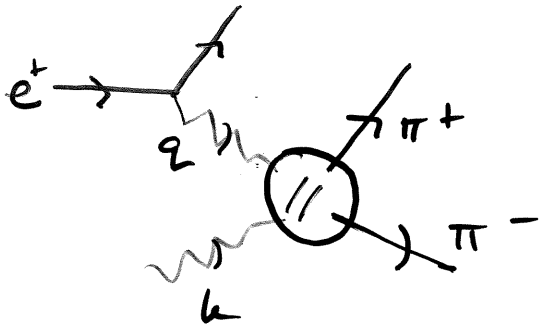
PRELIMINARY: > 90% Asymptotic W.F.

Same as  
seen in

$$F_{\gamma \rightarrow \pi^0}(Q^2)$$

D. Müller

Diehl  
Goussset  
Pire



Analyse in PQCD  
for fixed  $W^2 = m_\pi^2$   
and  $Q^2 \gg \Lambda_{QCD}^2$

$$T_{\mu\nu} = - \int_T^{\mu\nu} \sum_q \frac{e_q^2}{2} \int_0^1 dx \left[ \frac{1}{1-x} - \frac{1}{x} \right] \frac{\Phi_{q\pi\pi}^{\pi\pi}(x, \zeta, W^2)}{2\pi}$$

$$\frac{\Phi_{q\pi\pi}^{\pi\pi}(x, \zeta, W^2)}{2\pi} = \int \frac{dx^-}{2\pi} e^{-i x P^+ x^-}$$

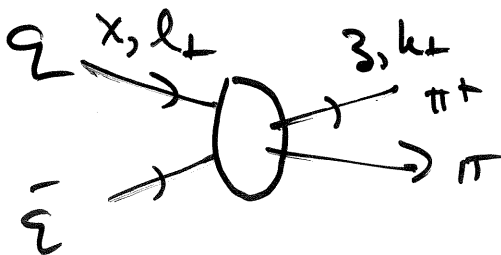
generalized  
distribution amplitude

$$\langle \pi(p^+) \pi(p^-) | \bar{q}(x^-) \delta^+ q(0) | 0 \rangle$$

(d.c.g.)

$$W^2 = \frac{k_\perp^2 + m_\pi^2}{z(1-z)}$$

also  $q\bar{q}$   
component



$q\bar{q}$  wavefunction  
of  $\pi\pi$  system

$$\Psi_{q\bar{q}}^{\pi\pi}(x, l_\perp, \lambda)$$

Diehl et al:

$$\gamma^* \gamma \rightarrow \pi^+ \pi^-, \pi^0 \pi^0$$

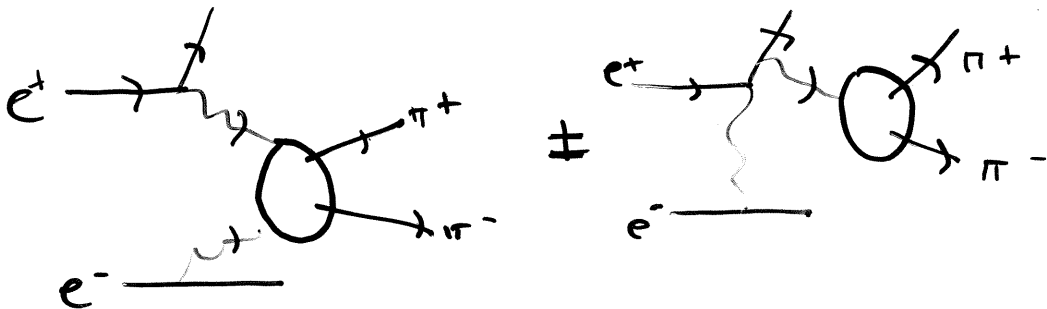
Dominance of  $A_{++}$  amplitude

Model estimate for dist. amp., rates

Cross sections

$$\sigma \sim \frac{1}{2} \text{nb} \text{ at Belle}$$

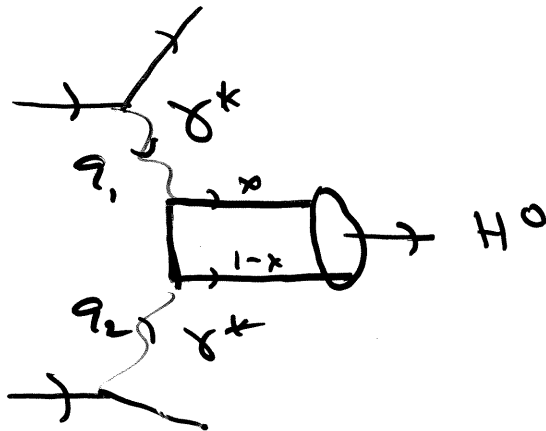
Interference with bremsstrahlung amplitude



$\pi^+$  vs  $\pi^-$  asymmetry



Extension to double-tagged

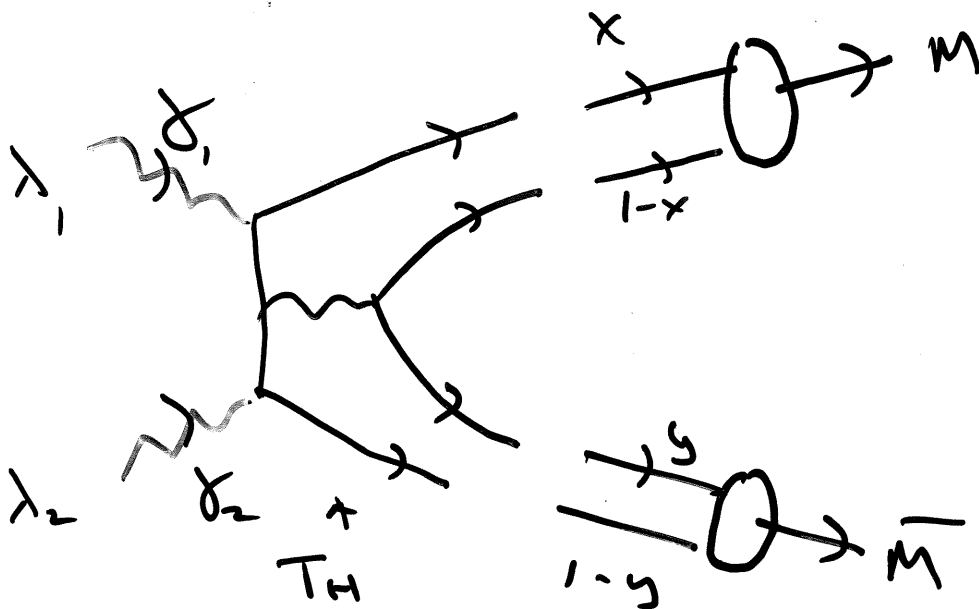


$$\frac{1}{(1-x)Q^2} \Rightarrow \frac{1}{(1-x)Q_1^2 + xQ_2^2}$$

Eng  
Walsh, et al  
Vest, et al

PQCD Factorization  
at large  $S, t$

G.P. Lepage  
+ SJB  
Vostok  
Cheng



no FSI!

$$M(\gamma_1, \gamma_2 \rightarrow M \bar{M})$$

$$= \int_0^1 dx \int_0^1 dy \cdot T_H(x, y; S, \theta_{cm})$$

$$\Phi_M(x, \tilde{q}) \Phi_{\bar{M}}(y, \tilde{q})$$

$$\Phi_M(x, \tilde{q}) = \int d^2k_{\perp} \Psi_{q\bar{s}/M}(x, k_{\perp}^2)$$

$$T_{\lambda, \lambda_2} = \begin{array}{c} \rho_{1, \lambda} \\ \rho_{2, \lambda} \end{array} \begin{array}{c} x \\ 1-x \\ y \\ 1-y \end{array} + \dots$$

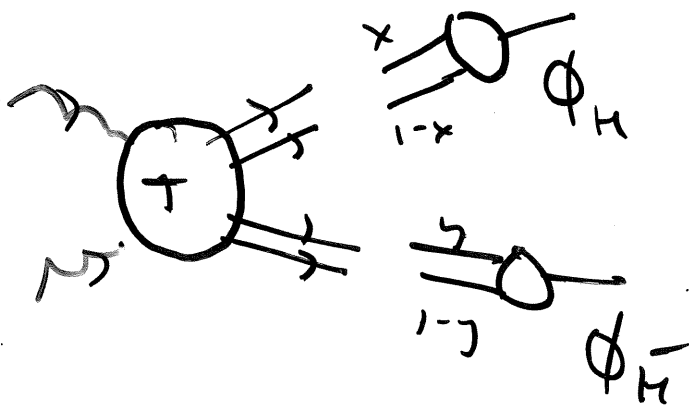
The diagram shows a series of Feynman diagrams representing the transition operator  $T_{\lambda, \lambda_2}$ . The first diagram has two incoming wavy lines from the left labeled  $\rho_{1, \lambda}$  and  $\rho_{2, \lambda}$ , and four outgoing lines to the right labeled  $x$ ,  $1-x$ ,  $y$ , and  $1-y$ . The second diagram is a similar tree-level diagram with a different internal structure. The third diagram is a tree-level diagram with a crossed internal line. The series continues with  $+\dots$ .

$$M_{\lambda, \lambda_2}(s, \theta_{cm}) = \int_0^1 dx \int_0^1 dy$$

$$\phi_H^*(x, Q) \phi_H^*(y, Q) T_{\lambda, \lambda_2}(x, y; s, \theta_{cm})$$

### Factorization theorem

\* Separates perturbative  $T_{\lambda, \lambda_2}$   
from non-perturbative  $\phi_H(x, Q)$



\*  $\phi_H$  universal; no FSI!

DLL

$$T_{\lambda_1 \lambda_2} \text{ for } \gamma \gamma_2 \rightarrow M \bar{M}$$

helicity  
zero mesons

leading twist

$$T_{++} = T_{--} = \frac{16\pi\alpha_s}{3S} \frac{32\pi\alpha}{x(1-x)y(1-y)} \frac{e_m^2 a}{1 - \cos^2\theta_m}$$

$$T_{+-} = T_{-+} = \dots \left[ \frac{e_m^2(1-a)}{1 - \cos^2\theta_m} + \frac{e_1 e_2 c}{s^2 - b^2 \cos^2\theta_m} \right]$$

$$a = (1-x)(1-y) + xy$$

$$b = (1-x)(1-y) - xy$$

$$c = (1-a) [y(1-y) + x(1-x)]$$

$$e_m = e_1 - e_2$$

Compare

$$T_{\gamma^* \rightarrow \gamma \bar{e}^-} = \frac{16\pi\alpha_s}{3S} \frac{1}{x(1-x)} \frac{1}{y(1-y)}$$

$$m_{++} = m_{--} = 16\pi\alpha F_M(s) \frac{e_m^2}{1 - \cos^2\theta_m}$$

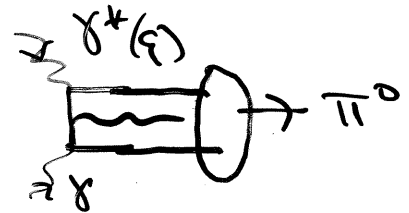
$$m_{+-} = m_{-+} = 16\pi\alpha F_M(s) \left[ \frac{e_m^2}{1 - \cos^2\theta_m} + 2e_1 e_2 g(\theta_m) \right]$$

$$g(\theta) = g(1-\theta)$$

# Prog of PQCD Factorization

Lepage + SJB  
 H.N. Li  
 Mueller, Duncan  
 ...

- 1. Decoupling of soft gluons to hard lines  
 (simplest in l.e.g.)



- 2. Decoupling of soft gluons to states with small color dipole moment
  - $\Rightarrow$  color transparency
  - $\Rightarrow$  suppression of FSI  
 (no violation of Watson's Theorem)

Mueller  
 SJB

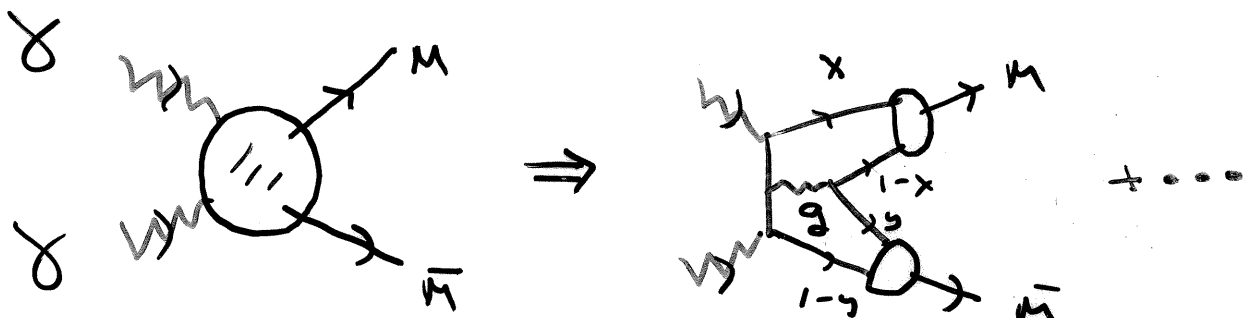
Watson's Theorem  
 Wolfenstein

- 3. End-point suppression  
 $x \rightarrow 1, k_{\perp}$  fixed (Feynman region)  
 from dynamical suppression of u.F.  
 and Sudakov suppression

# Issues in $\gamma\gamma$ Hard-Scattering

## Exclusive Processes

SJB  
G.P. Lepage



$$\frac{d\sigma}{dt} = \frac{\alpha^2 \alpha_s^2}{s^4} F\left(\frac{P_T^2}{s}, \ln \frac{P_T}{\Lambda}\right)$$

( $\lambda_H + \lambda_{\bar{H}} = 0$ )

$$\frac{\frac{d\sigma}{dt}(\gamma\gamma \rightarrow M\bar{M})}{\frac{d\sigma}{dt}(e^+e^- \rightarrow M\bar{M})} \approx F_{M\bar{M}}(\theta_{cm})$$

$$\Rightarrow \Phi_M(x, \tilde{Q})$$

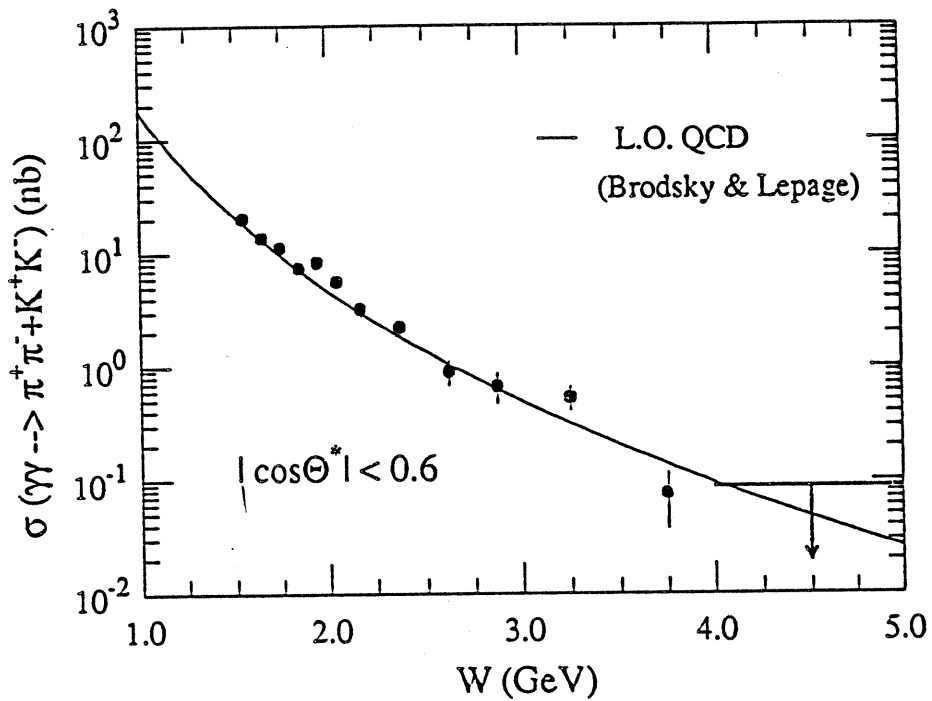
$$\alpha_s(Q) \Rightarrow \alpha_V(Q^*)$$

Commensurate  
Scale Relus

SJB, Ji, Pong, Robertson

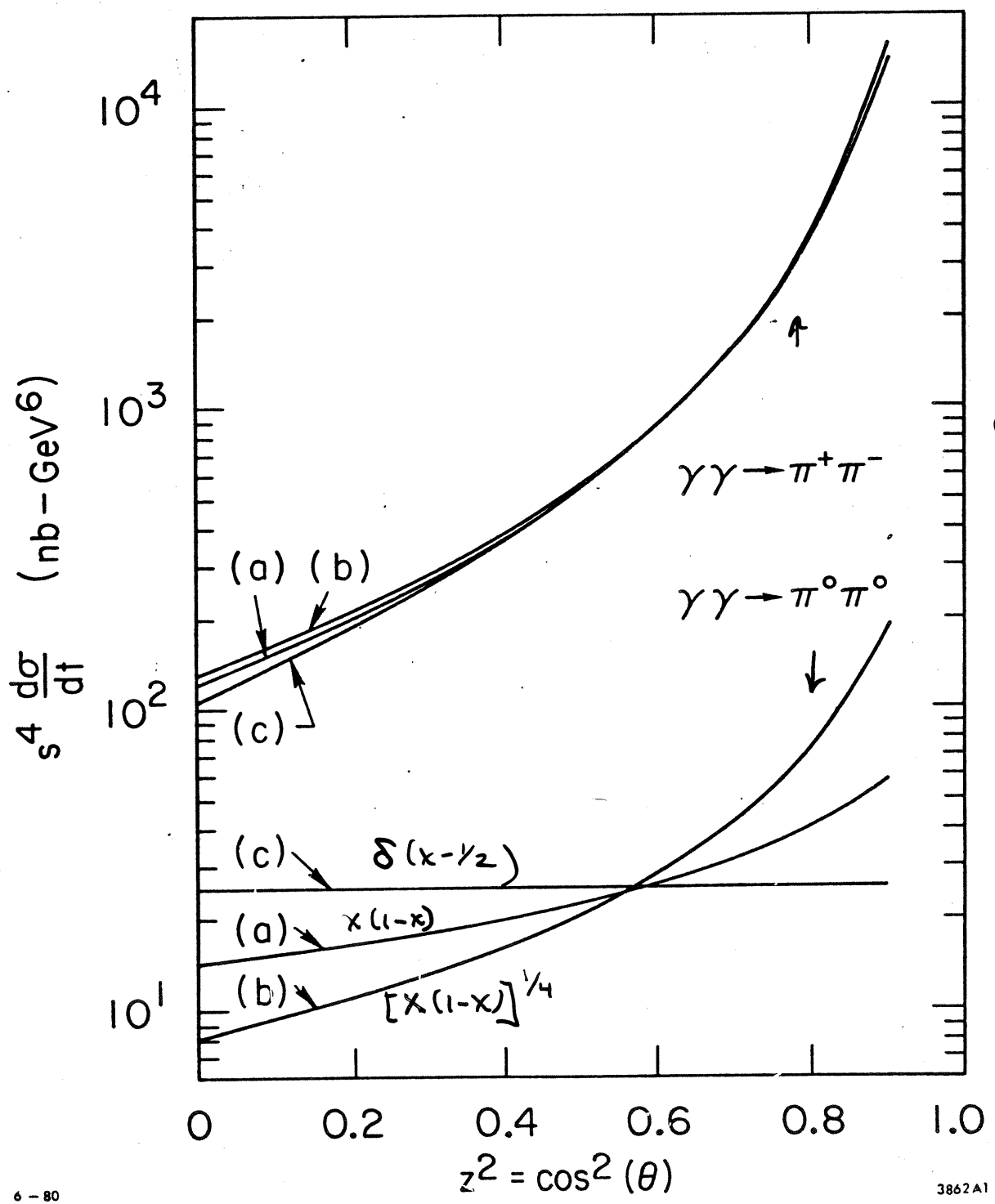
$\gamma\gamma \rightarrow \pi^+\pi^-, K^+K^-$

CLEO



$W_{\gamma\gamma}$

BHL



6-80

3862A1

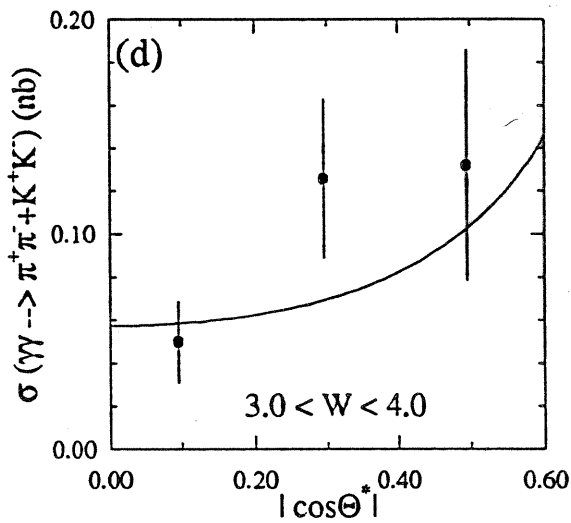
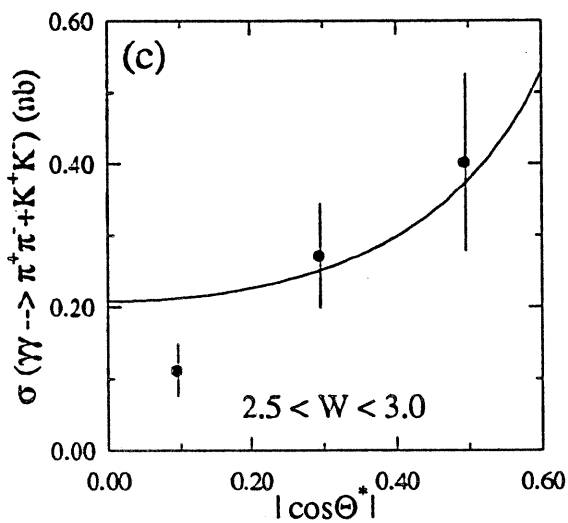
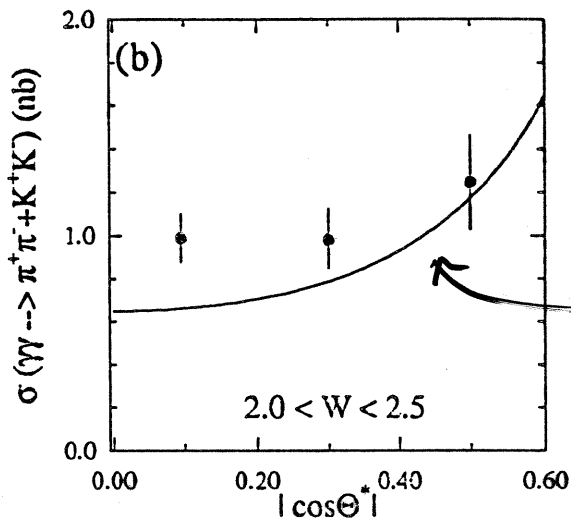
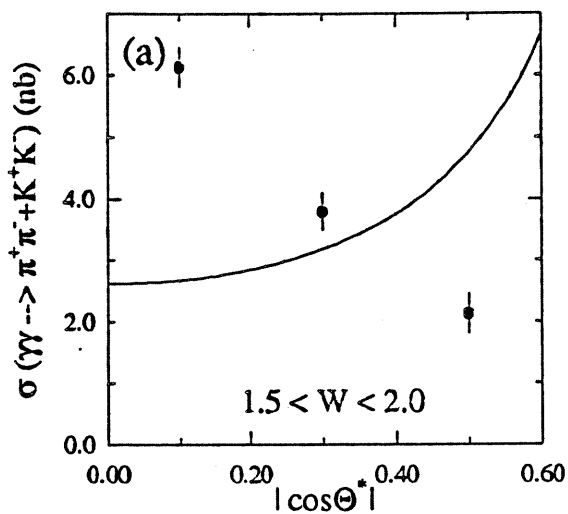
Fig. 3

Strong suppression  
 not true if  $\int \gamma\gamma \rightarrow \pi^0\pi^0$   
~~not true if~~  $\int \gamma\gamma \rightarrow \pi^0\pi^0$



$\gamma\gamma \rightarrow \pi^+\pi^-, K^+K^-$   
( $\theta^*$  dep)

CLEO



$|\cos\theta^*|$

Evidence for PQCD at  $W_{\gamma\gamma} \gtrsim 2.5$  GeV

DLV

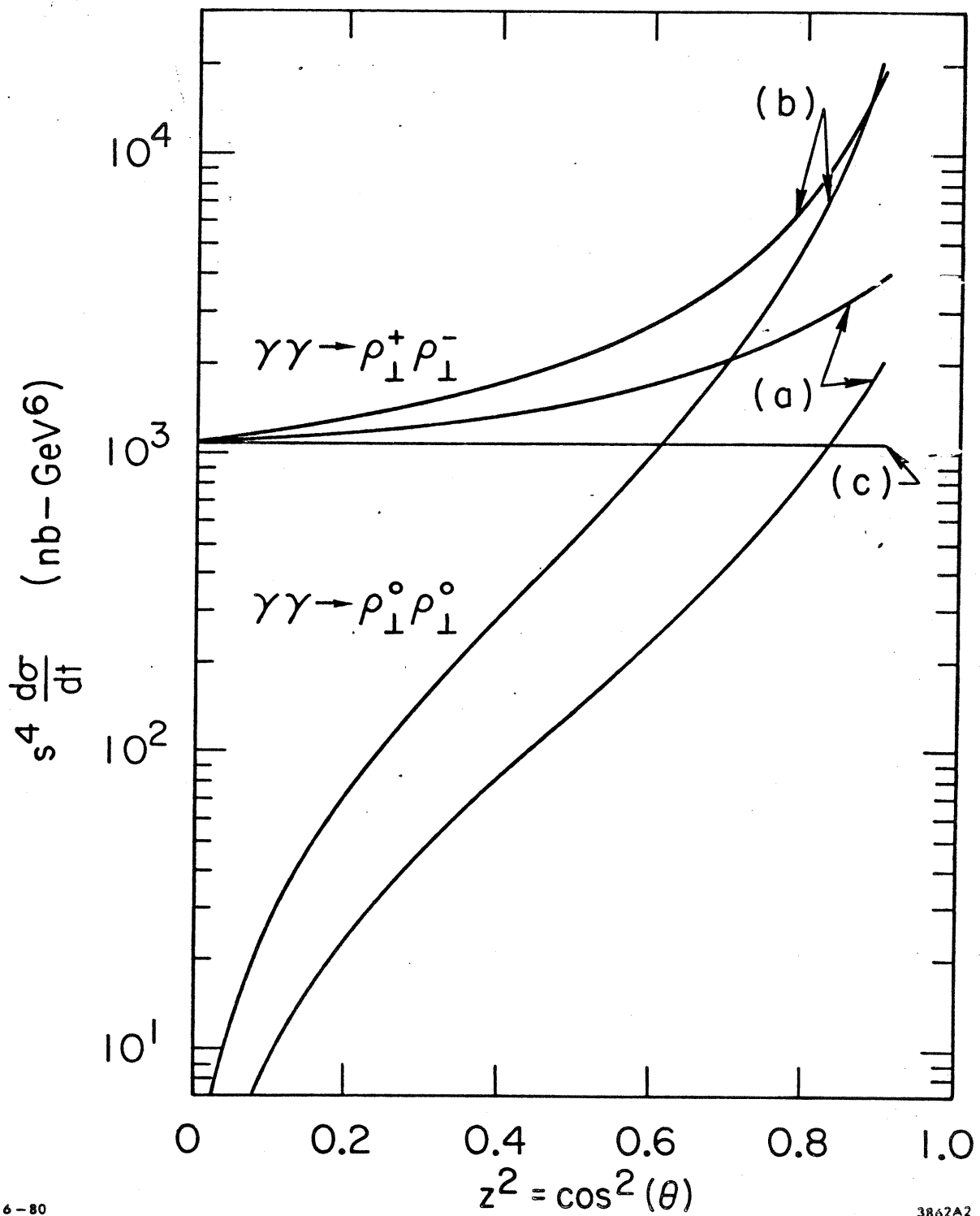
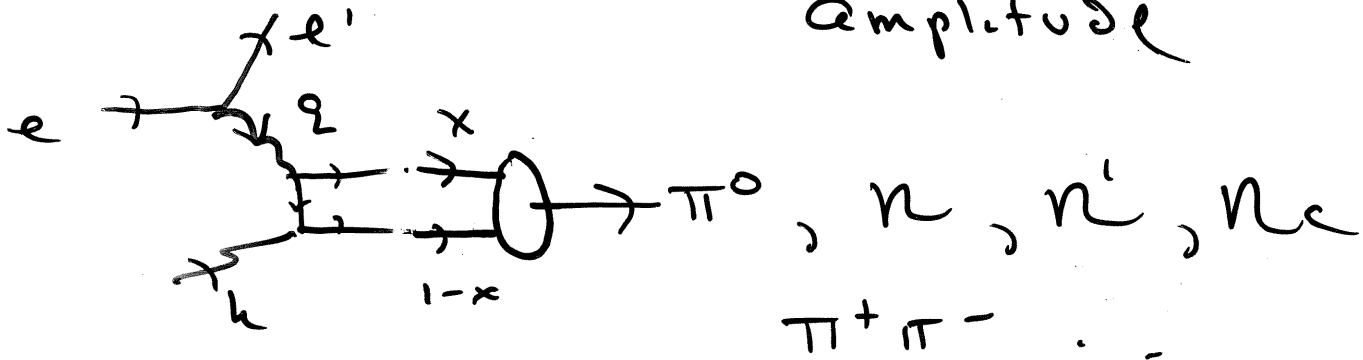


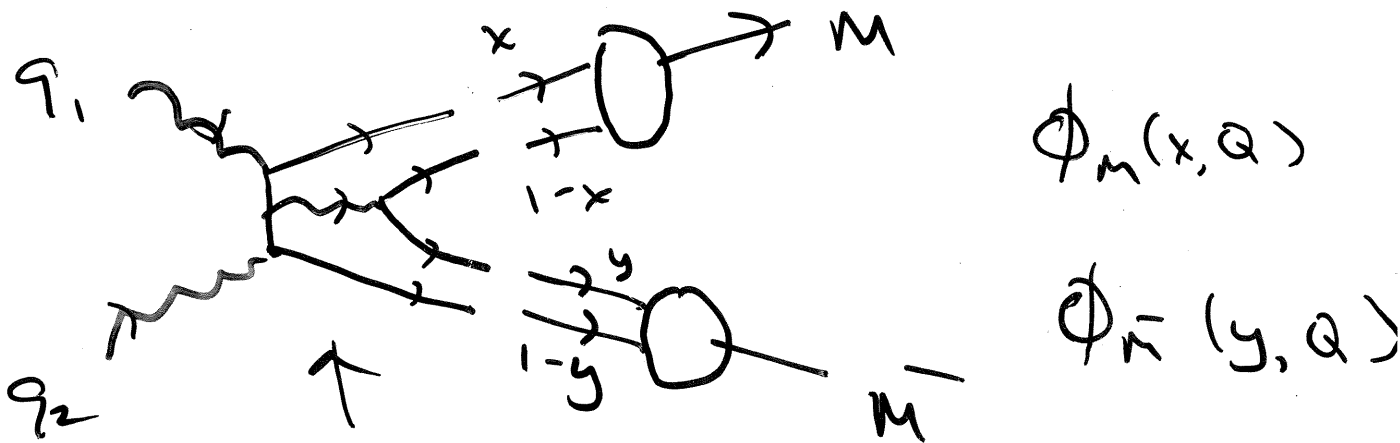
Fig. 5

\* Exclusive Two-Photon Reaction

measure  $\Phi_H(x, Q)$   
 distribution amplitude



$$k^2 \approx 0, \quad -q^2 \gg \Lambda_{QCD}^2$$



$$\Phi_M(x, Q)$$

$$\Phi_{\bar{M}}(y, Q)$$

$$T_H(x, y; s, \theta_{cm})$$

$s, t, u$  large (fixed  $\theta_{cm}$ )

For  $\gamma\gamma \rightarrow \pi^+\pi^-$  :

DLV

$$\frac{\frac{d\sigma}{dt}(\gamma\gamma \rightarrow \pi^+\pi^-)}{\frac{d\sigma}{dt}(\gamma\gamma \rightarrow \pi^+\pi^-)} = \frac{4 F_\pi^2(s)}{1 - \cos^2\theta_{cm}}$$

nearly insensitive to slope of  $\phi_\pi(x, \theta)$

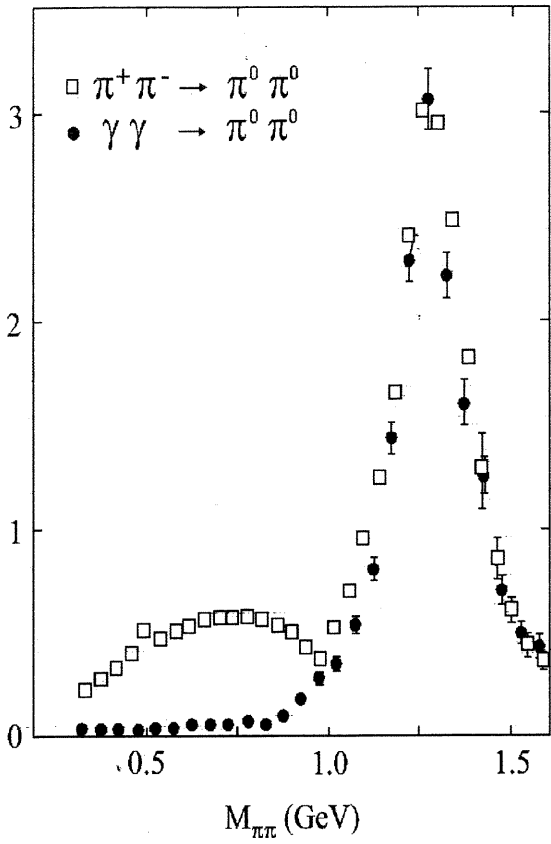
$$\int_0^1 dx \phi_m(x, \theta) = \frac{F_m}{2\sqrt{3}}$$

$$\frac{d\sigma}{dt}(\gamma\gamma \rightarrow k^+k^-) \approx 2 \frac{d\sigma}{dt}(\gamma\gamma \rightarrow \pi^+\pi^-)$$

$$k_L k_S \approx 0.3 \quad \pi^0\pi^0$$

$$\begin{matrix} \rho^+\rho^- \\ \lambda=0 \quad \lambda=0 \end{matrix} \approx 7.5 \quad \pi^+\pi^-$$

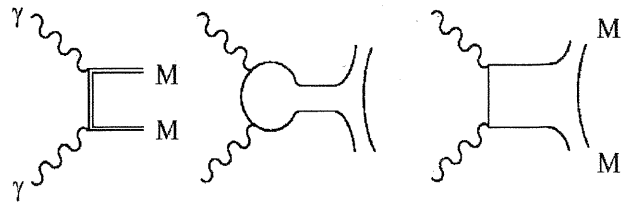
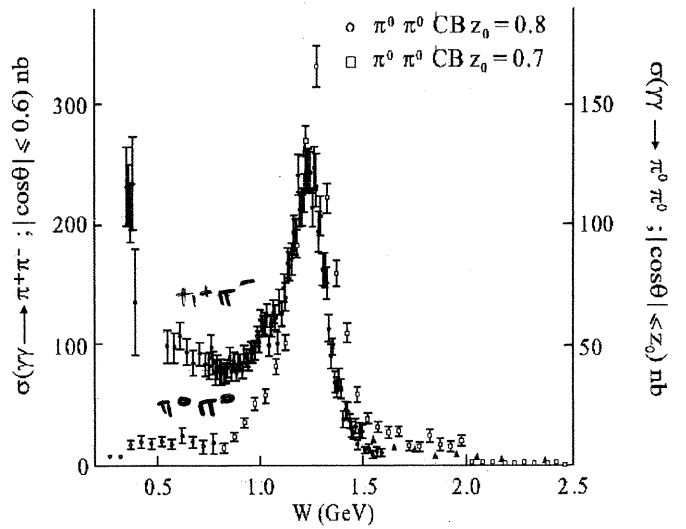
many such predictions of similar  $\phi_m$

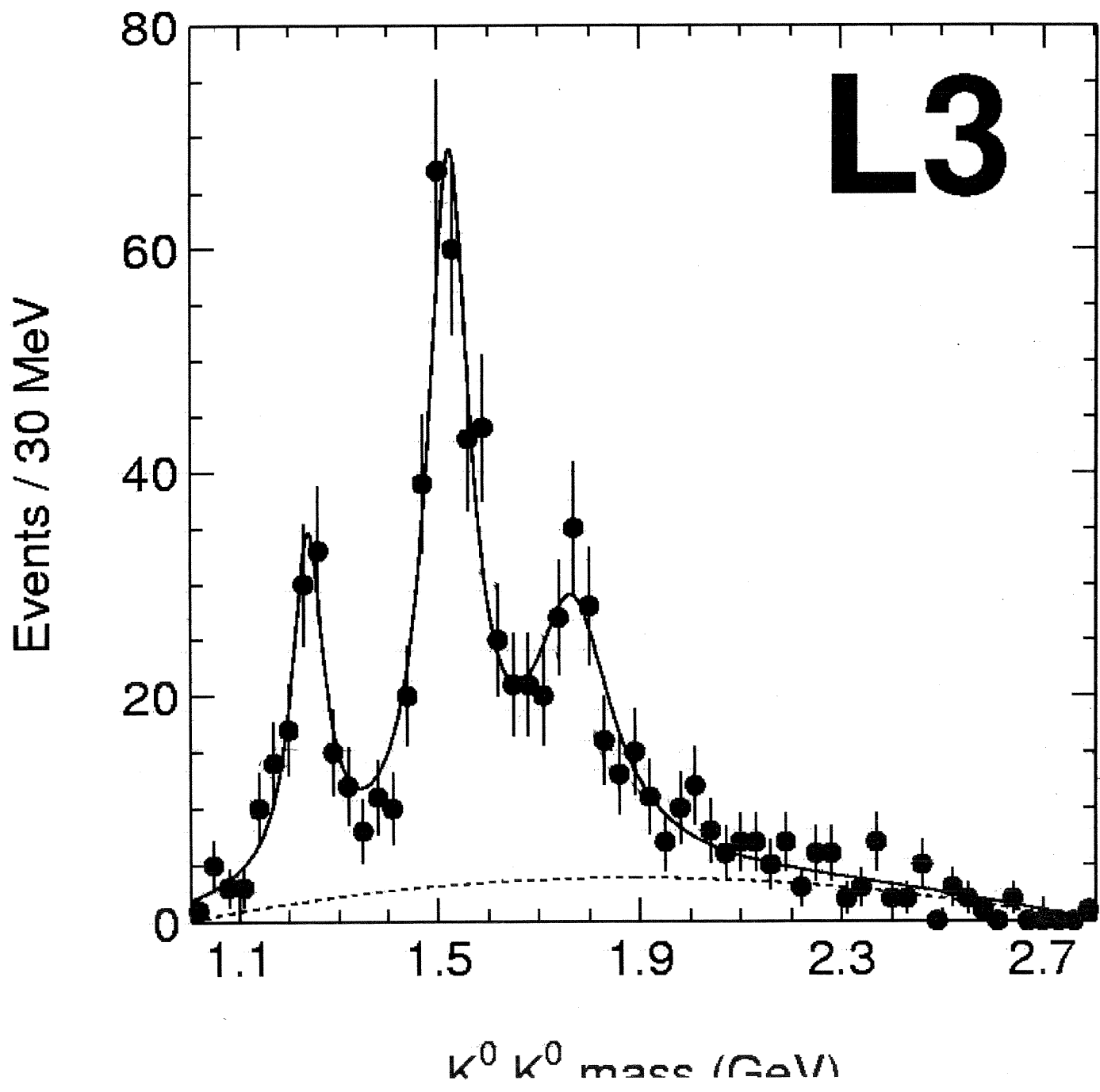


Pennington

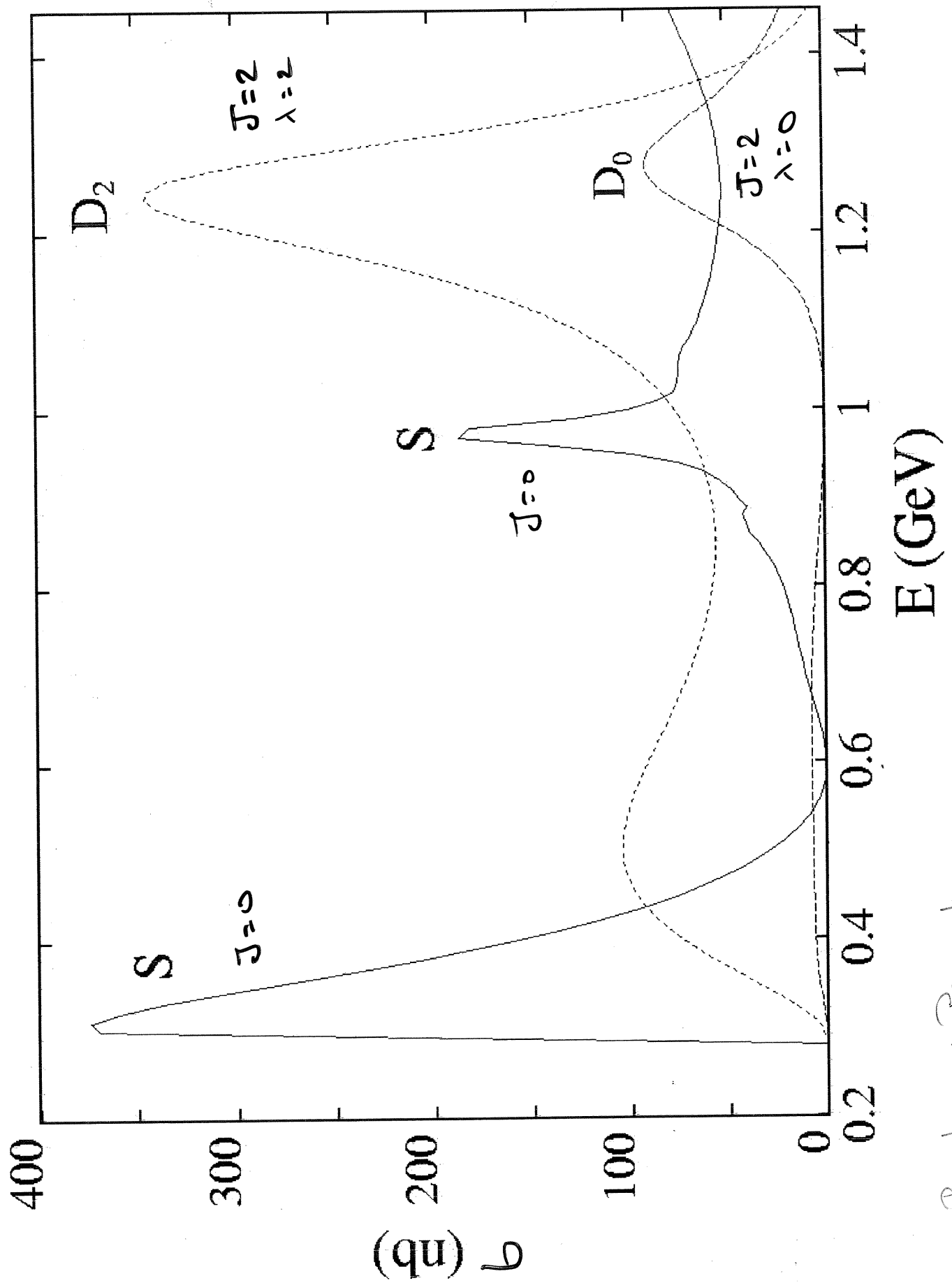
$\sigma$  resonance  $\Rightarrow$  glueball?  
 or gauge suppression?

$\bullet \pi^+ \pi^-$  Mark II, CLEO





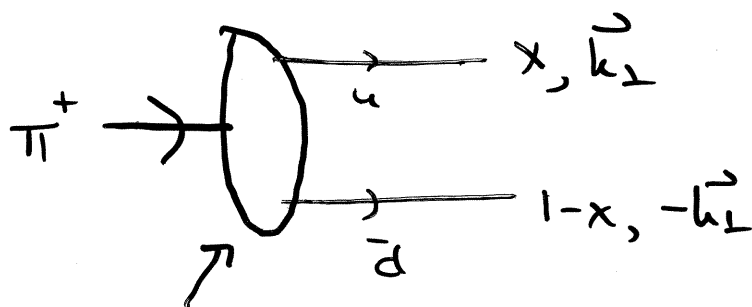
$\sigma(\gamma\gamma \rightarrow \pi\pi)$



Bogliione + Pennington

## Ingredients

\* 
$$\Phi_{\pi}(x, Q) = \int d^2k_{\perp} \Psi_{q\bar{q}}^{\pi}(x, \vec{k}_{\perp})$$



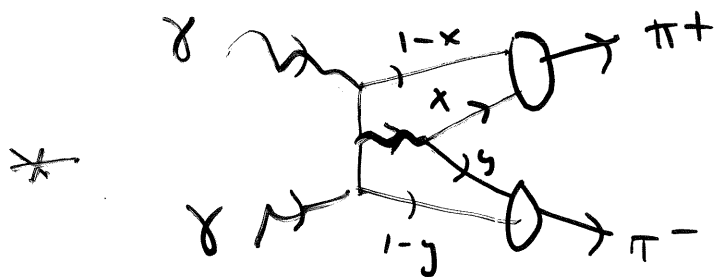
G.P.L.  
SJB

$$\Psi_{q\bar{q}}^{\pi}(x, \vec{k}_{\perp}) \quad (\tau = t - z/c = 0)$$

$$x = \frac{k^+}{p^+} = \frac{k^0 + k^z}{p^0 + p^z}$$

\*  $\alpha_S(Q^2)$  at low scales

e.g.:  $Q^2 = \hat{s} = xy s \sim \frac{s}{5}$



\* Color Transparency  
(Suppressed F.S.I.)

A.H. Mueller  
SJB



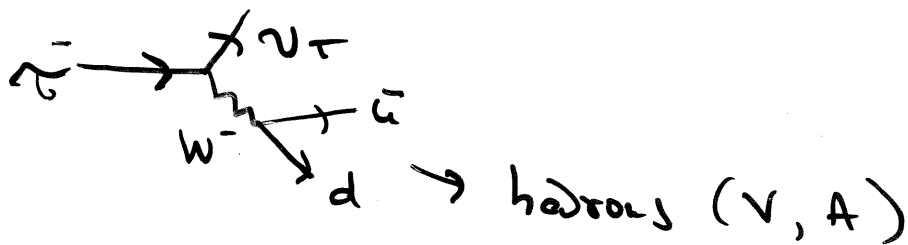
## Effective Charges:

Define  $\alpha_s(Q^2)$  from single observable  
Elim  $m_s$ : relate observables

e.g.:

$$R_{\text{ete}}(s) \equiv R_0(s) \left[ 1 + \frac{\alpha_R(s)}{\pi} \right]$$

$$* \text{BR}(\tau \rightarrow \text{hadrons} + \nu_\tau) \equiv (\text{BR})_0 \left[ 1 + \frac{\alpha_\tau(m_\tau^2)}{\pi} \right]$$

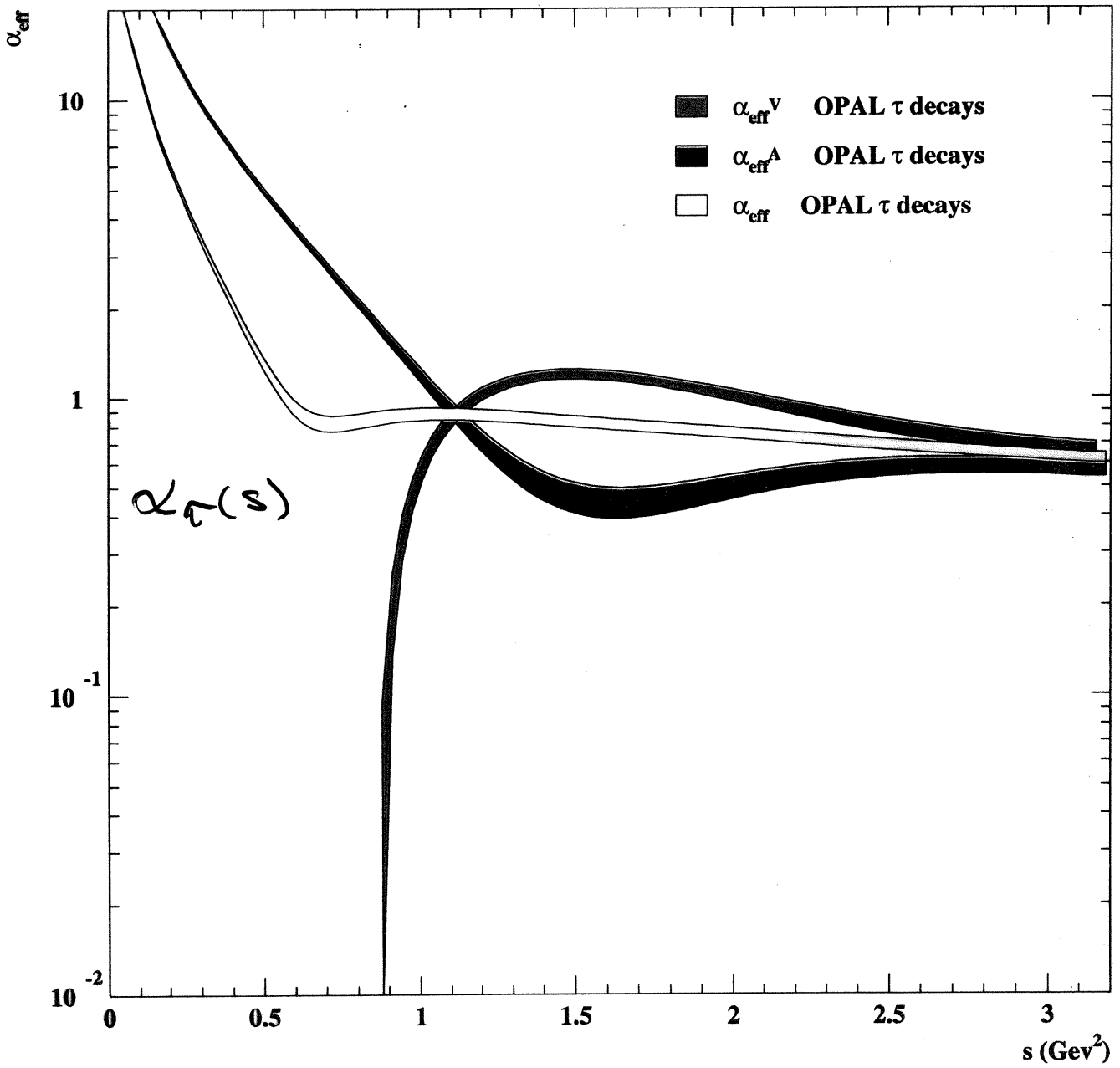


S. Menke: Isolate all channels  
(OPAL) separate  $V, A, S, \bar{u}$

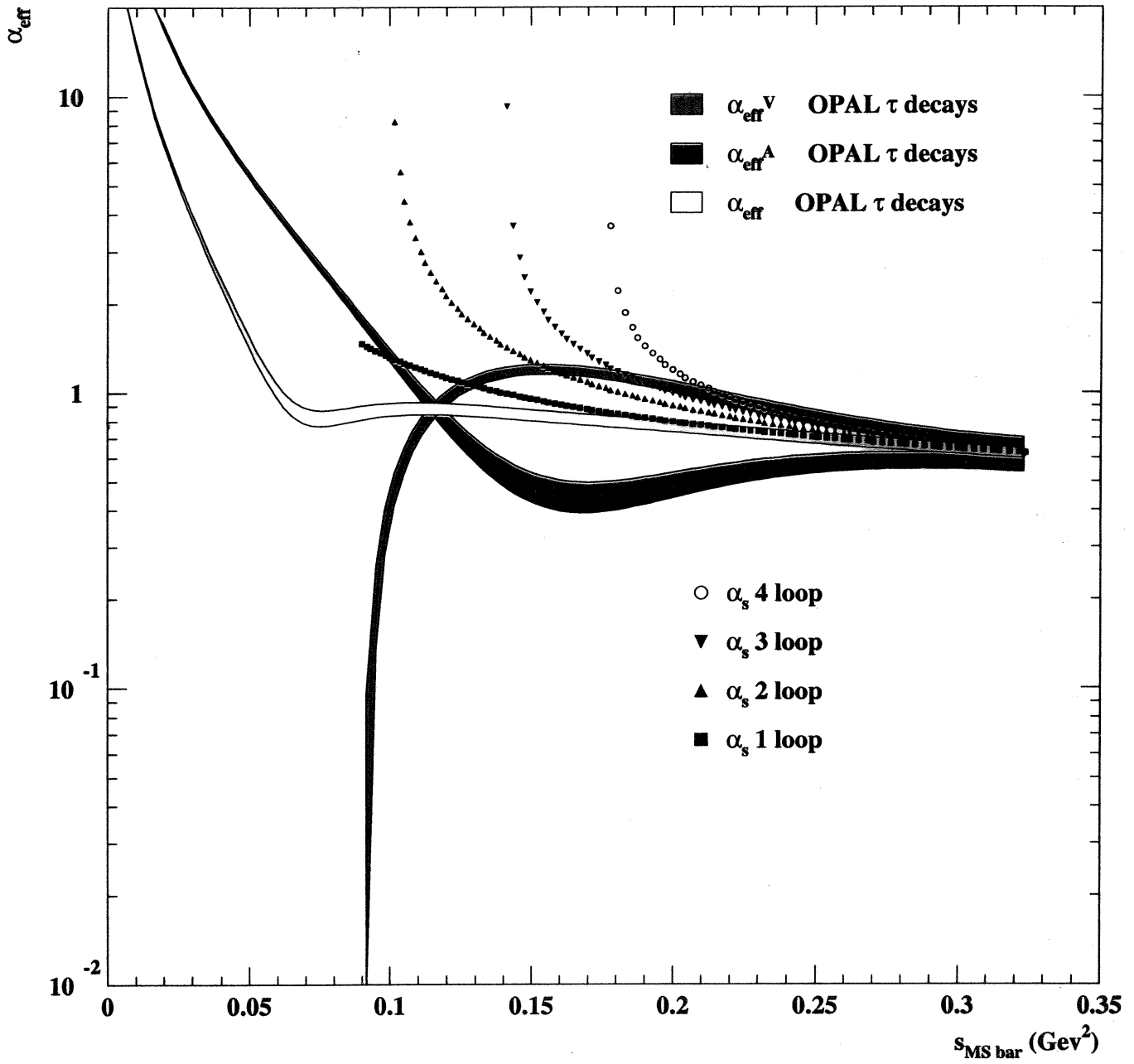
\*  $\alpha_\tau(s)$  Obeys standard RGE  
 $\text{BR}_\tau$ : known to 3-loops

Menke, Menno, SJB

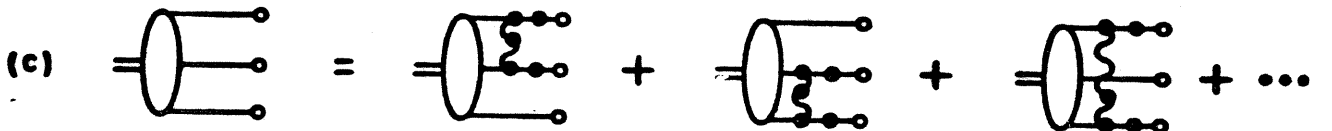
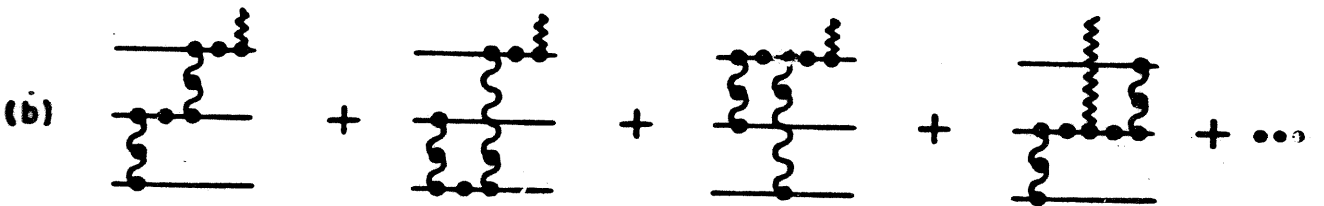
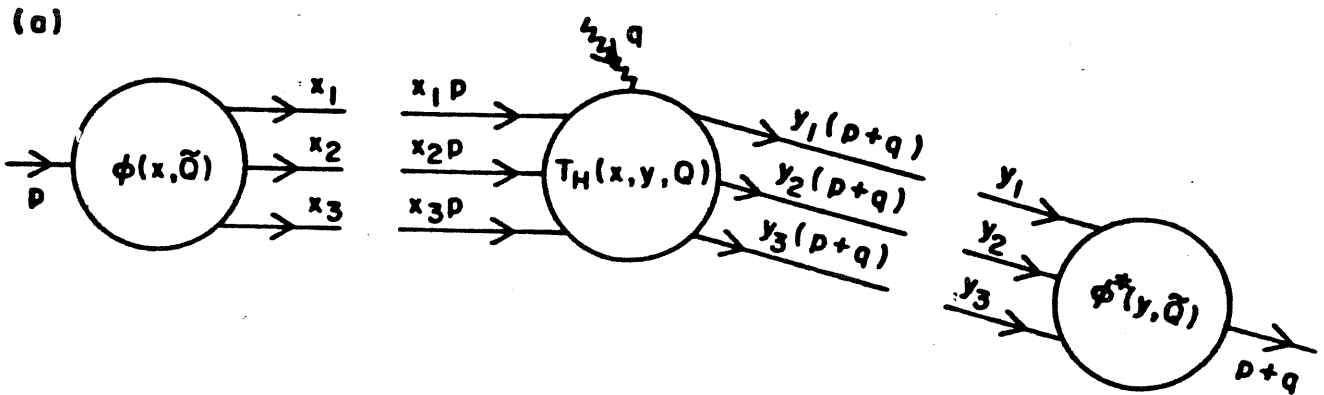
S. Meeke  
(pred.)



$\alpha_{\tau} \sim \text{flat}$   
 $\sim 1$   
not .3 to .4



high  $Q^2 = -q^2$   
 $Q^2 \gg \langle k_{\perp}^2 \rangle$



4-83

3793A13

Figure 19. (a) Factorization of the nucleon form factor at large  $Q^2$  in QCD. (b) The leading order diagrams for the hard scattering amplitude  $T_H$ . The dots indicate insertions which enter the renormalization of the coupling constant. (c) The leading order diagrams which determine the  $Q^2$  dependence of the distribution amplitude  $\phi(z, Q)$ .

Calculator of proton form factor in PQCD

Lepage + Brodsky  
 Chernyak + Zhitovitch  
 Radyushkin  
 Mueller + Duncan

Feynman  
 endpoint: Kroll et al

Fig from  
Kroll et al

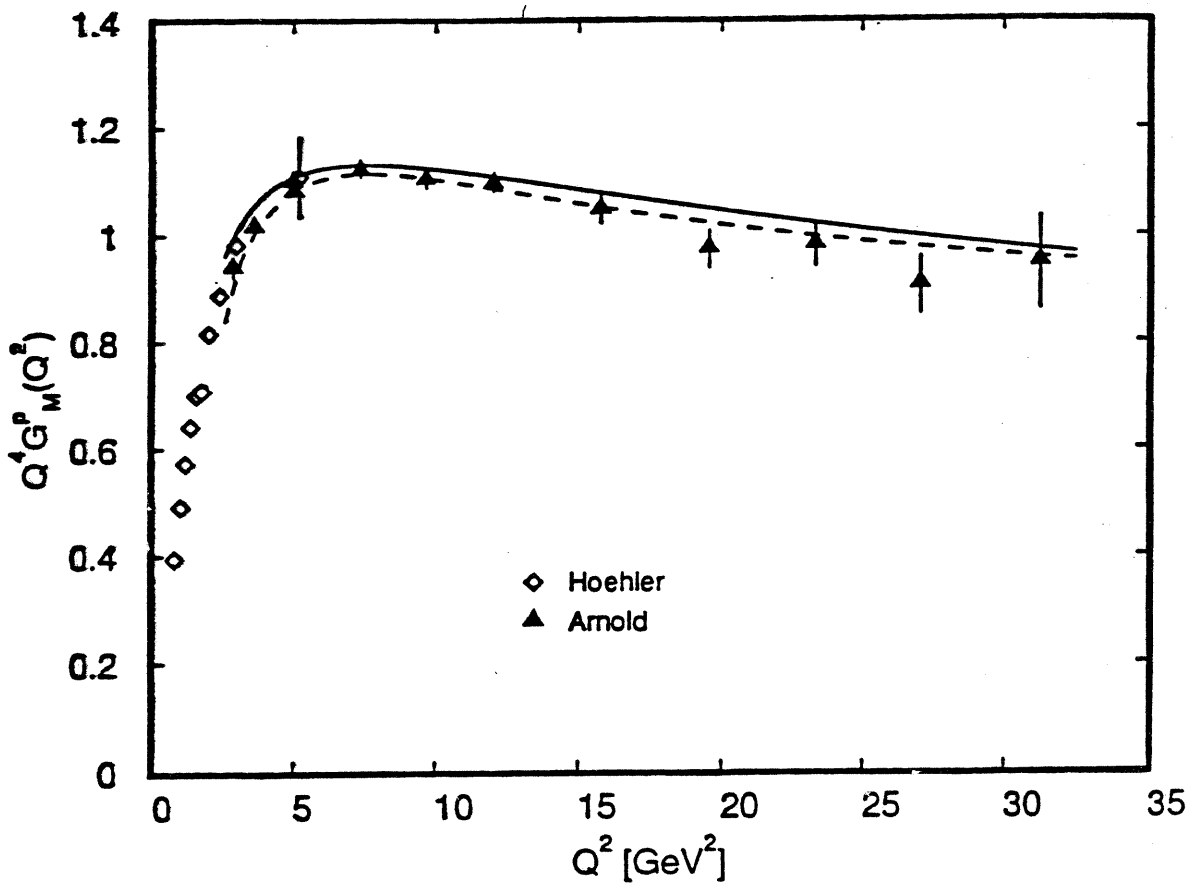


Fig. 4

PQCD  
predicts  $\frac{1}{Q^4}$  !

consistent with  
 $F_1(Q^2) \sim \frac{\alpha_s^2(Q^2)}{Q^4}$

Kroll, Pilsner,  
Schürmann, Schweiss

$$G_L + G_R$$

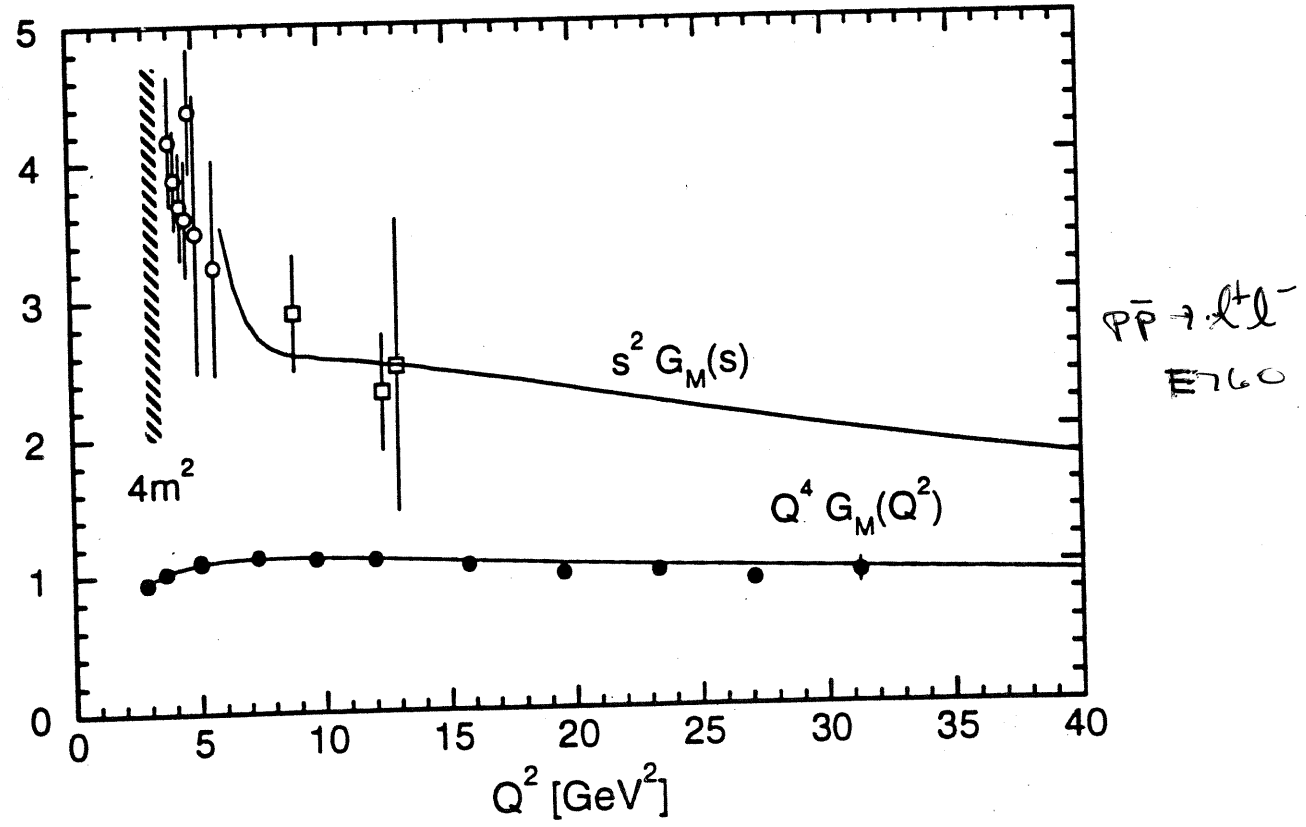
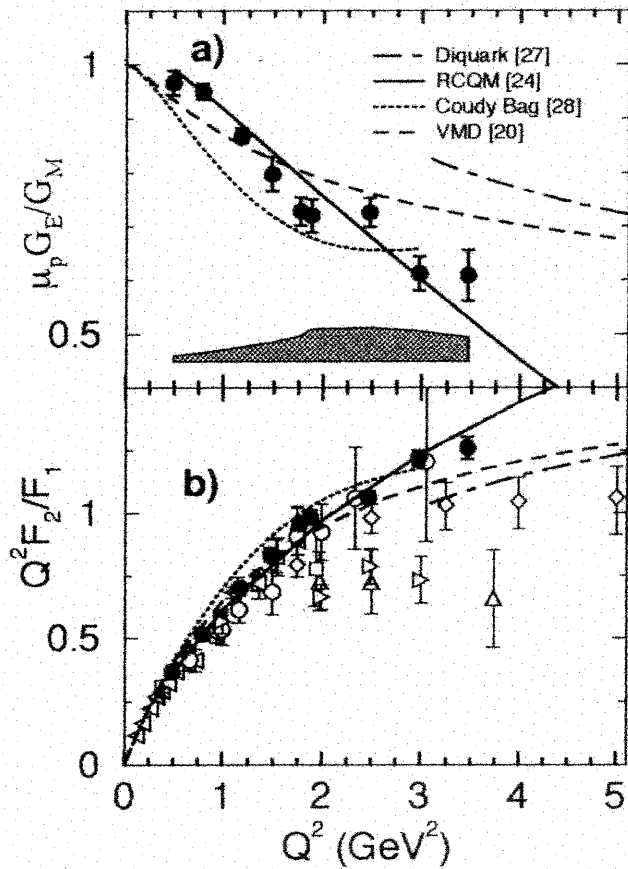


Fig. 1

Ji, Pang, Robertson, SJB :

Little room  
for variation  
of  $f_V(\frac{Q^2}{q})!$



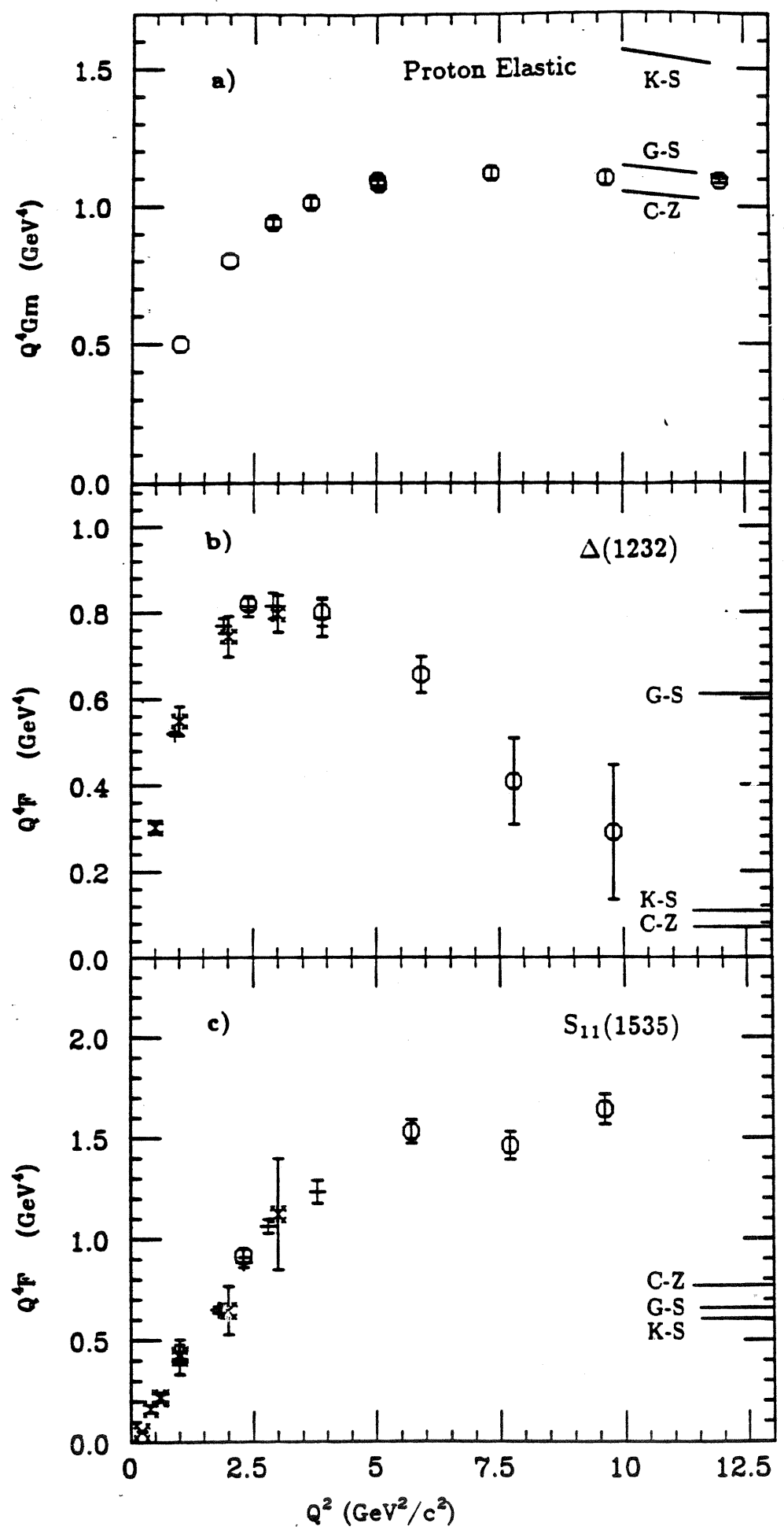
I Jefferson  
Lab

PQCD

$$\frac{F_2}{F_1} \sim \frac{1}{Q^2}$$

diquark  
model?

P. Stoler  
Carlson



$P \rightarrow P$

$P \rightarrow \Delta$

Suppress.  
from  
C-Z: s.r.

$P \rightarrow S_{11}$



# Near-Threshold Pion Production

P.V. Pobylitsa  
M.V. Polyakov  
M. Strikman

"Soft pion theorems  
for hard processes"

Example:

$$\gamma^* N \rightarrow (\pi N')$$

$$W - W_{Th} < m_\pi$$



$$W_{Th} = m_N + m_\pi$$

Also applicable to

$$\gamma\gamma, e^+e^- \rightarrow B(\bar{B}\pi)$$

continuum  
near threshold

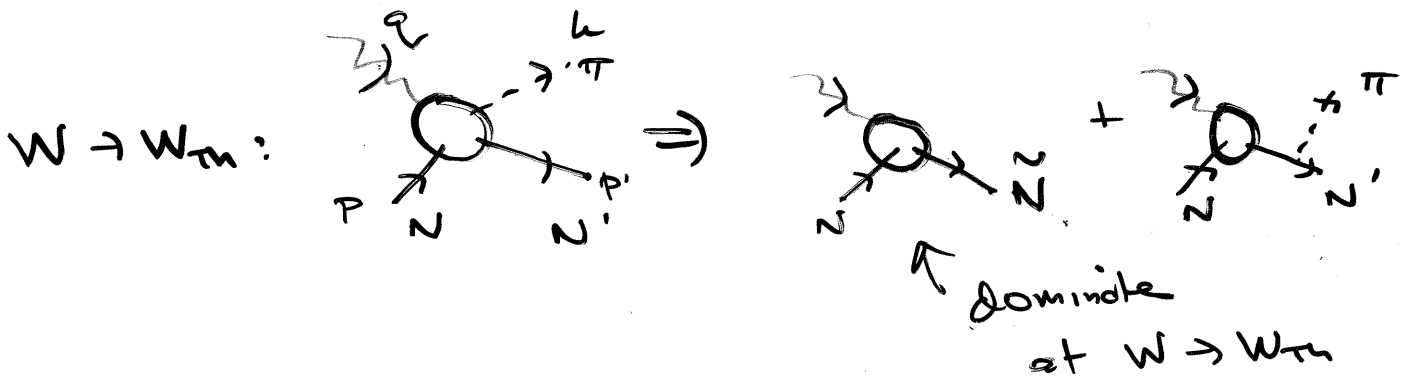
$$m_{\bar{B}\pi} - m_B - m_\pi < m_\pi$$

Weak decays, meson processes.

$$\gamma\gamma^* \rightarrow (\pi\pi)$$

(Tarasov)

# Soft pion reduction



$$M = \langle \pi N' | J^{\pi(0)} | N \rangle$$

$$\Rightarrow \frac{i}{f_\pi} \langle N' | [Q_5, J^{\pi(0)}] | N \rangle$$

$$+ \frac{i g_A}{4 f_\pi} \frac{p' \cdot k}{p' \cdot k} \bar{u}(p') \gamma_5 \tau u(p) \langle N' | J^{\pi(0)} | N \rangle$$

where  $2p' \cdot k = W^2 - m_p^2 - m_\pi^2 = (W - W_m)^2 + 2m_\pi m_p$

\* First term dominates at small  $\beta$ :

$$\beta = \sqrt{1 - \frac{(m_N + m_\pi)^2}{W^2}}$$

$[Q_5, J^{\pi(0)}]$ : chiral rotation of  $J$   
 $N \rightarrow N'$

Distribution amplitude of proton:

$$|P \uparrow\rangle = \frac{\phi_S(x_i)}{\sqrt{6}} [2u_{\uparrow\downarrow\uparrow}d_{\uparrow\downarrow\uparrow} - u_{\uparrow\downarrow\uparrow}d_{\uparrow\downarrow\uparrow} - d_{\uparrow\downarrow\uparrow}u_{\uparrow\downarrow\uparrow}]$$

$$+ \frac{\phi_A(x_i)}{\sqrt{2}} [u_{\uparrow\downarrow\uparrow}d_{\uparrow\downarrow\uparrow} - d_{\uparrow\downarrow\uparrow}u_{\uparrow\downarrow\uparrow}]$$

$$\phi_S(x_1, x_2, x_3) = \phi_S(x_3, x_2, x_1)$$

$$\phi_A(x_1, x_2, x_3) = -\phi_A(x_3, x_2, x_1)$$

Chiral rotator:

$$|P \uparrow \pi^0\rangle = \frac{\phi_S(x_i)}{2\sqrt{6}f_\pi} [6u_{\uparrow\downarrow\uparrow}d_{\uparrow\downarrow\uparrow} + u_{\uparrow\downarrow\uparrow}d_{\uparrow\downarrow\uparrow} + d_{\uparrow\downarrow\uparrow}u_{\uparrow\downarrow\uparrow}]$$

$$- \frac{\phi_A(x_i)}{3\sqrt{2}f_\pi} [u_{\uparrow\downarrow\uparrow}d_{\uparrow\downarrow\uparrow} - d_{\uparrow\downarrow\uparrow}u_{\uparrow\downarrow\uparrow}]$$

$$|n \uparrow \pi^+\rangle = \frac{\phi_S(x_i)}{\sqrt{12}f_\pi} [2u_{\uparrow\downarrow\uparrow}d_{\uparrow\downarrow\uparrow} - 3u_{\uparrow\downarrow\uparrow}d_{\uparrow\downarrow\uparrow} - 3d_{\uparrow\downarrow\uparrow}u_{\uparrow\downarrow\uparrow}]$$

$$- \frac{\phi_A(x_i)}{2f_\pi} [u_{\uparrow\downarrow\uparrow}d_{\uparrow\downarrow\uparrow} - d_{\uparrow\downarrow\uparrow}u_{\uparrow\downarrow\uparrow}]$$

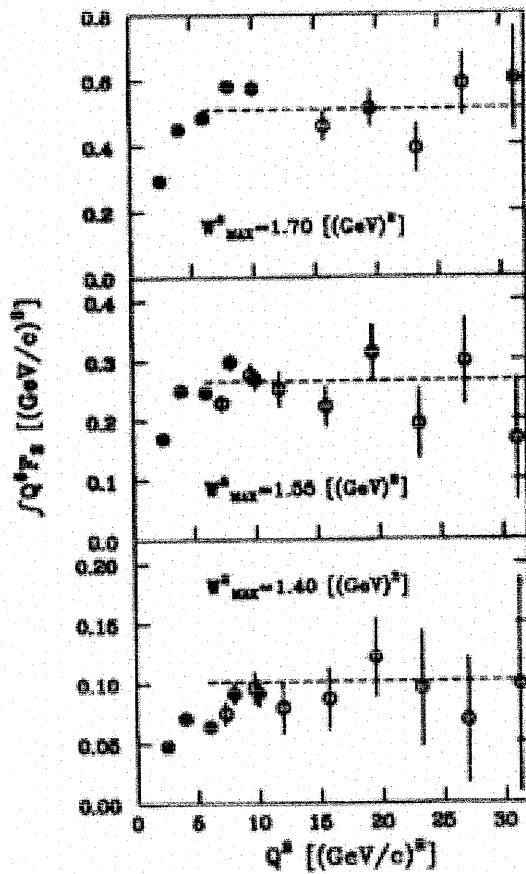
\*  $\infty$  Distributions amplitudes of  $(\pi N)$  system determined!



[16]  
Chiral Dynamics in Hard Processes  
Polyakov, M

E136

$$\int_{W_{th}^2}^{W_{MAX}^2} dW^2 Q^6 F_2(W, Q^2)$$

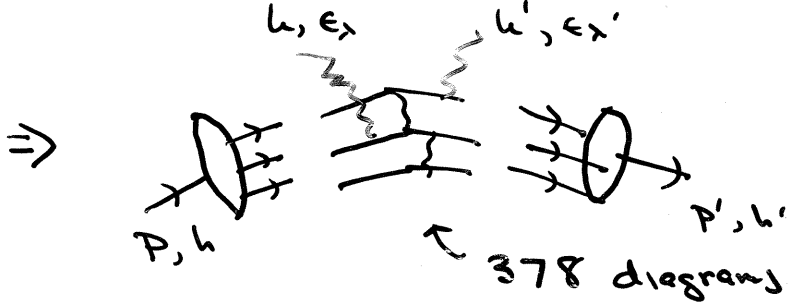
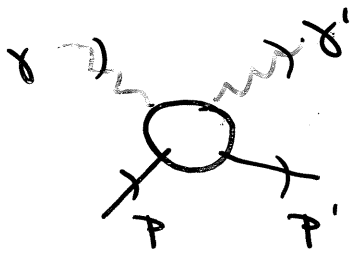


PQCD prediction  
+ soft pion theorem  
 $\phi_A = 0$

$\leftarrow W - W_{th} \sim W_{th}^2$   
MAX

Fig. 6

# Compton Scattering at Large Momentum Transfer



378 diagrams in Tx

SJB  
G.P. Ligon  
Nizic  
Kro-fed  
Brooks  
Dixon

$$M_{hh'}^{\lambda\lambda'}(s, t) = \sum^+ \int (dx/dy) \phi(x_i, Q) T_h(x_i, y_i; Q) \phi(y_j, Q)$$

$$* \quad \frac{d\sigma}{dt} (\gamma P_h \rightarrow \gamma' P_{h'}) \approx \frac{F_{hh'}^{\lambda\lambda'}(\theta_{cm})}{s^6} \quad \text{PQCD scaling}$$

$$\boxed{h = h'}$$

Hadron Helicity Conservation

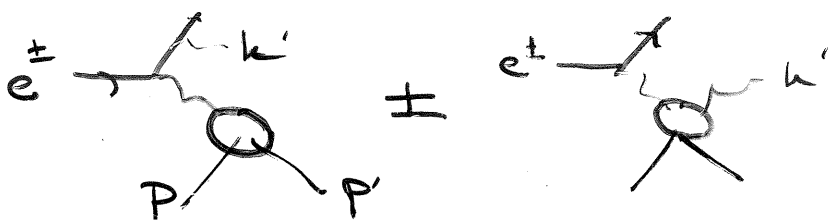
Detailed predictions for each helicity amplitude, phase

$$* \quad F_{hh'}^{\lambda\lambda'}(\theta_{cm}) \text{ depends on } \phi(x_i, Q) \text{ shape}$$

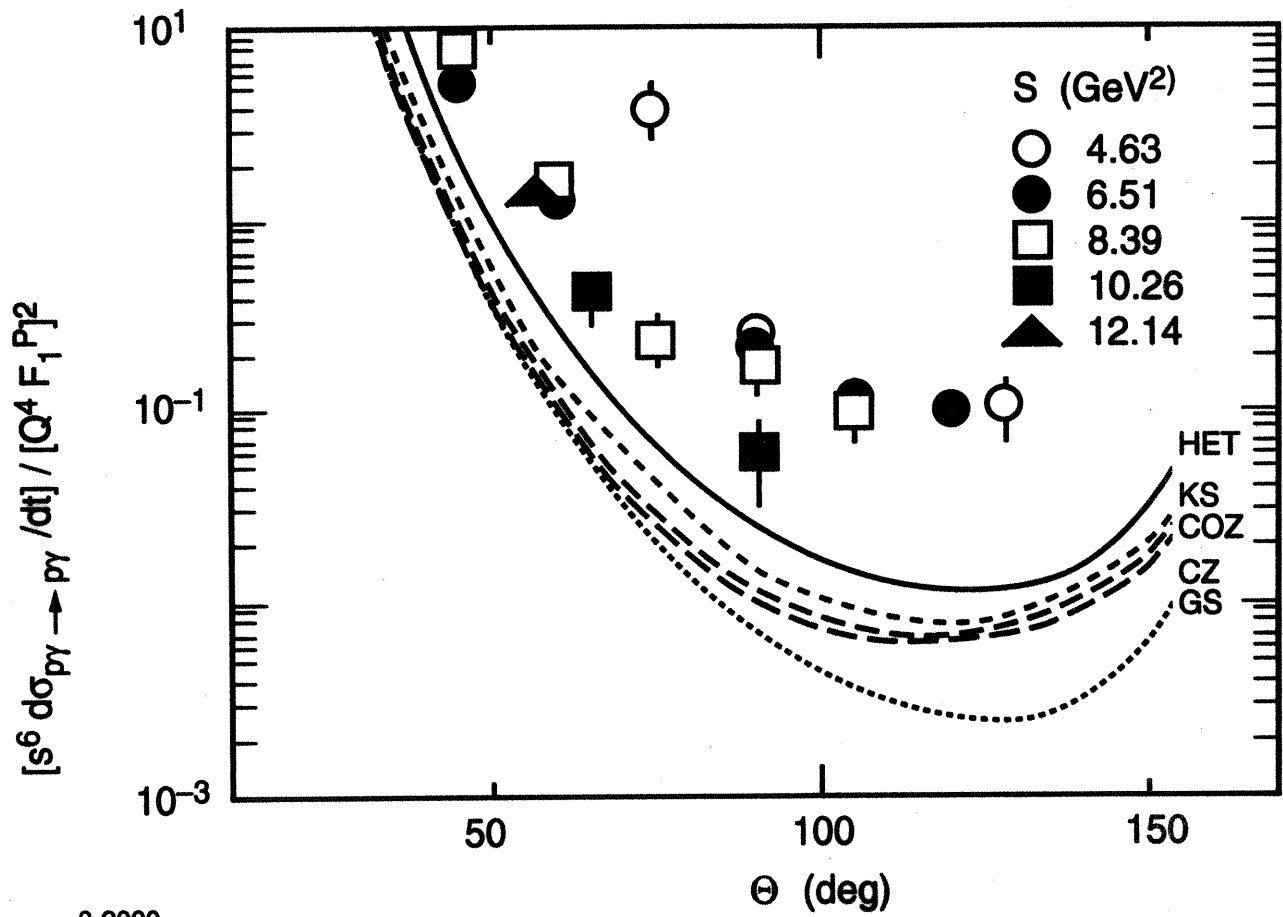
Use backscattered laser beam  $\gamma \uparrow P_r \rightarrow \gamma P$

Measure phase  $e^\pm P \rightarrow e^\pm P'$

Virtual Compton



Close  
to  
end

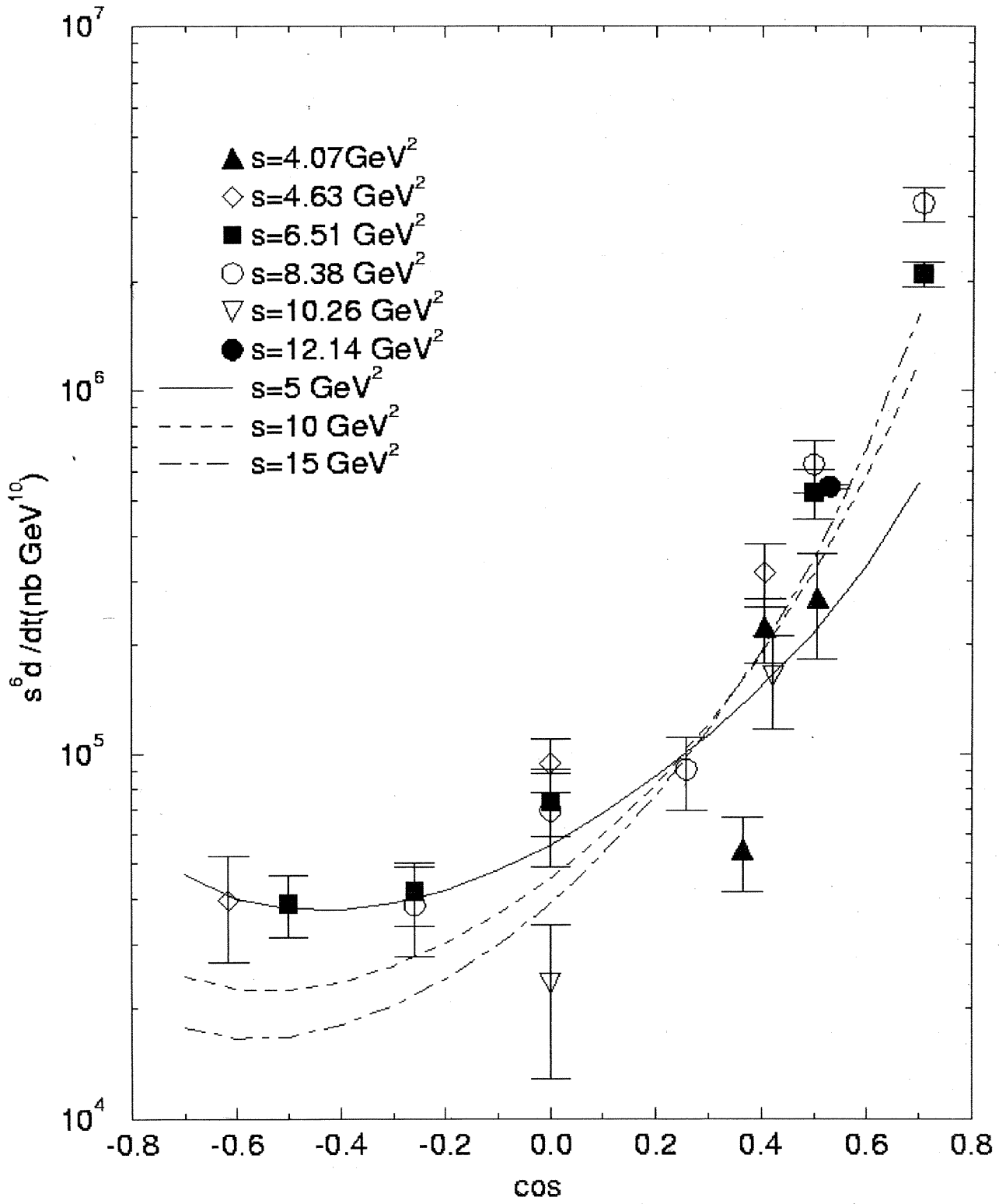


8-2000  
8561A15

Brooks + Dixon  
 $\alpha_s = .3$

# Real compton Scattering

*di quark  
model  
krall et al*



# Timelike Compton Processes

$$\begin{aligned}
 \gamma\gamma &\rightarrow P\bar{P}, n\bar{n}, \dots && \text{E760} \\
 &\rightarrow \Delta^+\Delta^-, S, S^* \dots && P\bar{P} \rightarrow \gamma\gamma \\
 &\rightarrow \Delta^{++}\Delta^{--} \\
 &\rightarrow P, \Delta^- \dots
 \end{aligned}$$

- Imparted QCD Laboratory

- PQCD  $\Rightarrow \Phi_N(x_i, Q)$  Feynman + Kramers Hqs  
 from ratios, angular distributions diquark model  
 crossing

- Threshold regime  $\Rightarrow$  novel physics?  
 Skyrmion - Anti-Skyrmion.

-  $\gamma^* \gamma \rightarrow \bar{P}P$  at fixed  $m^2$

- Virtual Compton amplitude

- Generalized distribution amplitudes



\* Normalization uncertainties cancel in

$$\frac{\mathcal{R}_{\gamma/e}}{\frac{d\sigma}{dt}(e_p \rightarrow e_p)} = \frac{\frac{d\sigma}{dt}(\gamma_p \rightarrow \gamma_p)}{\frac{d\sigma}{dt}(e_p \rightarrow e_p)} = F(\cos\theta_a)$$

T. Hye

\* Ratio highly sensitive to  $\phi_p(x_i)$  since

$$G_{mp} = 0 \quad \text{for} \quad \phi_p = x_1 x_2 x_3 \\ (\text{leading twist})$$

— Take:

$$\phi_{u \uparrow u \downarrow d \uparrow}^p(x_1, x_2, x_3)$$

$$= A [x_1 x_2 x_3 + C_1 (1 - 3x_3) + C_2 (x_1 - x_2)]$$

fit ratio to  $C_1, C_2$

Brooks  
Dixon

# Conformal Symmetry and

## Baryon Distribution Amplitudes

V. Braun, S. Derkachov,

A. Menesikov, G. Korchemsky

$$q \uparrow q \uparrow q \uparrow \Rightarrow \phi_{\Delta}^{\lambda=3/2}(x_i, \mu^2)$$

$$q \uparrow q \downarrow q \uparrow \Rightarrow \begin{cases} \phi_N^{\lambda=1/2}(x_i, \mu^2) \\ \phi_{\Delta}^{\lambda=1/2}(x_i, \mu^2) \end{cases}$$

$$\sum_{i=1}^3 x_i = 1$$

$$\phi_{\Delta}^{\lambda=1/2}(x_i, \mu^2) = x_1 x_2 x_3 \sum_{N=0}^{\infty} Q_N^{\mu_0} \left[ \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right]^{x_N}$$

Scalar diquark

$$\begin{pmatrix} \uparrow & \downarrow \\ 1 & 2 \end{pmatrix} \quad \uparrow \\ \quad \quad \quad 3$$

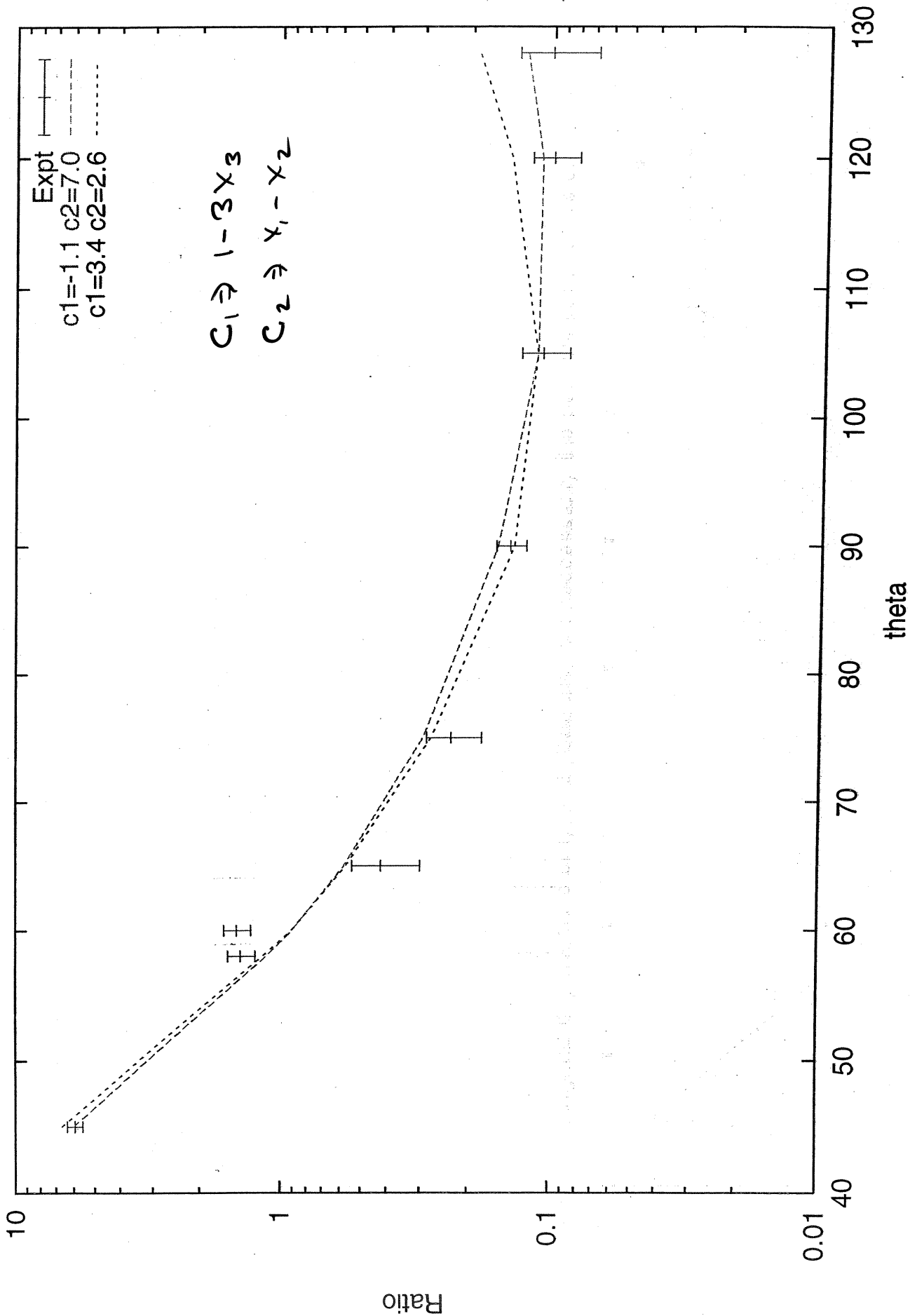
$$\times \left\{ P_N(1-2x_3) \pm P_N(1-2x_1) \right\}$$

Jacobi Polynomials  $P_N^{(1,2)}$

expansion in conformal polynomials

Brooks  
+  
Dixon  
  
(Predim)

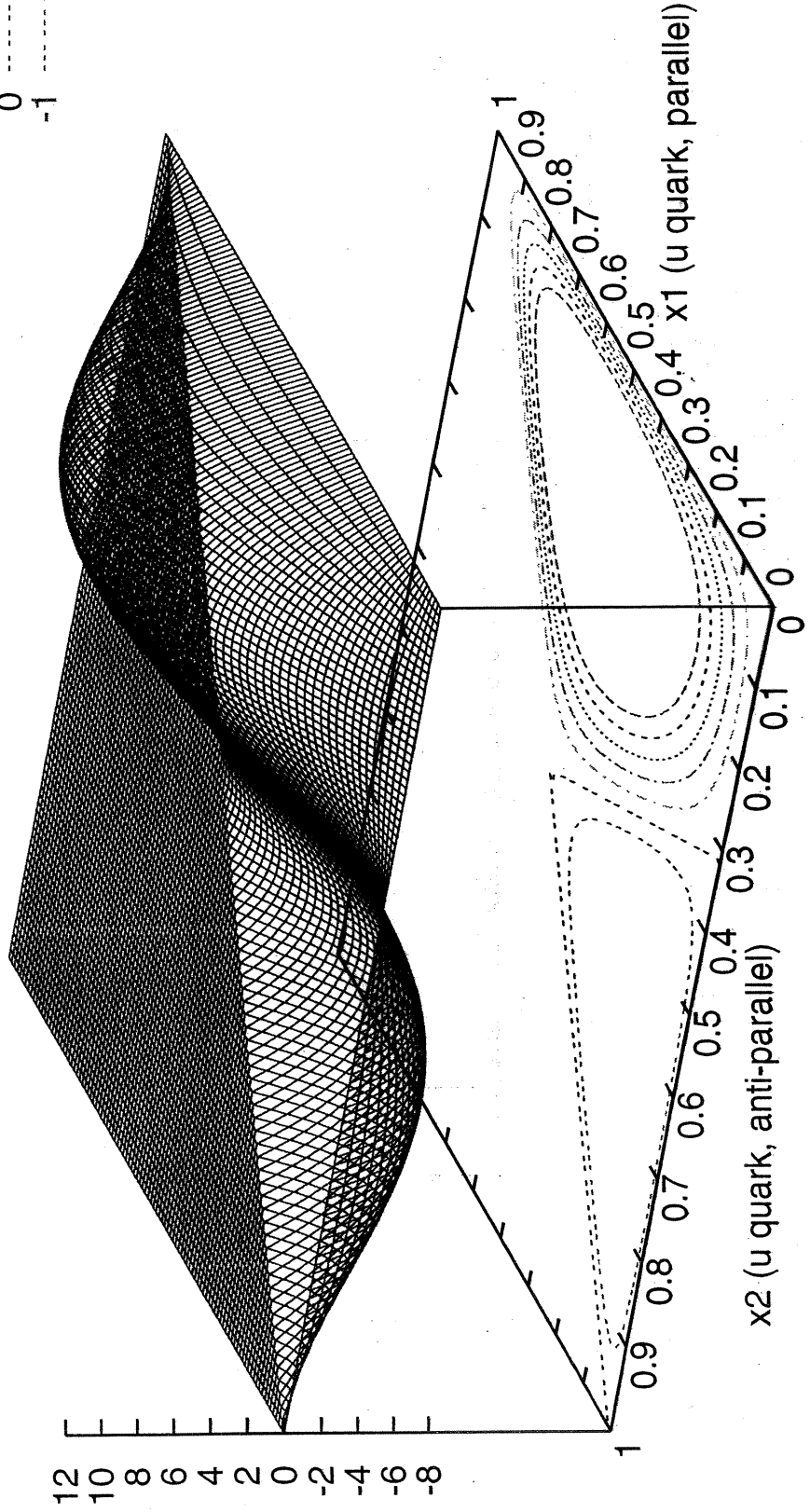
Angular dependence of typical good fits, not necessarily the best fits in the  $c_1$  vs  $c_2$  plane



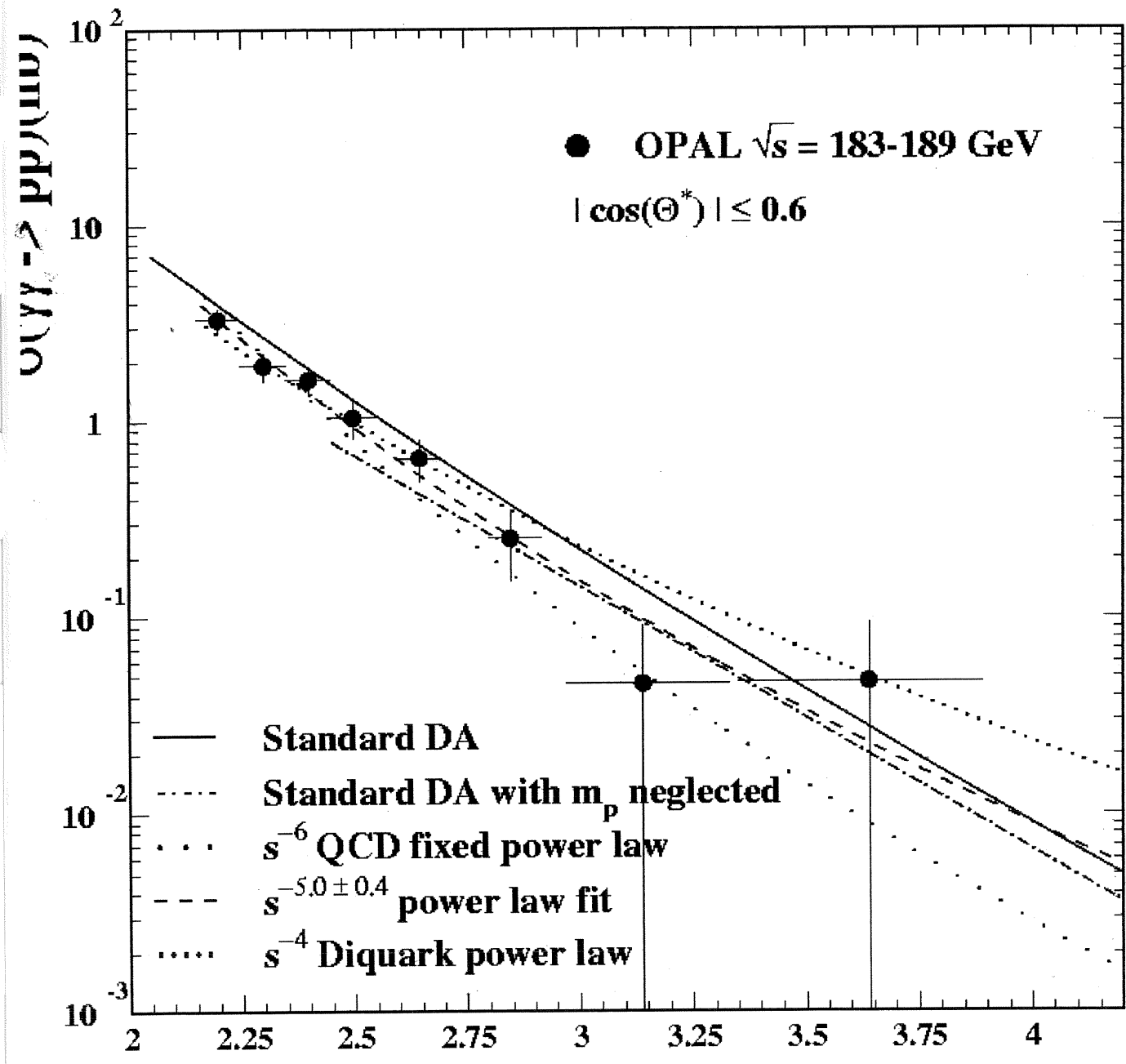
Brooks  
+  
Dron

Typical Distribution Amplitude  $c1=-1.1$   $c2=7.0$

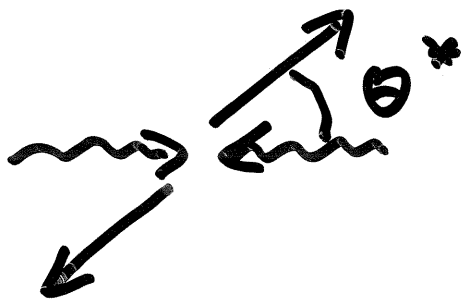
"da2.out"  
5  
4  
3  
2  
1  
0  
-1



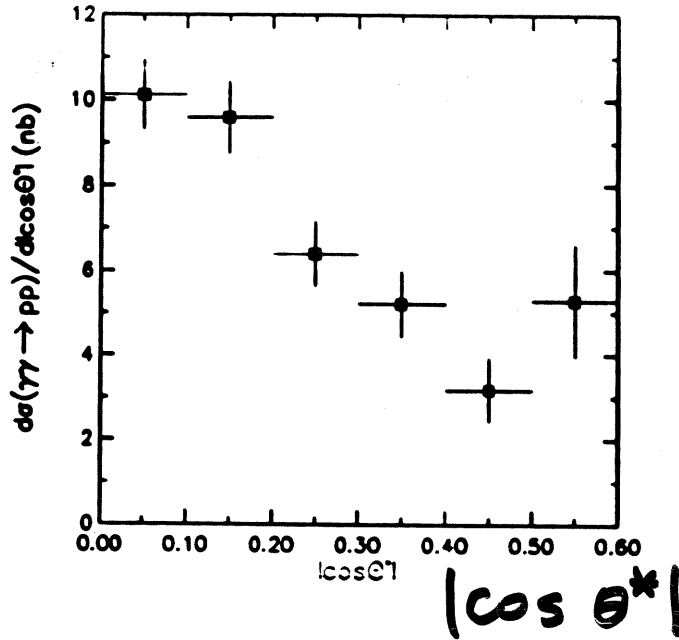
$\gamma\gamma \rightarrow P\bar{P}$



$\theta^*$ :  
(in  $\gamma\gamma$  CM)

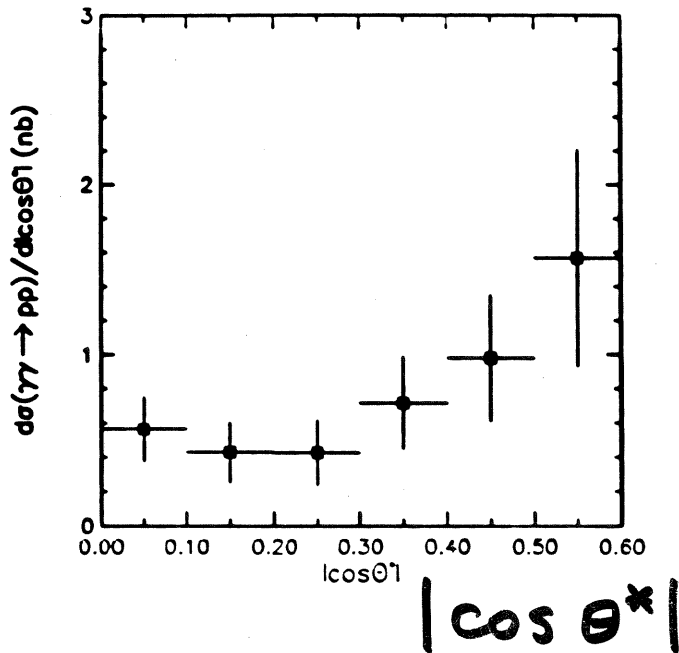


CLEO



$2 \leq W \leq 2.5$   
GeV

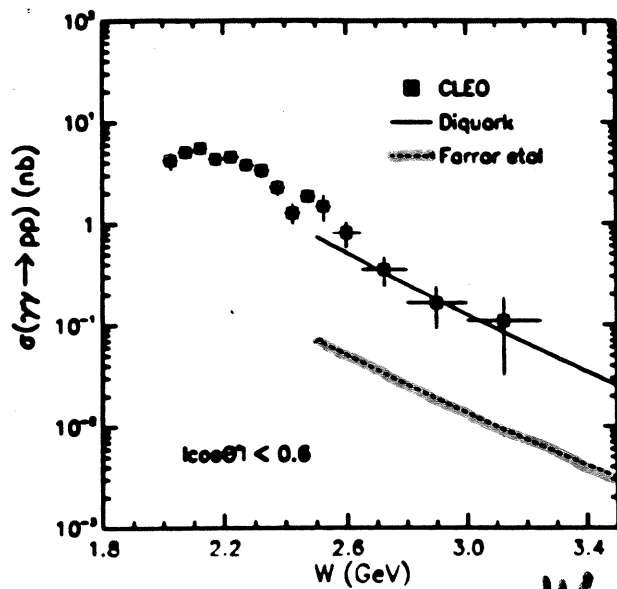
CLEO



$W > 2.5$   
GeV

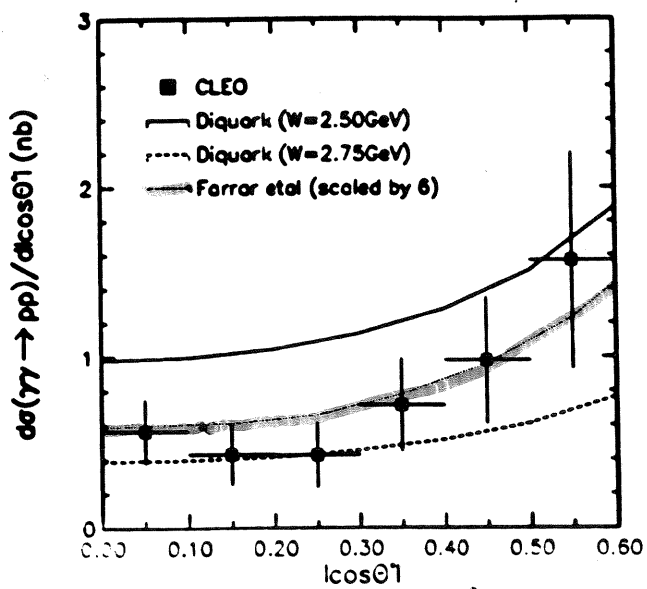
$$\underline{\gamma\gamma \rightarrow p\bar{p}}$$

CLEO



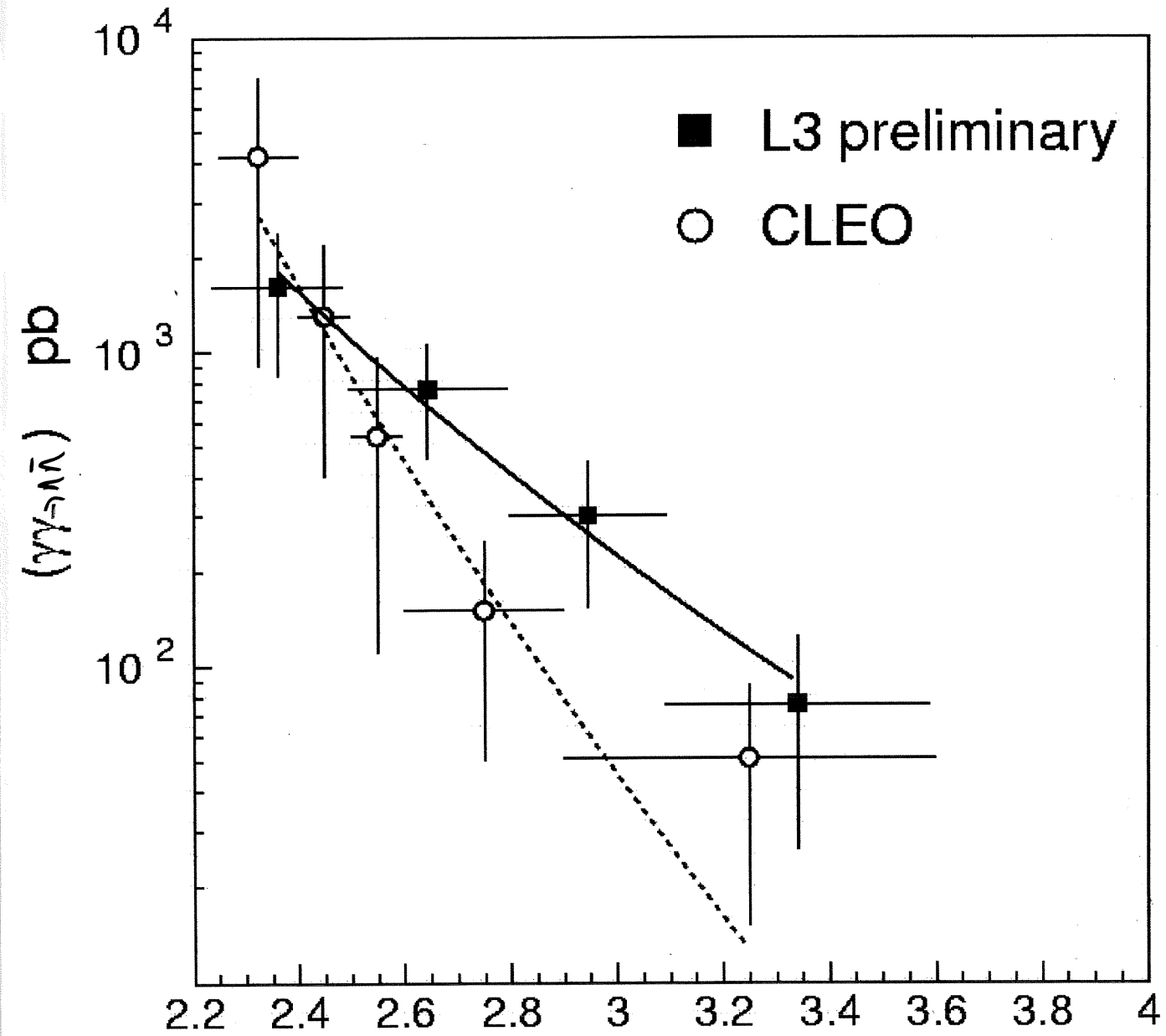
$W_{\gamma\gamma}$

CLEO



$|\cos\theta^*|$

$\gamma\gamma \rightarrow \Lambda \bar{\Lambda}$





Measurements of  
 $\Phi_H(x_i, Q)$   
 Central problem of QCD

$$\frac{d\sigma}{dt} \left[ \gamma\gamma \rightarrow H \bar{H} \right]_{\lambda_1 \lambda_2 \lambda_3 \lambda_4}$$

Scaling, helicity, angular structure

Retros critical [as ~ cancells]

$$\frac{\gamma\gamma \rightarrow \pi^0 \pi^0}{\gamma\gamma \rightarrow \pi^+ \pi^-} \Rightarrow \Phi_{\pi^0}(x, Q)$$

$$\frac{\gamma\gamma \rightarrow n \bar{n}}{\gamma\gamma \rightarrow p \bar{p}} \Rightarrow \Phi_N(x, Q)$$

CLEO, Babar, Belle, LEP, ...

opportunity for fundamental physics

# Summary

$\gamma\gamma \rightarrow$  Exclusive Channels

Experiment sees transition to  
angular distributions pred. by PQCD

Connections to Soft-physics

Analyticity

Soft-pion theorems

Soliton-anti-soliton formation at  $W=W_m$

Low energy theorems

Resonance phenomena

Compare  $\gamma\gamma \rightarrow X$  with  $J/\psi \rightarrow \gamma X$   
for glueball search

PQCD :  $\gamma\gamma, \gamma^*\gamma, \gamma^*\gamma^*$   
window to distribution amplitudes

$\Rightarrow$  critical for B-physics