Flame ~~Physics~~ Math(s) + pretty pictures (moving ones too)

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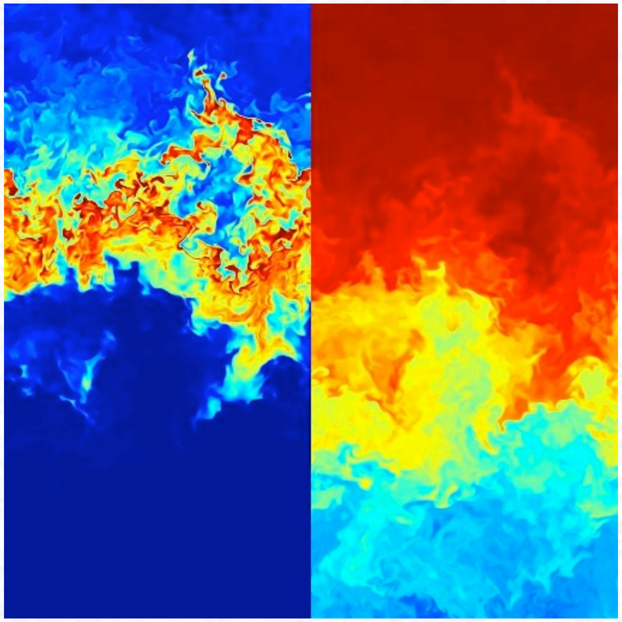
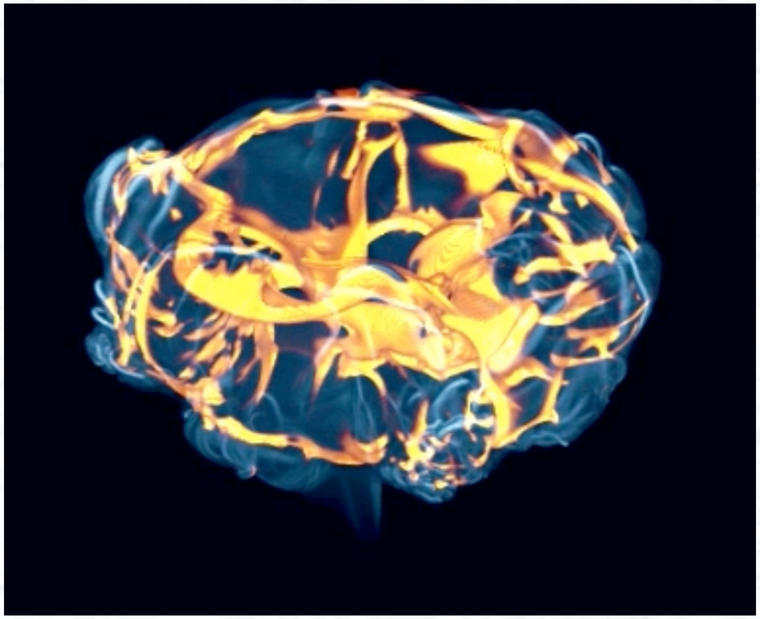
A game of two halves...



Turbulent flame modeling for
carbon-burning flames at large scales



Buoyant burning bubbles

First Half Objective

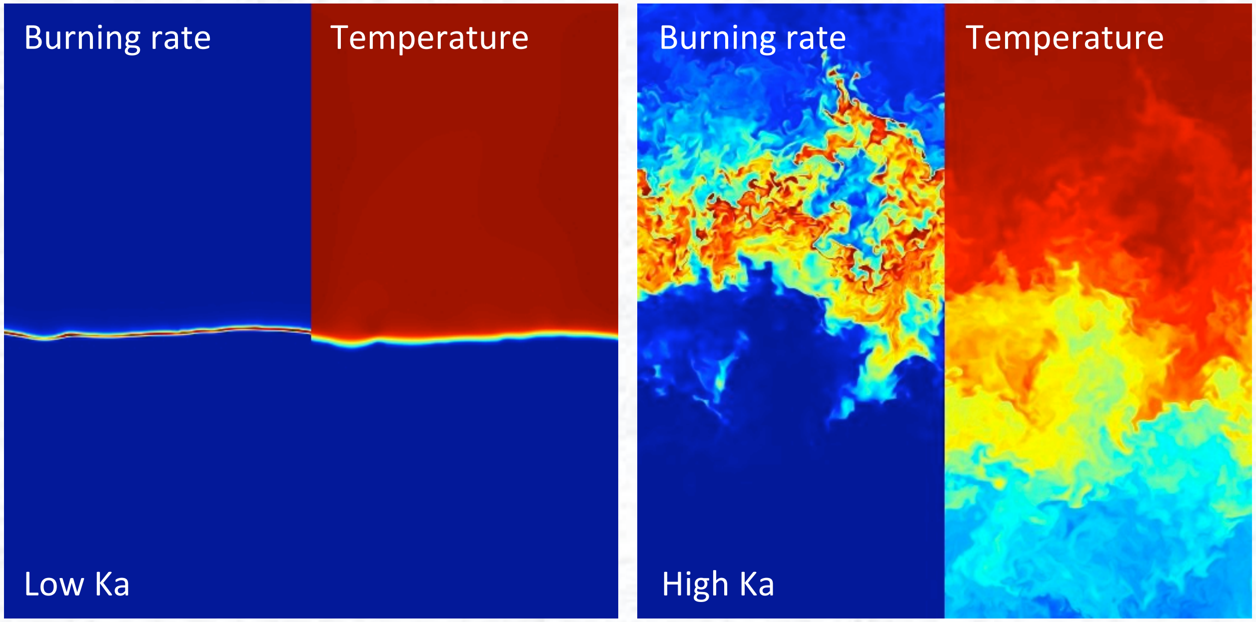
- Turbulent flame model for carbon-burning in SNe Ia
- Previous study (APS 2008) examined fully-resolved small-scale carbon-burning thermonuclear flames at high turbulence levels
- Present objective is to investigate much larger length scales
- Construct a turbulent flame model for large-scale distributed burning

First Half Outline

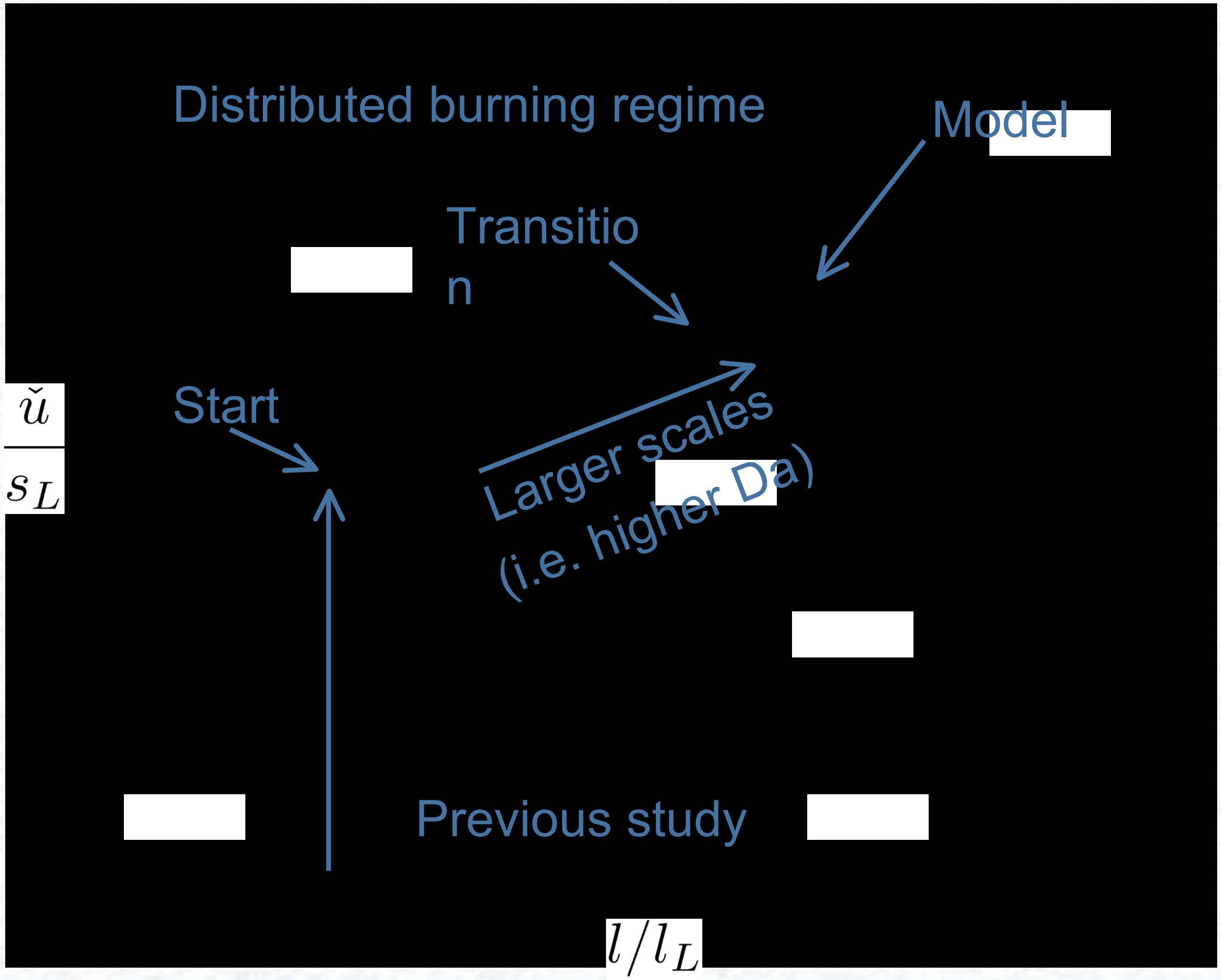
- Theoretical treatment of burning in distributed regime
- Based on Damköhler scaling (1940)
- Predict scaling relations for turbulent flame speed and width
- Three-dimensional simulations to test predictions
- Aspden, Bell, Woosley, *Distributed Flames in SNe Ia*, ApJ, **710**, 2010

Previous study

- Aspden, Bell, Day, Woosley and Zingale
  - *Turbulence-Flame Interactions in Type Ia Supernovae,* ApJ **689** (2008)
- At sufficiently high turbulence, a distributed flame was observed
- Mixing and transport are dominated by turbulence
- Turbulent flame speed much lower than turbulent intensity
- Turbulent flame width much larger than integral length scale



Regime Diagram



Turbulent intensity and integral length

$\check{u} \quad l$

Laminar flame speed and width

$s_L \quad l_L$

Karlovitz number

$$\mathrm{Ka}^2 = \frac{\check{u}^3}{s_L^3}\frac{l_L}{l}$$

Damköhler number

$$\mathrm{Da} = \frac{s_L}{\check{u}}\frac{l}{l_L}$$

Theory – Small Damköhler

- High Karlovitz number, small Damköhler number
- Damköhler (1940) "small-scale turbulence" regime
- Argued turbulence modifies transport – dominant mixing is turbulent
- Draw analogy with laminar flames
- Predict turbulent flame speed and width (in terms of diffusion and time scale)

$$s_T = \sqrt{\frac{\mathcal{D}_T}{\tau^T_{\mathrm{nuc}}}} \qquad l_T = \sqrt{\mathcal{D}_T \tau^T_{\mathrm{nuc}}}$$

- Time scale is inductive, longer than turbulence time scale, assumed constant
- Diffusion coefficient is due to turbulence (let's use a simple eddy viscosity)

$$\mathcal{D}_T = \alpha \check{u} l = \alpha {\varepsilon^*}^{1/3} l^{4/3} \qquad (\varepsilon^* = \check{u}^3/l)$$

- Gives scaling relations for fixed Karlovitz number (equivalent to a fixed energy dissipation rate)

$$s_T \sim l^{2/3} \qquad l_T \sim l^{2/3} \qquad \check{u} \sim l^{1/3} \qquad \tau_T = l/\check{u} \sim l^{2/3}$$

- Note for small Damköhler number

$$s_T < \check{u} \qquad l_T > l \qquad \tau^T_{\mathrm{nuc}} > \tau_T$$

Theory – Break Down

- Scaling breaks down when turbulence time scale comparable with turbulent nuclear burning time scale

$$\tau_T \approx \tau_{\rm nuc}^T$$

- Mixing no longer faster than burning
- Defining a turbulent Damköhler number

$$\mathrm{Da}_T = \frac{\tau_T}{\tau_{\rm nuc}^T} = \frac{s_T}{\check{u}}\frac{l}{l_T} = \sigma \mathrm{Da} \qquad \left(\sigma = \frac{\tau_{\rm nuc}^L}{\tau_{\rm nuc}^T}\right)$$

- Expect break down of the scaling relations at $\mathrm{Da}_T=1$
- Defining a turbulent Karlovitz number as

$$\mathrm{Ka}_T^2 = \frac{\check{u}^3}{s_T^3}\frac{l_T}{l}$$

- It can be shown that

$$\mathrm{Da}_T^2 \mathrm{Ka}_T^2 = 1 \qquad (\alpha = 1)$$

- So at the breakdown of the scaling relations

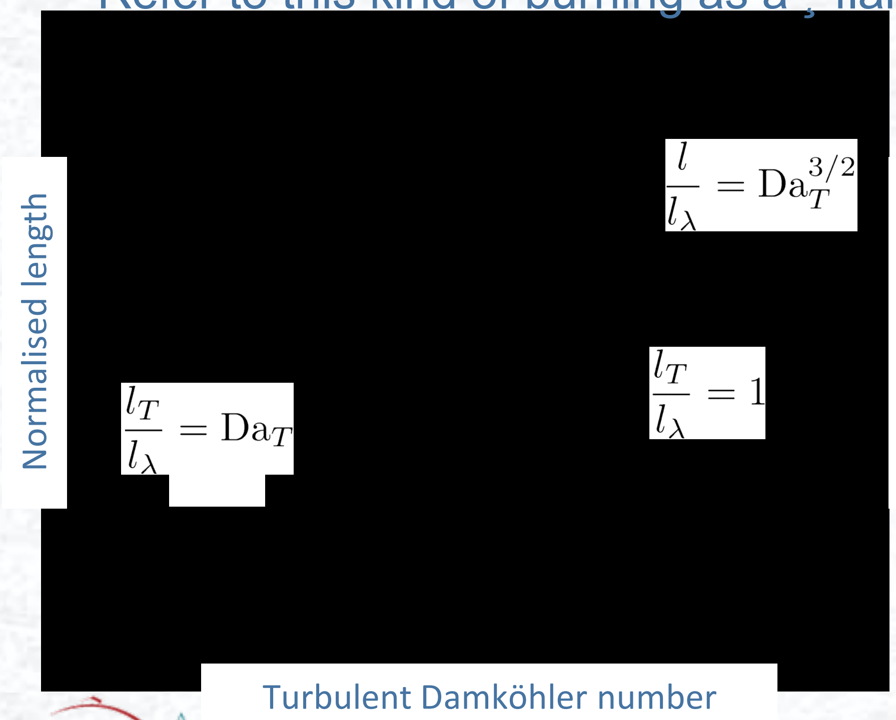
$$\mathrm{Da}_T = 1 \qquad \mathrm{Ka}_T = 1 \qquad s_T = \check{u} \qquad l_T = l$$

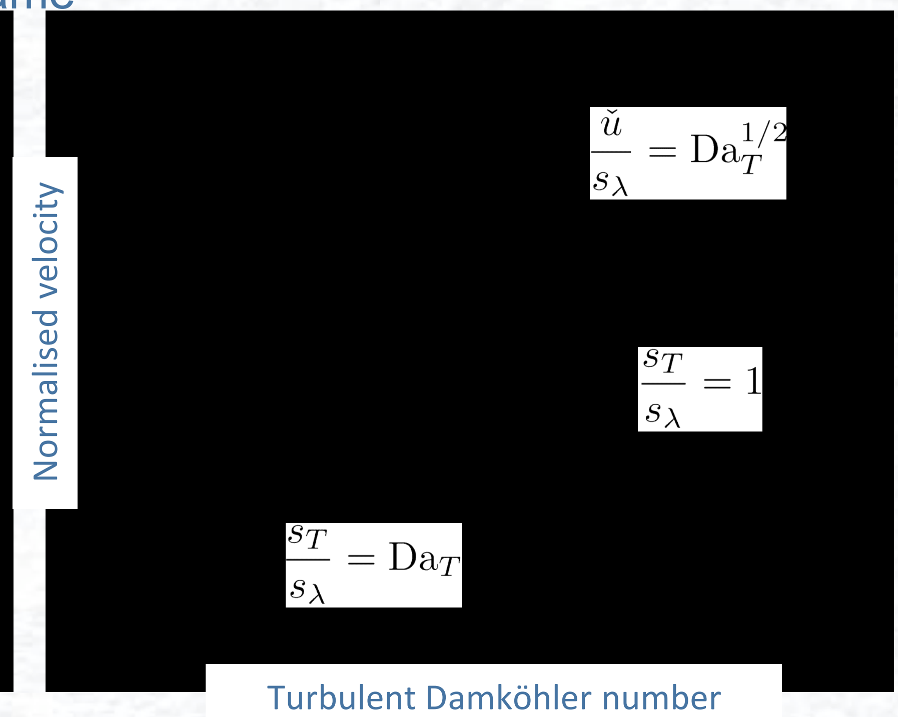
Theory – Large Damköhler

- For larger Da_T, turbulence cannot broaden the flame any further
- A limiting behaviour is reached
- Burns as a turbulently broadened effective unity Lewis number flame
- Local flame speed and width are constant (higher due to enhanced area)

$$s_T = s_\lambda \qquad l_T = l_\lambda$$ (on scale of l_λ)

- Refer to this kind of burning as a λ-flame





Theory – Lambda flames

- Can we predict the ¸-flame properties?
- Depend solely on turbulence intensity and burning time scale
- Dimensional analysis – four quantities in two units $\varepsilon^*,\, \tau_{\mathrm{nuc}}^T,\, s_\lambda,\, l_\lambda$
- Two dimensionless quantities

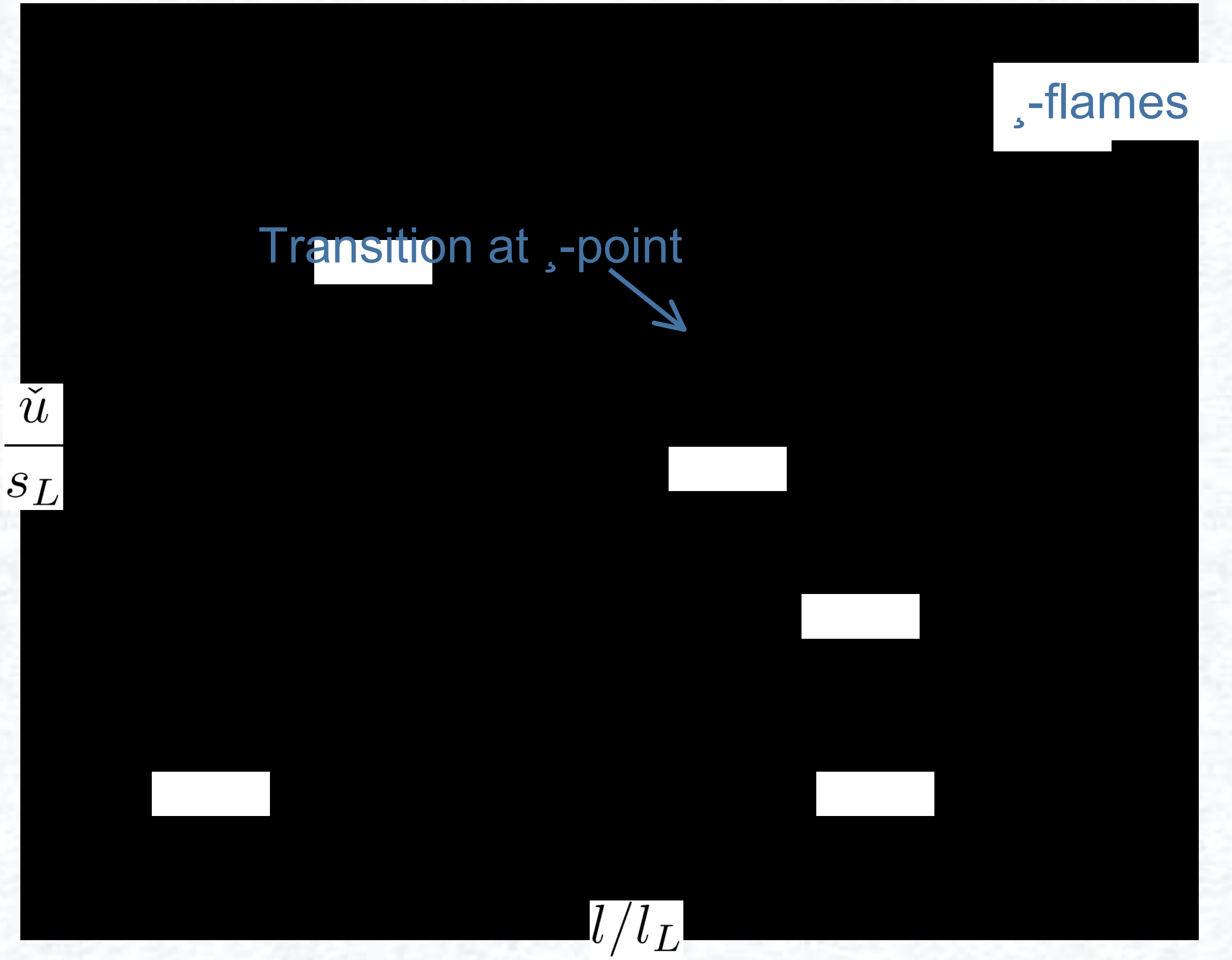
$$\Pi_1 = \frac{\varepsilon^* l_\lambda}{s_\lambda^3} \qquad \Pi_2 = \frac{\tau_{\mathrm{nuc}}^T s_\lambda}{l_\lambda}$$

- Both are identically equal to one, which implies

$$s_\lambda = \sqrt{\varepsilon^* \tau_{\mathrm{nuc}}^T} \qquad l_\lambda = \sqrt{\varepsilon^* {\tau_{\mathrm{nuc}}^T}^3}$$

- What is the turbulent nuclear burning time scale?
- Reference case with turbulent intensity and integral length $\check{u}_0,\, l_0$
- Measure the turbulent flame speed only s_T^0
- Use the relation $\mathrm{Da}_T^2 \mathrm{Ka}_T^2 = 1 \quad \Longrightarrow \quad l_T^0 = \dfrac{\check{u}_0 l_0}{s_T^0}$
- Then, by definition $\tau_{\mathrm{nuc}}^T = \dfrac{\check{u}_0 l_0}{{s_T^0}^2}$

Modified Regime Diagram



Numerical Solver

- Written at Center for Computational Sciences and Engineering
- Based on 3D variable-density incompressible Navier-Stokes solver
- Extended for low Mach number SNe flames
- Cartesian finite-volume discretisation
- Predictor-corrector approach
- Advection-diffusion and chemistry are operator split
- Approximate projection for divergence constraint
- Overall second-order accurate in space and time
- Adaptive mesh refinement to focus resolution on regions of interest
- Parallelised – performs well up to several thousand processors
- Capable of implicit LES calculations – don't need a turbulence model
    - Aspden *et. al,* CAMCoS **3** (2008)
- Further details can be found in Bell *et. al,* JCP **195** (2004)

Schematic

Three dimensional box

Fuel below ash

Propagates downwards

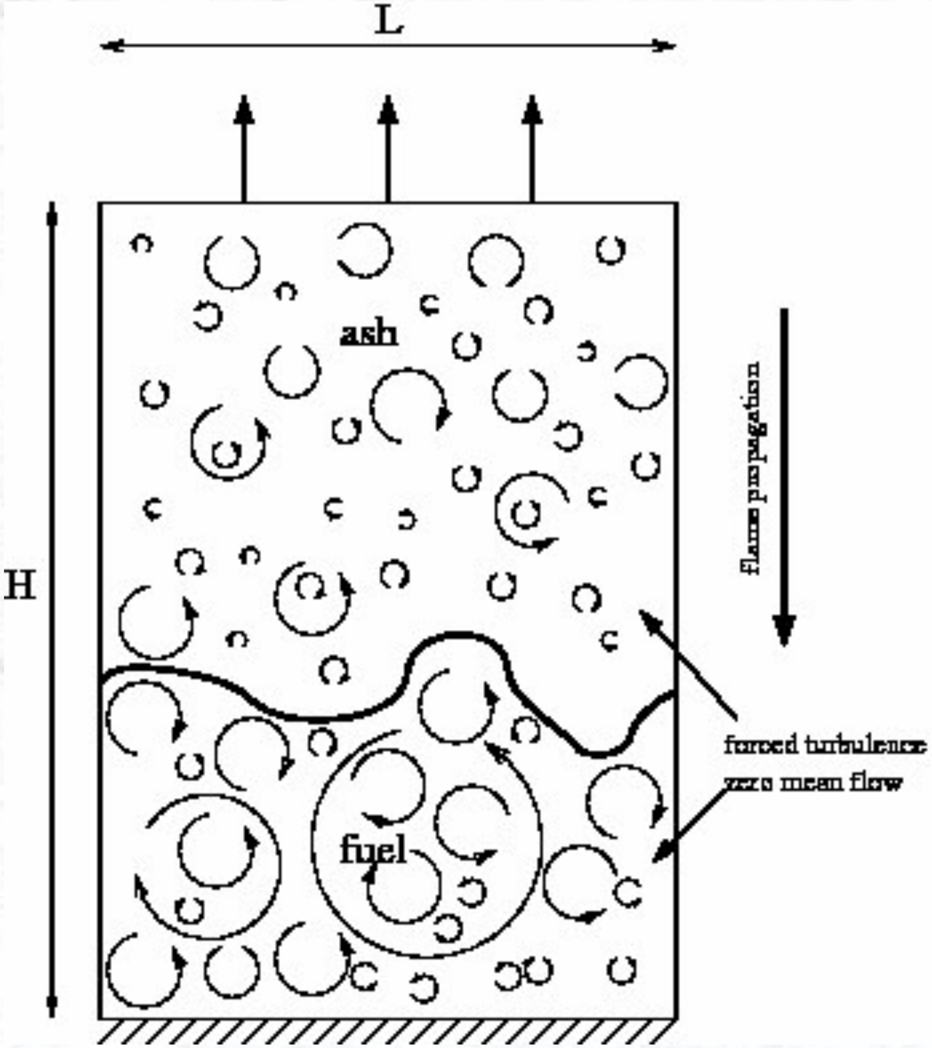
High aspect ratio

Forced throughout

Periodic sides

Solid base

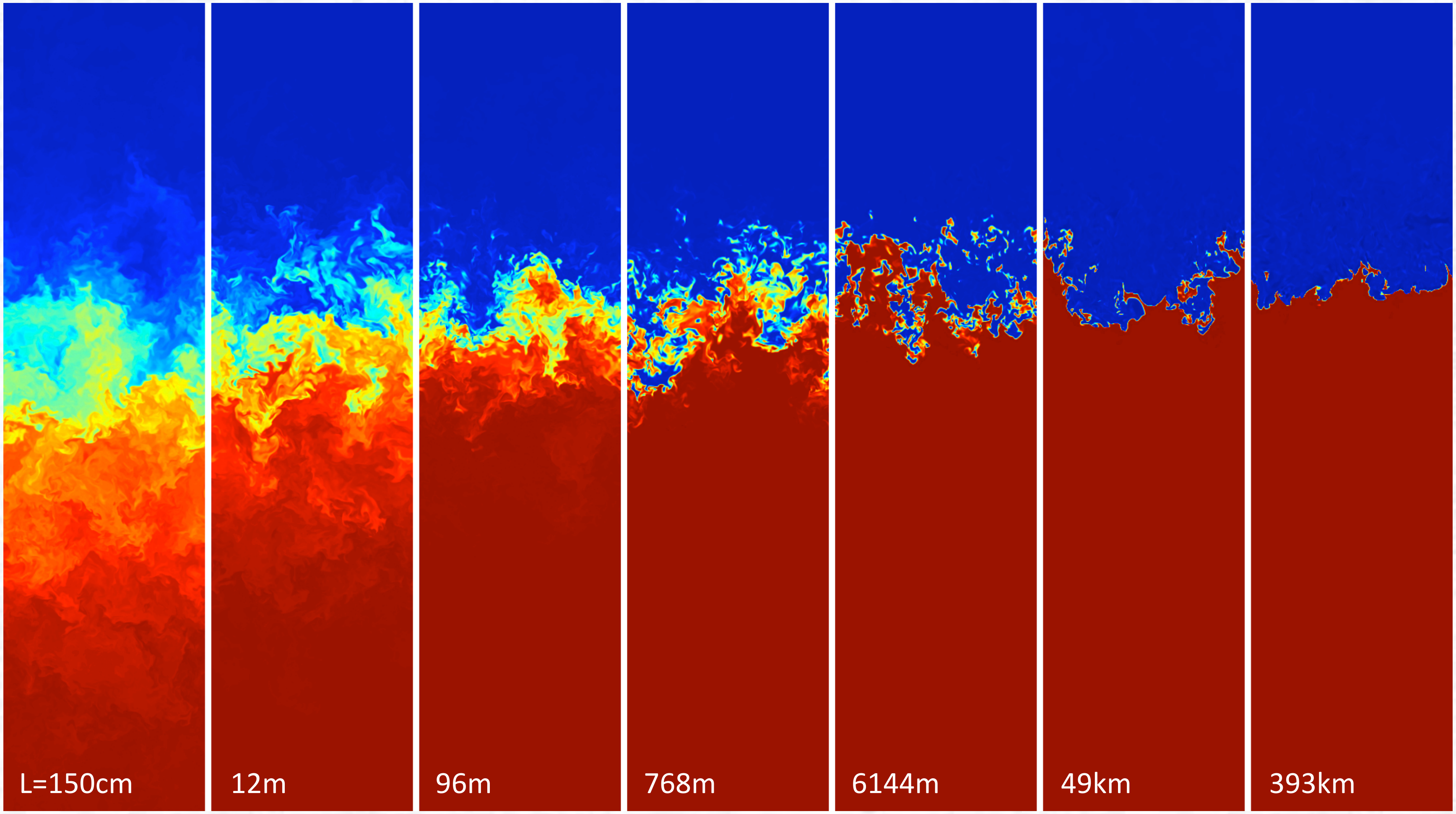
Outflow at top



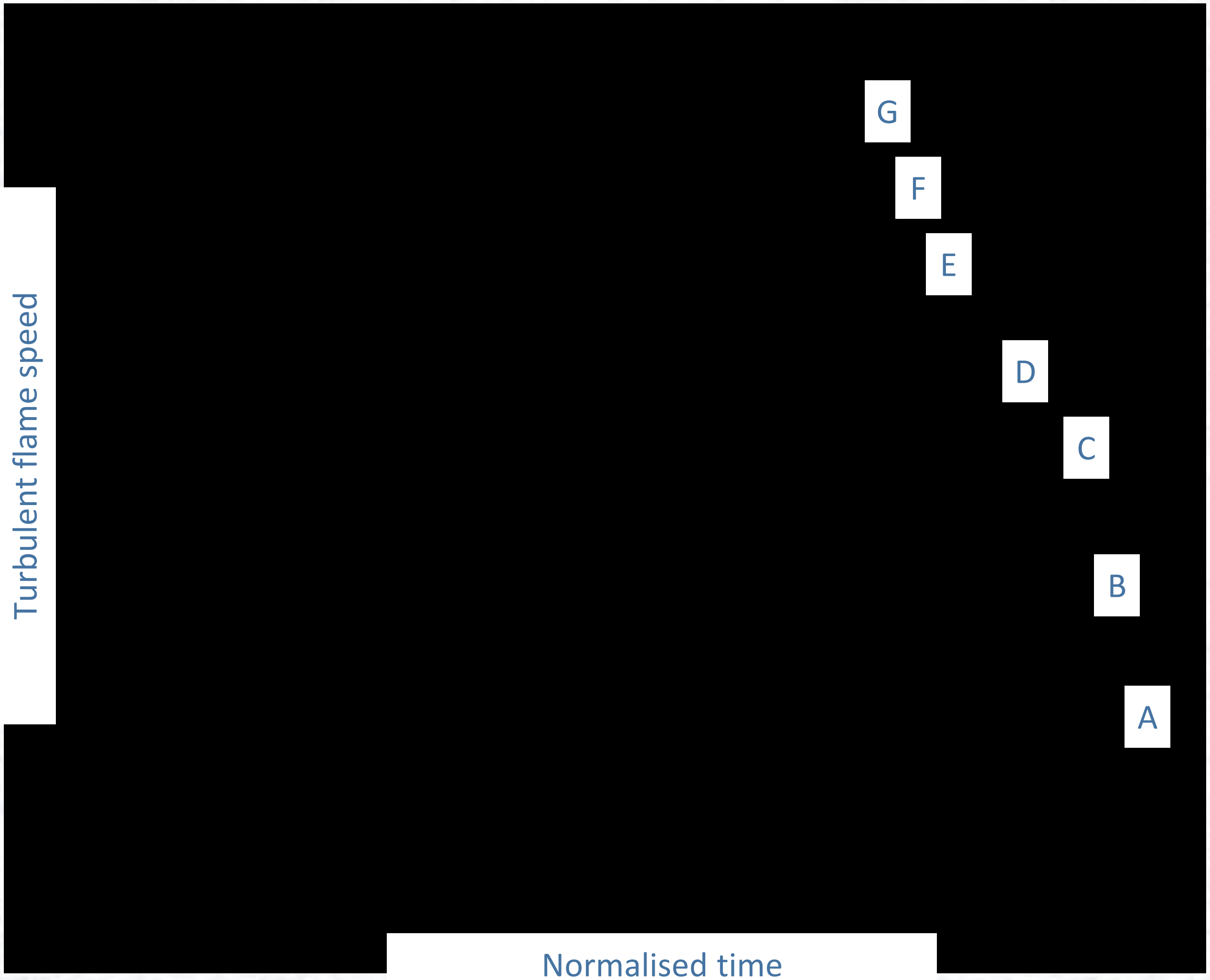
Procedure

- Aim is to simulate larger length scales (Da) keeping Ka fixed
- Resolution requirements become relaxed for distributed flames
- Mixing due to turbulence, relevant scales grow with integral length
- Start with high Karlovitz number case from ApJ paper (256x256x2048)
- Reduce resolution by a factor of 8 (32x32x256)
  - i.e. computational cell size 8 times larger
- Use turbulent flame speed as diagnostic check
- Use the new cell size to run in a domain 8 times larger
  - Adjust turbulent intensity accordingly to fix energy dissipation rate (Ka)
- Repeat
- Limited by Da_T – expect to be reasonably valid for $Da_T<1$
- For $Da_T>1$, relevant scales are fixed
- Seven cases A-G

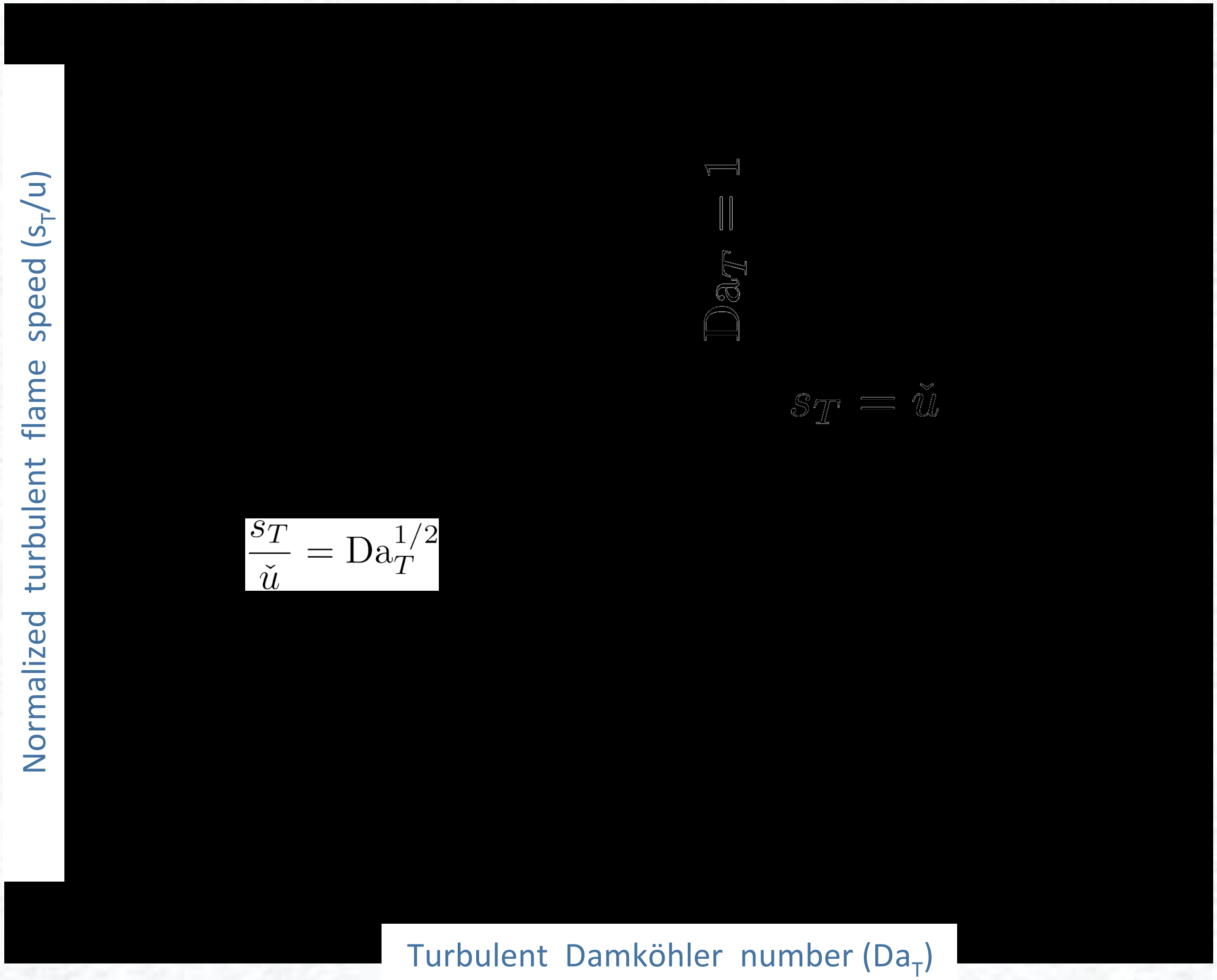
Slices of Density



Turbulent Flame Speeds



Turbulent Flame Speeds



First Half Conclusions

- Formulated and verified scaling relations at high Ka
  - More importantly extending to high Da
- Can predict constant local flame speed and width
- Perfectly suited to level set approach
  - Suggested an approach to describe the flame speed
- Tens of thousands times larger than original study in each dimension
- Overall flame speed is highly fluctuating
- Possible that these fluctuations may lead to run-away
- Aspden, Bell, Woosley, *Distributed Flames in SNe Ia*, ApJ, **710**, 2010

Second Half Outline

- Buoyant burning bubbles – first flames
  - Ignition leads to isolated burning bubbles that rise due to buoyancy
- Previous work has focused on early stages
- Here looking for late-time self-similar asymptotic behavior?
- Known in fluid dynamics literature as thermals (buoyant vortex rings)
- Theoretical approach based on Morton, Taylor, Turner (1956)
  - Entrainment assumption
- Numerical simulations to investigate this theory

Morton Taylor Turner Theory

- Idealized thermal
  - Represent thermal as sphere of radius b(t) at height $z_b(t)$
- Entrainment assumption
  - Fluid is entrained into the thermal at a rate proportional to the rise height
- Mixing is sufficiently fast that entrained fluid is mixed instantaneously
- Conservation equations for volume, momentum and buoyancy

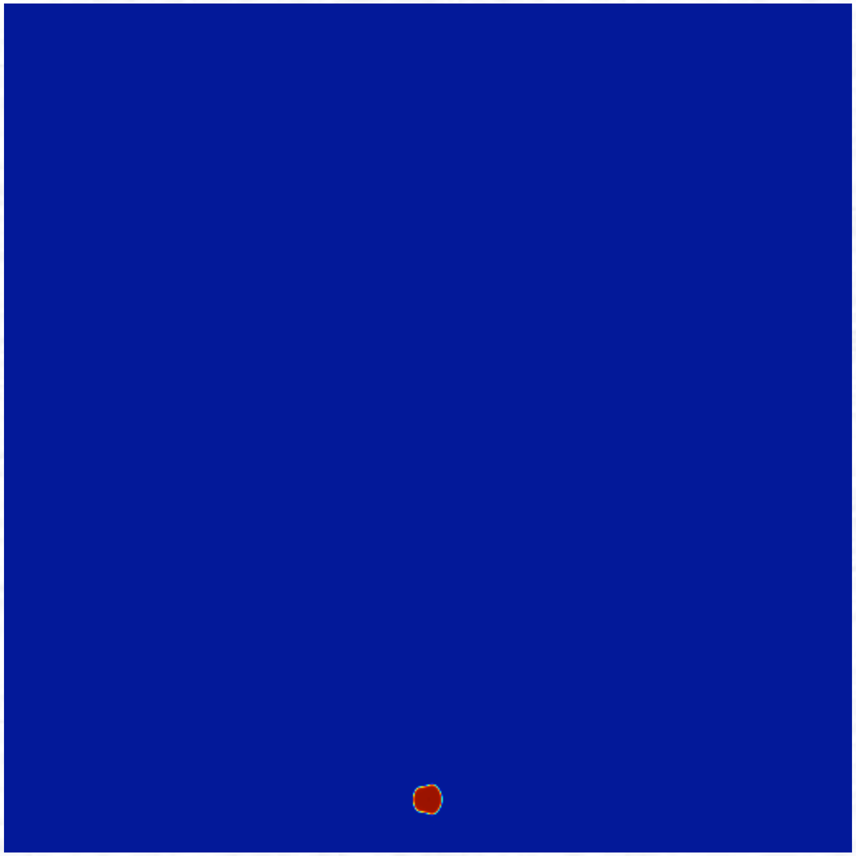
$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{4}{3}\pi b^3\right) &= 4\pi b^2 \alpha u_b \\
\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{4}{3}\pi b^3 \rho_i u_b\right) &= \frac{4}{3}\pi b^3 \left(\rho_e - \rho_i\right) g \\
\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{4}{3}\pi b^3 k g \frac{\rho_e - \rho_i}{\rho_0}\right) &= -\frac{4}{3}\pi b^3 u_b N^2
\end{aligned}
$$

α	entrainement coeff.
ρ_i	interior density
ρ_e	exterior density
ρ_0	reference density
u_b	rise velocity
N	buoyancy freq.

- Interesting property – evolves in a cone $b = \alpha z$

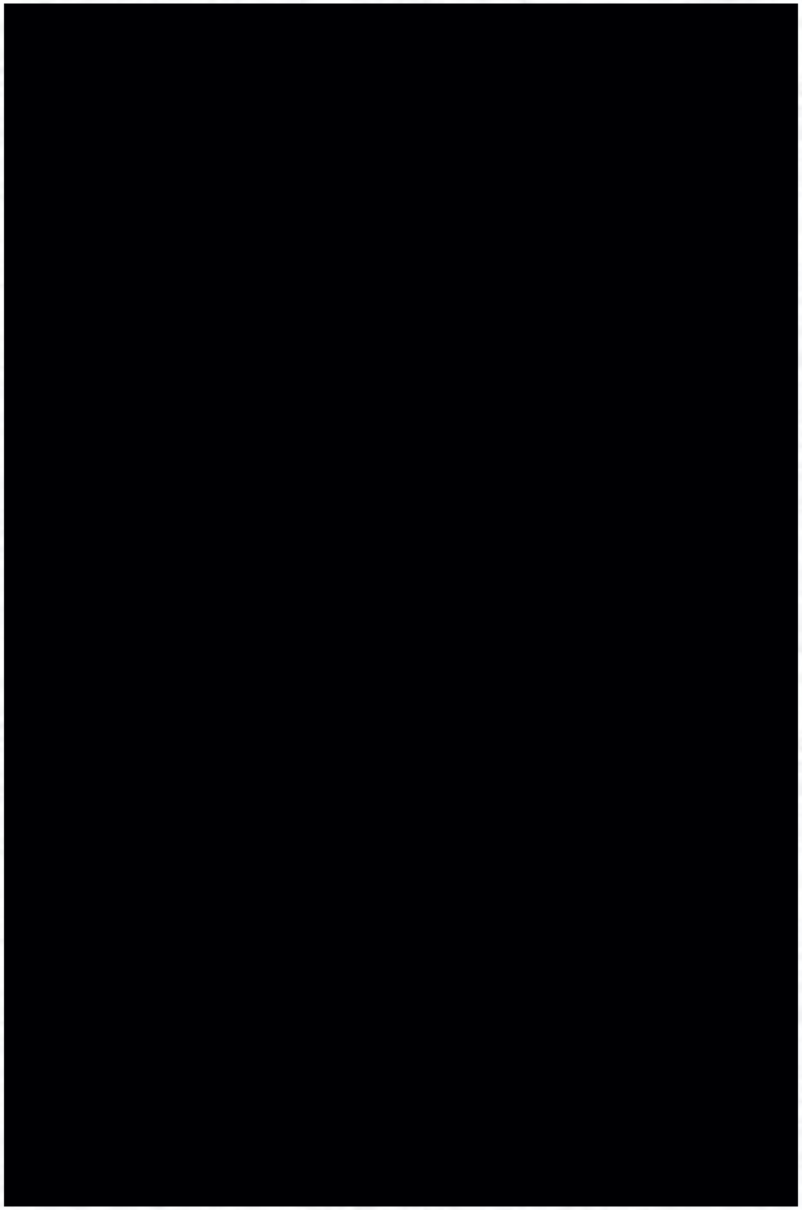
Initial Conditions

- 864 cm cube domain
- Resolutions up to 4096^3
    - Base grid 512^3 + 3 levels AMR
- Initial bubble radius about 14cm
- Perturbation to break symmetry
- Fuel density 1.5e7g/cm^3
- Gravity 10^9 cm/s^2
- Solid base, outflow elsewhere



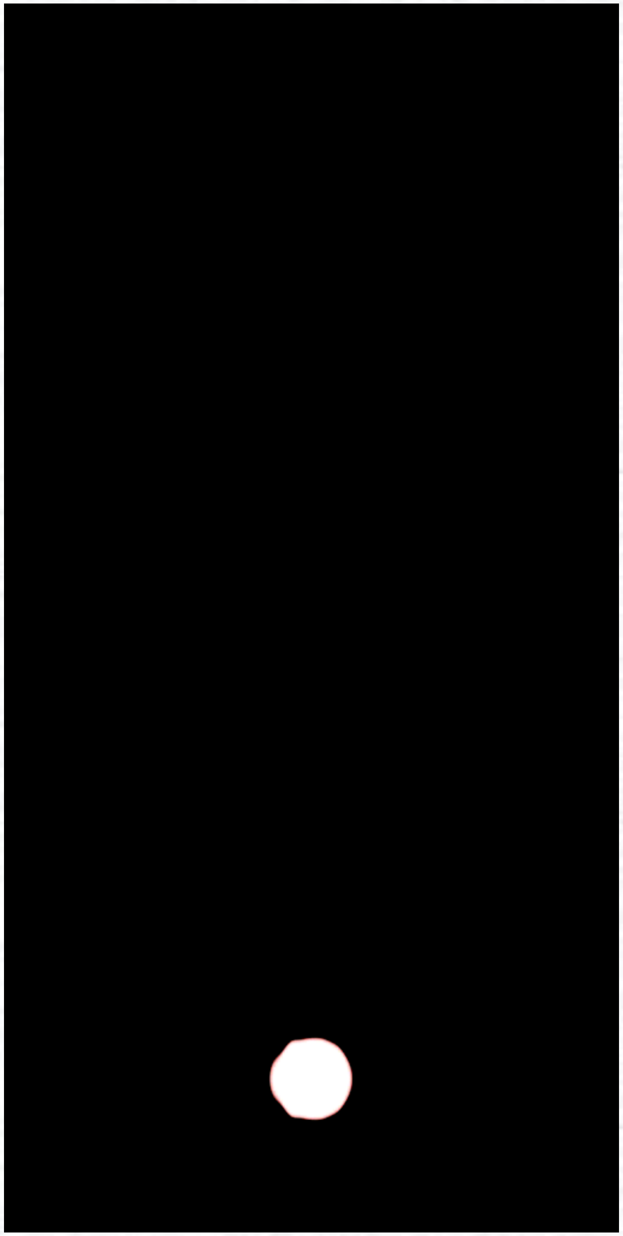
Inert Thermal

Vorticity
3d Rendering



Inert Thermal

Tracer Slices



MTT Theory Revisited

- How do we account for burning?
- Assume: Entrained fluid can be considered to burn instantly
- Have two discrete states – inside/outside
- Two constant densities) one conservation equation is redundant
- Conservation equations for volume and momentum

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{4}{3}\pi b^3\right) = 4\pi b^2 \sigma \alpha u_b \qquad \sigma = \frac{\rho_e}{\rho_i}$$

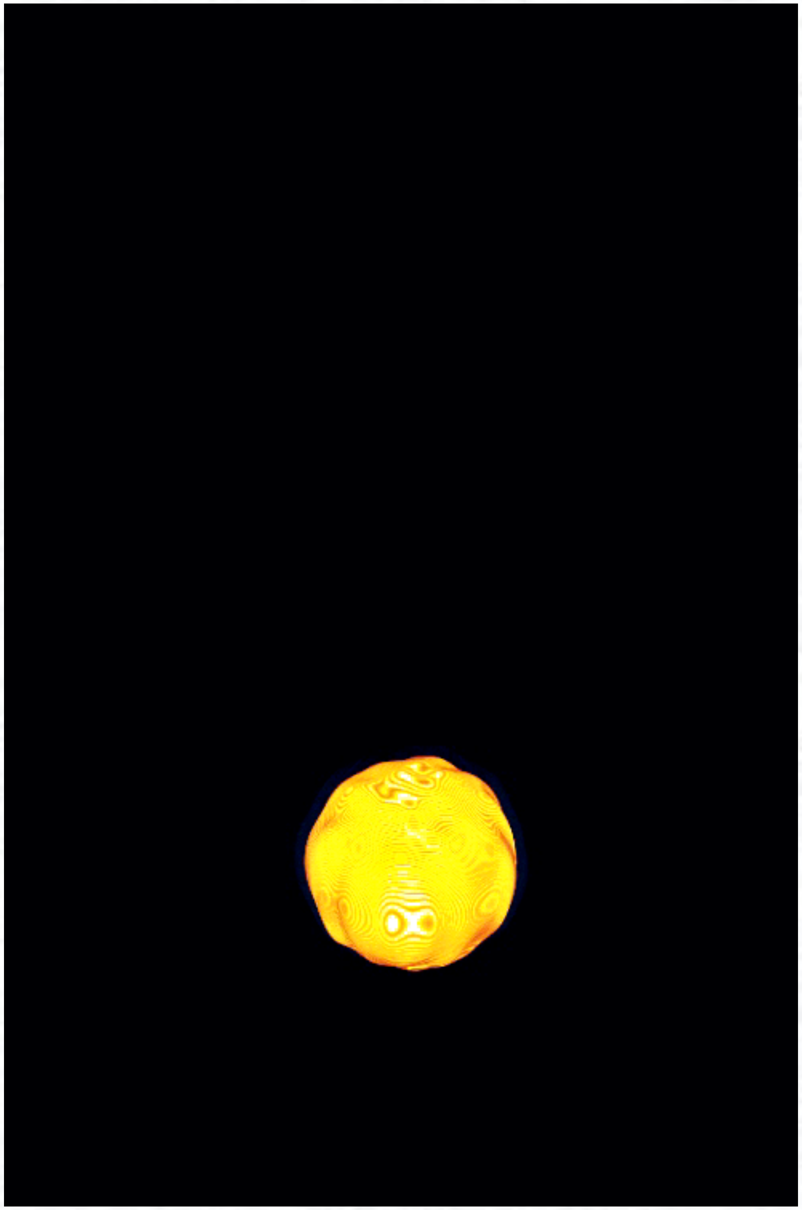
$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{4}{3}\pi b^3 u_b\right) = \frac{4}{3}\pi b^3 \sigma g' \qquad g' = \frac{\rho_e - \rho_i}{\rho_e} g$$

- Again conical: $b = \sigma \alpha z$
- Second-order non-linear ODE: $\frac{\mathrm{d}^2 z_b}{\mathrm{d}t^2} + \frac{3}{z_b}\left(\frac{\mathrm{d}z_b}{\mathrm{d}t}\right)^2 = \sigma g'$
- Which has the solution (for suitably defined virtual origin) $z_b = At^2$

Burning Bubble

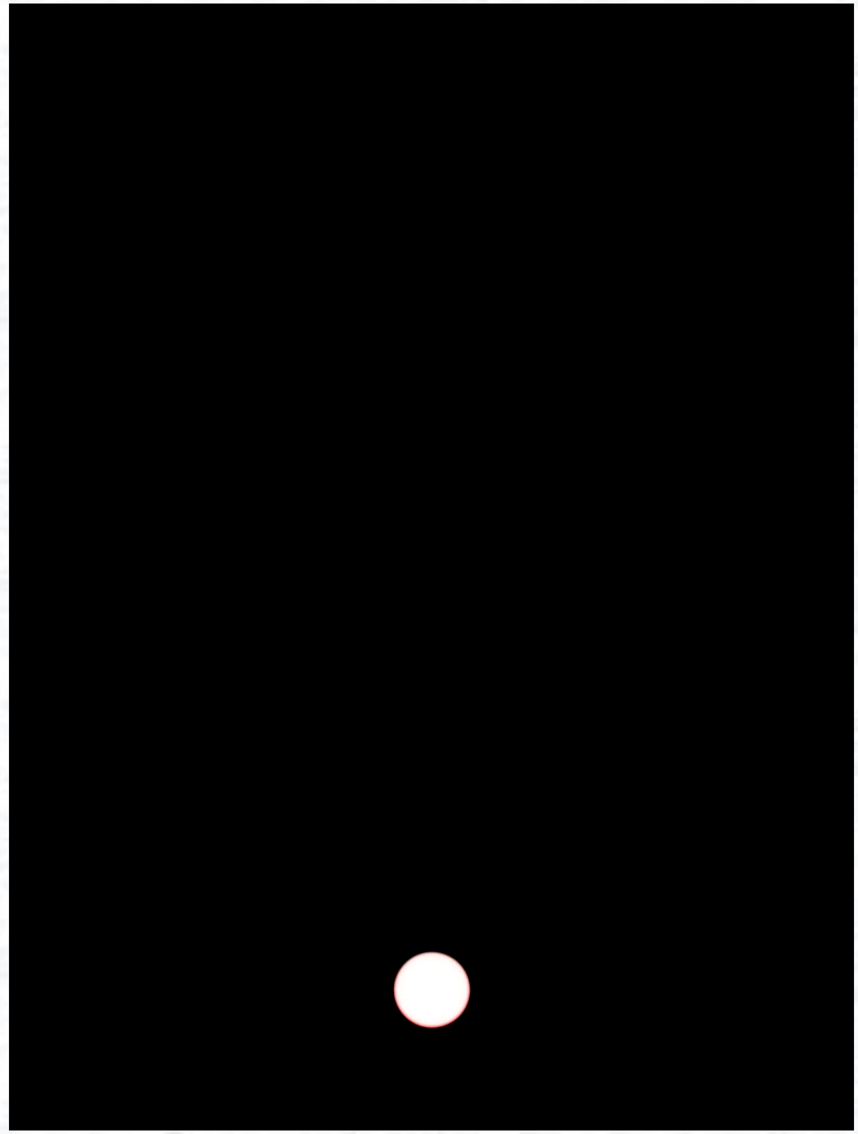
Vorticity and Burning

3d Rendering



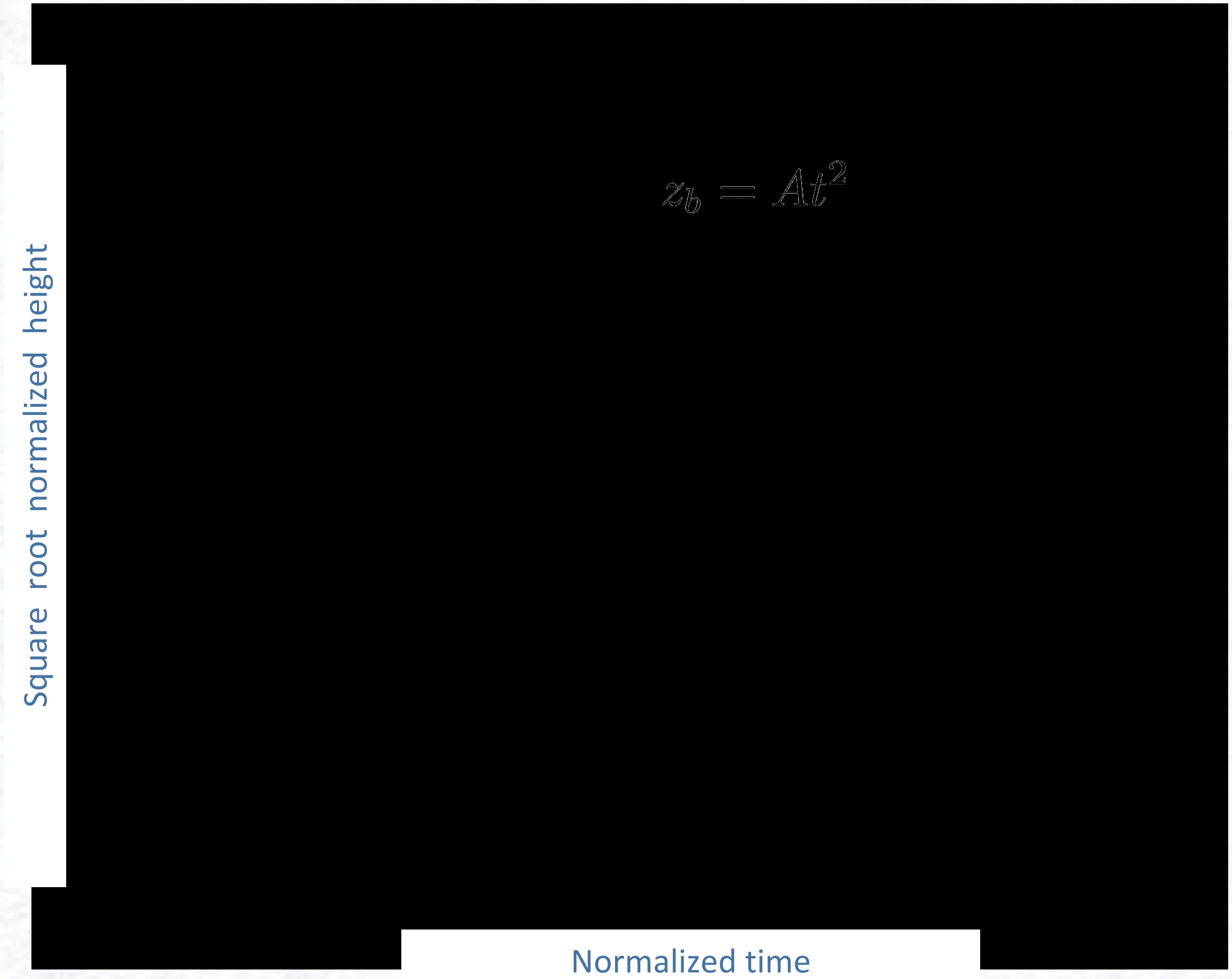
Buoyant Bubble

Tracer Slices



Bubble Height vs Radius (Cone)

Bubble Height vs Time



Second Half Conclusions

- Modified MTT theory to account for burning
- Appears to provide good predictions
  - But requires immense simulations
- Further work required for generality of entrainment coefficient
- Generalization of theory for application to full stars
  - Ambient stratification (variations in density and pressure)
  - Background turbulence
  - Straightforward to formulate one-dimensional system of ODEs