Kinematics of the Universe

- Robertson-Walker metric
- Metrical Tests
- Distance and Volume
- Time
- Velocity
- Acceleration
- Jerk
Fundamentals

★ Isotropy => homogeneity

★ Cosmic time \( t \)

★ Scale factor \( a(t_0) = 1 \)

★ Symmetry => Robertson-Walker metric

\[ ds^2 = dt^2 - a(t)^2 \left[ dr^2 + d(r)^2 (d \theta^2 + \sin^2 \theta \, d \phi^2) \right] \]

★ "radius" \( r \)

★ "(Comoving angular diameter) distance" \( d(r) \)
Maximally Symmetric 3-spaces

\[ d\left(r + dr\right)\alpha - d\left(r - dr\right)\alpha = dr\ \beta \]
\[ d\left(2r\right)\alpha = d\left(r\right)\beta \]
\[ d\left(2r\right) = 2d\left(r\right)d'\left(r\right) \]

\[ \star \implies d(r)=r, \ R_0 \sin(h)r/R_0. \]
**Distances**

Angular diameter distance
\[ d_A = \frac{x(\text{then})}{\alpha(\text{now})} = d\, a(t) \]

Luminosity distance
\[ d_L = [\frac{L(\text{then})}{4\pi F(\text{now})}]^{1/2} \]
Allow for redshift of photon energy, rate
\[ d_L = d/a(t) \]
**Distance Measurement**

- Parallax (Hipparcos) to 100pc
- LMC main sequence fitting and SN1987a rings – 50kpc
- Virgo cluster etc Cepheids ($L \sim P^{1.3}$) 15 Mpc
- Fisher-Tully for spirals ($L \sim V_c^4$)
- Type Ia SN, $L \sim 5 \times 10^9 L_\odot$; correct for rate of decline
- Type II supernova; modified black body formula with $R=Vt$

**Surface brightness fluctuations**  

**Systematic Errors!**
Surveying with Lenses

\[ \theta = \alpha \frac{d_{12}}{d_{02}} = \alpha R_{12}. \]

★ With many multiple images can solve for \( \alpha(\theta) \)

★ With many source redshifts can obtain \( R_{12}(z_2) \)

★ With many lenses can obtain \( R_{12}(z_1, z_2) \)
A2218
(Kneib et al)
~100 sources
~10 redshifts
Accurate astrometry and redshifts
Limited by model accuracy
**Metrical relationships**

\[ R_{12} + \frac{d_{01}}{d_{02}} R_{23} = R_{13} \]

\[ R_{12} + \frac{d_{01}}{d_{02}} = 1 \]

- **General relation for isotropic space**
  - from lenses

- **Relation for flat space**
  - from supernovae
Comoving Volume

\[ V = \Delta \Omega \int dr \, d^2 \]

for flat space

\[ V = \Delta \Omega \, d^3 / 3 \]

Relate to kinematics using \( dr = dt / a \)
Age of the Universe

- Radioactive dating using isotope pairs
- Stellar evolution in globular clusters
- White dwarf cooling
- Produces lower bounds but now consistent with indirect methods
Velocities

Define $H(t) = a'/a = a'_{\text{now}}$

$a = 1 - z + ... = 1 - H_0 (t - t_0) + ... \Rightarrow r = t_0 - t = z/H_0$. Hubble law
Gravitational Lenses

Source \( \xi \) \( \alpha \) Lens \( \psi \)

Observer

Time delay (relative to unperturbed ray)

\[ = \text{geometric delay} + \text{gravitational delay} \]

\[ = (1 + z) \left[ \xi \cdot \alpha / 2 \cdot c - \psi / c^3 \right], \]

along each perturbed ray.

Time \( \sim \) Age of Universe \( \times \alpha^2 \sim 1 \text{ yr} \sim H_0^{-1} \)

Frequency of occurrence \( \sim \Omega_{gal} \sim 0.003 \)
B1608+656

$Z_d = 0.63 \text{ E lens + merging companion}$

$Z_s = 1.394 \text{ E+A radio source}$

$t_{A,C,D} = 26, 33, 73d; \mu_{A,C,D} = 2, 1, 0.35$

*Accuracy of $H_0$ limited by model*
\[ q = -\frac{aa''}{a^2} \]

* **Flat space** \[ d = ad_L = r = \int \frac{dt}{a} = H_0^{-1} \left[ z - (1 - q_0/2)z^2 + ... \right] \]
Kinematics Summary

- Concordance of local kinematics with microwave background measurements
- Gravitational lenses provide excellent tool for surveying space but utility limited by model accuracy
- Natural kinematic models for $a(t)$ involve $j = a''' a^2 / a' ^3$
  - $\Lambda$CDM has $j=1$ and, in absence of credible dynamical model fit observations to $j$