

Measurement of V_{cb}

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Inclusive approaches

Exclusive approaches ($B \rightarrow D^ l \nu$, $B \rightarrow D l \nu$)*

Moments and Form factors (if time permits)

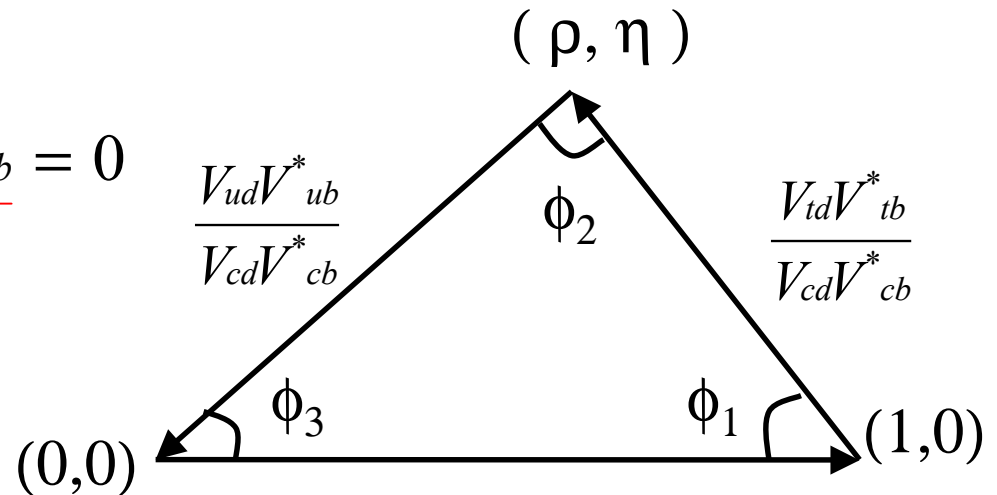
Conclusion

The V_{cb} element of the CKM matrix

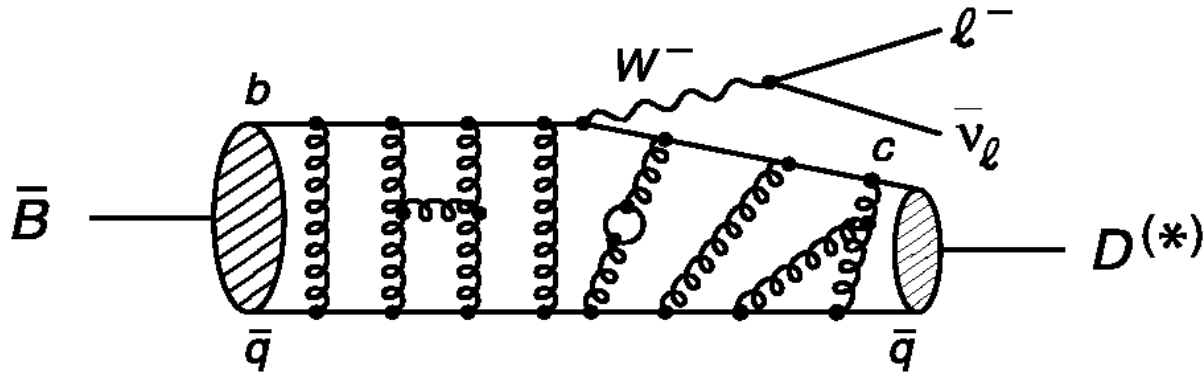
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

Determination of V_{cb} allows the determination of A , which is important for indirect constraints on the CKM triangle. For example, the CPV parameter ϵ_K is proportional to A^4 .

$$V_{td}V_{tb}^* + \underline{V_{cd}V_{cb}^*} + \underline{V_{ud}V_{ub}^*} = 0$$



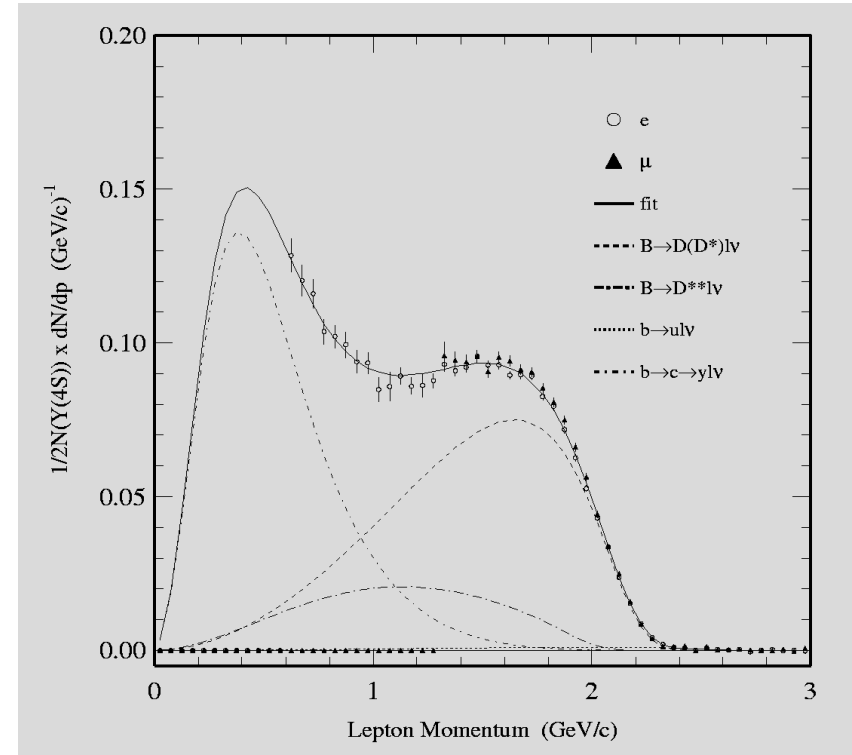
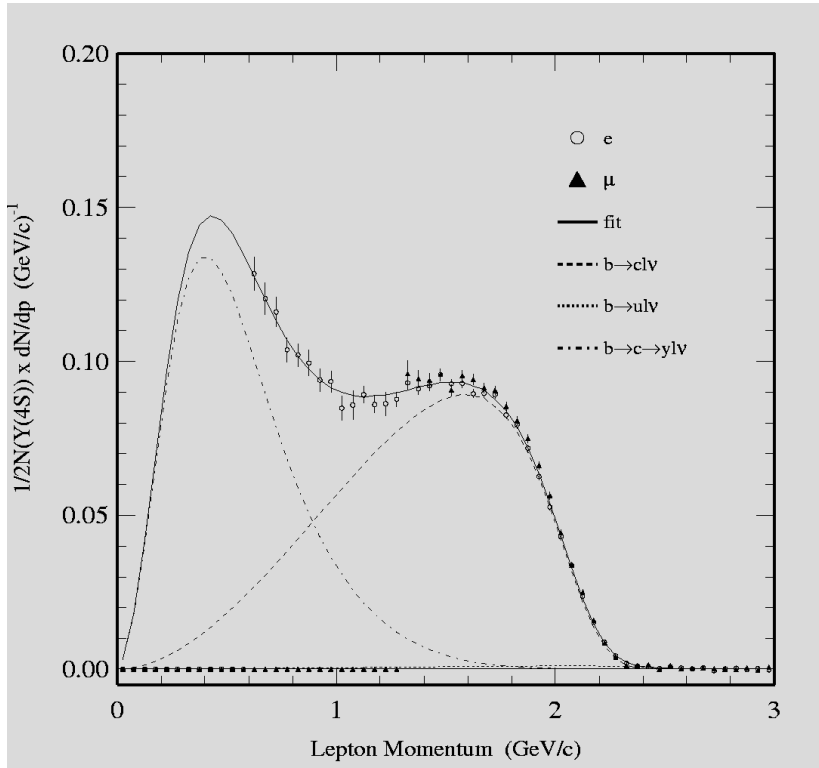
Inclusive Approach to $b \rightarrow c$



Hadronic effects are restricted to the lower part of the graph.

Sum over all final states: *only detect the final state lepton.*

Inclusive Approach to $b \rightarrow c$ (single leptons)



ACCM model (inclusive)

ISGW** model (exclusive)

Either cut at $p_L^ > 1.4 \text{ GeV}$ and accept large model dependence or fit: the correlation between $b \rightarrow c$ (direct) and $b \rightarrow c \rightarrow s l v$ (cascade) component leads to model dependence e.g. BF: 10.42% to 10.98%*

Inclusive Approach to $b \rightarrow c l \bar{\nu}$ (dileptons)

To overcome this problem, ARGUS introduced a new approach using dileptons.

Idea: tag with one high momentum lepton $p^* > 1.4$ GeV (to guarantee that it is from a primary $b \rightarrow c$ decay). Then examine a second lepton with $p^* > 0.6$ GeV.

The charge correlation (**opposite for direct leptons** and **same for cascade leptons**) eliminates correlation problem. **The angular correlation** between the two leptons removes the background due to leptons from the same B meson.

Application of the dilepton method to $b \rightarrow c l \nu$

Example from Belle

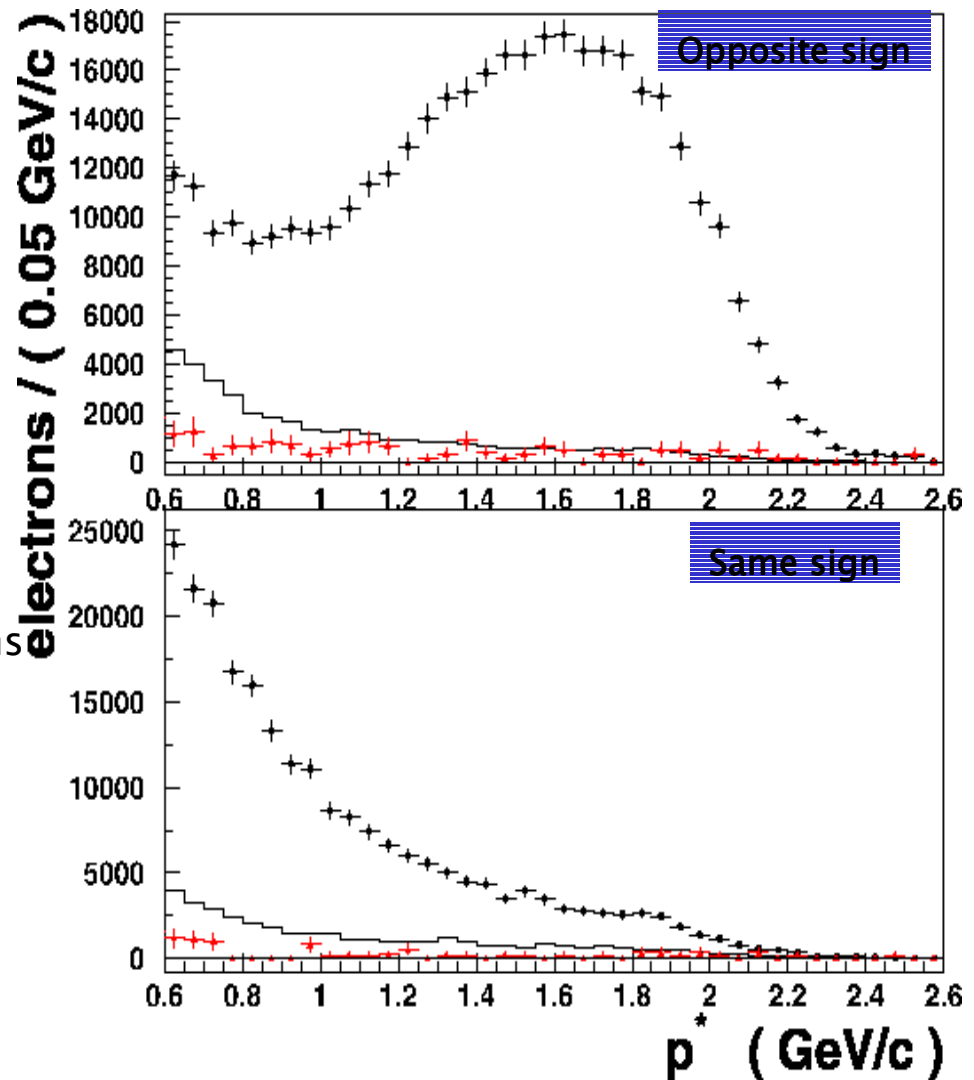
High momentum lepton tag:
 $1.4 < p_l < 2.2 \text{ GeV}$

Require an additional electron
(p_e down to 0.6 GeV)

Divide the sample into opposite/
same sign events.

Suppress secondary opposite sign leptons
with $p_e + \cos \theta_{le} > 1.2$
($\cos \theta_{le}$ = opening angle b/w leptons)

Estimate # of electrons by fitting E/p
in momentum bins.



Application of the dilepton method to $b \rightarrow c l \nu$

$$\begin{pmatrix} N_{+-}(p^*) \\ N_{\pm\pm}(p^*) \end{pmatrix} = N_{tag} \begin{pmatrix} \varepsilon_1(1-\chi) & \varepsilon_1\chi \\ \varepsilon_2\chi & \varepsilon_2(1-\chi) \end{pmatrix} \begin{pmatrix} \frac{dB\Gamma(b \rightarrow evX)}{dp^*} \\ \frac{dB\Gamma(b \rightarrow c \rightarrow evX)}{dp^*} \end{pmatrix}$$

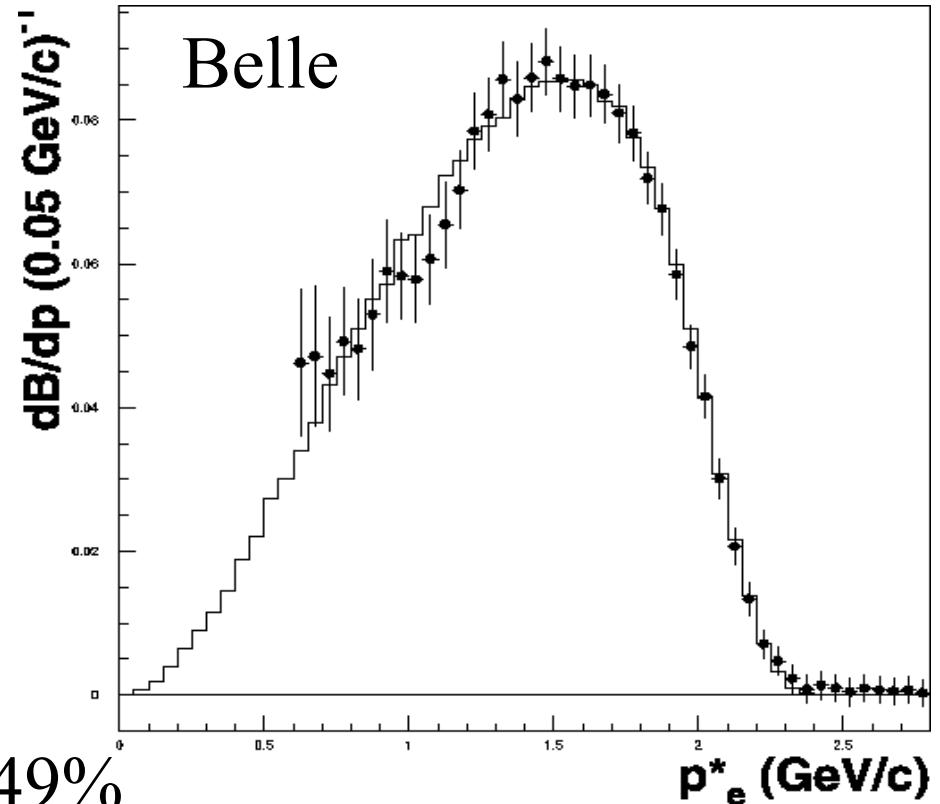
$$\chi = \chi_0 f_{00} = 0.0843 \pm 0.0060$$

mixing parameter(PDG)

ε_1 : eff. for opposite sign

ε_2 : eff. for same sign

Correct for B-Bbar mixing and the small fraction of signal (6.1%) below 0.6 GeV.



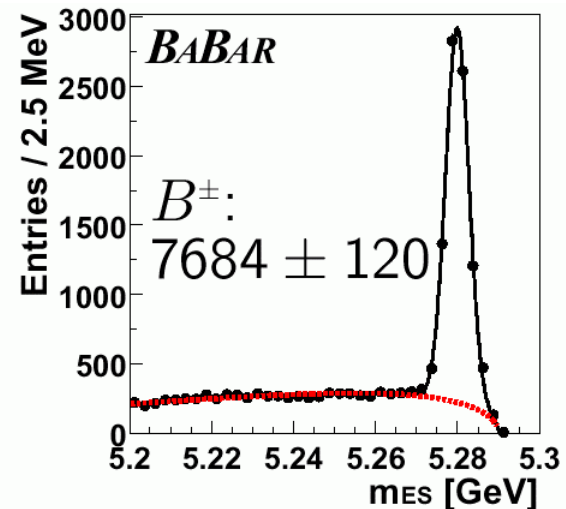
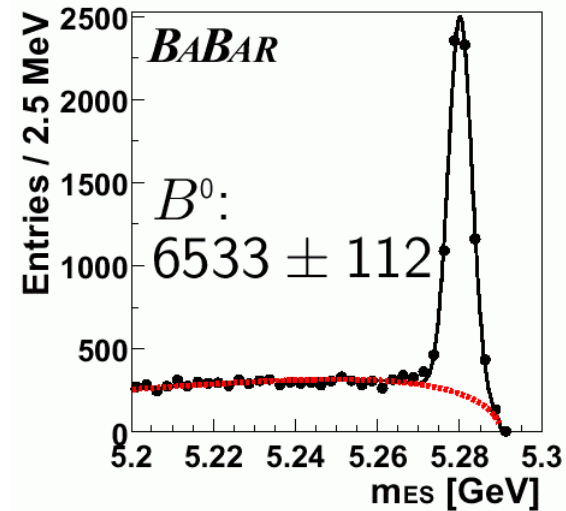
$$BF(b \rightarrow c l \nu) = 10.90 \pm 0.12 \pm 0.49\%$$

$B \rightarrow c l \nu$ with Fully Reconstructed B Tags

- Fully reconstructed hadronic B decays:

$$B \rightarrow D^{(*)}\pi, D^{(*)}\rho, D^* a_1 \\ \rightarrow J/\psi K^{(*)}, \psi(2S) K^{(*)}$$

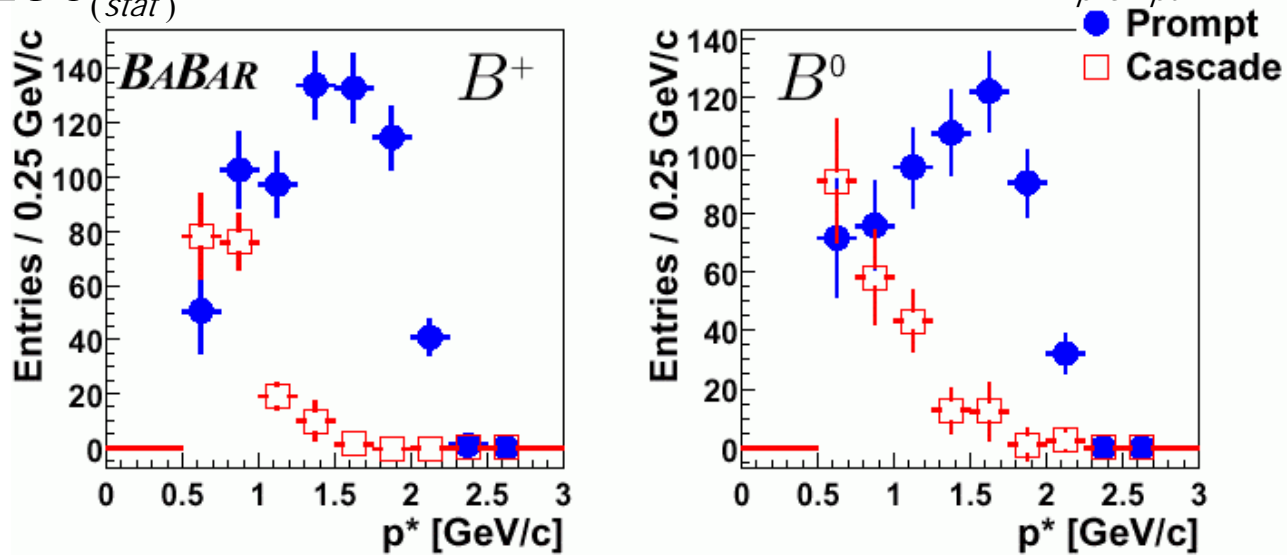
Check whether spectator quark effects are important – compare B_{SL} for B^+ and B^0 decays.



BaBar: semileptonics from B tags

$$N_{prompt}^+ = 597 \pm 38_{(stat)}$$

$$N_{prompt}^0 = 674 \pm 34_{(stat)}$$



- Branching fractions (preliminary) :
 - $\text{BF}(B^+ \rightarrow X e \nu) = (10.3 \pm 0.6_{stat} \pm 0.5_{sys})\%$
 - $\text{BF}(B^0 \rightarrow X e \nu) = (10.4 \pm 0.8_{stat} \pm 0.5_{sys})\%$


$$\text{BF}(B \rightarrow X e \nu) = (10.4 \pm 0.5_{stat} \pm 0.5_{sys})\%$$

$$\text{BF}(B^+ \rightarrow X e \nu) / \text{BF}(B^0 \rightarrow X e \nu) = 0.99 \pm 0.10_{stat} \pm 0.04_{sys}$$

Determinations of $BF(b \rightarrow c l \nu)$

Expt.	BR	stat	syst
CLEO	10.49	± 0.17	± 0.43
BELLE (ℓ Tag)	10.90	± 0.12	± 0.49
BABAR (e Tag)	10.87	± 0.18	± 0.30
AVERAGE	10.63	± 0.19	± 0.16

Eid, tracking,
and low p
background
subtractions



Expt.	BR	stat	syst	model
ALEPH	10.70	± 0.10	± 0.23	± 0.26
DELPHI	10.70	± 0.08	± 0.21	$+0.44$ -0.30
L3	10.85	± 0.12	± 0.38	± 0.26
L3 (double tag)	10.16	± 0.13	± 0.20	± 0.22
OPAL	10.83	± 0.10	± 0.20	$+0.20$ -0.13
AVERAGE	10.63	± 0.09	± 0.15	± 0.18

Good agreement between Upsilon(4S) and LEP results.

Conversion of $BF(b \rightarrow c l \nu)$ to V_{cb}

$$\rightarrow \boxed{\gamma_c} |V_{cb}|^2 \cong Br(B \rightarrow X l \nu) / \tau_B$$

Using the Belle result

Model	$ V_{cb} \times 10^{-2}$
ACCMM	$4.10 \pm 0.10 \pm 0.40$
ISGW2	$4.00 \pm 0.10 \pm 0.40$
M.Shifman	$4.04 \pm 0.10 \pm 0.20$
P.Ball	$3.95 \pm 0.09 \pm 0.19$

*This type of determination also assumes **quark-hadron duality** i.e. that the inclusive quark-level rate reproduces the sum of a few exclusive states ($B \rightarrow D l \nu$, $B \rightarrow D^* l \nu$, $B \rightarrow D^{**} l \nu$)*

OPE Expansion and Moments

$$\Gamma_{\text{sl}} = \frac{G_F^2 |V_{cb}|^2}{192 \pi^3} m_B^5 c_1 \left\{ 1 - c_2 \frac{\alpha_s}{\pi} + \frac{c_3}{m_B} \bar{\Lambda} \left(1 - c_4 \frac{\alpha_s}{\pi} \right) + \frac{c_5}{m_B^2} (\bar{\Lambda}^2 + c_6 \lambda_1 + c_7 \lambda_2) \right. \\ \left. + O\left(\frac{1}{m_B^3}\right) + O\left(\frac{\alpha_s^2}{\pi}\right) \dots \right\}$$

$\Lambda, \lambda_1, \lambda_2$ are non-perturbative parameters.

λ_1 (-) kinetic energy of the b-quark (a.k.a μ_π^2)

λ_2 hyperfine splitting from B^*-B mass difference, $\lambda_2 = 0.12 \text{ GeV}^2$ (a.k.a μ_C^2)

$\Lambda = m_B - m_b + (\lambda_1 - 3 \lambda_2) / 2m_B \dots$ (energy of “light DOF”)

–Additional parameters enter at higher orders ($\rho_1, \rho_2, \tau_1, \tau_2, \tau_3, \tau_4$); use theoretical estimates

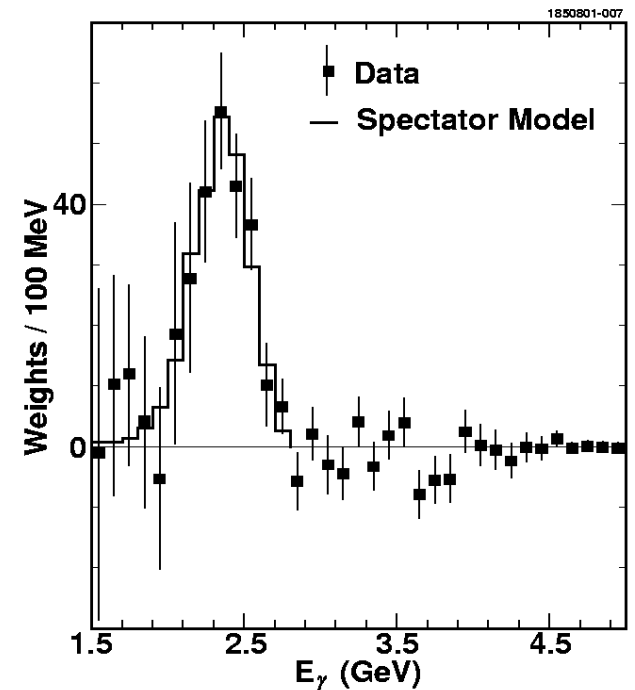
$\Lambda, \lambda_1, \lambda_2$ are the main sources of theoretical uncertainty in inclusive V_{cb} (calibrate γ_c).

Conversion of $\text{BF}(b \rightarrow c \ell \nu)$ to $|V_{cb}|$

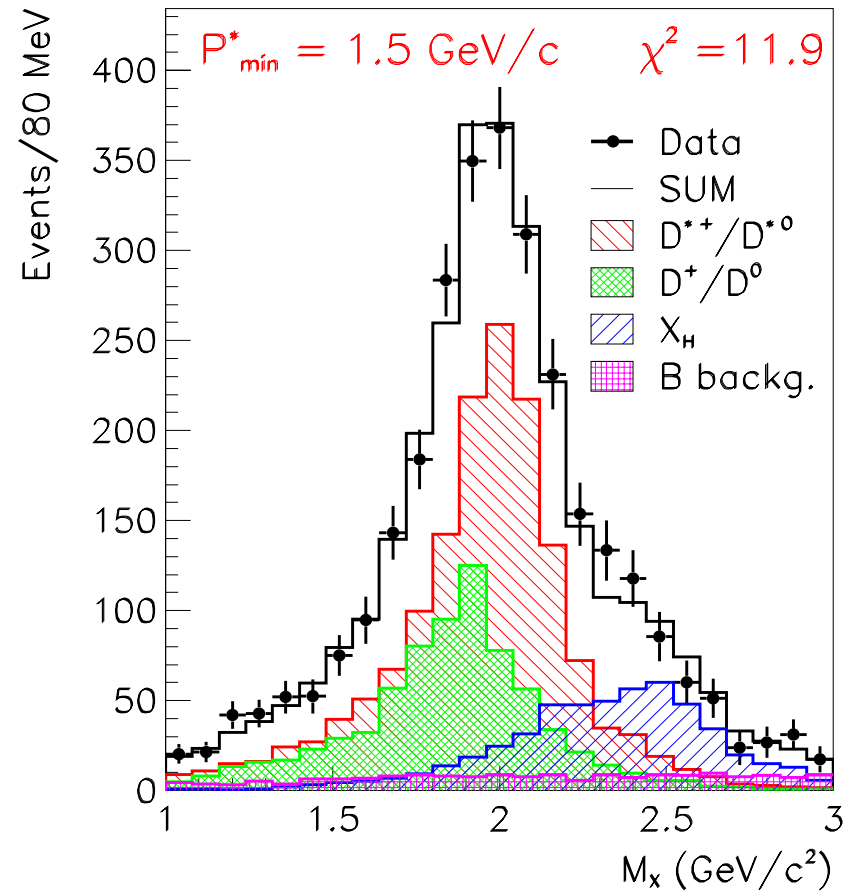
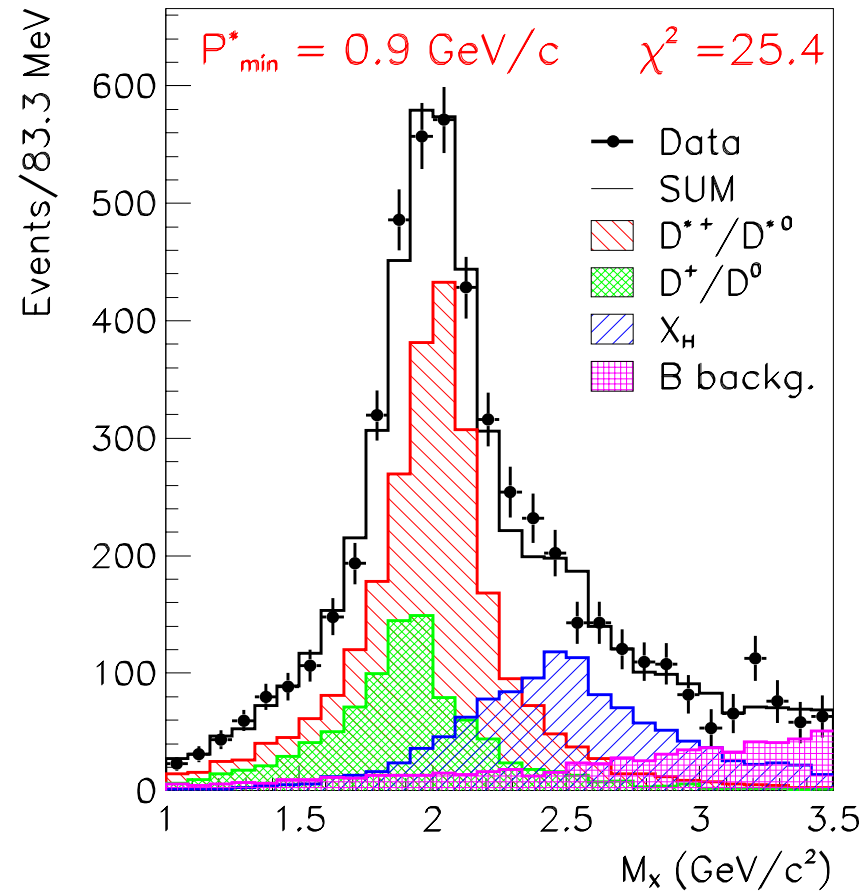
Idea: The non-perturbative parameters Λ , λ_1 , λ_2 can be determined from other experimental measurements.

$b \rightarrow s \gamma$ photon energy moments,
 $b \rightarrow c \ell \nu$ lepton energy moments,
and $b \rightarrow c \ell \nu$ hadronic mass moments.

For example: $\langle E_\gamma \rangle$ and $\langle E_\gamma \rangle^2 - \langle E_\gamma^2 \rangle$ in $b \rightarrow s \gamma$



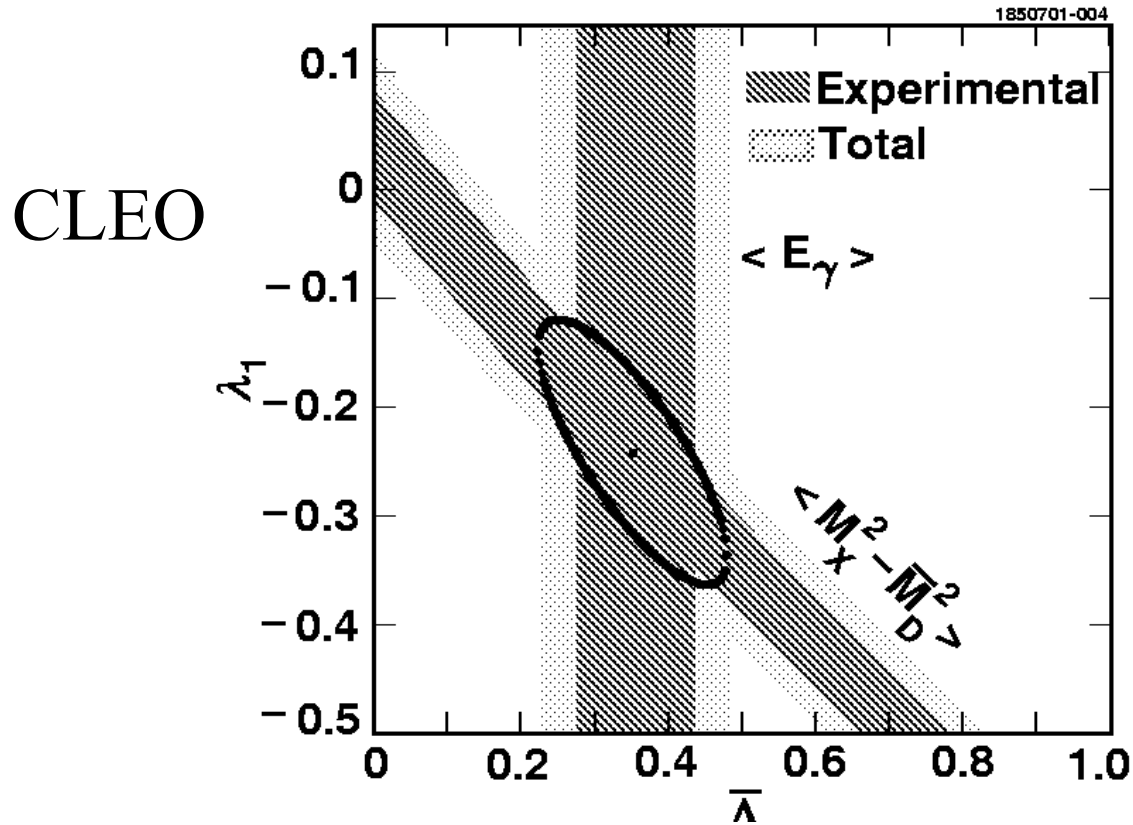
BaBar: Moments of the M_X Distribution



Binned χ^2 fit to M_X Distribution: 4 Contributions

$$D = f_D P_D + f_{D^*} P_{D^*} + f_{HX} P_{HX} + f_{BG}(\text{fixed}) P_{BG}$$

OPE parameters determined from two sets of moments:



$$\Lambda = 0.35 \pm 0.07 \pm 0.10 \text{ GeV}, \quad \lambda_1 = 0.236 \pm 0.071 \pm 0.078 \text{ GeV}^2$$

BaBar: Problem with E_L cut dependence of moments.

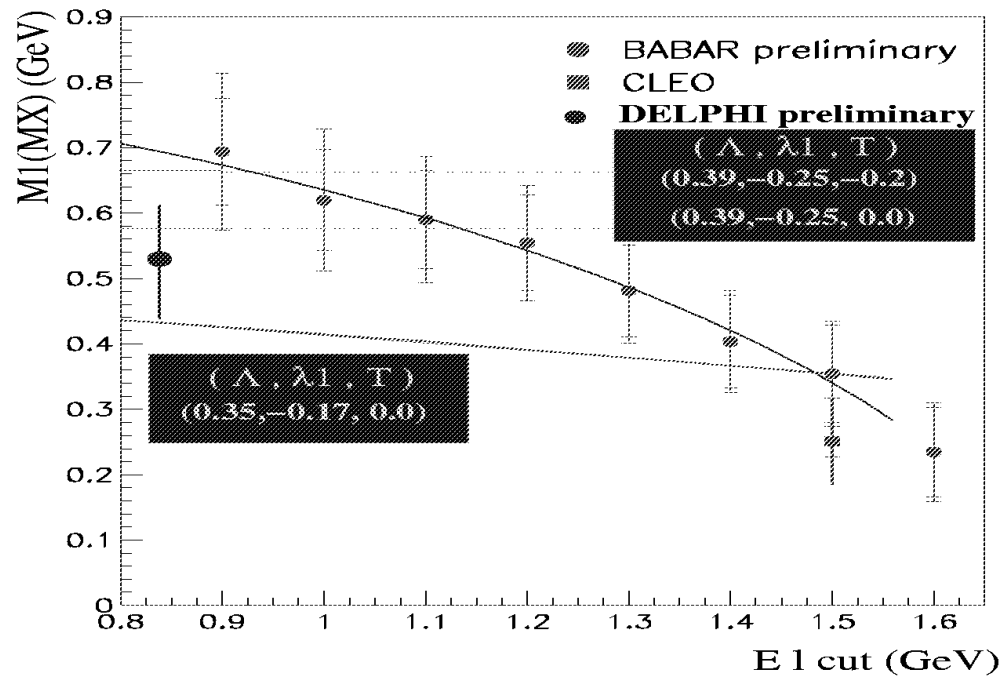
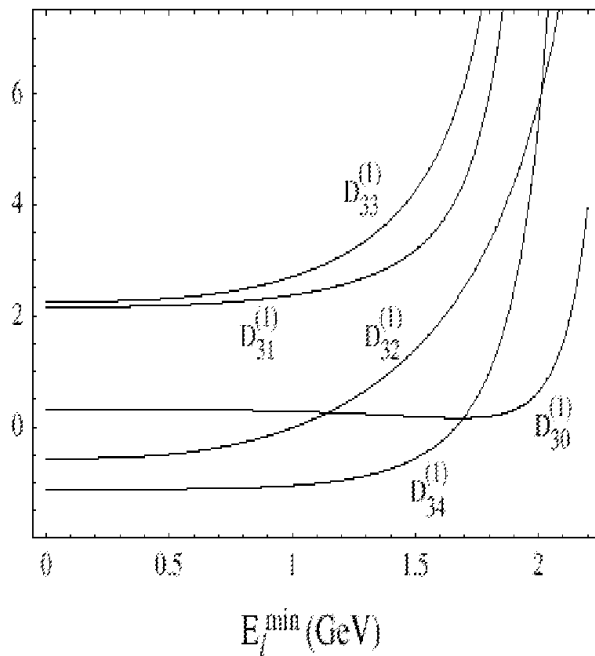
CLEO $E_\ell > 1.5$ GeV DELPHI $E_\ell \geq 0.5$ GeV

BABAR $0.8 \text{ GeV} < E_\ell < 1.6$ GeV

$1/m_b^3$ Coefficients vs. E_ℓ^{cut}
Falk and Luke

$M_1(M_X)$ vs. E_ℓ^{cut}

CLEO + BABAR + DELPHI PRELIMINARY



Using the measured OPE parameters, can determine a more precise V_{cb} value.

$$V_{cb} = (40.4 \pm 0.5(\text{exp}) \pm 0.5(\lambda_1, \Lambda) \pm 0.8(\text{theo})) \times 10^{-3}$$

V_{cb} from $B \rightarrow D^{(*)} l \nu$ at zero recoil

The differential rate for $B \rightarrow D^{()} l \nu$ at zero recoil is related to V_{cb} :*

$$\frac{d\Gamma}{d\omega} \propto |V_{cb}|^2 F_{D^{(*)}}^2(\omega)$$

ω : relativistic γ factor of the D^* in the B rest frame

($\omega = E_{D^*}/m_{D^*}$)

$\omega = v \cdot v'$, the dot product of B and $D^{(*)}$ 4-velocities.

Other notations are used including: w , y

$$\omega = (m_B^2 + m_{D^*}^2 - q^2) \frac{1}{2m_B m_{D^*}}$$

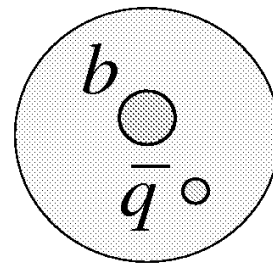
V_{cb} from $B \rightarrow D^{(*)} l \nu$ at zero recoil

$$\frac{d\Gamma}{d\omega} \propto |V_{cb}|^2 F_{D^{(*)}}^2(\omega)$$

In the zero-recoil configuration, HQET symmetry implies $F(\omega = 1) = 1$ with small theoretical corrections.

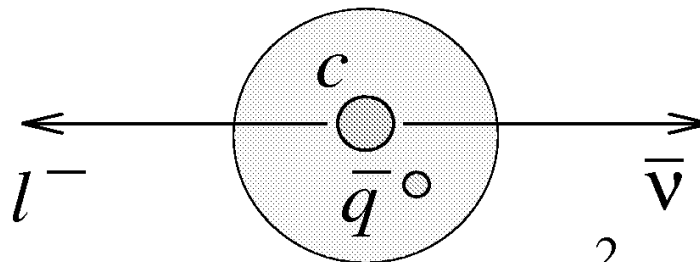
Luke's theorem: the corrections to 1 are $O(1/m_b^2)$ for $B \rightarrow D^{*} l \nu$ in HQET. From models and lattice, $F(1) = 0.91 \pm 0.04$ (PDG)

Initial B meson



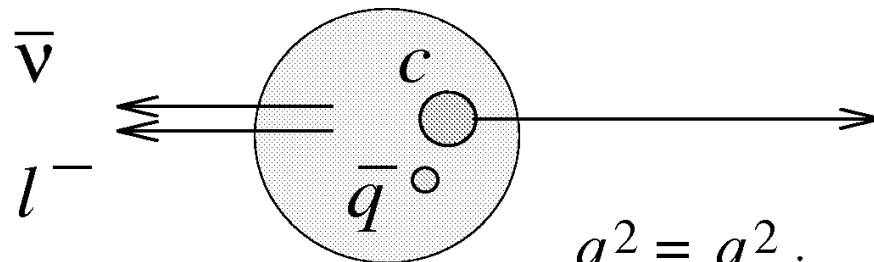
(a)

Light degrees of freedom are not disturbed $\rightarrow F(1)=1$



(b)

$$q^2 = q_{\max}^2 \quad \omega = 1$$



(c)

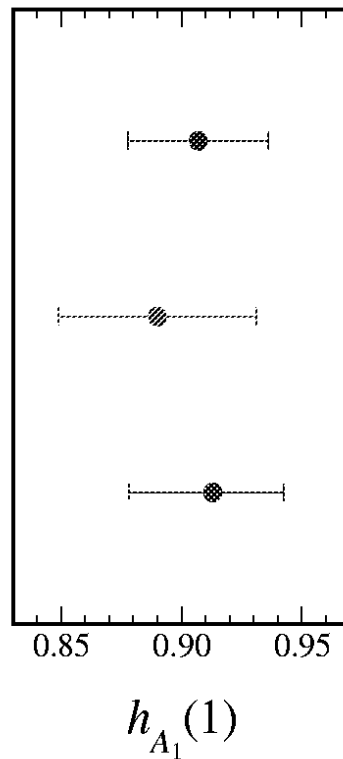
$$q^2 = q_{\min}^2 \quad \omega = 1.5$$

V_{cb} from $B \rightarrow D^{(*)} l \nu$ at zero recoil (theory)

$\mathcal{F}_{B \rightarrow D^*}(1) = 0.913$	$+0.024$ -0.017	± 0.016	$+0.003$ -0.014	$+0.000$ -0.016	$+0.006$ -0.014
	stat	match	a	m_q	quench



State of the art
Lattice calculation
from Kronfeld et al.



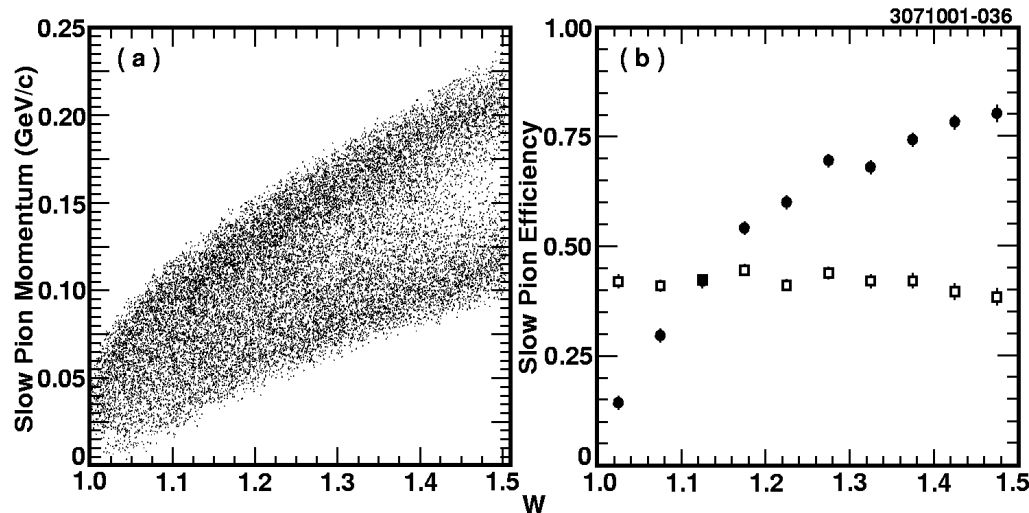
See Lepage's talk

(Still quenched)

Theory uncertainties still at the 3-4% level

Experimental issues for zero recoil:

The D^ is nearly at rest i.e. slow pion from $D^* \rightarrow D \pi$ has a very low momentum.*



But at LEP
eff is flat.

Rate is proportional to p_{D^*} . (At zero recoil there is no rate). *Measure the rate near zero recoil, then assume a functional form for $F(\omega)$ and extrapolate.*

CLEO 2002: $B \rightarrow D^{(*)} l \nu$ signals

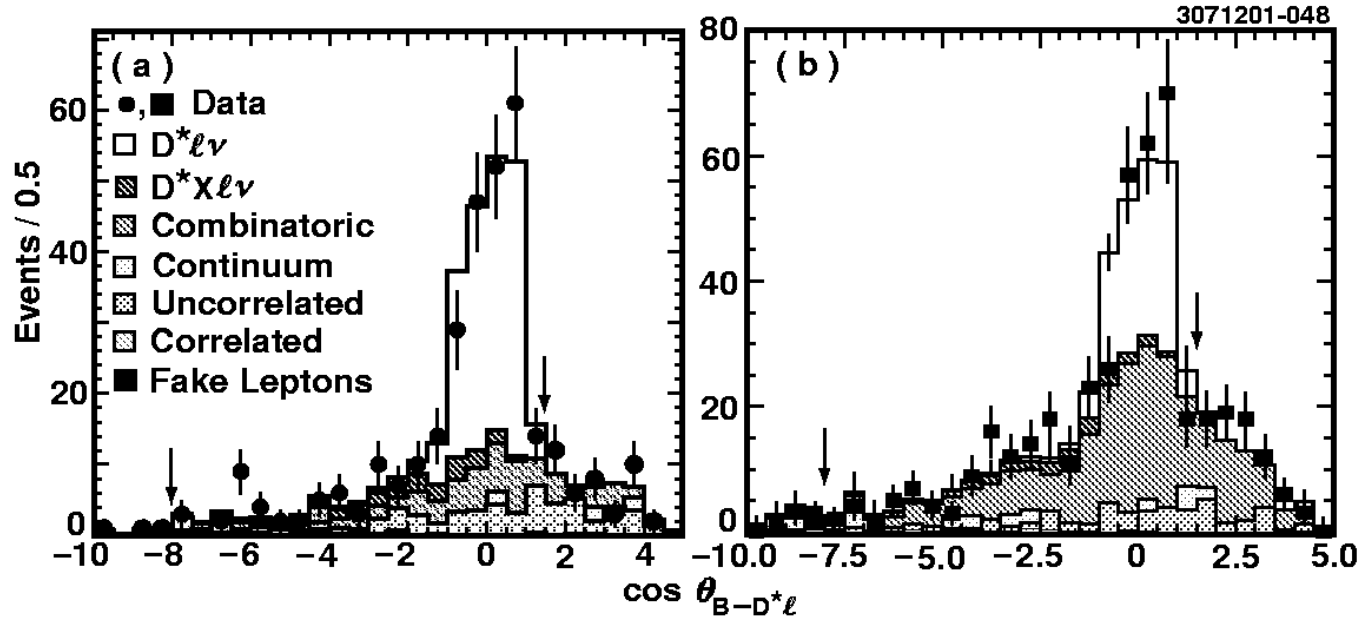


FIG. 1. The candidate yields for $1.10 < w < 1.15$ bin with the results of the fit superimposed for (a) $D^{*+} \ell^{-} \bar{\nu}$ and for (b) $D^{*0} \ell^{-} \bar{\nu}$. The fit uses the region between the arrows.

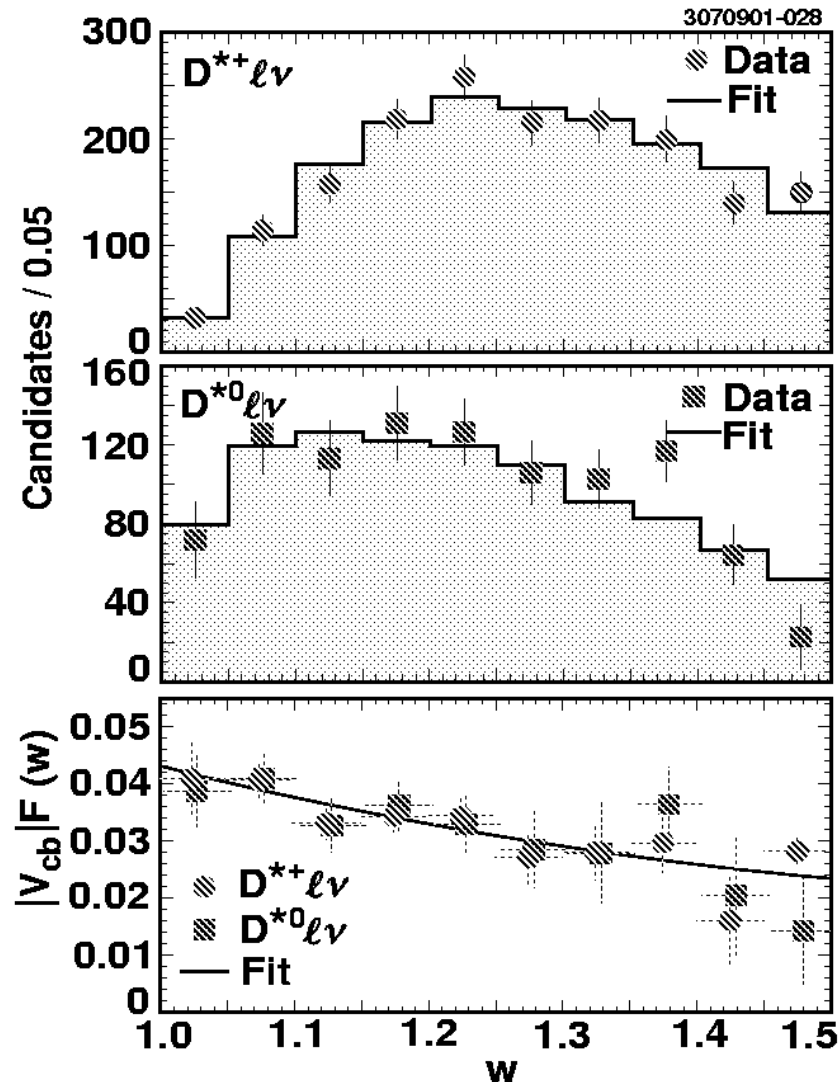
$$\cos \theta_{B-Xl} \equiv \frac{2E_B E_{Xl} - M_B^2 - M_{Xl}^2}{2|\vec{p}_B||\vec{p}_{Xl}|}$$

$|\cos_{B-Xl}| < 1$ if X and lepton come from signal

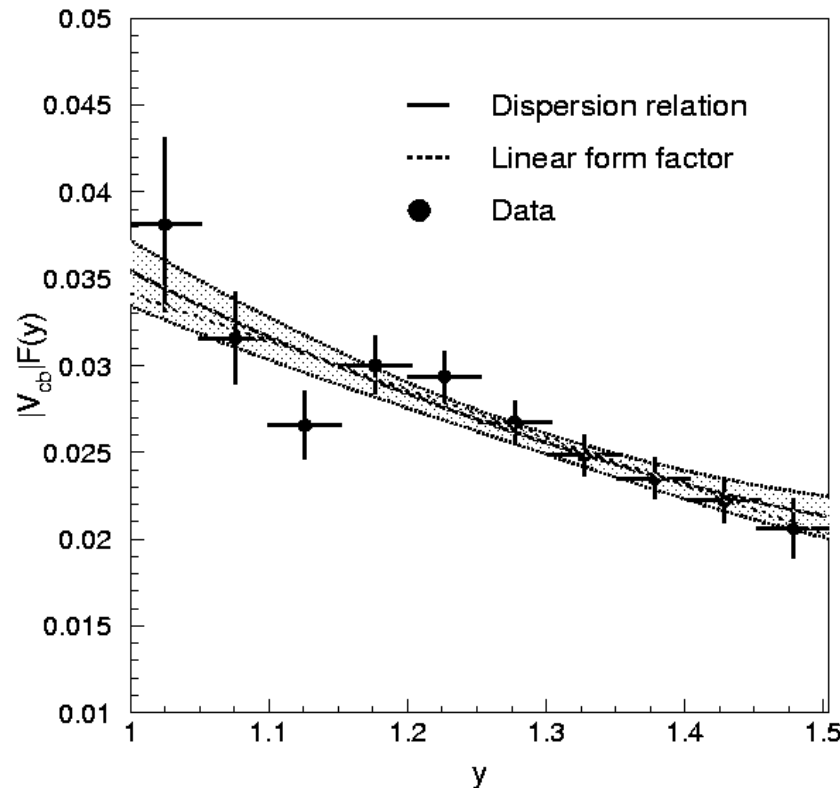
CLEO 2002: V_{cb} from $B \rightarrow D^{(*)} l \nu$ at zero recoil

Note higher efficiency near $w \sim 1$ 

Use both $D^{+} \rightarrow D^0 \pi^+$
and $D^{*0} \rightarrow D^0 \pi^0$*



Belle: Sensitivity of $|V_{cb}|$ to FF parameterization.



$|V_{cb}|F(1)$ 0.0342-0.0358
 $\rightarrow V_{cb}$ range 5%

Summary of fit results according to different form factor (FF) parameterizations, where the errors are statistical only. Our main analysis uses values of $R_1(1)$, $R_2(1)$ from QCD sum rules.

FF shape & $R_1(1), R_2(1)$	$ V_{cb} F(1) \cdot 10^2$	$\rho_{A_1}^2$	ρ_F^2	$\mathcal{B}(\bar{B}^0 \rightarrow D^{*+} e^- \bar{\nu})$	χ^2/ndf
Dispersive & QCD sum rules	3.54 ± 0.19	1.35 ± 0.17	.	$(4.59 \pm 0.23)\%$	3.38/8
Dispersive & CLEO value	3.58 ± 0.19	1.45 ± 0.16	.	$(4.60 \pm 0.23)\%$	3.79/8
Linear & Heavy quark limit	3.42 ± 0.17	.	0.81 ± 0.12	$(4.57 \pm 0.24)\%$	2.23/8

$B \rightarrow D^* l \nu$ Form Factor Parametrization Issues I

$$\frac{d\Gamma(B \rightarrow D^* l \nu)}{dw} = \frac{G_F^2}{48\pi^3} M_{D^*}^3 (M_B - M_{D^*})^2 \sqrt{w^2 - 1} (w+1)^2 g(w) |V_{cb}|^2 F_{D^*}^2(w)$$

where $g(w) = \left[1 + \frac{4w}{(1+w)} \frac{1-2wr+r^2}{(1-r)^2} \right]$, $r = M_{D^*}/M_B$

First round of measurements assumed a linear functional form for $F(y)$.

$$F(\omega) = F(1) [1 - \rho^2 (\omega-1) + O(\omega^2)]$$

B → D* l ν Form Factor Parametrization Issues II

$$\frac{d\Gamma(B \rightarrow D^* l \nu)}{dw} = \frac{G_F^2}{48\pi^3} M_{D^*}^3 (M_B - M_{D^*})^2 \sqrt{w^2 - 1} (w+1)^2 g(w) |V_{cb}|^2 F_{D^*}^2(w)$$

Using the axial vector form factor $A_1(y)$, form factor ratios $R_1(y)$ and $R_2(y)$

$$g(w)F_{D^*}^2(w) = \left[2 \left(\frac{1 - 2wr + r^2}{(1-r)^2} \right) \left(1 + R_1^2(w) \frac{w-1}{w+1} \right) + \left(1 + (1 - R_2(w)) \frac{w-1}{1-r} \right)^2 \right] A_1^2(w)$$

Dispersion relations give model-independent bounds on form factors

(Caprini, NP B530 (1998))

$$R_1(w) \sim R_1(1) - 0.12(w-1) + 0.05(w-1)^2$$

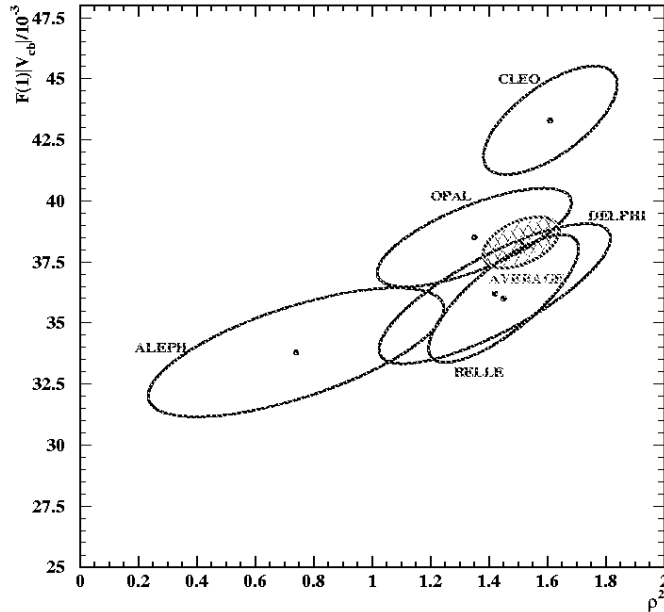
$$R_2(w) \sim R_2(1) - 0.11(w-1) - 0.06(w-1)^2$$

$$A_1(w) \sim A_1(1)[1 - 8r^2z + (53r^2 - 15)z^2 - (231r^2 - 91)z^3]$$

$$z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$$

V_{cb} from the end point of $B \rightarrow D^{(*)} 1 \nu$

Note strong V_{cb} , ρ^2 correlation 

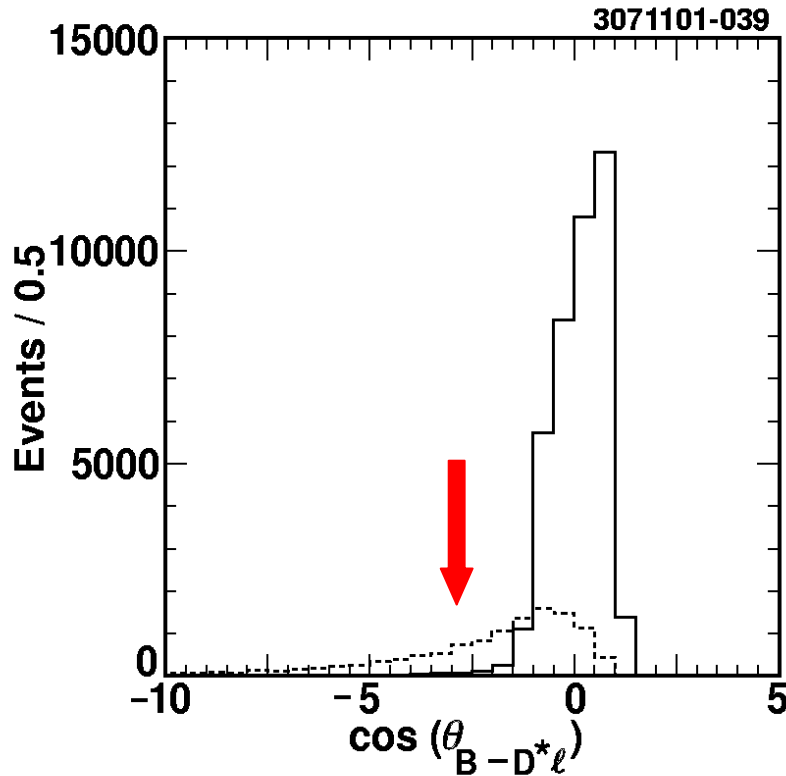


experiment	$\mathcal{F}(1) V_{cb} (\times 10^3)$	ρ^2	Corr _{stat}
ALEPH	$33.8 \pm 2.1 \pm 1.6$	$0.74 \pm 0.25 \pm 0.41$	94%
DELPHI	$36.1 \pm 1.4 \pm 2.5$	$1.42 \pm 0.14 \pm 0.37$	94%
OPAL	$38.5 \pm 0.9 \pm 1.8$	$1.35 \pm 0.12 \pm 0.31$	89%
Belle	$36.0 \pm 1.9 \pm 1.8$	$1.45 \pm 0.16 \pm 0.20$	90%
CLEO	$43.3 \pm 1.3 \pm 1.8$	$1.61 \pm 0.09 \pm 0.21$	86%
World average	$38.3 \pm 0.5 \pm 0.9$	$1.51 \pm 0.05 \pm 0.12$	86%

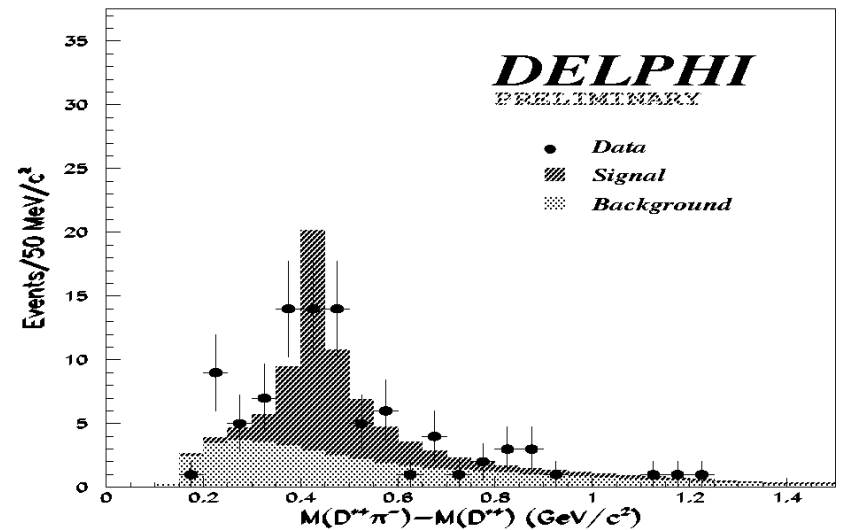
Artuso and Barbieri.

Exp error is only 2.7%, limited by theory

Uncertainty in $B \rightarrow D^{**} 1 \nu$ background



Explains the difference between CLEO and other results.



$$\text{BF}(B \rightarrow D^{*} \pi 1 \nu) = (0.64 \pm 0.08 \pm 0.09) \%$$

$B \rightarrow D l \nu$ versus $B \rightarrow D^* l \nu$ for V_{cb}

(+) *Detection efficiency for $B \rightarrow D$ is higher than $B \rightarrow D^*$ (no slow π).*

(-) *Background is worse for $B \rightarrow D$ (no D^* mass constraint) .*

(-) *$B \rightarrow D l \nu$ has a p_D^3 suppression while $B \rightarrow D^* l \nu$ has only a p_{D^*} suppression*

(-) *Luke's theorem does not apply to $B \rightarrow D l \nu$. There are $O(1/m_b)$ corrections. (See Ligeti's talk).*

Variables for exclusive semileptonic analysis via ν reconstruction (used for $B \rightarrow D l \nu$)

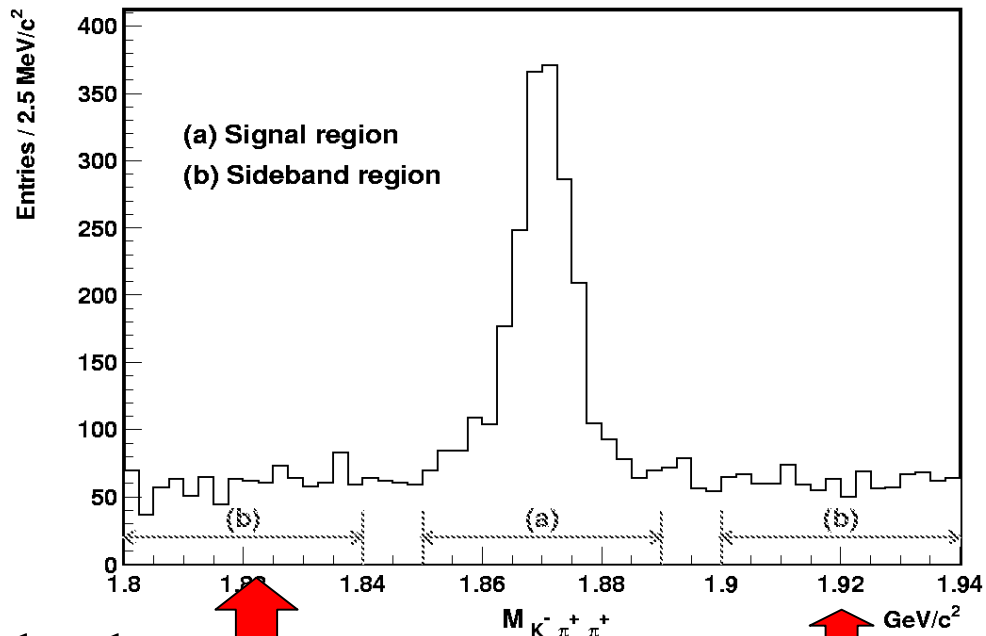
$$\begin{aligned} p_{miss} &= - \sum p_i \\ E_{miss} &= 2 E_{beam} - \sum E_i \\ M_{miss}^2 &= E_{miss}^2 - P_{miss}^2 \\ p_\nu &= (p_{miss} , | p_{miss} |) \end{aligned}$$

Can then form ΔE and M_B

$$\begin{aligned} \Delta E &\equiv E_{beam} - (E_\pi + E_l + E_\nu) \\ M_B &\equiv \sqrt{E_{beam}^2 - |p_\pi + p_l + p_\nu|^2} \end{aligned}$$

where $E_{beam} = 5.29 \text{ GeV}$

Belle: $B \rightarrow D l \nu$



sideband

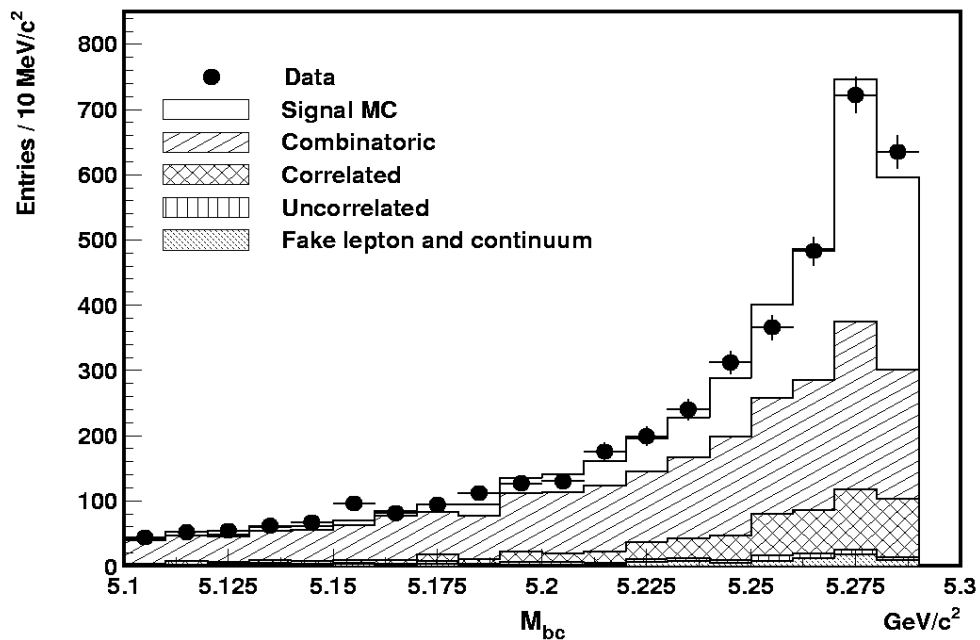
Signal : 1099 ± 57

Corr: 983 ± 22

Uncorr: 35 ± 4

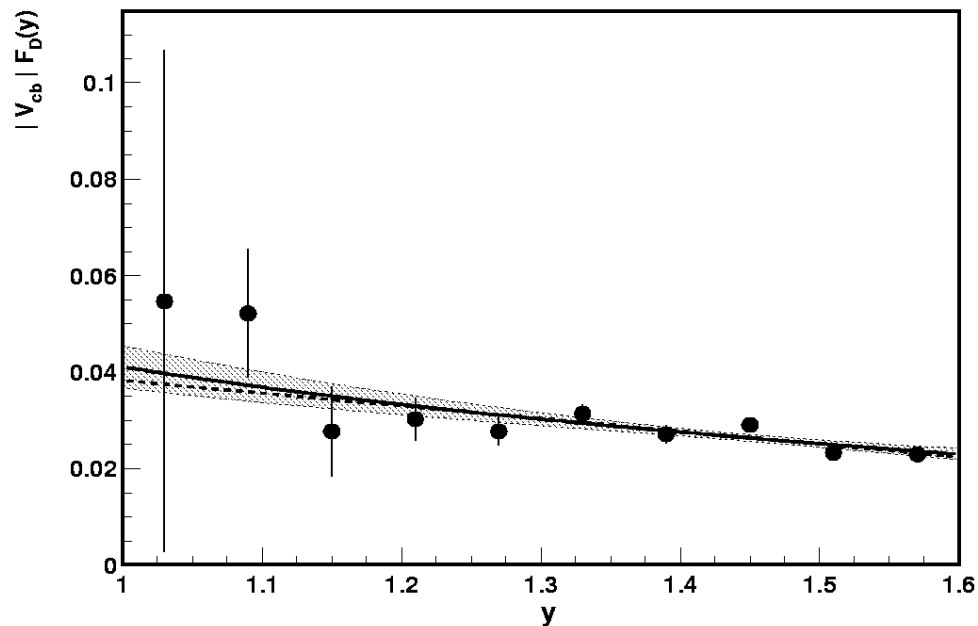
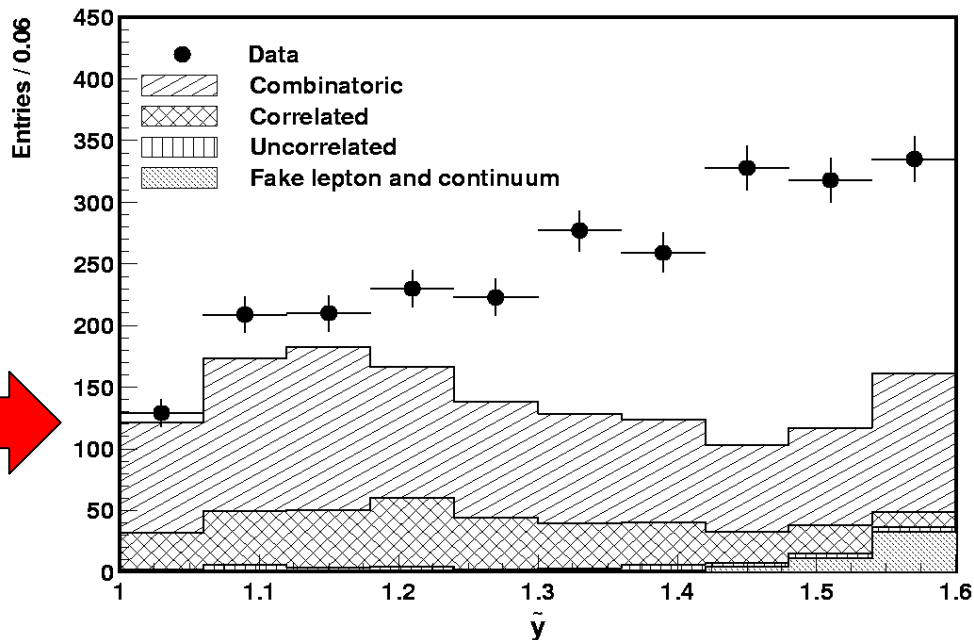
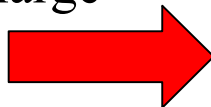
Fakes: 9 ± 1

Cont: 43 ± 7



Belle: $B \rightarrow D 1 \nu$

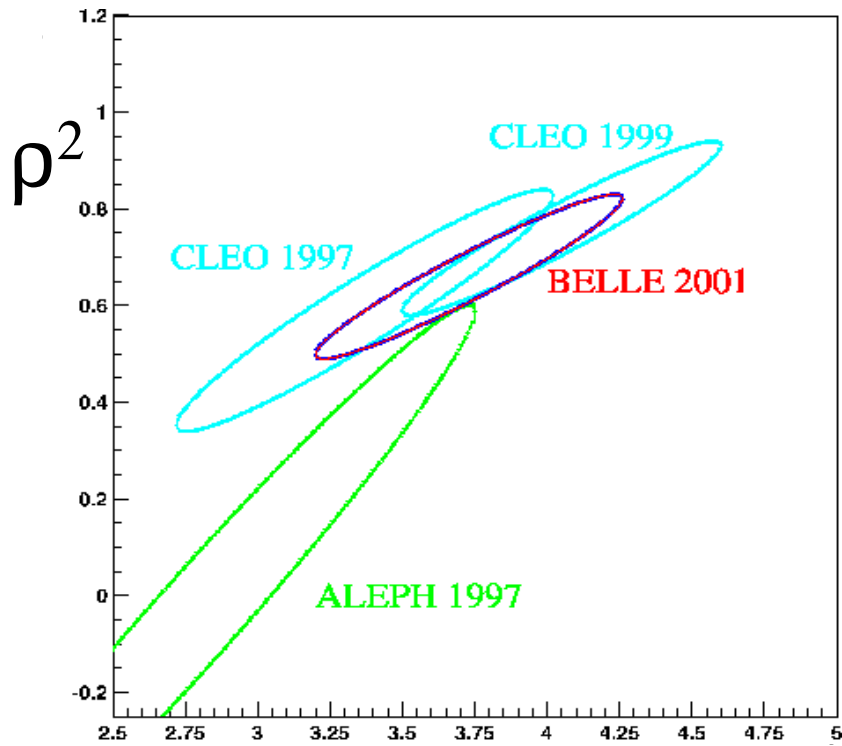
Note high efficiency but large backgrounds.



Results on V_{cb} and ρ^2 from $B \rightarrow D^{(*)} l \nu$

$$B^0 \rightarrow D^- l^+ \nu$$

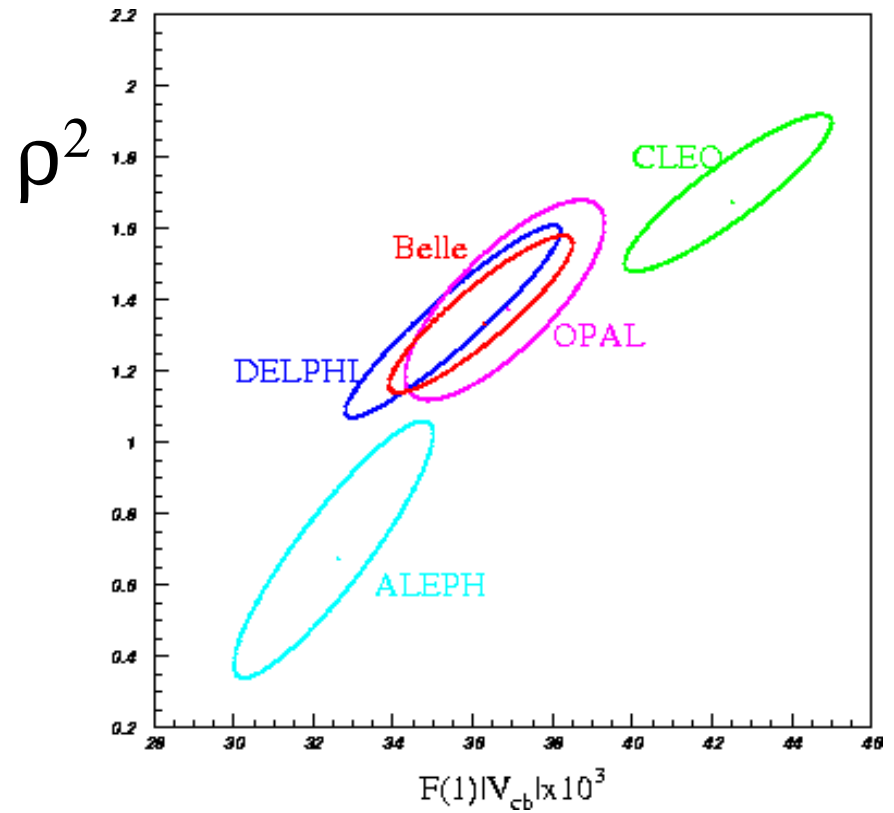
All using a linear model



$$F(1)|V_{cb}| \times 10^2$$

$$B^0 \rightarrow D^{*-} l^+ \nu$$

All using a dispersion relation



$$F(1)|V_{cb}| \times 10^2$$

Test HQET: $B \rightarrow D \ell \nu$ versus $B \rightarrow D^* \ell \nu$

From the results of $\bar{B}^0 \rightarrow D^{*+} e^- \bar{\nu}$ analysis at Belle, the ratio of $F_D(1)$ and $F_{D^*}(1)$ and the difference between $\hat{\rho}_D^2$ and $\hat{\rho}_{D^*}^2$ are measured to be

$$\frac{F_D(1)}{F_{D^*}(1)} = \begin{cases} 1.12 \pm 0.12 \pm 0.12 \text{ (Linear form factor)} \\ 1.16 \pm 0.14 \pm 0.12 \text{ (Caprini } et al. \text{ form factor)}, \end{cases}$$

$$\hat{\rho}_D^2 - \hat{\rho}_{D^*}^2 = \begin{cases} -0.12 \pm 0.18 \pm 0.13 \text{ (Linear form factor)} \\ -0.23 \pm 0.29 \pm 0.20 \text{ (Caprini } et al. \text{ form factor)}, \end{cases}$$

The size of $O(1/m_Q)$ corrections is not large.

With more data, we will start to observe these corrections to HQET.

Motivation for $B \rightarrow D^{(*)} l \nu$ form factor analysis

HQET which is used to extract V_{cb} also predicts ratios of form factors.

Form factors are a major source of uncertainty in $|V_{ub}|$ analysis.

Imagine $B \rightarrow D l \nu$ and $B \rightarrow D^* l \nu$ as two body $B \rightarrow D W$ and $D^* W$ decays. The first is a p-wave while the latter can be either a s, p or d-wave

Definition of angles for $B \rightarrow D^* l \nu$ FF analysis

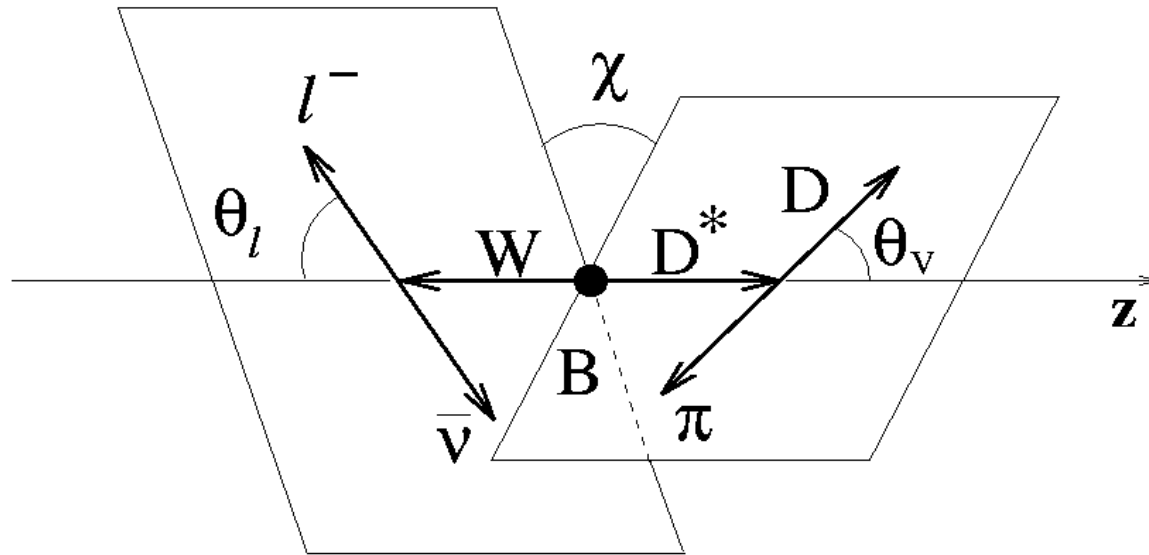


Fig. 34. Definition of the angles θ_V , θ_ℓ , and χ in the decay $\bar{B} \rightarrow D^* \ell^- \bar{\nu}$. (These angles are used for any $P \rightarrow V \ell \nu$ in which the vector meson decays into two pseudoscalars.) The lepton and neutrino are drawn back to back because they are shown in the W^* rest frame. Similarly, the D and the π are shown in the D^* rest frame. The angle θ_ℓ is thus measured in the W rest frame, while θ_V is measured in the D^* rest frame. The azimuthal angle χ is measured between the W and D^* decay planes. In the literature, the angle θ_ℓ is sometimes defined as the direction between the charged lepton and the recoiling vector meson, measured in the $\ell \nu$ rest frame.

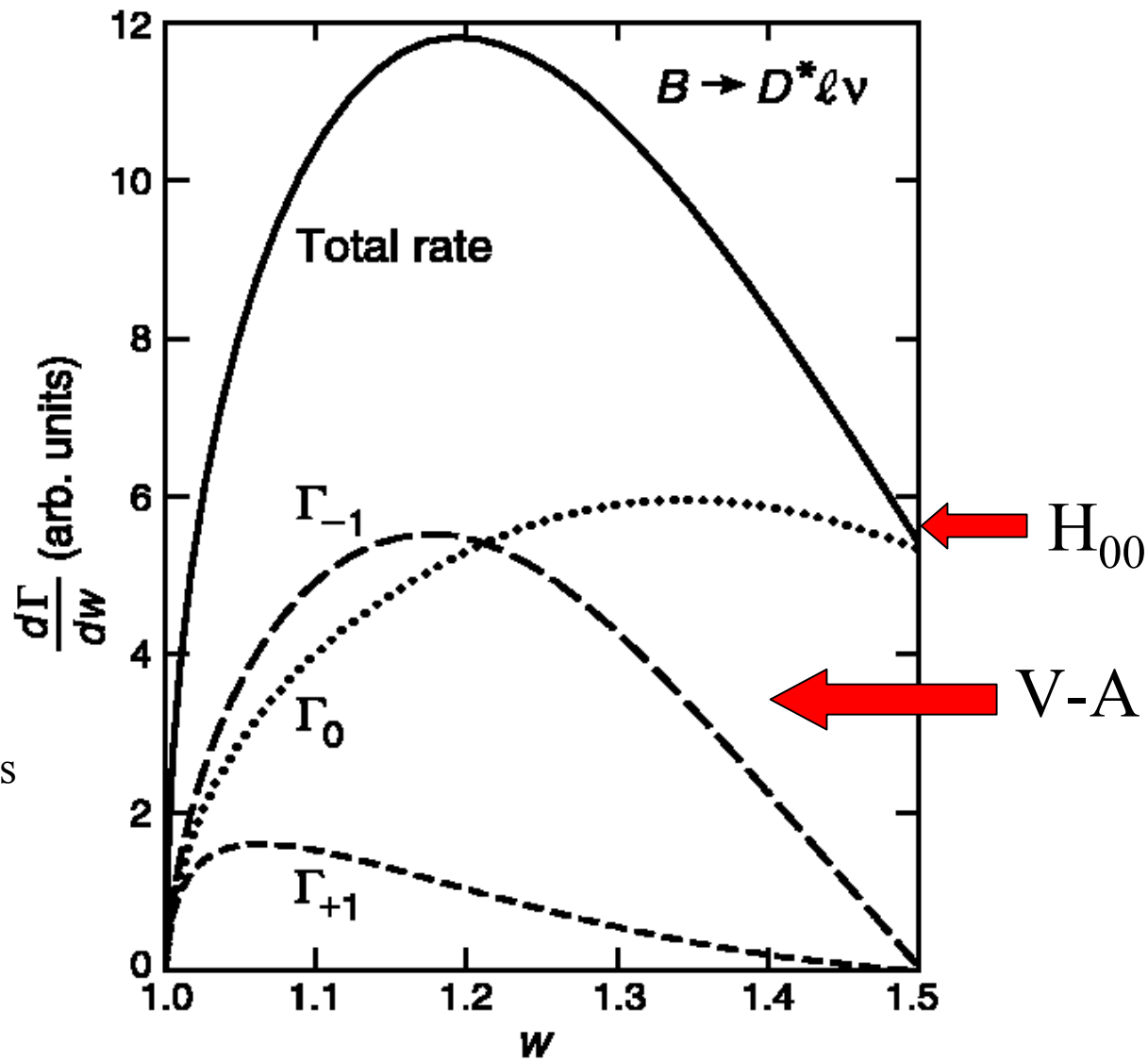
Differential rate for $B \rightarrow D^* l \nu$

The differential decay rate for $P(Q\bar{q}) \rightarrow V(q'\bar{q})\ell^-\bar{\nu}$, $V \rightarrow P_1P_2$ can be expressed in terms of these four kinematic variables q^2 , θ_ℓ , θ_V and χ (Gilm90, Korn90b):

$$\begin{aligned}
 \frac{d\Gamma(P \rightarrow V\ell\nu, V \rightarrow P_1P_2)}{dq^2 d\cos\theta_V d\cos\theta_\ell d\chi} &= \frac{3}{8(4\pi)^4} G_F^2 |V_{q'Q}|^2 \frac{p_V q^2}{M^2} \mathcal{B}(V \rightarrow P_1P_2) \\
 &\times \left\{ (1 - \eta \cos\theta_\ell)^2 \sin^2\theta_V |H_+(q^2)|^2 \right. \\
 &+ (1 + \eta \cos\theta_\ell)^2 \sin^2\theta_V |H_-(q^2)|^2 \\
 &+ 4 \sin^2\theta_\ell \cos^2\theta_V |H_0(q^2)|^2 \\
 &- 4\eta \sin\theta_\ell (1 - \eta \cos\theta_\ell) \sin\theta_V \cos\theta_V \cos\chi H_+(q^2) H_0(q^2) \\
 &+ 4\eta \sin\theta_\ell (1 + \eta \cos\theta_\ell) \sin\theta_V \cos\theta_V \cos\chi H_-(q^2) H_0(q^2) \\
 &\left. - 2 \sin^2\theta_\ell \sin^2\theta_V \cos 2\chi H_+(q^2) H_-(q^2) \right\}, \tag{113}
 \end{aligned}$$

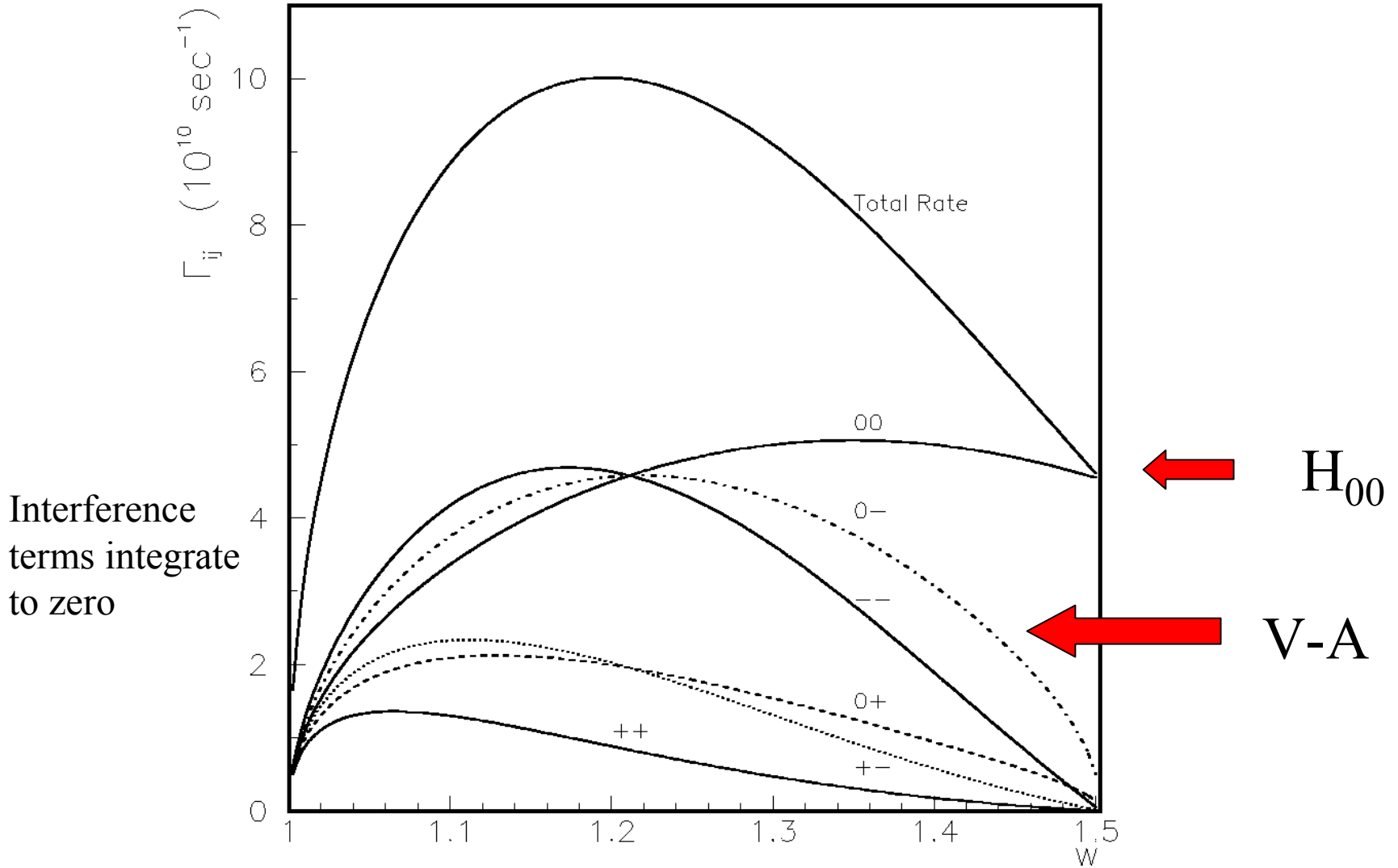
Involves three form factors $H_+(q^2)$, $H_-(q^2)$, $H_0(q^2)$ corresponding to the three possible W helicities.

The w dependence of $B \rightarrow D^* \ell \nu$ FFs.

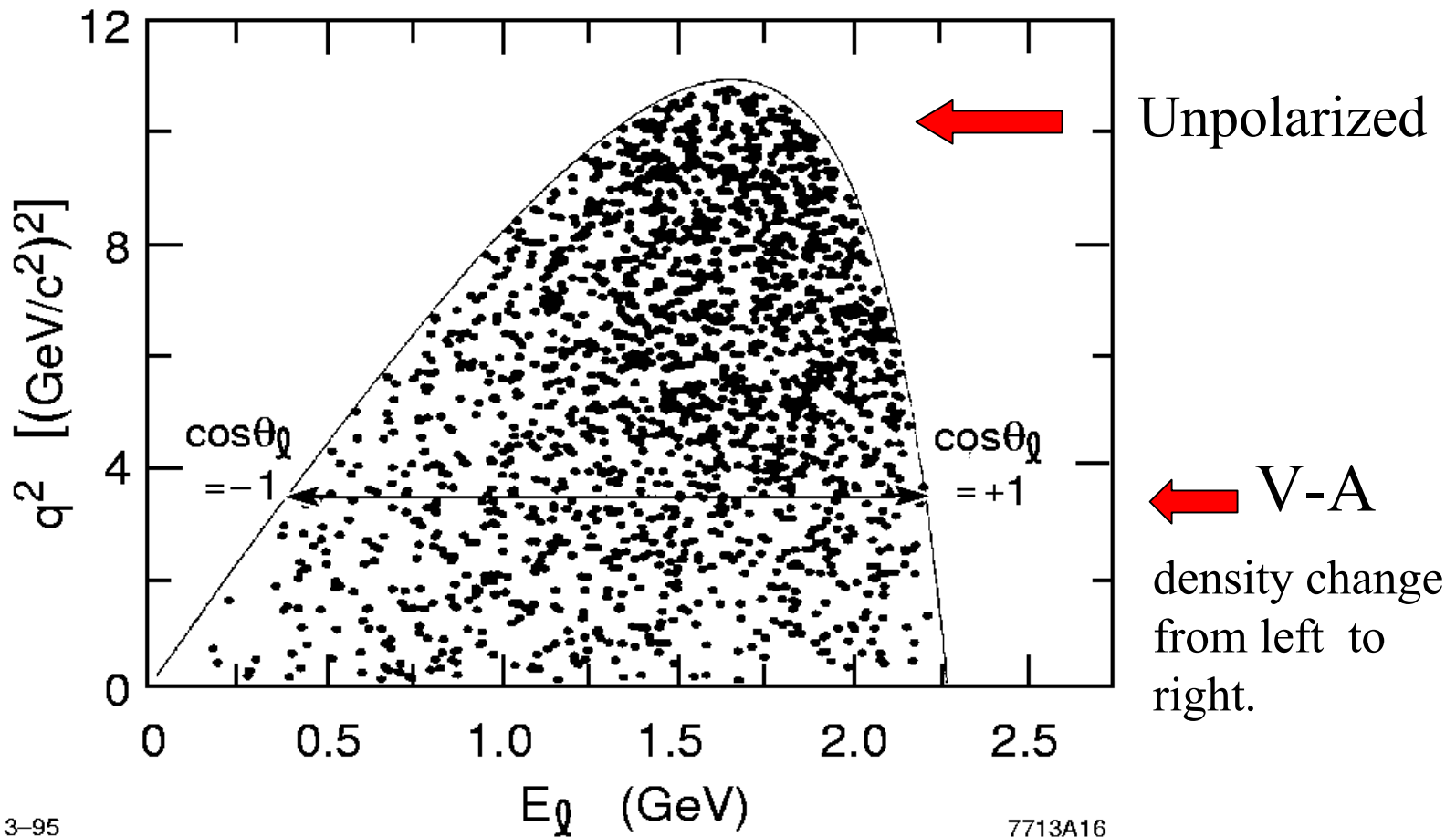


Interference terms
integrate to zero

The w dependence of $B \rightarrow D^* l \nu$ FFs including interference.




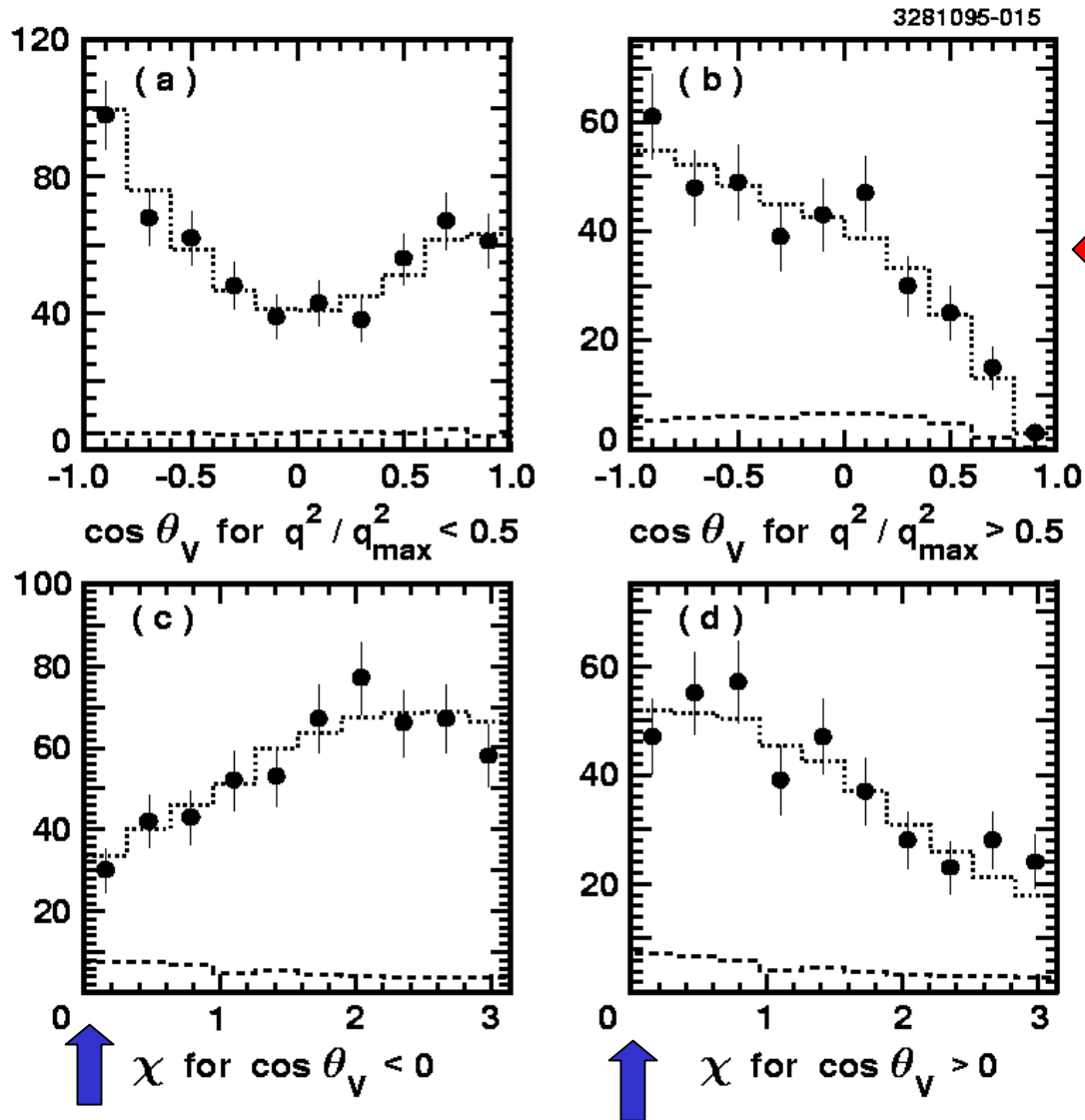
MC simulation of $B \rightarrow D^* l \nu$ Dalitz plot



CLEO: $B \rightarrow D^* 1 \nu$ FF analysis

H_{00} 

 $H_{00}, H_{\pm 1, \pm 1}$



Interference term $H_{-1,-1} H_{00}$

Helicity amplitudes and HQET $B \rightarrow D^* 1 \nu$ FF

$$H_0(w) = (m_B - m_{D^*}) \sqrt{\frac{m_B m_{D^*}}{q^2(w)}} (w + 1) h_{A_1}(w) \\ \times \left[1 + \left(\frac{w - 1}{1 - r} \right) (1 - R_2(w)) \right]$$

$$H_{\pm}(w) = (m_B - m_{D^*}) \sqrt{\frac{m_B m_{D^*}}{q^2(w)}} (w + 1) h_{A_1}(w) \\ \times \frac{\sqrt{1 - 2wr + r^2}}{1 - r} \left[1 \mp \sqrt{\frac{w - 1}{w + 1}} R_1(w) \right],$$

Measure

$$R_1(w) \equiv \frac{h_V(w)}{h_{A_1}(w)} = \left[1 - \frac{q^2}{(M + m_V)^2} \right] \frac{V(q^2)}{A_1(q^2)}$$

$$R_2(w) \equiv \frac{h_{A_3}(w) + (m_V/M) h_{A_2}(w)}{h_{A_1}(w)} \\ = \left[1 - \frac{q^2}{(M + m_V)^2} \right] \frac{A_2(q^2)}{A_1(q^2)}.$$

$R_1 = R_2 = 1$ in the
infinite mass
limit

Measure ratios, since h_{A_1} determines overall norm.

Helicity amplitudes and usual $B \rightarrow D^* 1 \nu$ FF

The helicity amplitudes can, in turn, be related to the two axial-vector form factors, $A_1(q^2)$ and $A_2(q^2)$, and the vector form factor, $V(q^2)$, which appear in the hadronic current (Eq. 7.47):

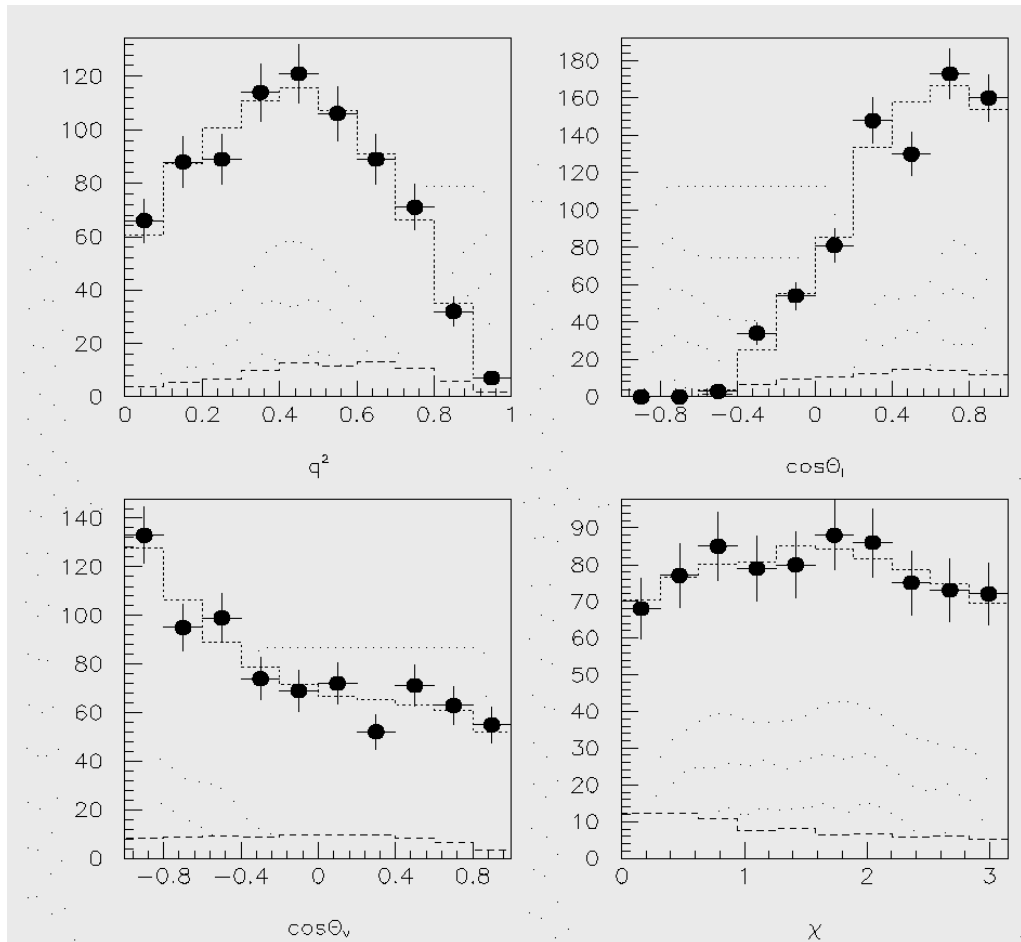
$$H_0(q^2) = \frac{1}{2m_V \sqrt{q^2}} [(M^2 - m_V^2 - q^2)(M + m_V)A_1(q^2) - 4 \frac{M^2 p_V^2}{M + m_V} A_2(q^2)] \quad (7.59)$$

and

$$H_{\pm}(q^2) = (M + m_V)A_1(q^2) \mp \frac{2Mp_V}{M + m_V} V(q^2). \quad (7.60)$$

CLEO

R_2 →



← R_1

R_2 →

← R_1

A 4-dimensional fit is used to find the ratios R_1, R_2

	$R_1(w = 1)$	$R_2(w = 1)$
CLEO II	$1.18 \pm 0.30 \pm 0.12$	$0.71 \pm 0.22 \pm 0.07$
Neubert	1.3 ± 0.1	0.8 ± 0.2
Close & Wambach	1.15	0.91
ISGW2	1.27	1.01

← Test corrections to HQET

Conclusions on V_{cb}

Inclusive approaches give high precision results (1.2% exp) limited by errors on OPE parameters, theory and quark-hadron duality.

$$V_{cb} = (40.4 \pm 0.5(\text{exp}) \pm 0.5(\lambda_1, \Lambda) \pm 0.8(\text{theo})) \times 10^{-3}$$

HQET based approach to $B \rightarrow D^ l \nu$ gives increasingly precise measurements (2.7% exp) also limited by theory errors.*

$$V_{cb} = (38.3 \pm 0.5(\text{exp}) \pm 0.9(\text{theo})) \times 10^{-3}$$

Check with complementary measmts (FFs, $B \rightarrow D l \nu, \dots$)