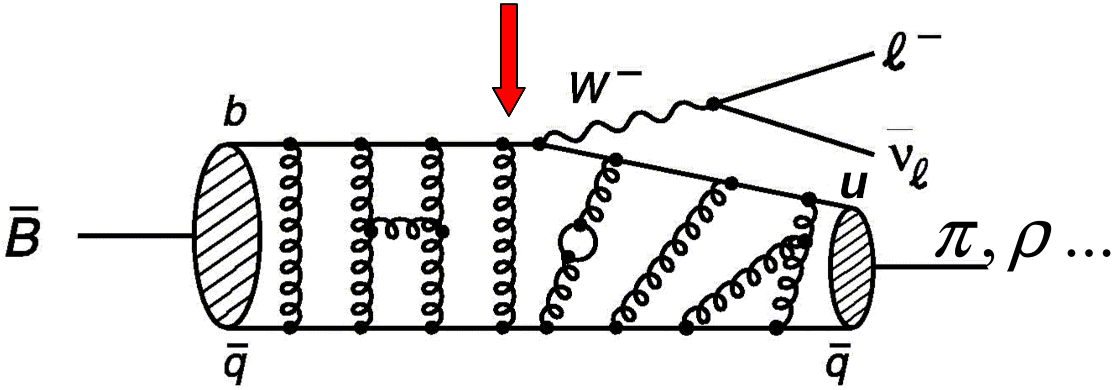
Measurement of $|V_{ub}|$

Tom Browder (University of Hawaii)

Inclusive approaches (endpoint, M_X , q^2)

Exclusive approaches ($B \rightarrow \pi\, l\, \nu$, $B \rightarrow \rho\, l\, \nu$)

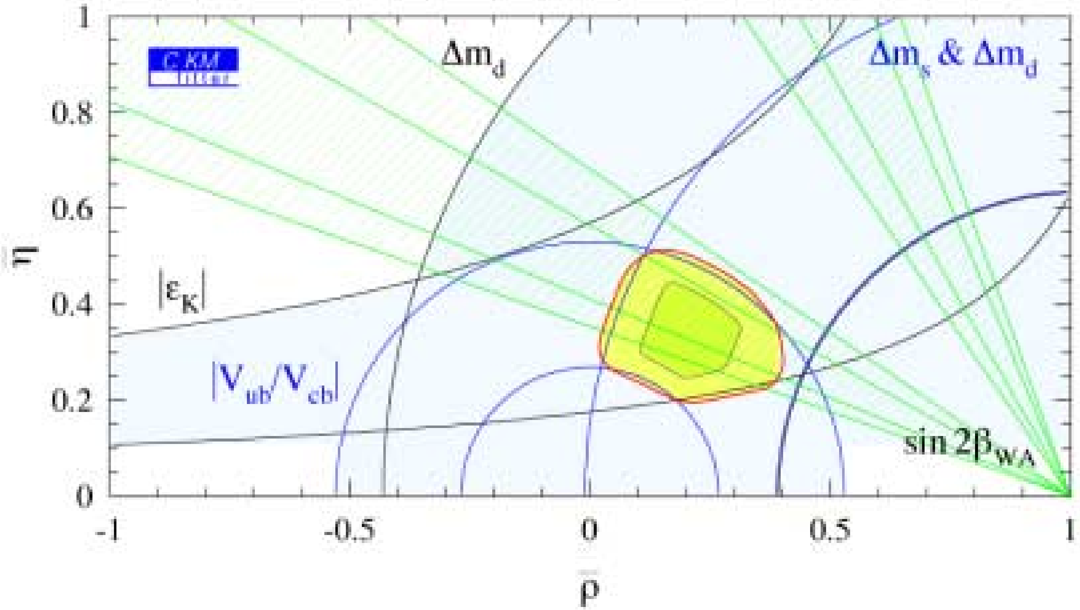
Conclusion



The V_{ub} element of the CKM matrix

$$\begin{pmatrix} Vud & Vus & Vub \\ Vcd & Vcs & Vcb \\ Vtd & Vts & Vtb \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & \boxed{A\lambda^3(\rho-i\eta)} \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$|V_{ub}|$ determines a circle of radius^2 = $\rho^2 + \eta^2$ for the apex of the Bjorken triangle. *Very important for indirect constraints on the CKM triangle and for detecting New Physics.*

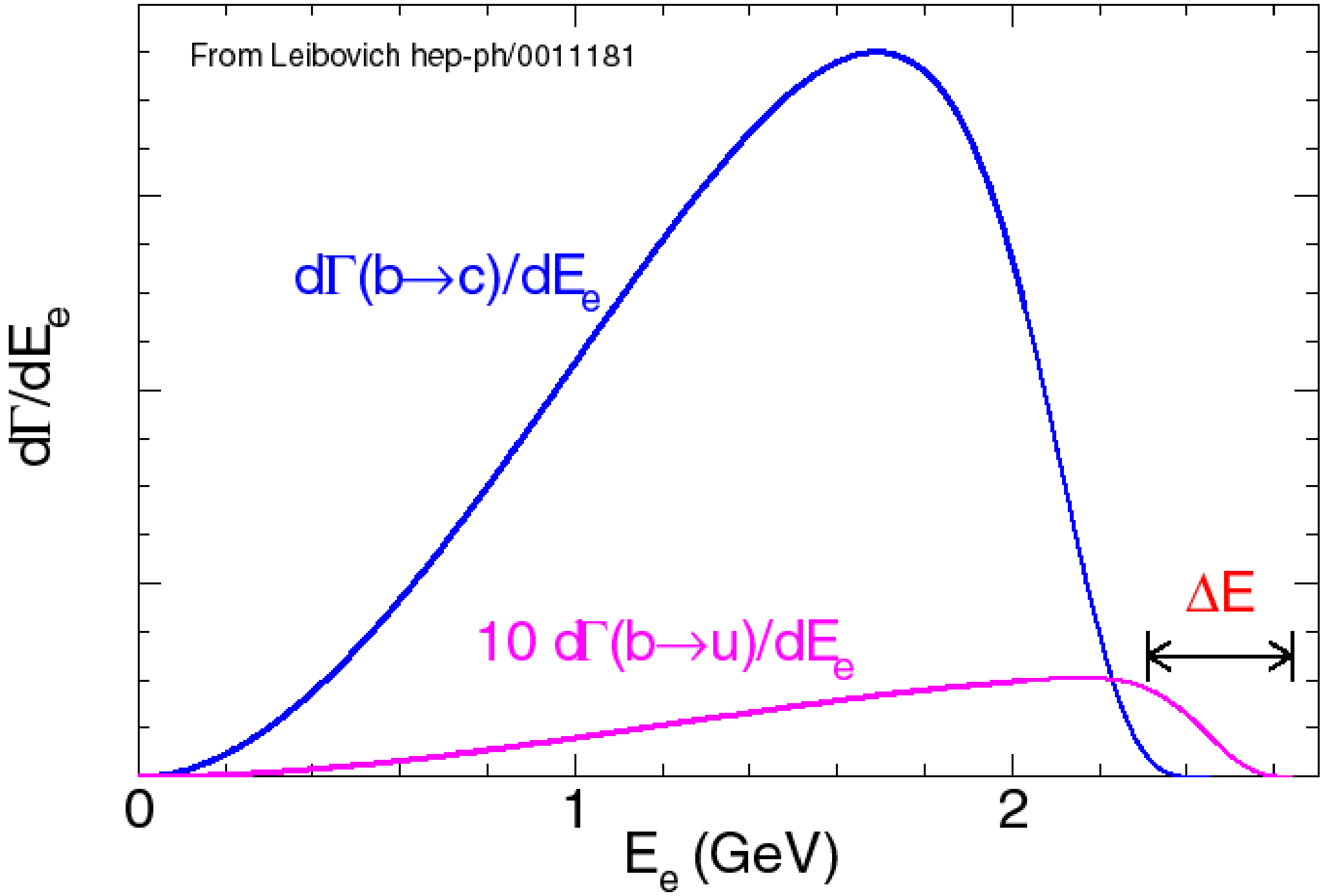


How are $|V_{ub}|$ measurements different from $|V_{cb}|$ measurements?

Cannot observe the whole spectrum of b →u l v unlike b →c l v. Backgrounds are too large hence restricted to small portions of phase space.

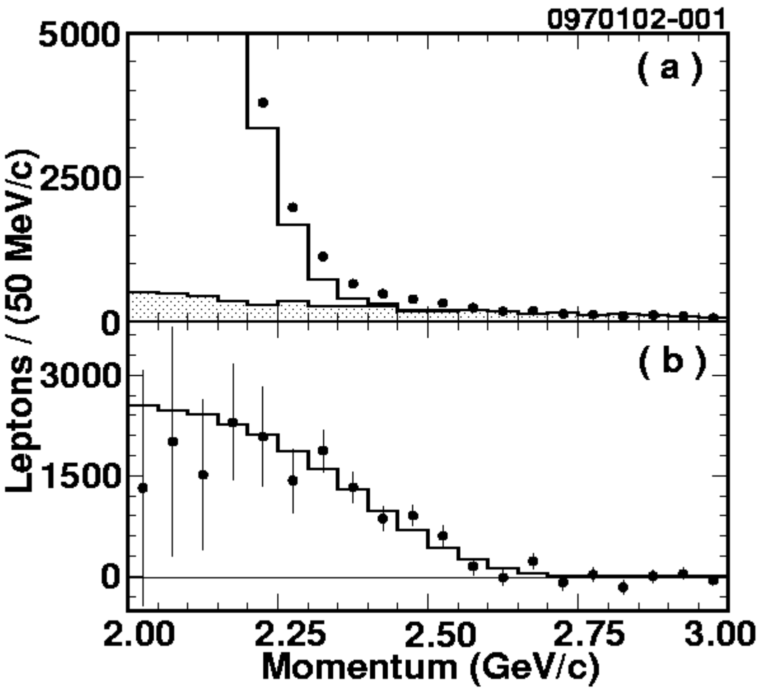
Heavy →light FF not Heavy →Heavv FF: very little simplification from HQET. W or q^2 *range is much larger (dependence on FF is much greater).*

$|V_{ub}|$ from the Lepton Endpoint



A large theoretical extrapolation is required to obtain |Vub|

CLEO 2001: $|V_{ub}|$ from leptons beyond the b→c endpoint



1901±127±256

Large cont subtr.

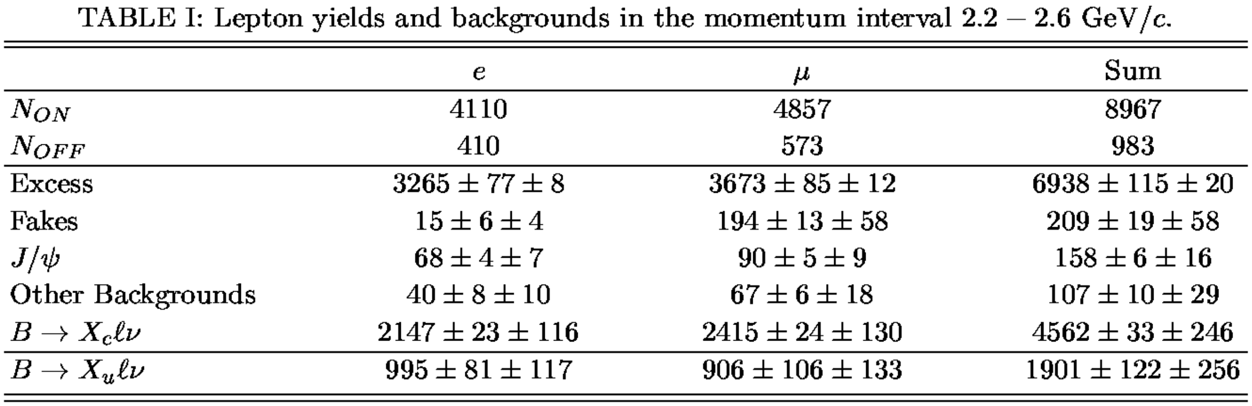
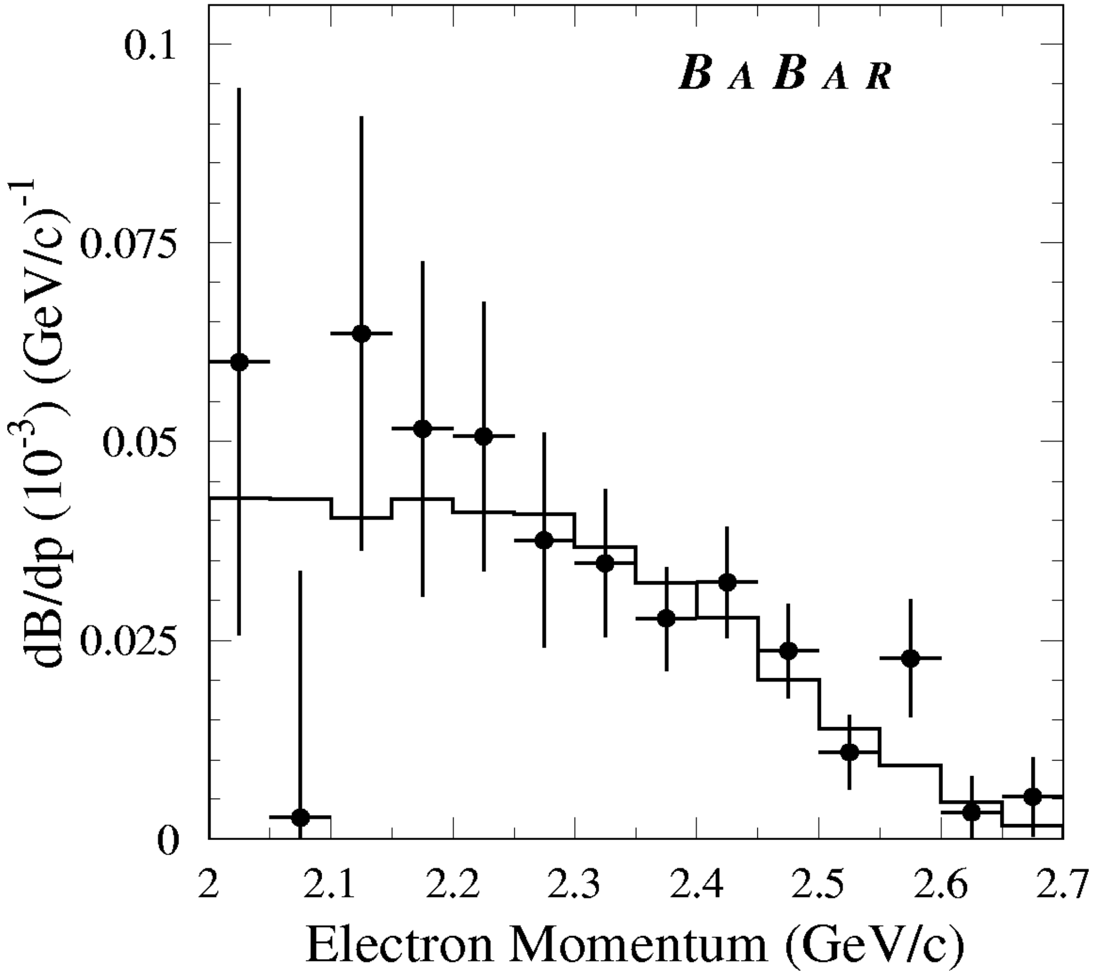


TABLE I: Lepton yields and backgrounds in the momentum interval 2.2 – 2.6 GeV/c.

	e	μ	Sum
N_{ON}	4110	4857	8967
N_{OFF}	410	573	983
Excess	$3265 \pm 77 \pm 8$	$3673 \pm 85 \pm 12$	$6938 \pm 115 \pm 20$
Fakes	$15 \pm 6 \pm 4$	$194 \pm 13 \pm 58$	$209 \pm 19 \pm 58$
J/ψ	$68 \pm 4 \pm 7$	$90 \pm 5 \pm 9$	$158 \pm 6 \pm 16$
Other Backgrounds	$40 \pm 8 \pm 10$	$67 \pm 6 \pm 18$	$107 \pm 10 \pm 29$
$B \to X_c \ell \nu$	$2147 \pm 23 \pm 116$	$2415 \pm 24 \pm 130$	$4562 \pm 33 \pm 246$
$B \to X_u \ell \nu$	$995 \pm 81 \pm 117$	$906 \pm 106 \pm 133$	$1901 \pm 122 \pm 256$

BABAR 2002: $|V_{ub}|$ from leptons beyond the endpoint

1696±133



Bkg subtracted yield and ISGW2 model

BF(b→u l v, 2.3-2.6 GeV)= (0.152 ±0.014±0.014) x 10^{-3}

Agrees well with CLEO (0.143±0.010±0.014) x 10^{-3}

Model Dependence in $|V_{ub}|$ from inclusive decay.

CLEO 1993: Model dependence

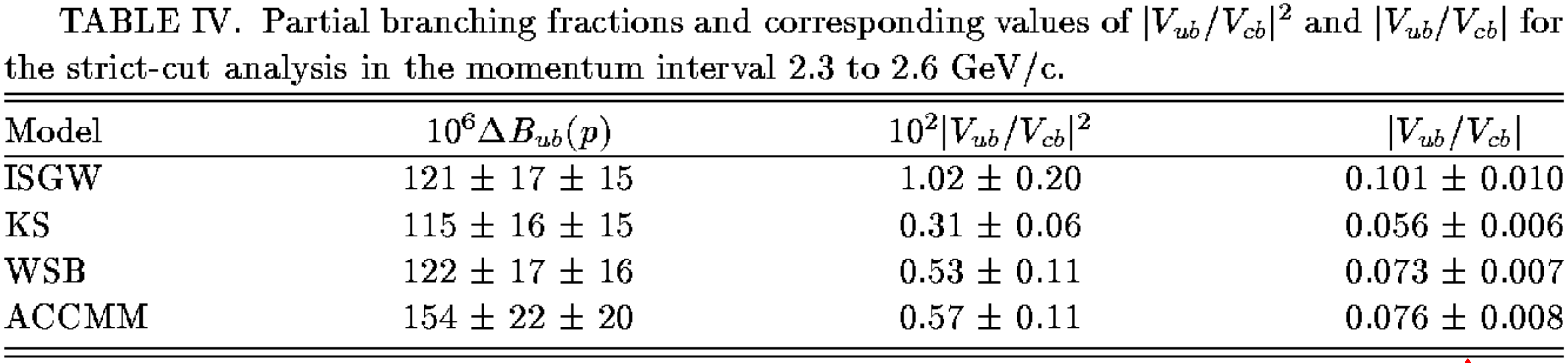


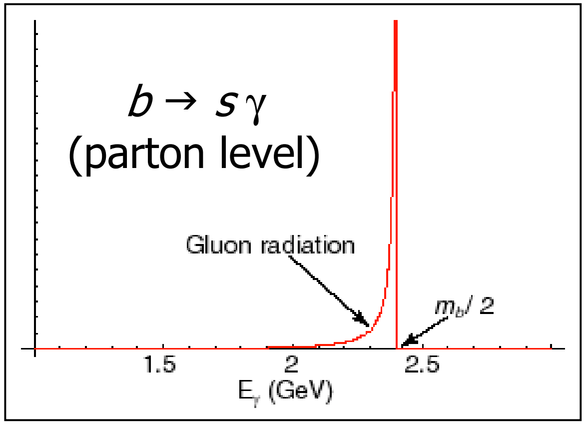
TABLE IV. Partial branching fractions and corresponding values of $|V_{ub}/V_{cb}|^2$ and $|V_{ub}/V_{cb}|$ for the strict-cut analysis in the momentum interval 2.3 to 2.6 GeV/c.

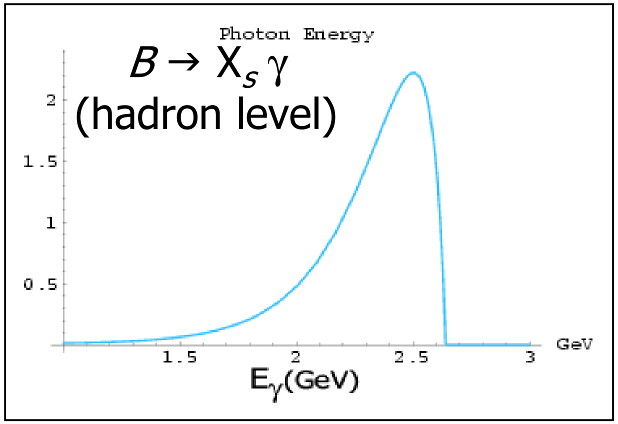
Model	$10^6 \Delta B_{ub}(p)$	$10^2 \lvert V_{ub}/V_{cb}\rvert^2$	$\lvert V_{ub}/V_{cb}\rvert$
ISGW	121 ± 17 ± 15	1.02 ± 0.20	0.101 ± 0.010
KS	115 ± 16 ± 15	0.31 ± 0.06	0.056 ± 0.006
WSB	122 ± 17 ± 16	0.53 ± 0.11	0.073 ± 0.007
ACCMM	154 ± 22 ± 20	0.57 ± 0.11	0.076 ± 0.008

CLEO 2001, BABAR 2002: No longer use a model for extrapolation. Instead rely on the $b \rightarrow s\, \gamma$ shape function..

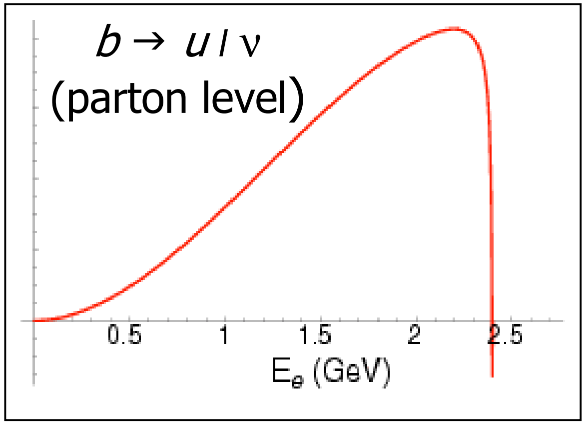
(Discussed in lectures by Ligeti)

B → light quark shape function, SAME (to lowest order in Λ_{QCD}/m_b) for $b \to s\gamma$ ($B \to X_s\gamma$) and $b \to u\, l\nu$ ($B \to X_u\, l\nu$).



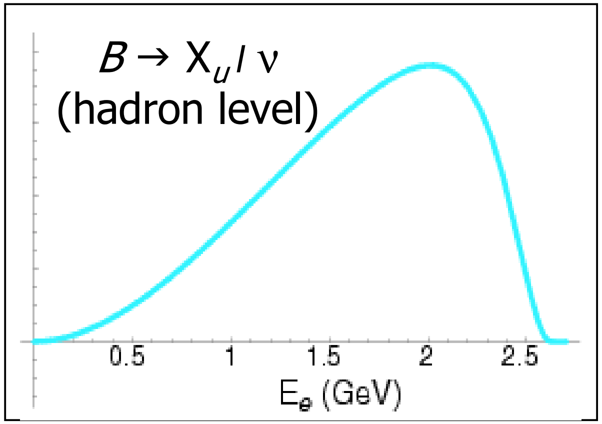


Convolute with light cone shape function.



Fraction of $b \to u\ell\nu$ spectrum above 2.2 is

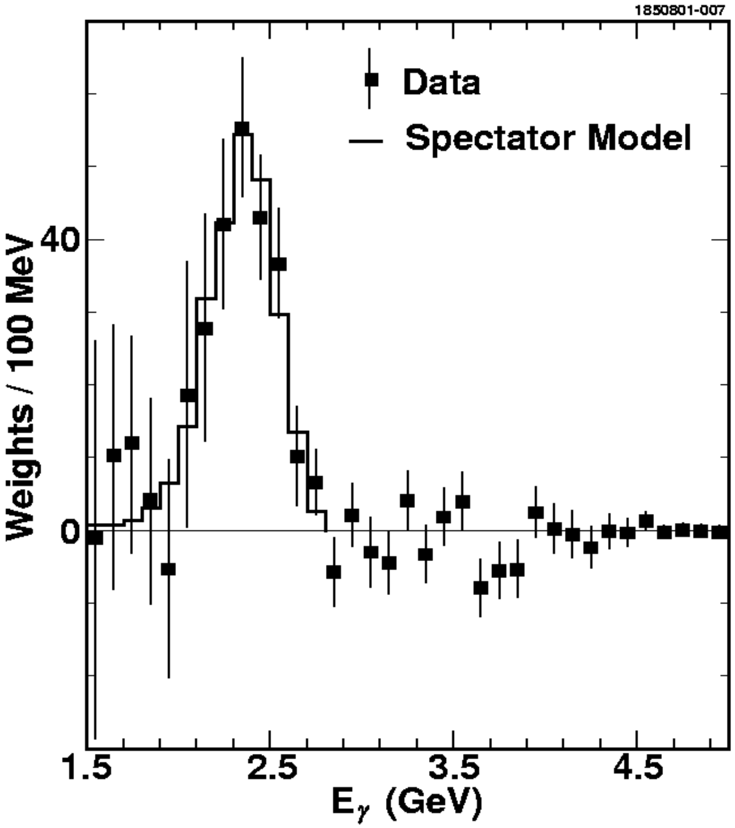
0.13 ± 0.03



Extrapolation from endpoint region using b→s γ

Idea: Use the shape function in b→s γ to determine the b→u l ν shape function. Then get the fraction of events in the b→u l υ endpoint region.

CLEO



Idea introduced by Neubert; Leibovich, Low, and Rothstein.

f_u (2.2-2.6 GeV)

= 0.130±0.024±0.05

V_{ub} Extrapolation from endpoint region using b→s γ

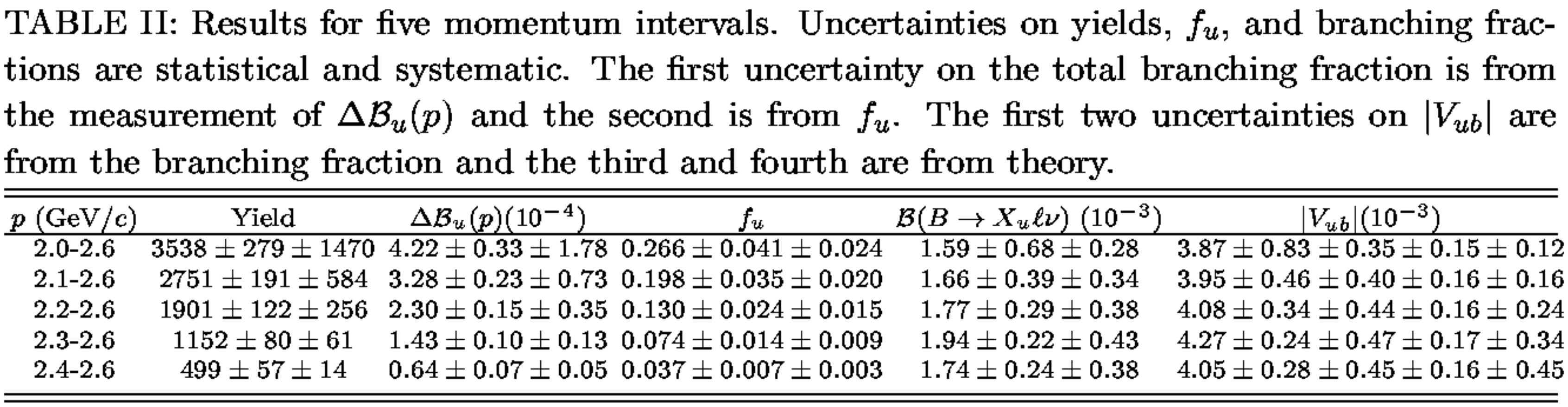


TABLE II: Results for five momentum intervals. Uncertainties on yields, f_u, and branching fractions are statistical and systematic. The first uncertainty on the total branching fraction is from the measurement of $\Delta\mathcal{B}_u(p)$ and the second is from f_u. The first two uncertainties on $|V_{ub}|$ are from the branching fraction and the third and fourth are from theory.

p (GeV/c)	Yield	$\Delta\mathcal{B}_u(p)(10^{-4})$	f_u	$\mathcal{B}(B \to X_u \ell\nu)$ (10^{-3})	$\lvert V_{ub}\rvert(10^{-3})$
2.0-2.6	$3538 \pm 279 \pm 1470$	$4.22 \pm 0.33 \pm 1.78$	$0.266 \pm 0.041 \pm 0.024$	$1.59 \pm 0.68 \pm 0.28$	$3.87 \pm 0.83 \pm 0.35 \pm 0.15 \pm 0.12$
2.1-2.6	$2751 \pm 191 \pm 584$	$3.28 \pm 0.23 \pm 0.73$	$0.198 \pm 0.035 \pm 0.020$	$1.66 \pm 0.39 \pm 0.34$	$3.95 \pm 0.46 \pm 0.40 \pm 0.16 \pm 0.16$
2.2-2.6	$1901 \pm 122 \pm 256$	$2.30 \pm 0.15 \pm 0.35$	$0.130 \pm 0.024 \pm 0.015$	$1.77 \pm 0.29 \pm 0.38$	$4.08 \pm 0.34 \pm 0.44 \pm 0.16 \pm 0.24$
2.3-2.6	$1152 \pm 80 \pm 61$	$1.43 \pm 0.10 \pm 0.13$	$0.074 \pm 0.014 \pm 0.009$	$1.94 \pm 0.22 \pm 0.43$	$4.27 \pm 0.24 \pm 0.47 \pm 0.17 \pm 0.34$
2.4-2.6	$499 \pm 57 \pm 14$	$0.64 \pm 0.07 \pm 0.05$	$0.037 \pm 0.007 \pm 0.003$	$1.74 \pm 0.24 \pm 0.38$	$4.05 \pm 0.28 \pm 0.45 \pm 0.16 \pm 0.45$

Optimal interval is $2.2 < p_L < 2.6$ GeV

BF(2.2-2.6 GeV) = $(2.30\pm0.15\pm0.35) \times 10^{-4}$

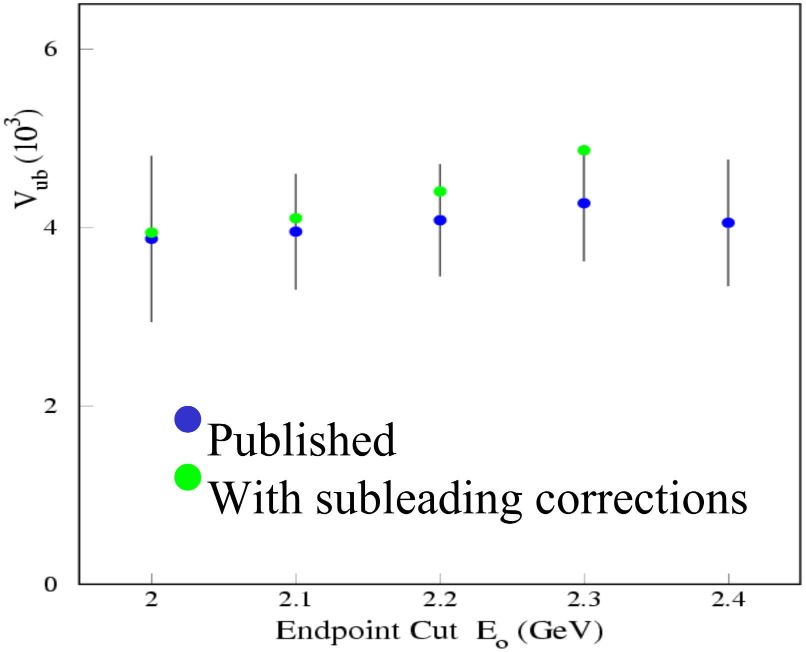
f_u (2.2-2.6 GeV)= $0.130\pm0.024\pm0.05$

$$|V_{ub}| = (3.07 \pm 0.12) \times 10^{-3} \times \left[\frac{\mathcal{B}(B \to X_u e\nu)}{0.001} \frac{1.6\text{ ps}}{\tau_B}\right]^{\frac{1}{2}}.$$

CLEO: |V_{ub}| from Lepton Endpoint (using $b \to s\gamma$)

$$|Vub| = (4.08 \pm 0.34 \pm 0.44 \pm 0.16 \pm 0.24) \times 10^{-3}$$

stat b→s γ Γ theory Λ/M_B theory



- *Subleading corrections large*
  C. Bauer, M. Luke, T. Mannel
  A. Leibovich, Z. Ligeti, M. Wise

- Method for partial inclusion of subleading corrections: Neubert

- *Quark-hadron duality ?*

How can we improve our knowledge of $|V_{ub}|$ from inclusive decays ?

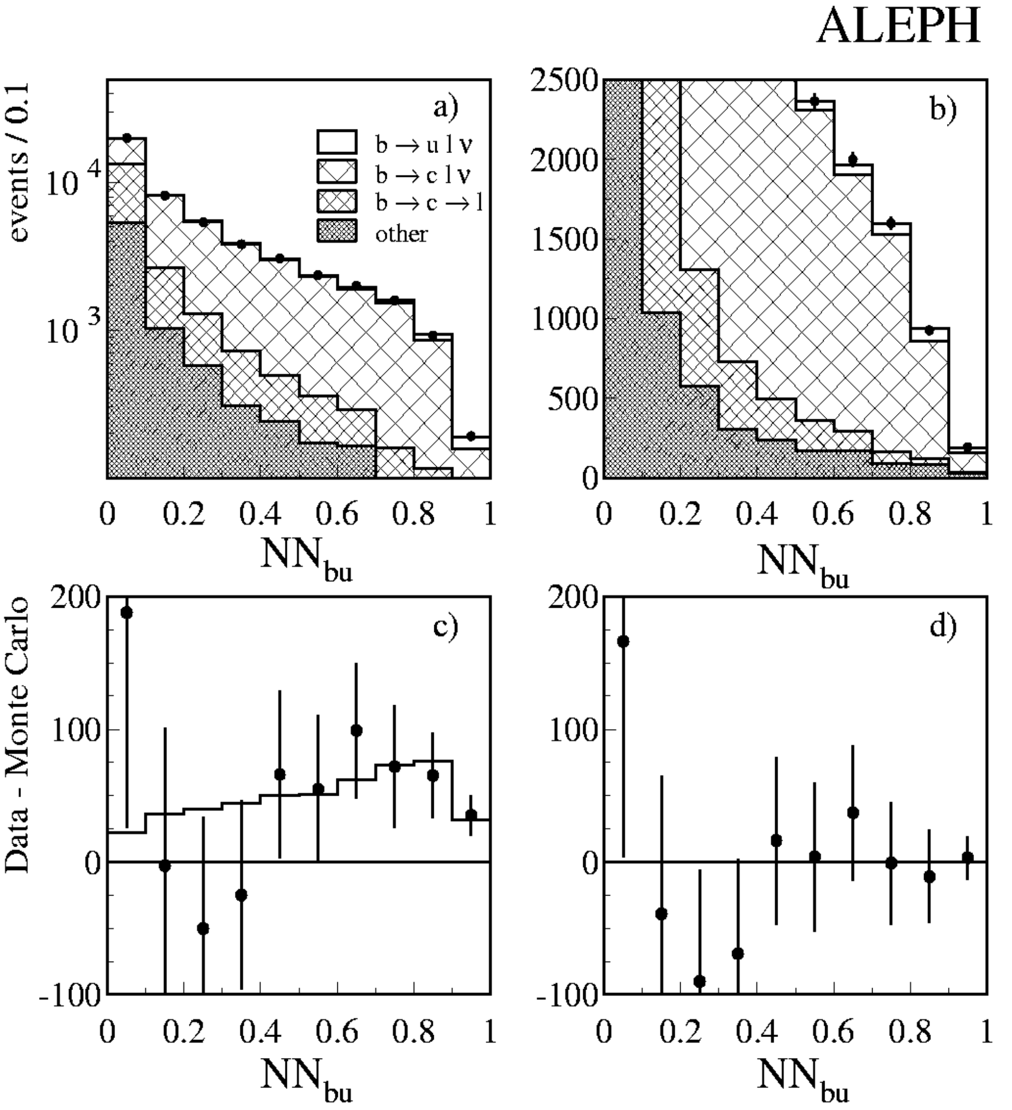
Use all of phase space (LEP)

Use favorable regions of M_X *or* q^2 *(DELPHI, CLEO) and new theoretical strategies*

Fraction with E_L >2.2 GeV (~15%); fraction with $M_X < M_D$ (~70 %), fraction with $q^2 > (M_B - M_D)^2$ (~20%)

ALEPH $|V_{ub}|$ Measurement

S/B=0.07

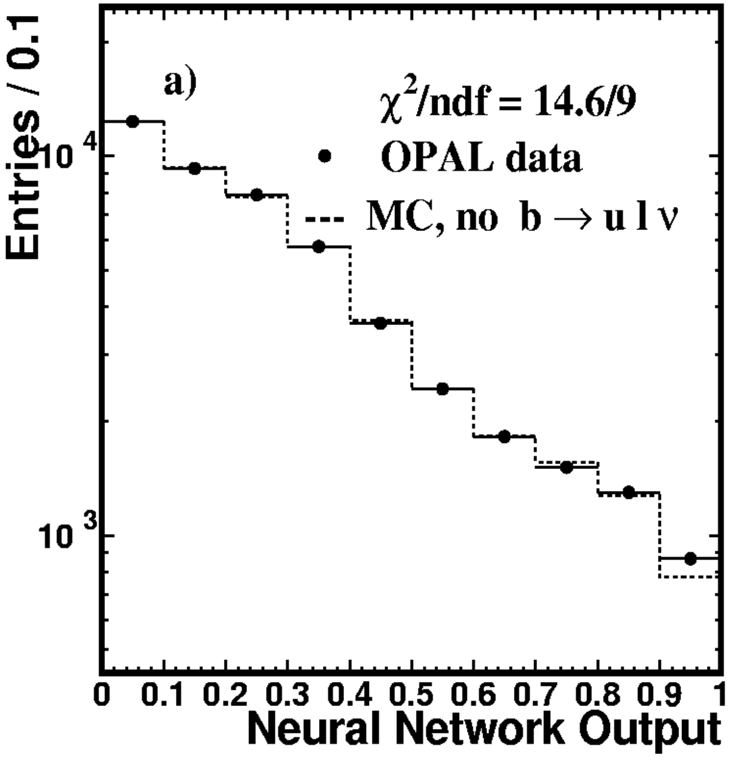


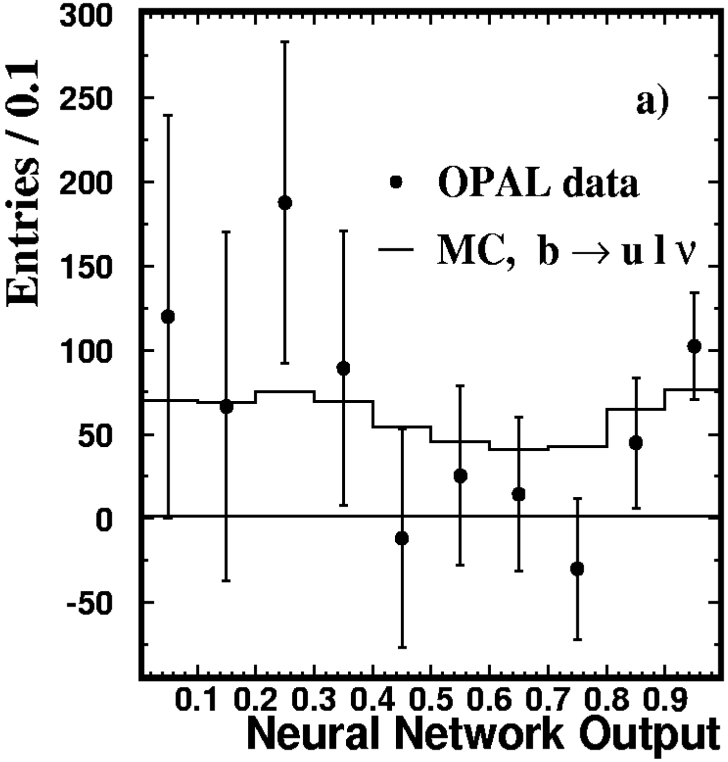
b→u enriched

b→u depleted

20 kinematic variables in a NN

OPAL $|V_{ub}|$ Measurement

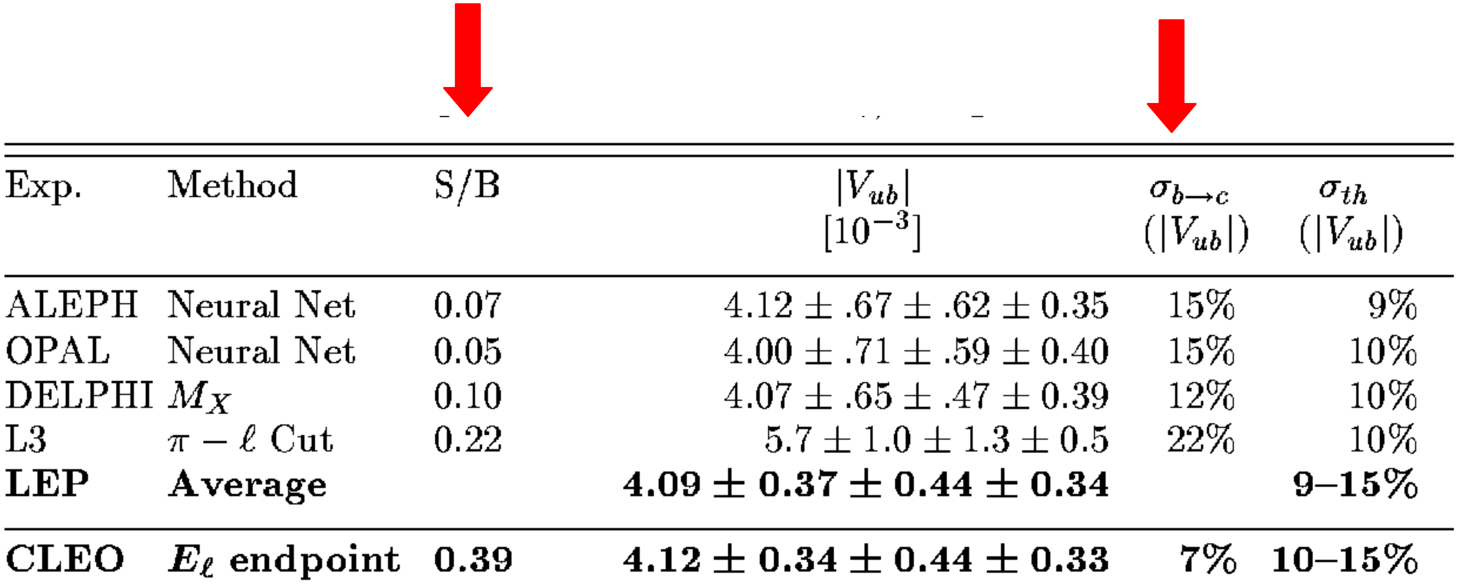




Huge background suppressed with 7 variable Neural Net

Small signal extraction depends on $b \to c\ell\nu$ model! $S/B = 0.05$

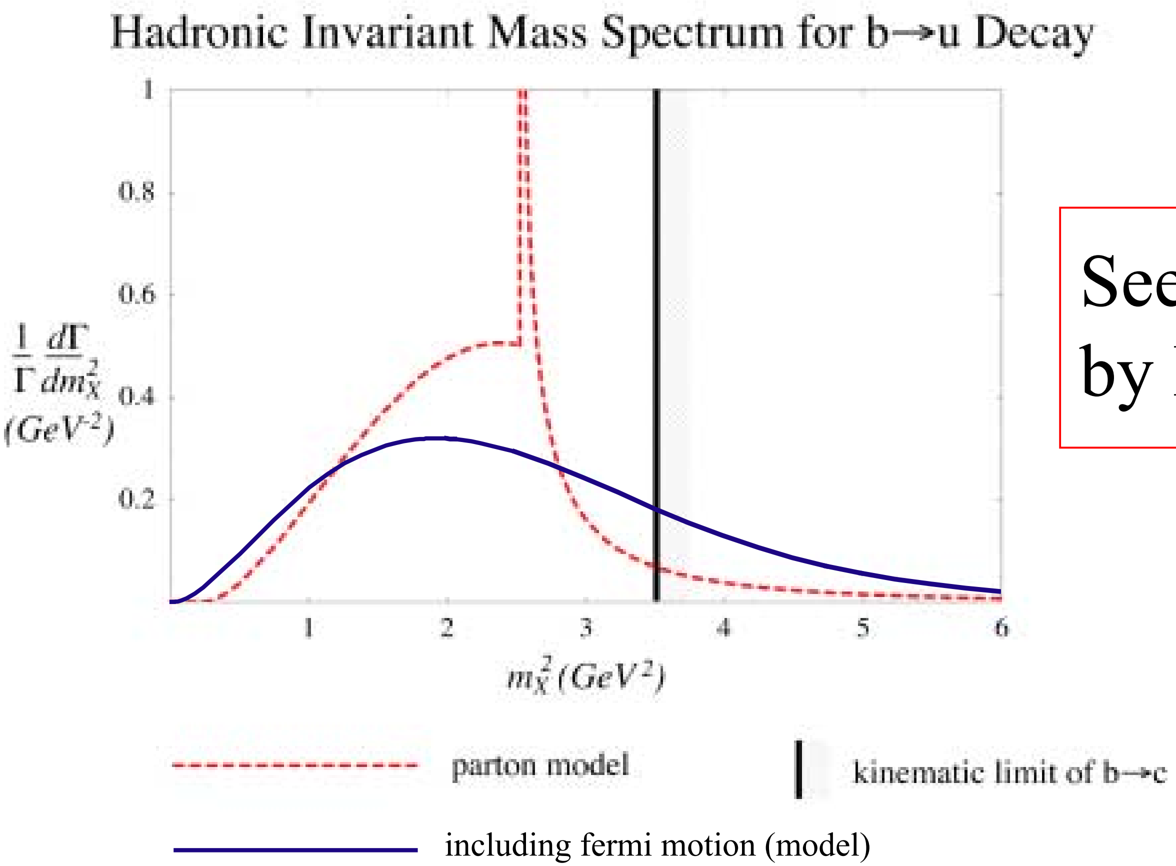
Published Inclusive |Vub| Determinations



LEP

Exp.	Method	S/B	$\lvert V_{ub}\rvert$ $[10^{-3}]$	$\sigma_{b\to c}$ $(\lvert V_{ub}\rvert)$	σ_{th} $(\lvert V_{ub}\rvert)$
ALEPH	Neural Net	0.07	$4.12 \pm .67 \pm .62 \pm 0.35$	15%	9%
OPAL	Neural Net	0.05	$4.00 \pm .71 \pm .59 \pm 0.40$	15%	10%
DELPHI	M_X	0.10	$4.07 \pm .65 \pm .47 \pm 0.39$	12%	10%
L3	$\pi - \ell$ Cut	0.22	$5.7 \pm 1.0 \pm 1.3 \pm 0.5$	22%	10%
LEP	**Average**		$\mathbf{4.09 \pm 0.37 \pm 0.44 \pm 0.34}$		**9–15%**
CLEO	**E_ℓ endpoint**	**0.39**	$\mathbf{4.12 \pm 0.34 \pm 0.44 \pm 0.33}$	**7%**	**10–15%**

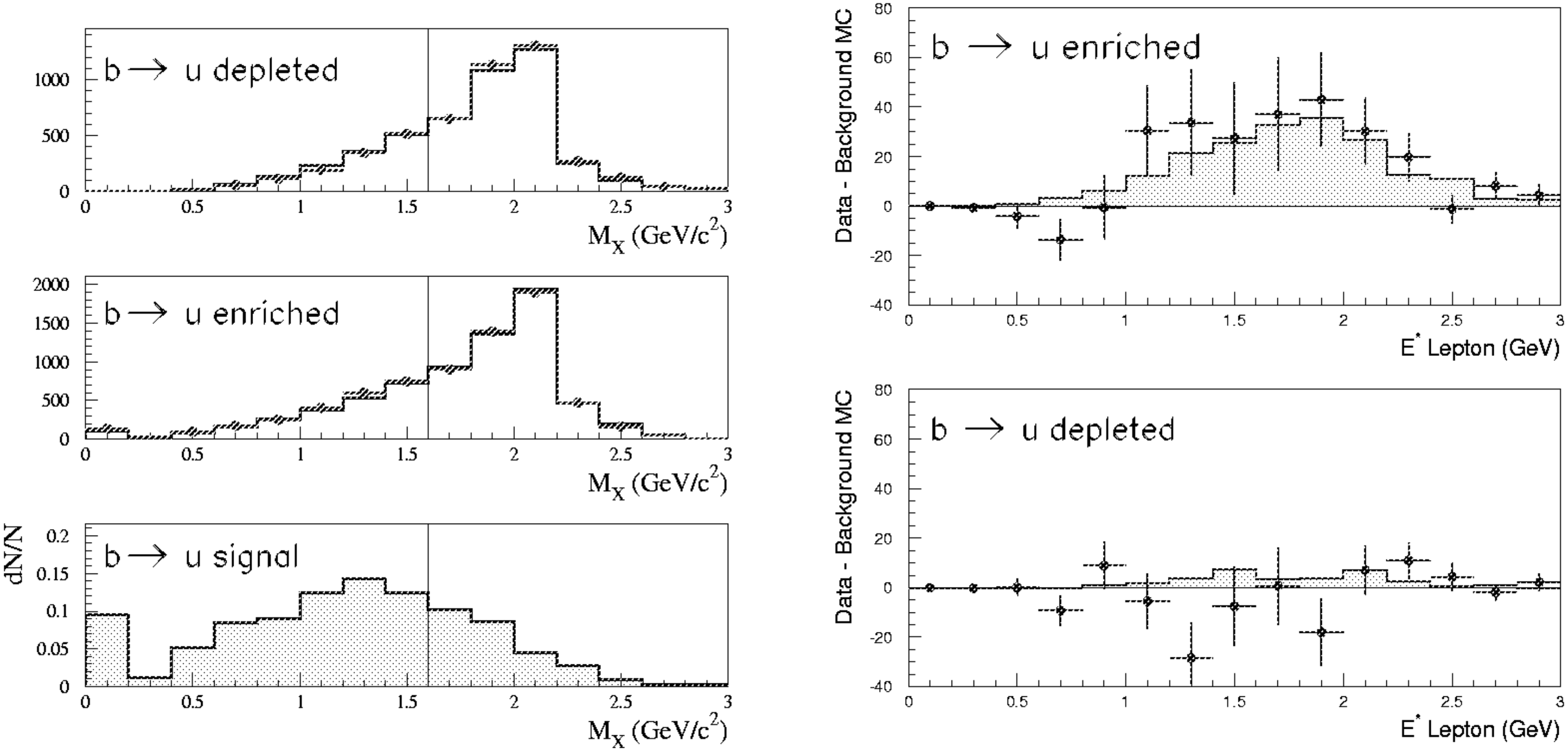
Battaglia and Gibbons



See lectures by Ligeti

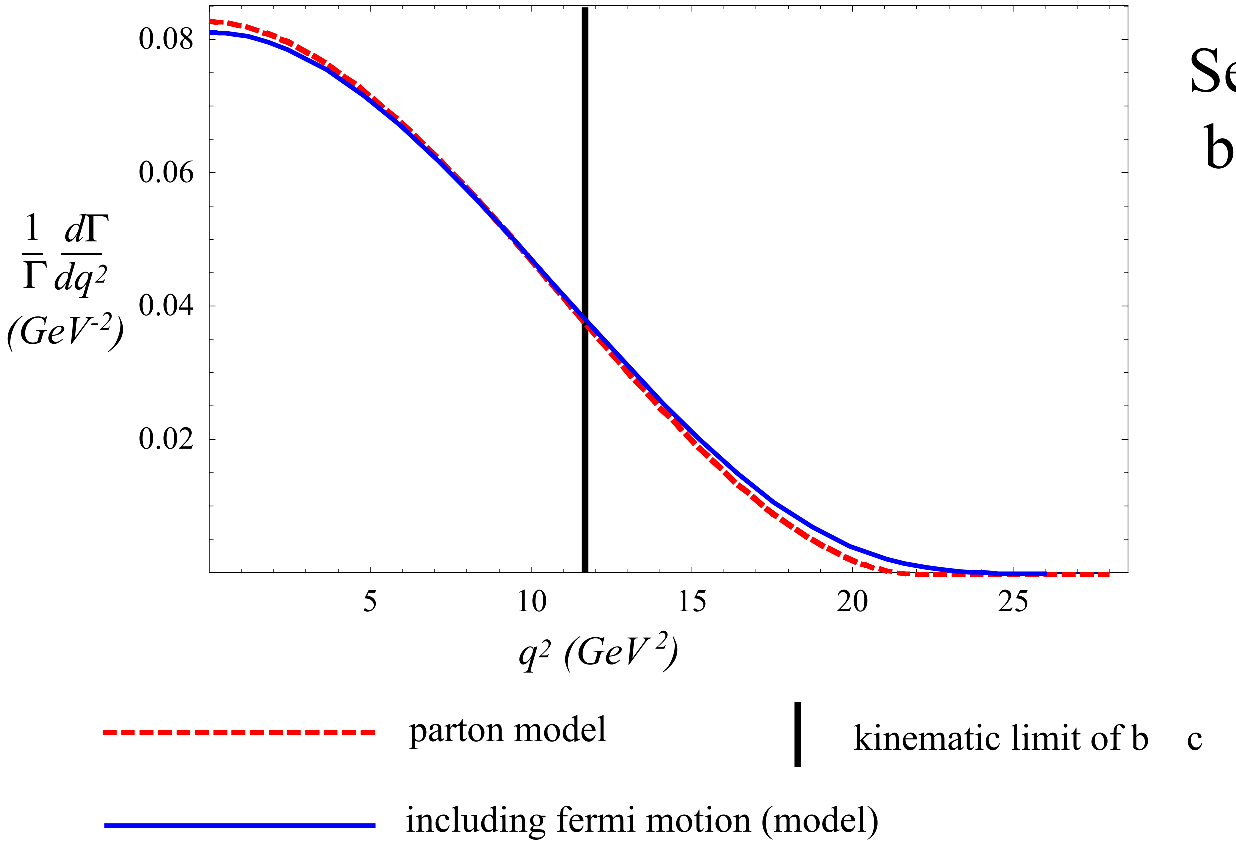
- singularity is smeared out by b quark light-cone distribution function $f(k_+)$
- rate is sensitive to details of $f(k_+)$ unless $m_X^2 >> \Lambda_{QCD} m_b$ (bad for $m_X < m_D$!) - introduces model dependence unless we know $f(k_+)$

DELPHI 2000: Analysis with M_X <1.6 GeV



$$|V_{ub}| = (4.07 \pm 0.65(\text{exp}) \pm 0.47(b \rightarrow c) \pm 0.39(\text{theo})) \times 10^{-3}$$

Lepton Invariant Mass Spectrum for b u Decay



See lectures by Ligeti

Lepton q^2 spectrum is insensitive to Fermi motion (and has less model dependence.)

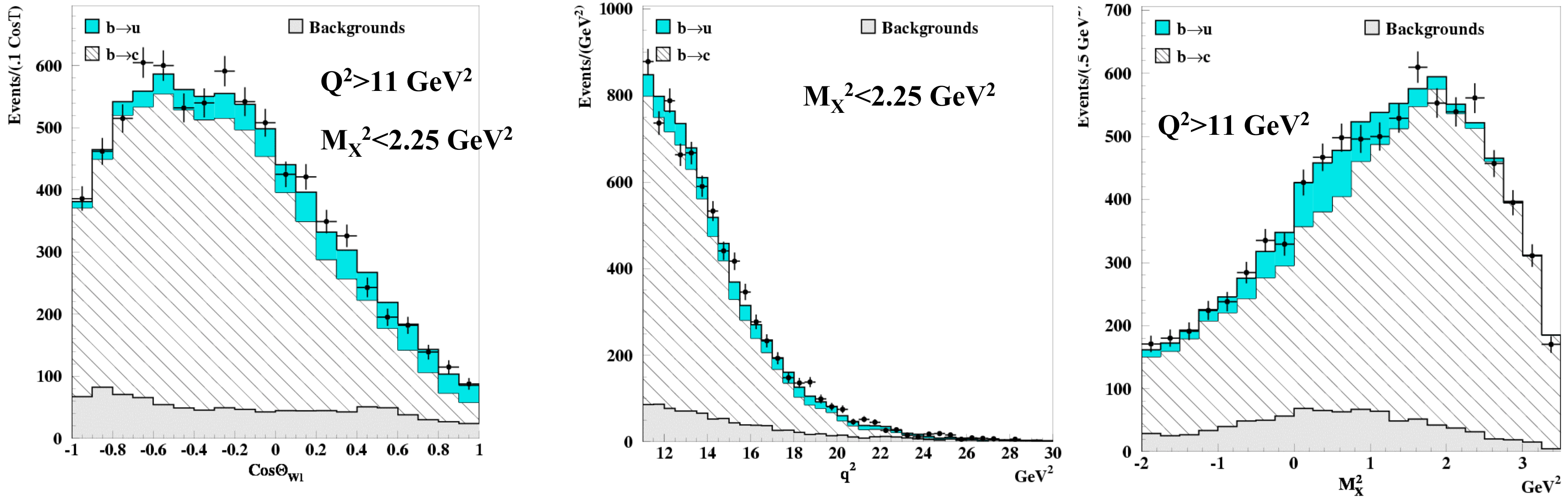
M.Luke:

Representative cuts:

(a)	$q^2>6$ GeV^2, $m_X<m_D$	46% of rate
(b)	$q^2>8$ GeV^2, $m_X<1.7$ GeV	33% of rate
(c)	$q^2>11$ GeV^2, $m_X<1.5$ GeV	18% of rate

Uncertainty	Size (in V_{ub})	Improvement?
Δm_b	±80 MeV: 7%, 8%, 10% ±30 MeV: 3%, 3%, 4%	RG improved Υ sum rules, moments of B decay spectra, lattice
α_s	2%, 3%, 7%	full two-loop calculation
$1/m_b^3$ (weak annihilation)	3%, 4%, 8%	compare $B^{\pm}$, B^0 compare S.L. width of D^0, D_S, lattice

CLEO: B → X lν with Neutrino Reconstruction



$$|V_{ub}| = (4.05 \pm \underset{\text{stat}}{0.18} \pm \underset{\text{sys}}{0.58} \pm \underset{b\rightarrow c}{0.25} \pm \underset{b\rightarrow u}{0.21} \pm \underset{\text{theory}}{0.56})\ 10^{-3}$$

$|V_{ub}|$ from M_X or q^2 with fully reconstructed B tags (MC)

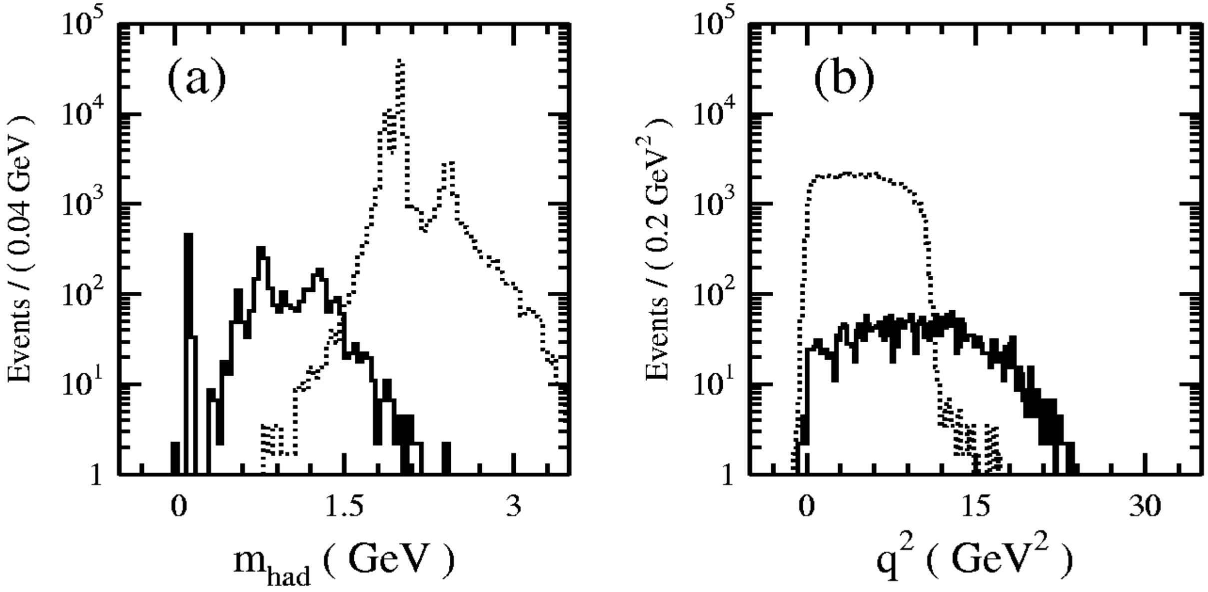


FIG. 1: (a) Hadronic mass ($m_{\rm had}$) distribution for 1000 fb^{-1} data found with CLEO III fast MC. The solid histogram is the $m_{\rm had}$ distribution of $b \to u\ell\nu$, and the dashed histogram is the $m_{\rm had}$ distribution of $b \to c\ell\nu$. (b) q^2 distribution for 1000 fb^{-1} data found with CLEO III fast MC. The solid histogram is the q^2 distribution of $b \to u\ell\nu$, and the dashed histogram is the q^2 distribution of $b \to c\ell\nu$.

		$m_{\rm had}$					q^2				
year	$\mathcal{L}_{int}(fb^{-1})$	S	B	$\delta V_{\rm ub}^{\rm expt.}$ (%)			S	B	$\delta V_{\rm ub}^{\rm expt.}$ (%)		
				stat.	sys.	tot.			stat.	sys.	tot.
2002	100	335	127	3.2	2.2	3.9	127	7	4.6	3.0	5.5
2005	500	1675	635	1.5	1.5	2.1	635	36	2.0	1.2	2.3
2010	2000	6700	2540	0.7	1.5	1.7	2538	144	1.0	1.2	1.6

Shipsey
andLee

Exclusive Approaches to $|V_{ub}|$

Measure BF($B \rightarrow \pi\, l\, \nu$), BF($B \rightarrow \rho\, l\, \nu$) or BF($B \rightarrow \omega\, l\, \nu$) .

With more statistics, can then measure $d\Gamma/dq^2$ ($B \rightarrow \pi\, l\, \nu$) or even form factors for BF($B \rightarrow \rho\, l\, \nu$).

A key experimental ingredient is the use of detector hermiticity to deduce the ν momentum

Variables for ν reconstruction of exclusive semileptonic decays (used for B→ π (ρ) l ν)

$$p_{miss} = -\sum p_i$$

$$E_{miss} = 2E_{beam} - \sum E_i$$

$$M^2_{miss} = E^2_{miss} - P^2_{miss}$$

$$p_\nu = (p_{miss}, |p_{miss}|)$$

$$\Delta E \equiv E_{beam} - (E_\pi + E_l + E_\nu)$$

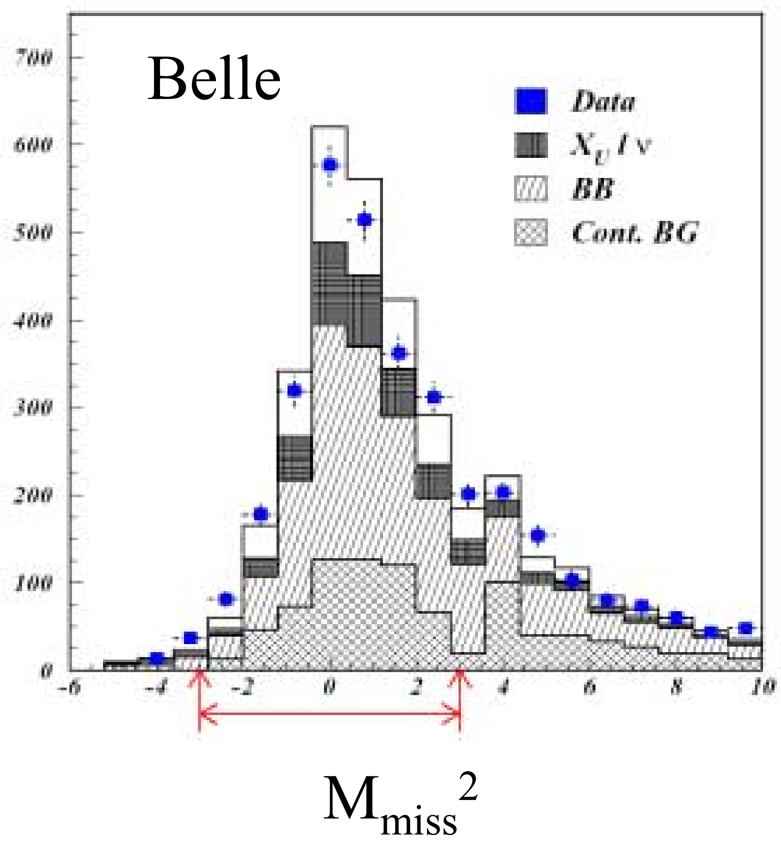
$$M_B \equiv \sqrt{E^2{}_{beam} - |p_\pi + p_l + p_\nu|^2}$$

where Ebeam = 5.29GeV

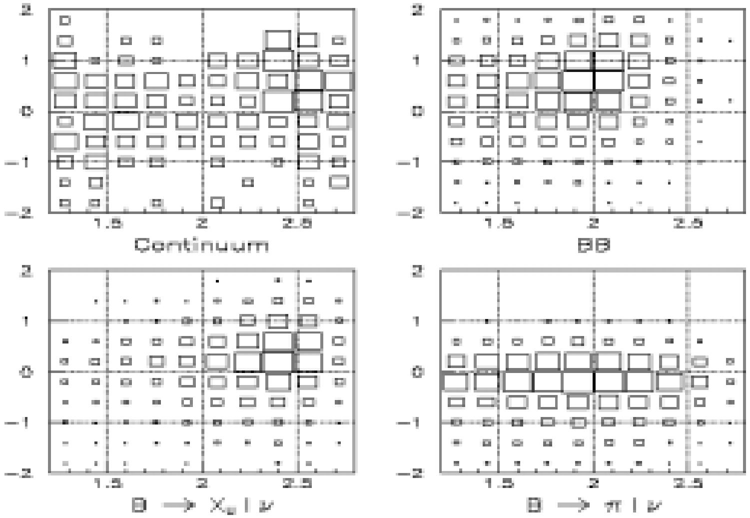
Detector hermiticity requirements (cont'd)

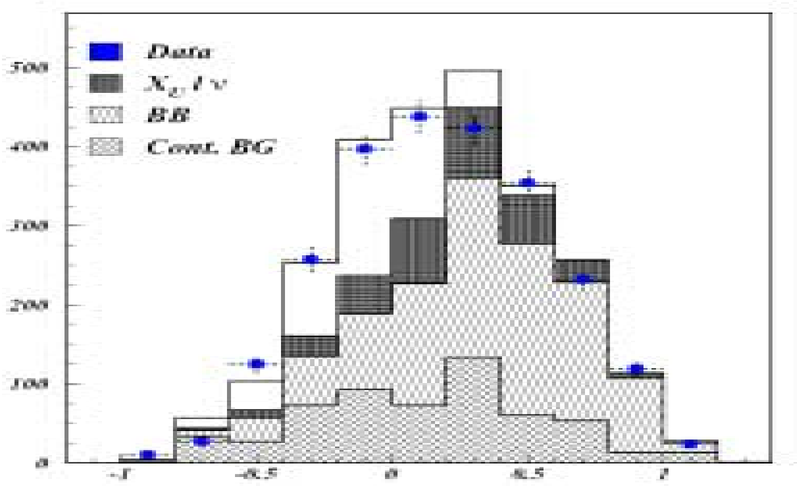
e.g. Only one lepton and $|Q_{tot}| = \pm 2$. Also require $|M_{miss}|^2 < 3.0\ \mathrm{GeV}^2$

Fiducial cut on p_{miss} is important at the B factories. E.g. $17^0 < \theta_{miss} < 150^0$



Belle: $B^0 \rightarrow \pi^+ l^- \nu$ signal





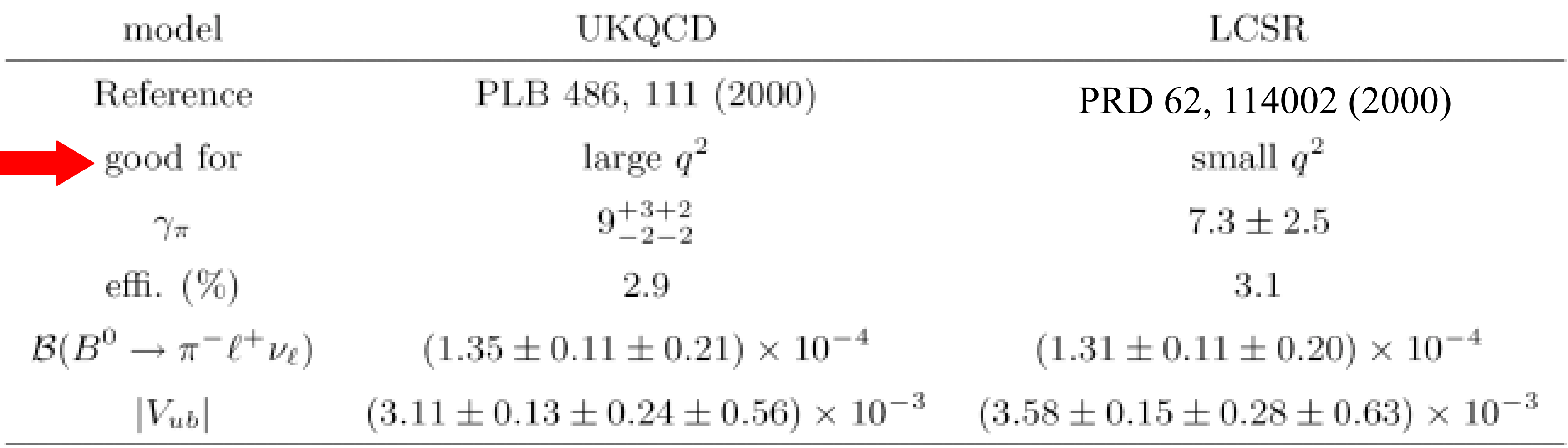
ΔE

Signal is extracted from a 2D fit to ΔE and p_L

Determination of $|V_{ub}|$ from BF($B^0 \rightarrow \pi\, l\, \nu$)

Models or lattice calculations are needed to determine detection efficiency as well as convert the BF into a value of $|V_{ub}|$

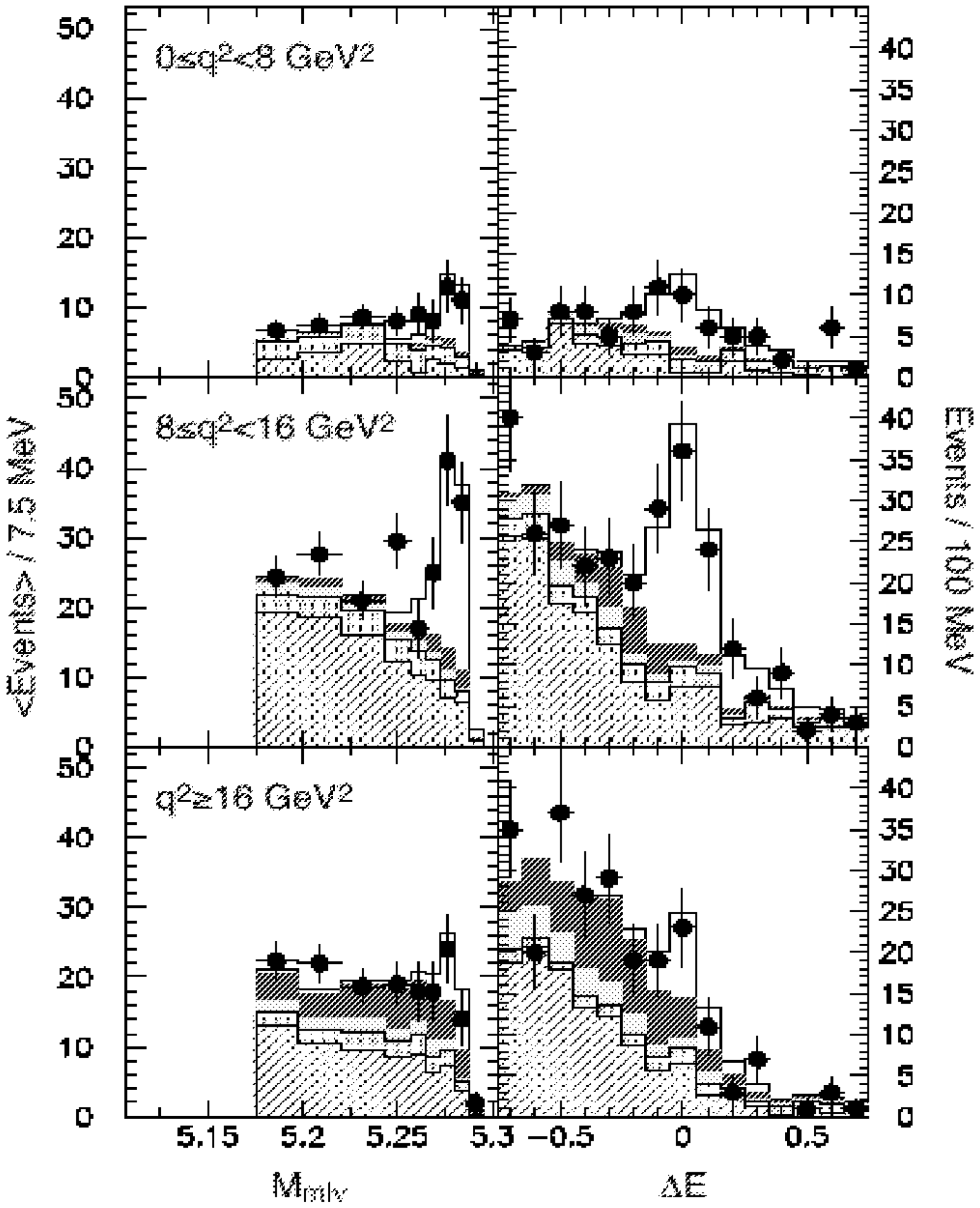
(Khodjamirian *et al.*)



model	UKQCD	LCSR
Reference	PLB 486, 111 (2000)	PRD 62, 114002 (2000)
good for	large q^2	small q^2
γ_π	9^{+3+2}_{-2-2}	7.3 ± 2.5
effi. (%)	2.9	3.1
$\mathcal{B}(B^0 \rightarrow \pi^- \ell^+ \nu_\ell)$	$(1.35 \pm 0.11 \pm 0.21) \times 10^{-4}$	$(1.31 \pm 0.11 \pm 0.20) \times 10^{-4}$
$\|V_{ub}\|$	$(3.11 \pm 0.13 \pm 0.24 \pm 0.56) \times 10^{-3}$	$(3.58 \pm 0.15 \pm 0.28 \pm 0.63) \times 10^{-3}$

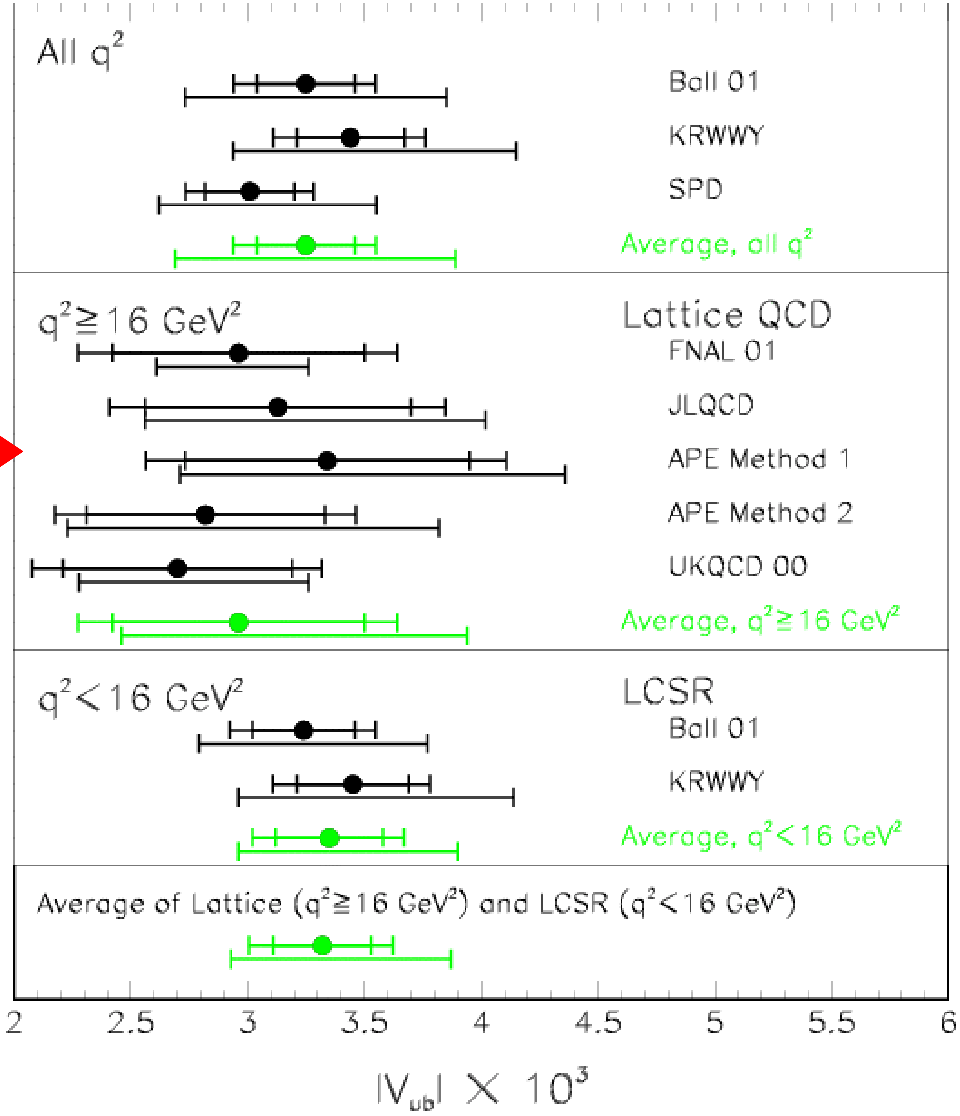
CLEO 2002:

$B^0 \rightarrow \pi^+ l^- \nu$ signal in bins of q^2 .

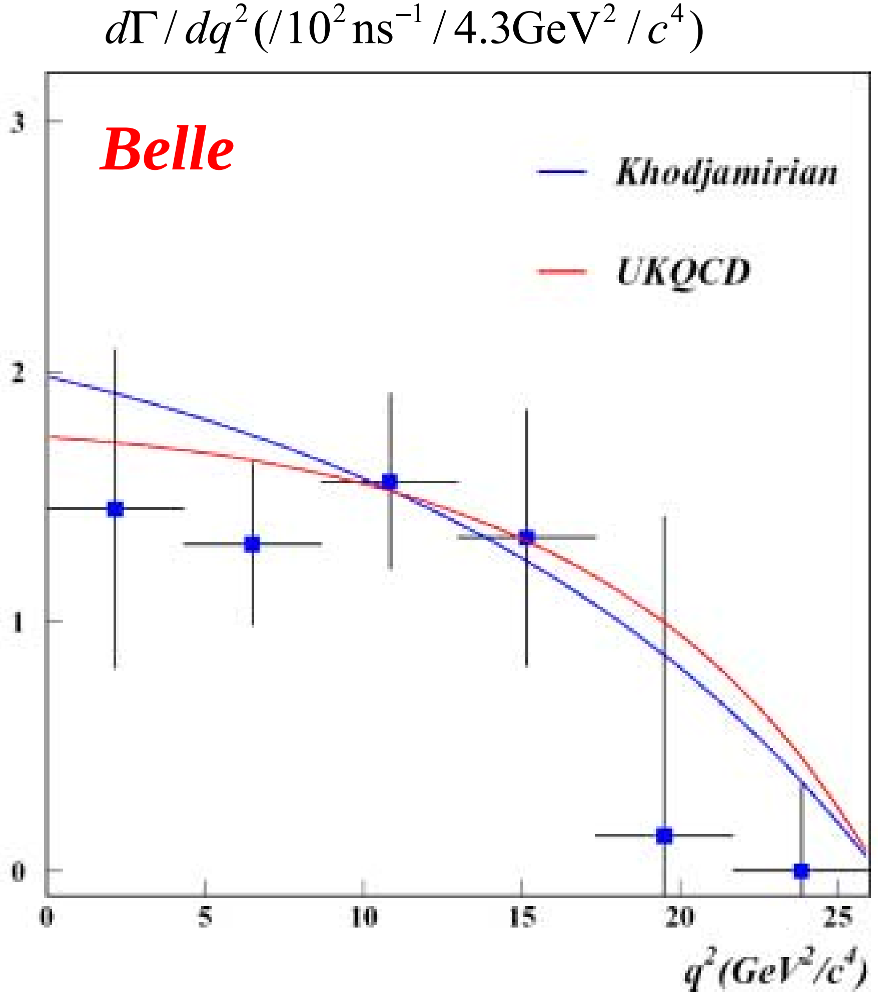


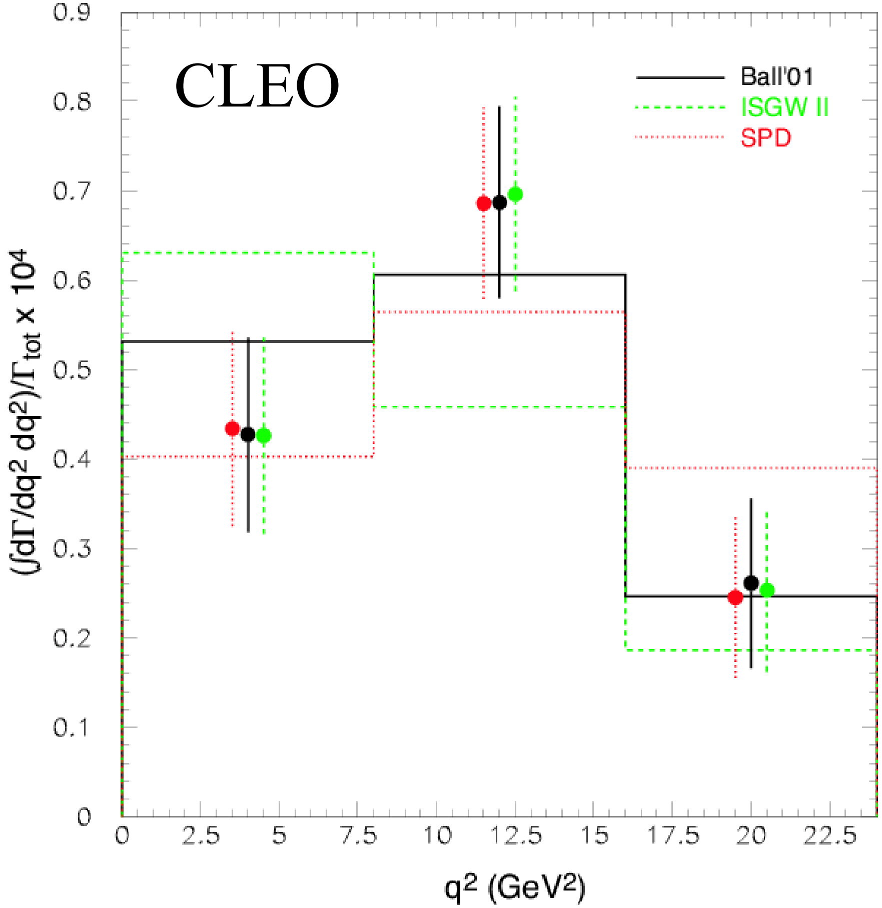
CLEO:Determination of $|V_{ub}|$ from BF($B^0 \rightarrow \pi\, l\, \nu$)

Large errors



q^2 distribution of $B \to \pi\, l\, \nu$



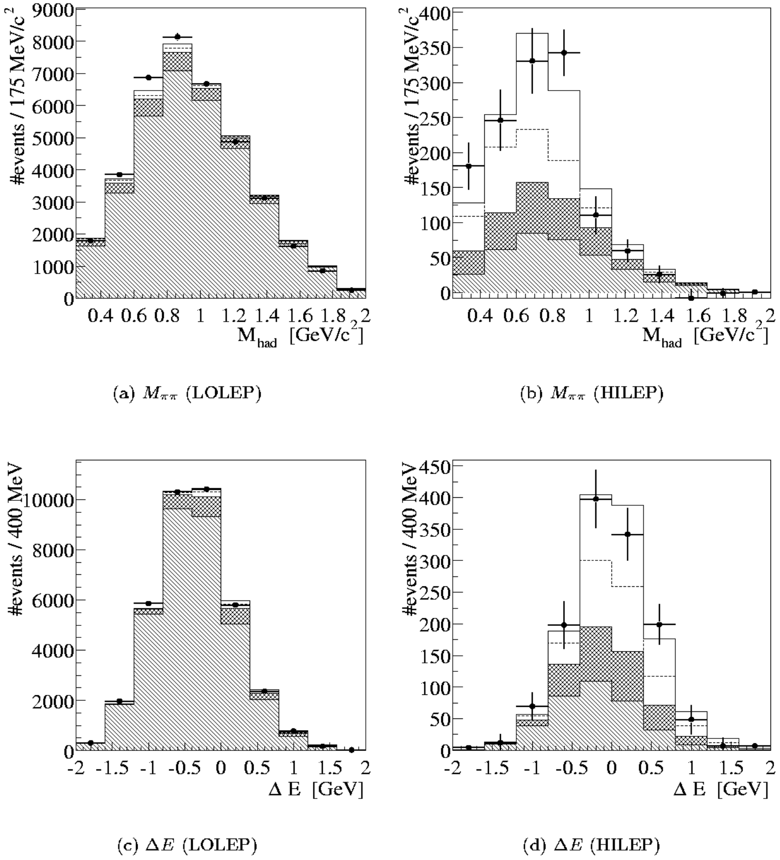


Find prob(ISGWII) ~1%

BABAR $B \rightarrow \rho^{0(+)} l \nu$ signal

LOLEP

$2.0 < p_L < 2.3$ GeV



(a) $M_{\pi\pi}$ (LOLEP)

(b) $M_{\pi\pi}$ (HILEP)

(c) ΔE (LOLEP)

(d) ΔE (HILEP)

Figure 3: Continuum-subtracted projections of the ISGW2 fit result for the $B^+ \to \rho^0 e^+$ channels in the LOLEP and HILEP electron energy regions; the contributions are the dire and crossfeed components of the signal (unhatched region, above and below the dashed lin respectively); the background from $b \to u e \nu$ other than $B \to \rho e \nu$ and $B \to \omega e \nu$ modes (doubl hatched region); the background from $b \to c e \nu$ and other backgrounds (single-hatched region)

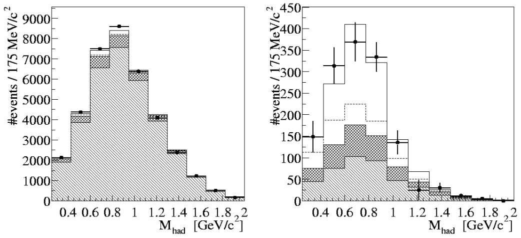
HILEP

$2.3 < p_L < 2.7$ GeV

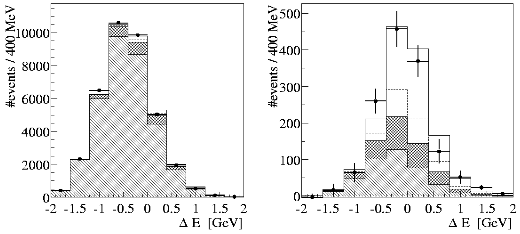
BABAR B→$\rho^{0(+)}$ l ν signal

LOLEP

2.0<p_L <2.3 GeV



(a) $M_{\pi\pi}$ (LOLEP)   (b) $M_{\pi\pi}$ (HILEP)



(c) ΔE (LOLEP)   (d) ΔE (HILEP)

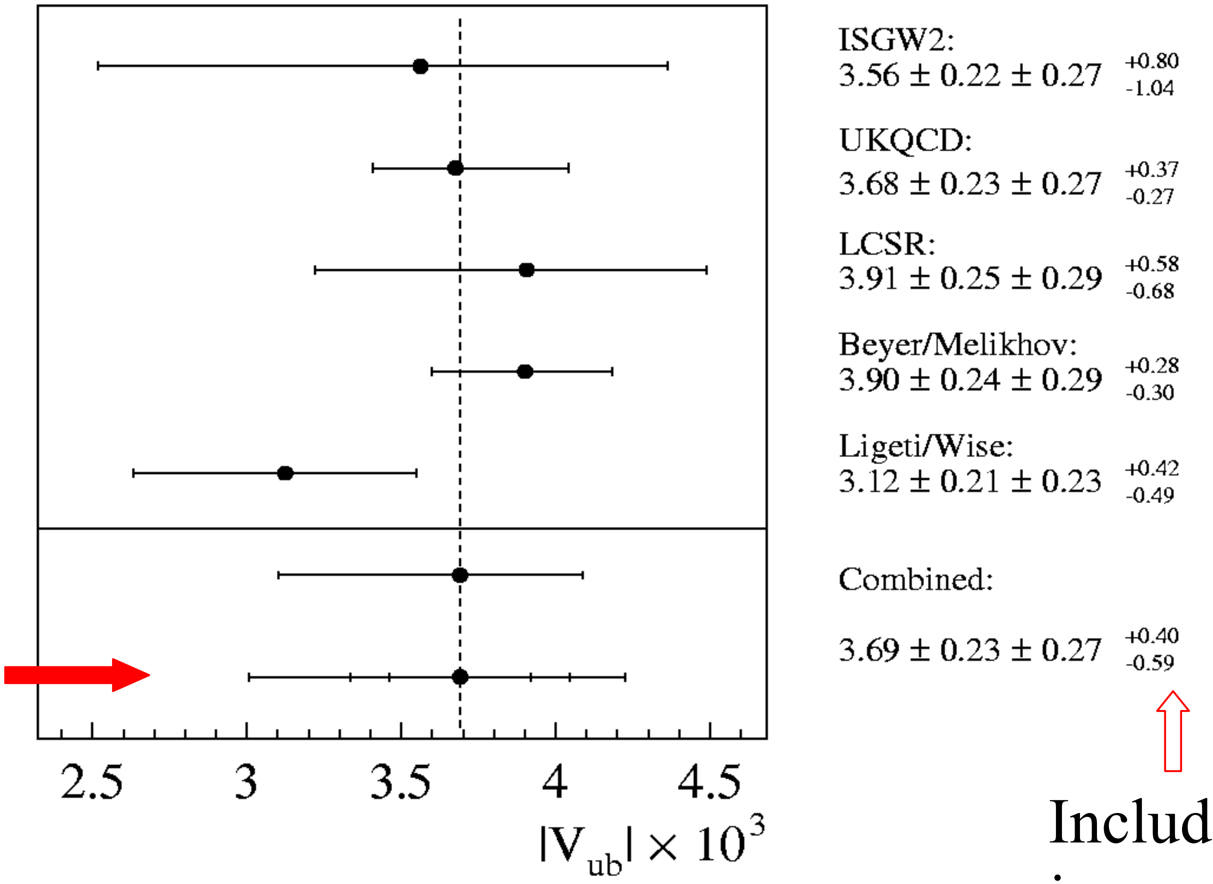
HILEP

2.3<p_L <2.7 GeV

Table 2: Summary of data yields for the $B^0 \to \rho^- e^+ \nu$ and $B^+ \to \rho^0 e^+ \nu$ modes with electron energies between 2.3 and 2.7 GeV (HILEP), and between 2.0 and 2.3 GeV (LOLEP). The yields presented in this table were obtained using the ISGW2 form-factor. The downfeed background includes all $B \to X_u e \nu$ modes except for ρ, ω, and π. The crossfeed signal contribution corresponds to events from the other signal modes with ρ^0, ω, or π and is constrained to the signal in the fit. All errors are statistical only.

	$B^0 \to \rho^- e^+ \nu$		$B^+ \to \rho^0 e^+ \nu$	
	HILEP	LOLEP	HILEP	LOLEP
On-resonance yield	2302	39349	2213	40155
Direct signal	510 ± 63	718 ± 89	324 ± 40	440 ± 55
Crossfeed signal	262 ± 32	538 ± 73	363 ± 42	725 ± 86
Downfeed	203 ± 55	2278 ± 403	226 ± 92	2435 ± 430
$b \to c e \nu$	414 ± 5	33859 ± 438	367 ± 5	34366 ± 458
$e^+e^- \to q\bar{q}$	917 ± 73	1928 ± 106	912 ± 73	2063 ± 110
Fake electrons	12 ± 3	80 ± 9	18 ± 4	76 ± 9

Model dependence in $|V_{ub}|$ BABAR $B \rightarrow \rho\, l\, \nu$ signal



Weighted mean

Includes spread in models

Does it make sense to take the average of models ?

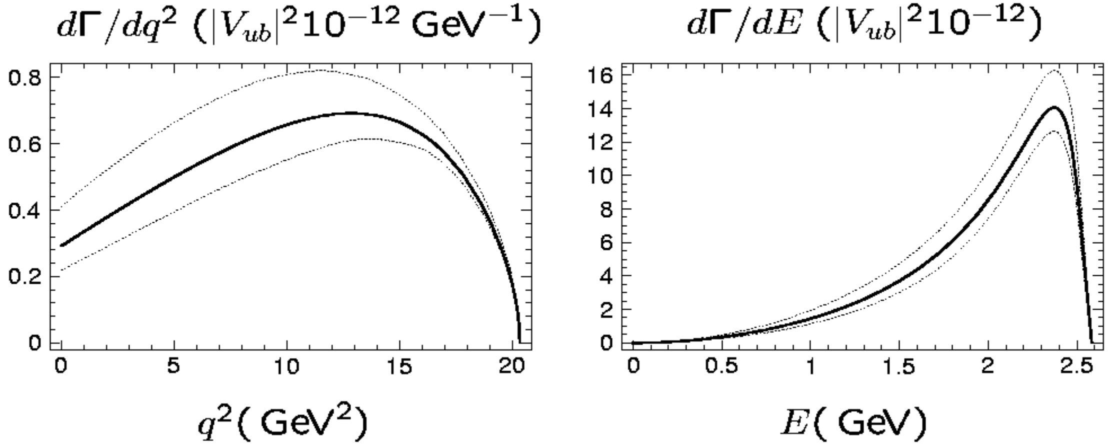
How can we improve our knowledge of V_{ub} from exclusive decays ?

A considerable amount of model dependence is due to FF uncertainties.

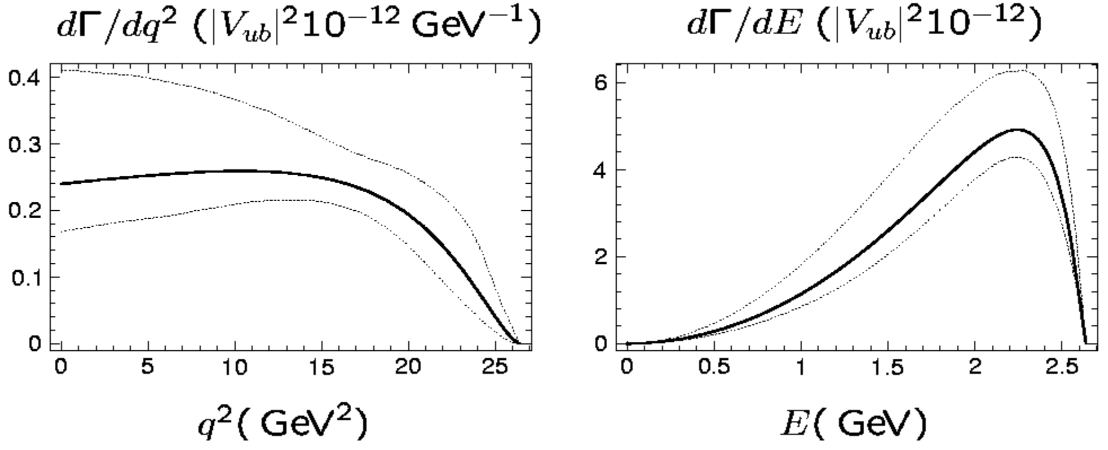
Examples:

J. Flynn

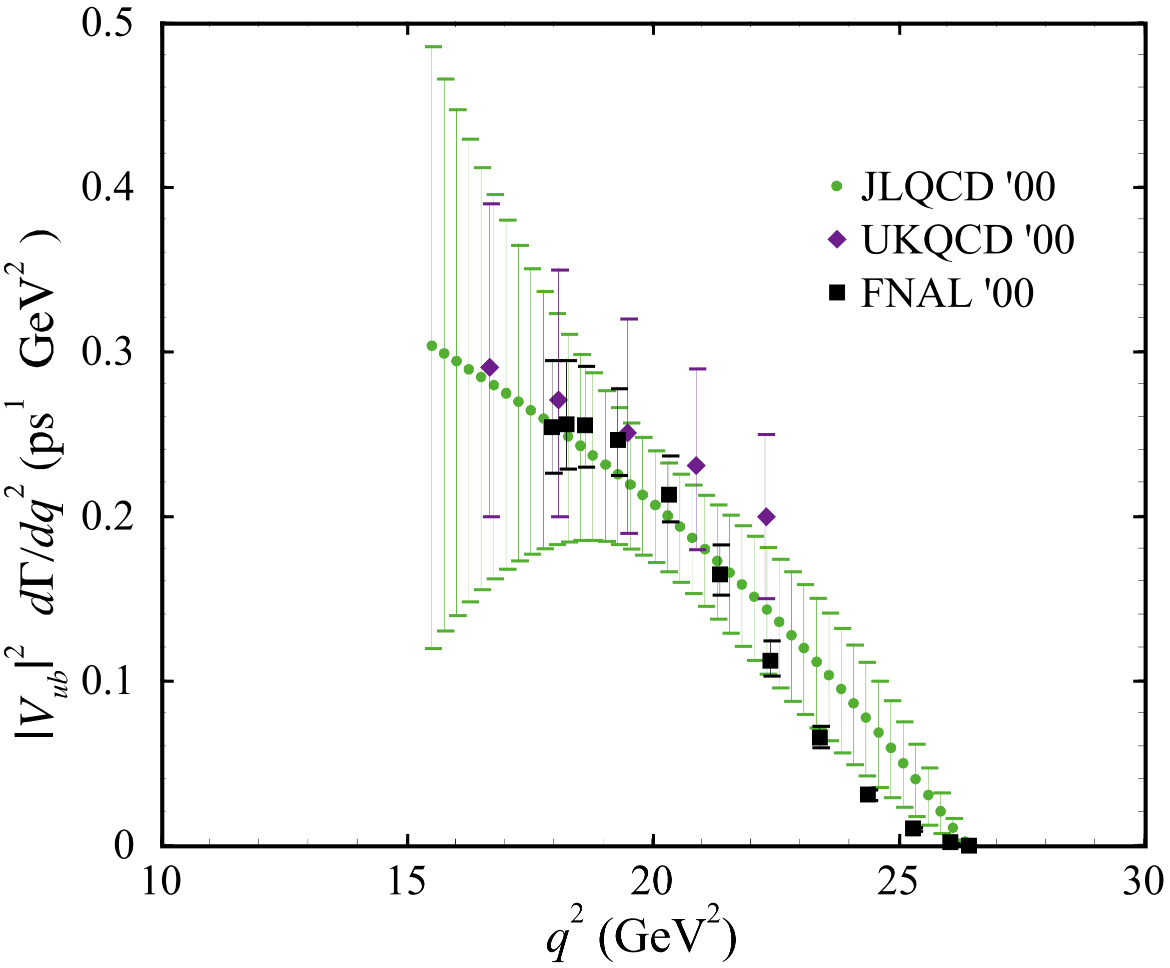
$$\boxed{\bar{B}^0 \to \rho^+ l^- \bar{\nu}_l}$$



$$\boxed{\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l}$$



Lattice Calculations of B→ π l ν Form Factor



(from A. Kronfeld, hep-ph/0010074)

Need to measure $d\Gamma/dq^2$ for $B \rightarrow \pi\, l\, \nu$ at high q^2/low p_π

Future Improvements in $|V_{ub}|$ from the Lattice

$$|V_{ub}|^2 = \frac{12\pi^2}{G_F^2 m_B} \frac{1}{\boxed{T_B(p_{\min}, p_{\max})}} \int_{p_{\min}}^{p_{\max}} dp \frac{d\Gamma_{B\to\pi\ell\nu}}{dp}$$

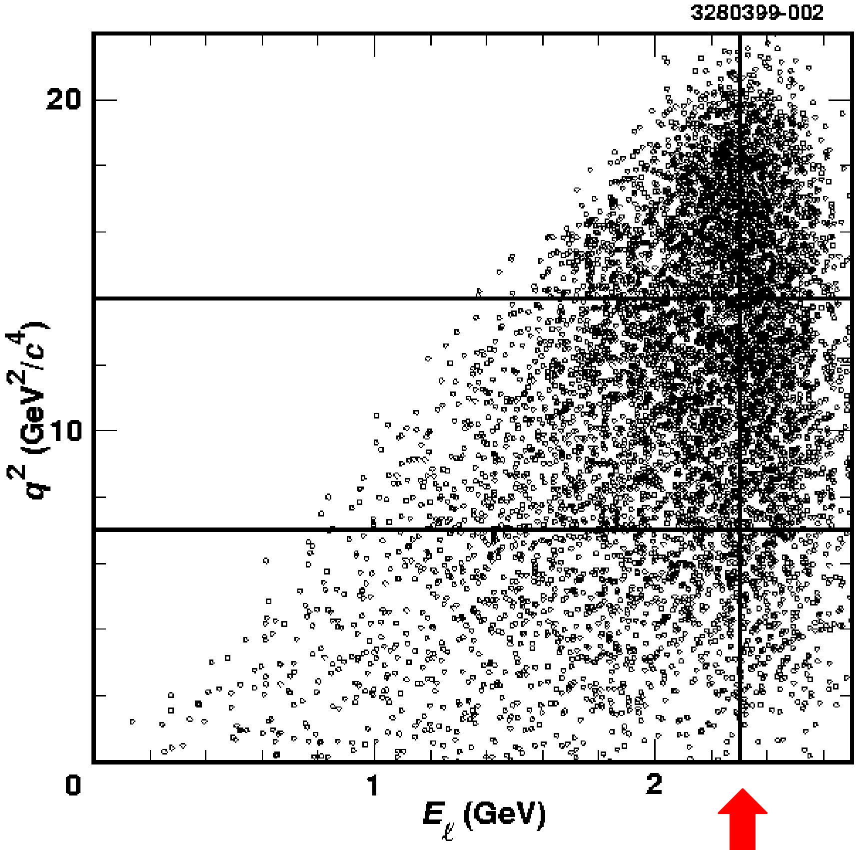
$$T_B(0.4\ \text{GeV},\ \ 1.0\ \text{GeV}) = 0.55 \underbrace{{}^{+.15}_{-.05}}_{\text{statistical}} \underbrace{{}^{+.09}_{-.12}}_{\text{chiral extrapolation}} \underbrace{{}^{+.09}_{-.02}}_{\text{lattice spacing}} \pm \underbrace{.06}_{\text{matching}} \pm \underbrace{.09}_{\text{misc. (lattice units, …)}}\ \text{GeV}^4$$

$\Delta V_{ub} \approx 15\text{-}18\% + \text{quenching error}$

(A. El-Khadra et. al., PRD64, 014502)

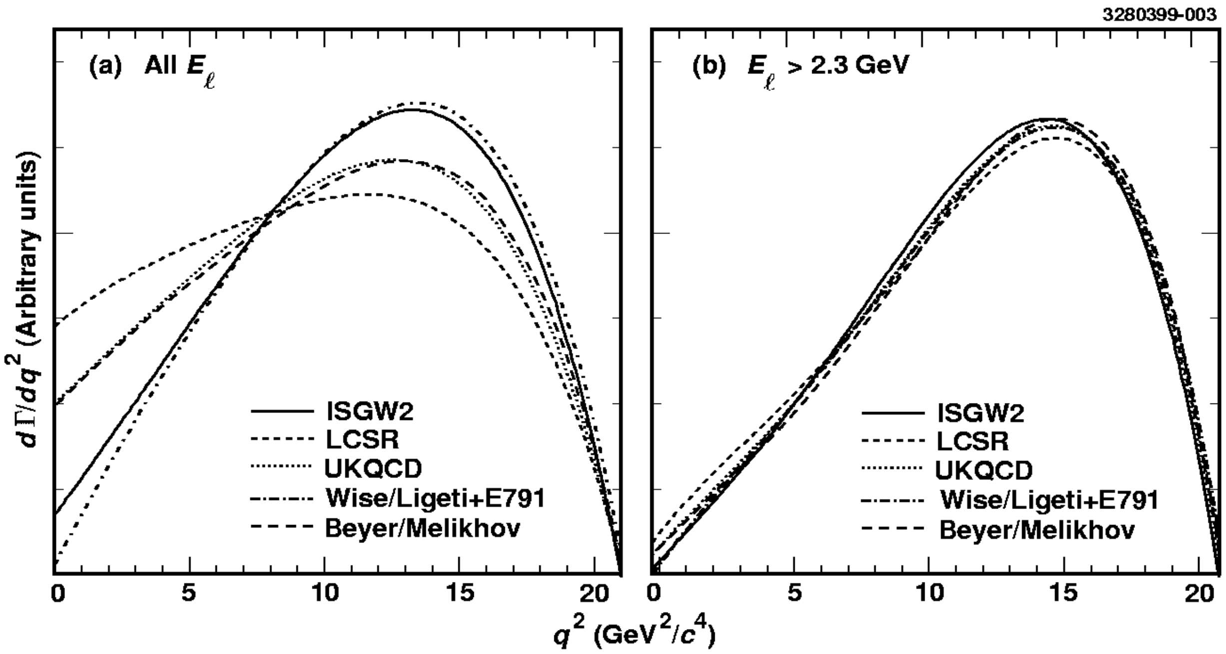
To understand these errors: $\boxed{\textit{see Lectures by P. Lepage}}$

MC simulation of the B→ρ l ν Dalitz plot

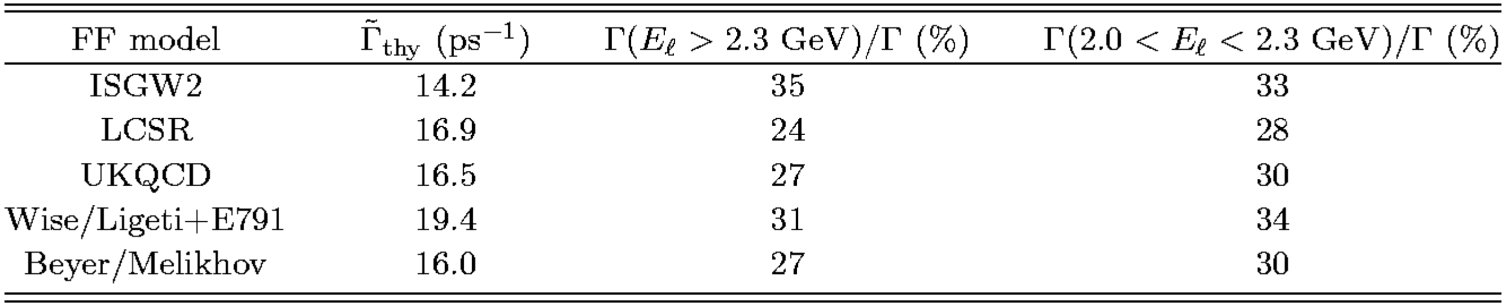


P_1 cut

Model dependence in B→ρ l ν form factors



A tight p_L cut makes it difficult to distinguish models.



FF model	$\tilde{\Gamma}_{\text{thy}}$ (ps^{-1})	$\Gamma(E_\ell > 2.3$ GeV$)/\Gamma$ (%)	$\Gamma(2.0 < E_\ell < 2.3$ GeV$)/\Gamma$ (%)
ISGW2	14.2	35	33
LCSR	16.9	24	28
UKQCD	16.5	27	30
Wise/Ligeti+E791	19.4	31	34
Beyer/Melikhov	16.0	27	30

Model dependence of B→ρ l ν form factors

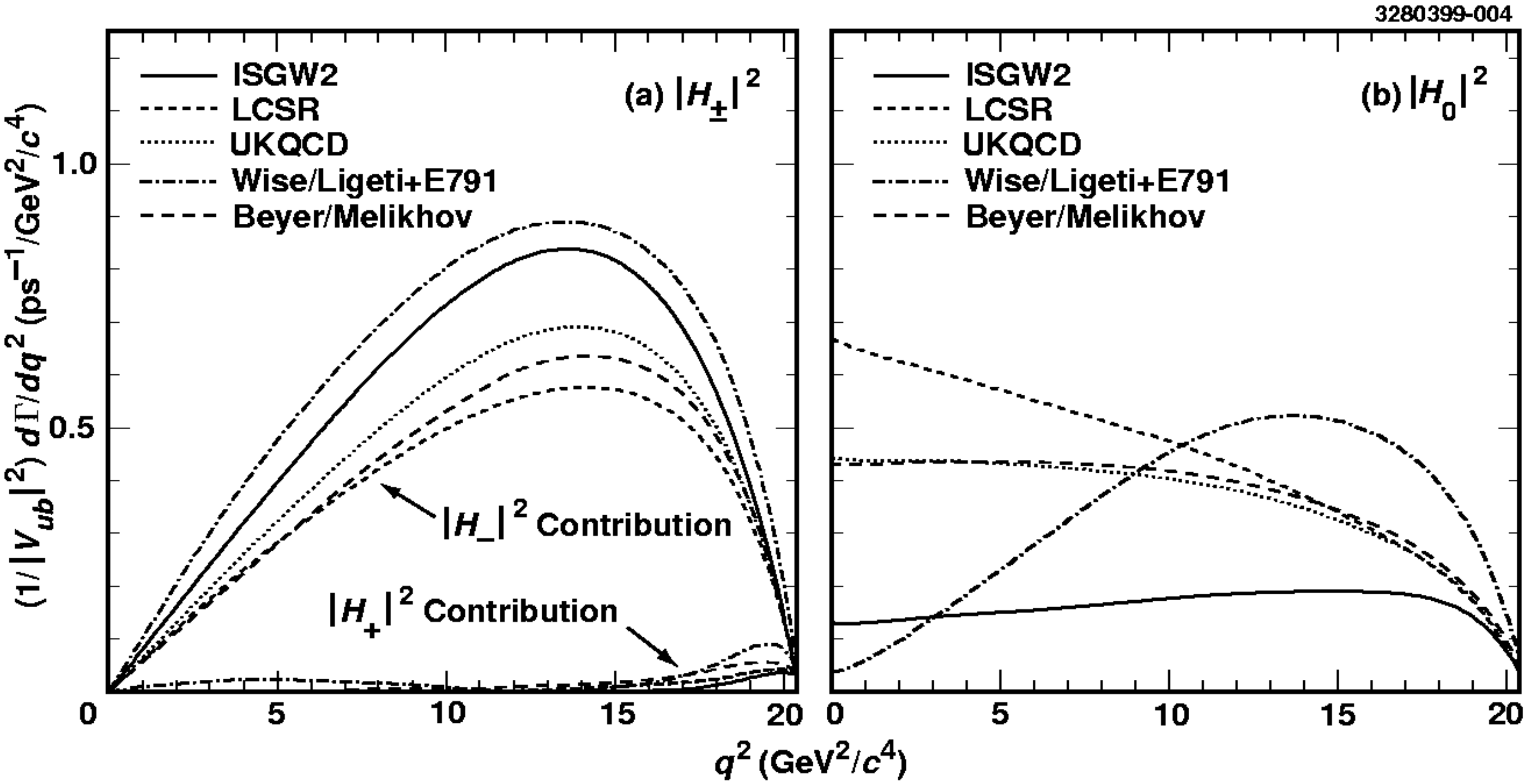
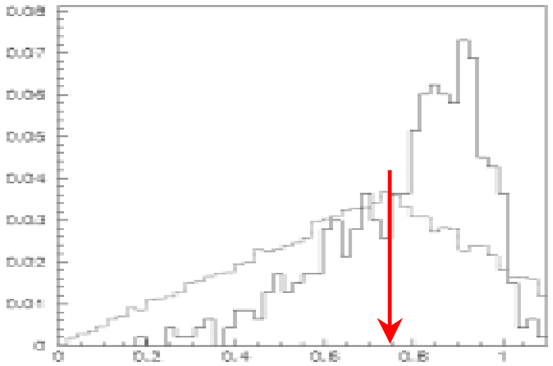


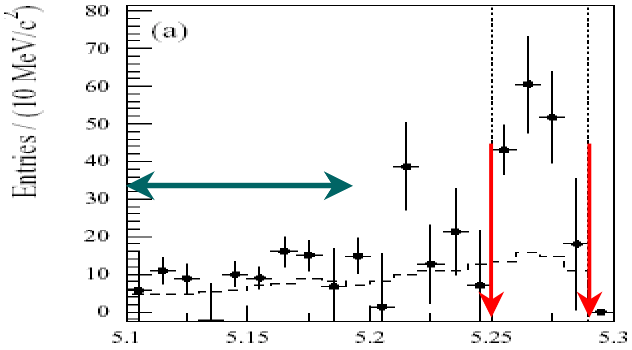
FIG. 4. $d\Gamma/dq^2$ distributions for each of the three terms in Eq. 9: (a) the terms proportional to $|H_-|^2$ and $|H_+|^2$ and (b) the $|H_0|^2$ term.

BELLE: $B^0 \rightarrow \omega e^+ \nu$ signal selection

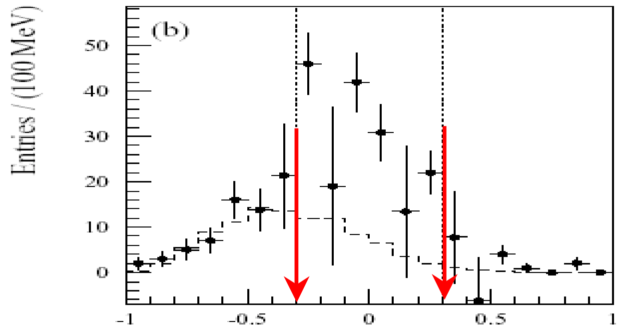
Lattice calcs of $B \rightarrow \rho$ FFs cannot handle the finite width of the ρ



$|P_{\pi+} \times P_{\pi-}|$



M_B



ΔE

Less exp problem with $B \rightarrow \pi\pi \, l \, \nu$

Belle-CONF-0242

BELLE: $B^0 \rightarrow \omega e^+ \nu$ signal

- N(events) in the signal region

  with $0.76 < m(3\pi) < 0.81$

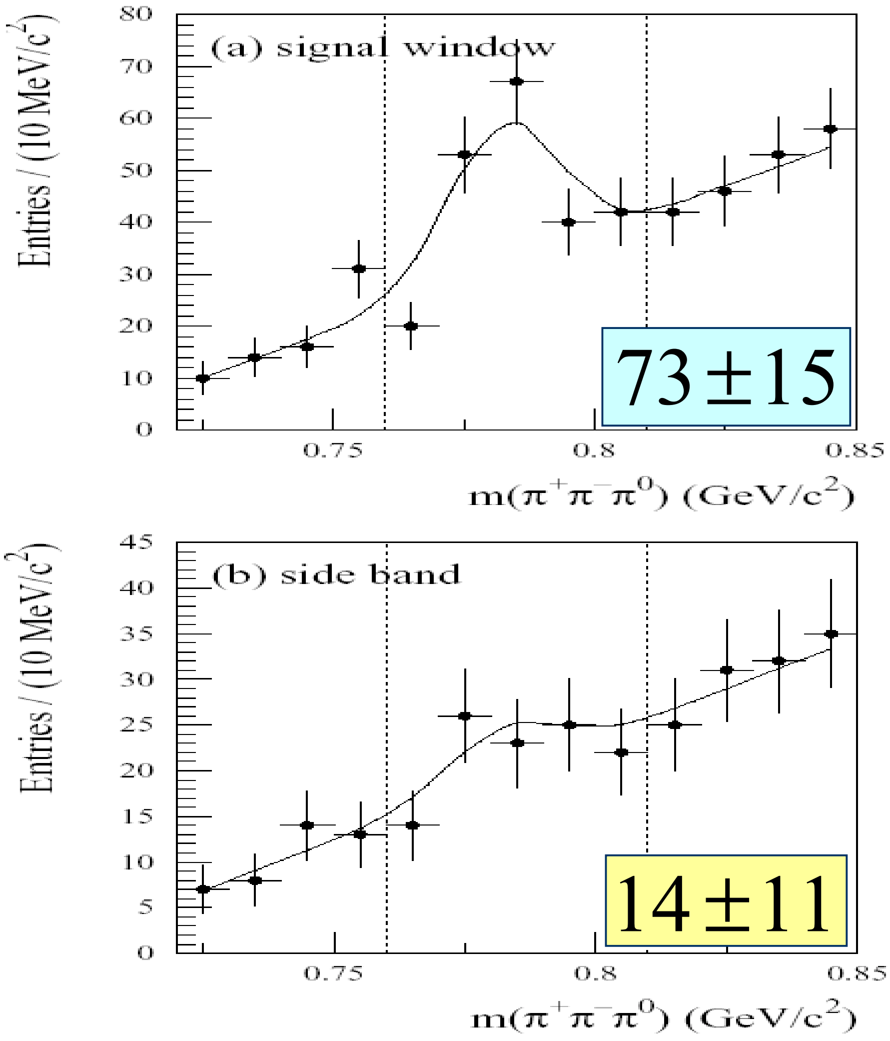
$$\begin{aligned} &222 \pm 15 \ \ \text{(total)} \\ &\left.\begin{aligned} 48 \pm 10 \ \ &(b \rightarrow c) \\ 2 \pm 2 \ \ &\text{(fake)} \\ 47 \pm 21 \ \ &\text{(cont.)} \end{aligned}\right\} \end{aligned}$$

MC est.

- Excess in $m(3\pi)$

  after side-band subtraction

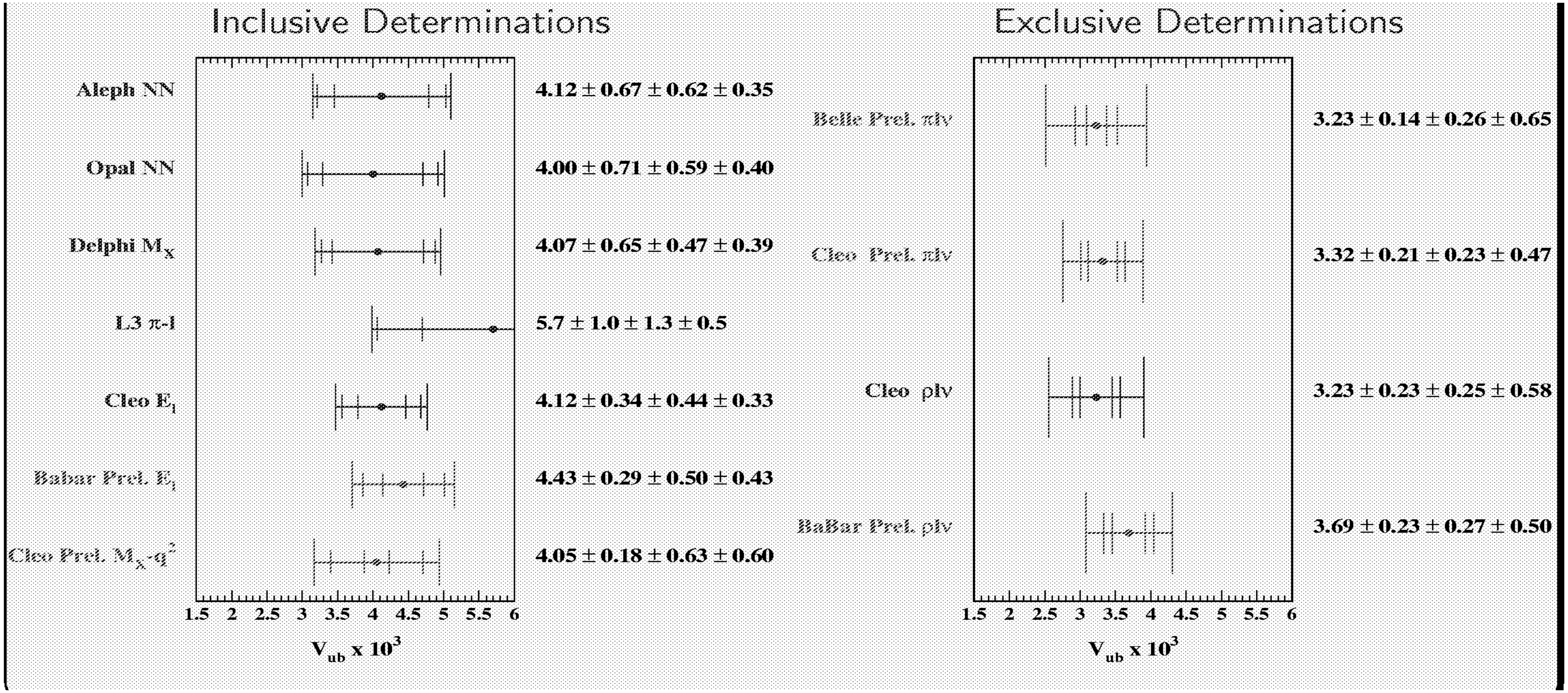
  $= 59 \pm 15$ events



73 ± 15

14 ± 11

Summary of recent $|V_{ub}|$ determinations



Review by
Battaglia

Conclusions

An improved method for $|V_{ub}|$ *determination using leptons in the endpoint region has been introduced. The uncertainty in the extrapolation is reduced by using the shape function measured in* $b \rightarrow s\,\gamma$.

An inclusive method using optimized cuts on q^2 and M_X appears promising.

Prospects for improved $|V_{ub}|$ *in* $B \rightarrow \pi\, l\, \nu$ *using high statistics measurements of* $d\Gamma/dq^2$ *and FFs determined from the lattice appear good.*

Acknowledgments

Many of the pedagogical plots in the discussion of CKM matrix elements are taken from Prof. J.D. Richman’s lectures at Les Houches. He was the first person to give a comprehensible presentation of many of the issues related to semileptonic form factors. I have also benefitted and borrowed extensively from previous B physics reviews by P. Drell, S.Stone, E.Thorndike and M.S. Witherell. Numerous plots were taken from talks by K.Ecklund, M.Luke and others.