High-Precision Nonperturbative QCD

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Why High-Precision and Nonperturbative?

Essential for Standard Model

E.g., CKM weak interaction parameters
ρ and η from:

B-$\bar{B}$ mixing

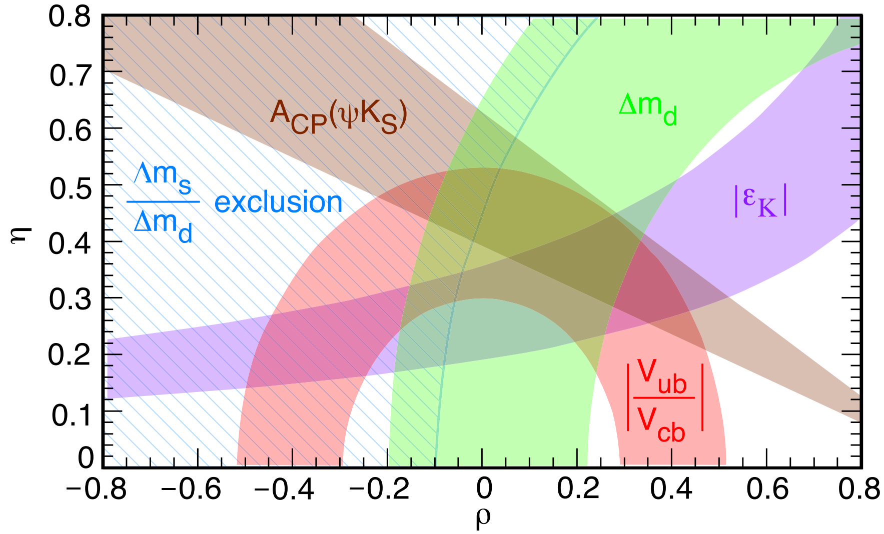
$B \to \pi l \nu$

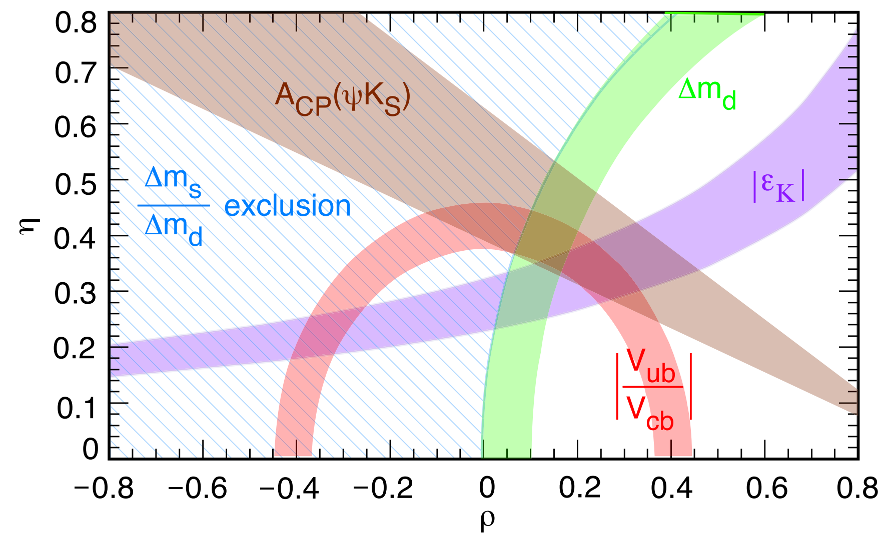
K-$\bar{K}$ mixing

$\ldots$

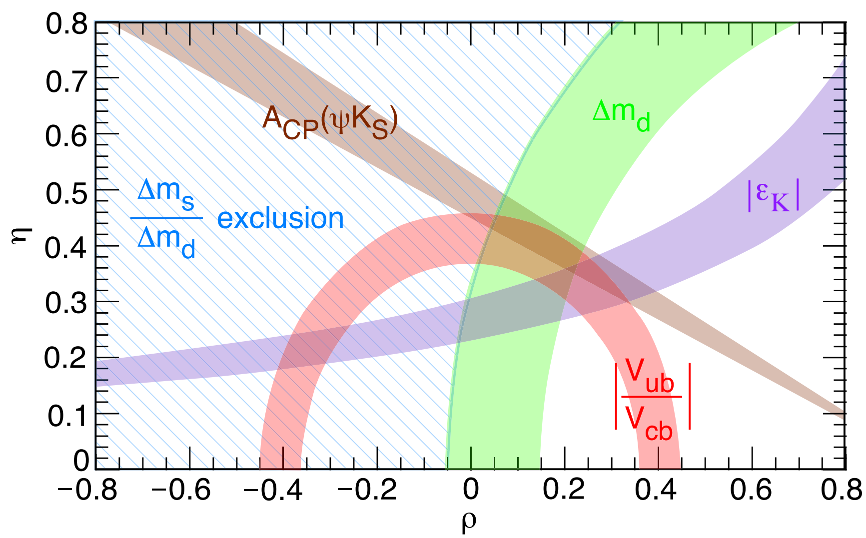
Nonpert've QCD Part $\times$ Weak Int'n Part

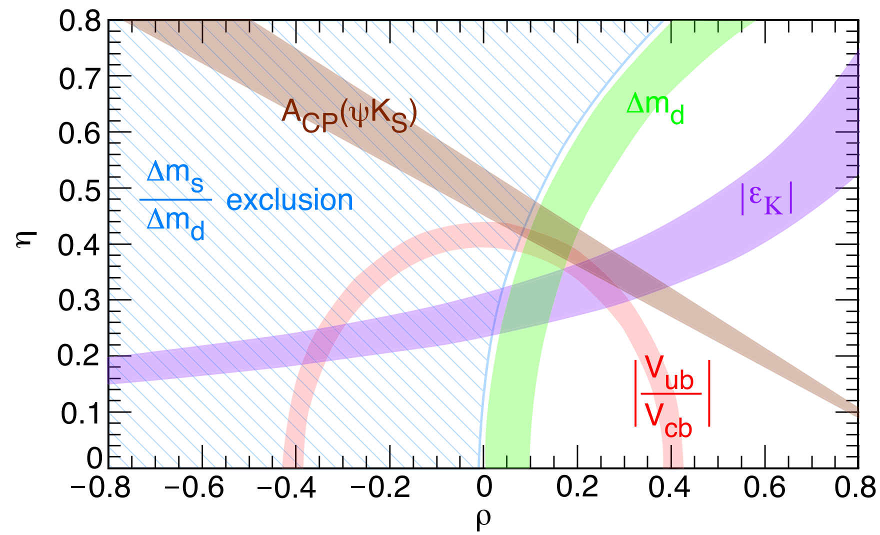
CKM today . . .   . . . and with 2–3% theory errors.





And with B Factories . . .





95% confidence levels; CLEO-c (2001).

Essential Beyond the S.M.?

Strongly coupled field theories are an outstanding challenge to all theoretical physics.

- Field theory is generic; weak coupling is not.

2 of 3 known interactions are strongly coupled: QCD, gravity.

Asymptotic freedom + logarithmic evolution

$\Rightarrow$ Strong coupling at low E and large mass hierarchies.

- E.g., in QCD:

$$\alpha_s(M_{\rm planck}) = 0.02 \quad \text{and} \quad \alpha_s(m_{\rm hadron}) \approx 1$$

$$\Rightarrow \quad m_{\rm hadron}/M_{\rm planck} \approx 10^{-19}.$$

$\Rightarrow$ Strong coupling is *natural* in particle physics.

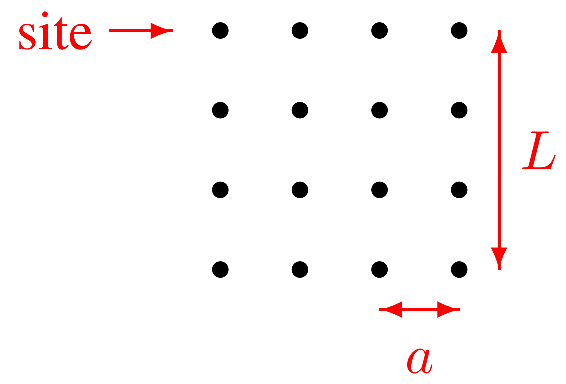
Strong coupling is possible (likely?) at the LHC and/or beyond.

- Generic at low energies in non-abelian gauge theories . . .
- . . . unless gauge symmetry spontaneously broken $\Rightarrow$ dynamical symmetry breaking $\Rightarrow$ strong coupling.
- Critical near-term need for reliable, generic techniques for strong coupling.

What is Lattice QCD?

Lattice Approximation

Continuous
Space & Time $\longrightarrow$



$\Rightarrow$ Fields $\psi(x)$, $A_\mu(x)$ specified only at grid sites; interpolate for other points.

⇒ QCD → multidimensional integration.

$$\int \mathcal{D}A_\mu \ \ldots \ \mathrm{e}^{-\int L \mathrm{d}t} \longrightarrow \int \prod_{x_j \,\epsilon\, \mathrm{grid}} \mathrm{d}A_\mu(x_j) \ \ldots \ \mathrm{e}^{-a \sum L_j}.$$

⇒ Millions of integration variables.

⇒ Numerical Monte Carlo integration.

N.B. Cost $\propto (1/a)^\omega$ where $\omega \geq 6$ implies must keep a as large as possible!

Fall & Rise of LQCD

- Invented in 1974; "explains" confinement.
- Stalls for almost 20 years.
  - Ken Wilson declares it dead! (1989)
- Renaissance in 1990's.
  - Perturbation theory fixed.
  - Effective field theories for c, b's.
  - Improved discretizations $\Rightarrow$ larger a's.
  - Unquenching! (2000)
- First high-prescision nonperturbative results.
  - $\alpha_s(M_Z)$, $M_b \ldots$ to few %.
  - Ken Wilson retracts. (1995)

QCD Revolution

Traditional wisdom $\Rightarrow$ need $a \leq 0.05\,\mathrm{fm}$.

New simulation results $\Rightarrow$ $a = 0.1$–0.4 fm works.

Simulation cost $\propto (1/a)^6$

$\Rightarrow$ new simulations cost 10^2–10^6 times less!

Quantum Field Theory on a Lattice

Approximate Derivatives

Numerical Analysis $\Rightarrow$

$$\frac{\partial \psi(x_j)}{\partial x} = \Delta_x \psi(x_j) + \mathcal{O}(a^2)$$

$$\frac{\psi(x_j + a) - \psi(x_j - a)}{2a}$$

$\Rightarrow$ uses only ψ's at grid sites.

N.B. Errors $\propto (pa)^n \Rightarrow$ want $p < \mathcal{O}(1/a)$.

Large $a \Rightarrow$ need *improved discretizations*.

E.g.

$$\frac{\partial\psi}{\partial x} = \Delta_x\psi - \frac{a^2}{6}\Delta_x^3\psi + \mathcal{O}(a^4)$$

10–15% for $a = 0.4\,\mathrm{fm}$

1–2% for $a = 0.4\,\mathrm{fm}$

$\Rightarrow a = 0.4\,\mathrm{fm}$ okay?

N.B. Need smaller as for large p.

Ultraviolet Cutoff

$\lambda_{\text{min}} = 2a$ is smallest wavelength.

E.g.) $\psi = \;+1 \quad -1 \quad +1 \quad -1 \quad +1$

$\qquad\qquad\;\; \bullet \quad\;\; \bullet \quad\;\; \bullet \quad\;\; \bullet \quad\;\; \bullet$

$\Rightarrow$ all quark and gluon states with $p > \pi/a$ are excluded by the lattice since $p = 2\pi/\lambda$.

$\Rightarrow$ lattice QCD $\equiv$ QCD + lattice UV regulator
$\equiv$ “real” QCD.

But $\forall\, p$s important in quantum field theory! (Consider ultraviolet divergences.)

Renormalization Theory $\Rightarrow$ mimic effects of $p > \pi/a$ excluded states by adding extra a-dependent *local* terms to the field equations, Lagrangian, currents, etc.

$$\Rightarrow \qquad \partial\psi \;\rightarrow\; \Delta\psi + c(a)\, a^2\, \Delta^3\psi + \cdots$$

where

$$c(a) \;=\; -\frac{1}{6} + \begin{array}{l}\text{Contribution for}\\ p > \pi/a \text{ physics}\end{array}$$

Numerical Analysis

Theory & context specific $\Rightarrow$ not universal!

Bad News: Need a^2 corrections when a large, but *Numerical Recipes* won't tell you values of $c(a)\ldots$

Good News: $p > \pi/a$ QCD is perturbative if a *small* enough (asymptotic freedom).

$\Rightarrow$ compute $c(a)\ldots$ using perturbation theory.

Perturbation theory fills in gaps in lattice;
$\Rightarrow$ continuum results without $a \to 0$!

E.g.,

Renormalization constant.

$$\mathcal{L}^{(a)} = Z(a)\,\overline{\psi}(\Delta\cdot\gamma - m(a))\psi$$
$$+c(a)\,a^2\,\overline{\psi}\Delta^3\cdot\gamma\psi$$
$$+\cdots$$

Finite-a correction.

where

$$c(a) = -\frac{1}{6} + c_1\,\alpha_s(\pi/a) + \cdots$$

Numerical Analysis

Mimics effects of $p > \pi/a$ states excluded by grid.

Asymptotic freedom in QCD $\Rightarrow$

- short-distance physics simple (perturbative);
- long-distance physics difficult (nonperturbative).

Lattice separates "short" from "long":

- $p > \pi/a$ QCD $\rightarrow$ corrections $\delta\mathcal{L}$ computed in perturbation theory (determines a);
- $p < \pi/a$ QCD $\rightarrow$ nonperturbative, numerical Monte Carlo integration.

Perturbation Theory

Improved discretizations and larger as — old ideas.

But perturbation theory is essential.

- $\Rightarrow$ a must be small enough so that $p \approx \pi/a$ QCD is perturbative.
- $\Rightarrow$ Before 1992: $a < 0.05\,\mathrm{fm}$.
- $\Rightarrow$ After 1992: $a < 0.4\,\mathrm{fm}$ works.

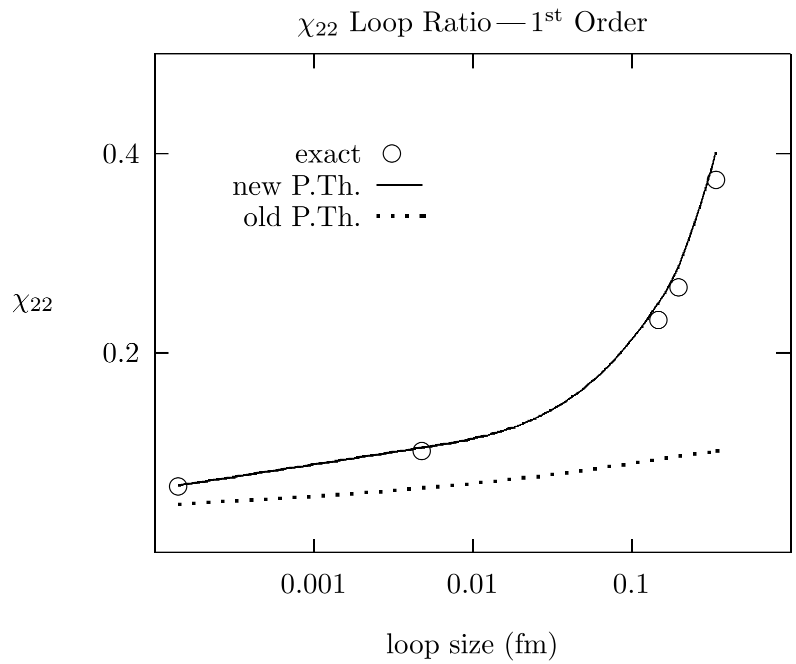
Test by comparing short-distance quantities from:

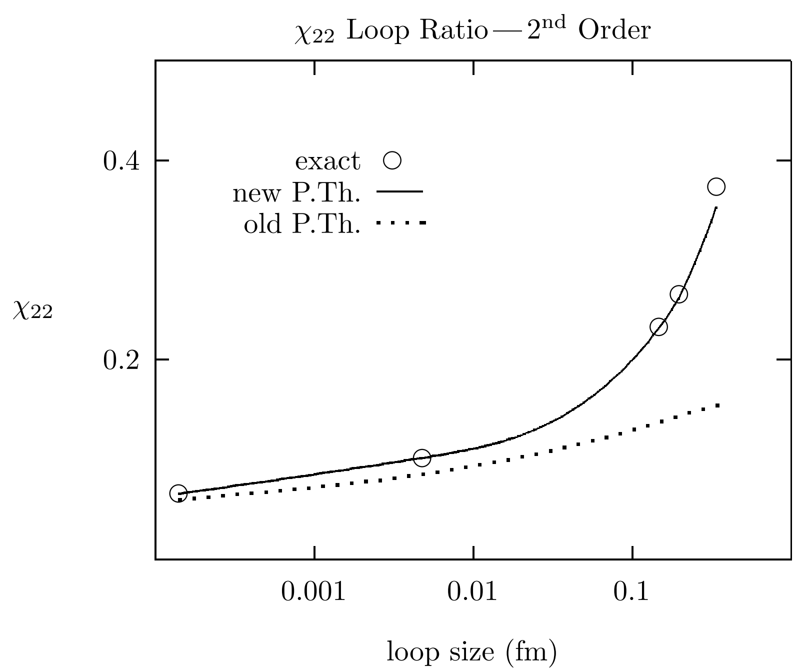
- perturbation theory;
- numerical Monte Carlo integration ($\Rightarrow$ exact result).

E.g., Wilson loops:

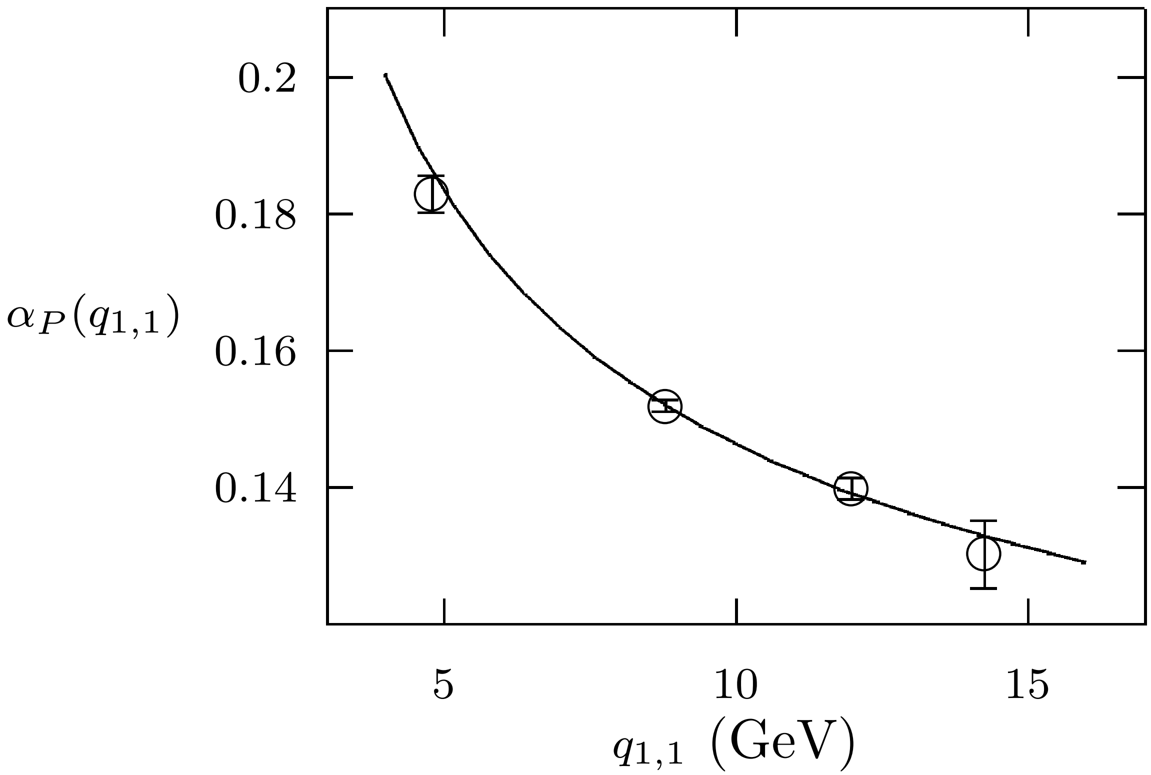
$$W(\mathcal{C}) \equiv \langle 0| \frac{1}{3} \,\mathrm{Re}\ \mathrm{Tr}\ \mathrm{P}\, \mathrm{e}^{-ig\oint_{\mathcal{C}} A\cdot \mathrm{d}x} |0\rangle,$$

$\mathcal{C}$ = small, closed path.


χ_{22} Loop Ratio — 1st Order
exact
new P.Th.
old P.Th.
χ_{22}
0.4
0.2
0.001
0.01
0.1
loop size (fm)


χ_{22} Loop Ratio — 2nd Order
exact
new P.Th.
old P.Th.
χ_{22}
0.4
0.2
0.001
0.01
0.1
loop size (fm)

Running coupling constant:



LQCD Tour

Recap

Expect errors $<$ few $\%$ from simulations with $a < 0.4\,\mathrm{fm}$.

Improved discretizations essential for speed and precision.

Questions:

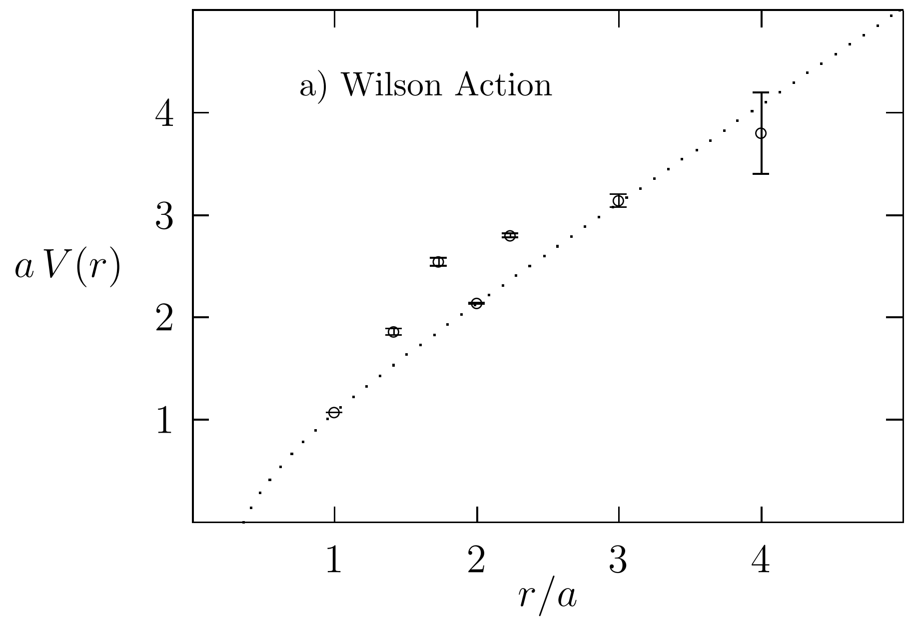
- Do the improvements work?
- Are the simulations faster?

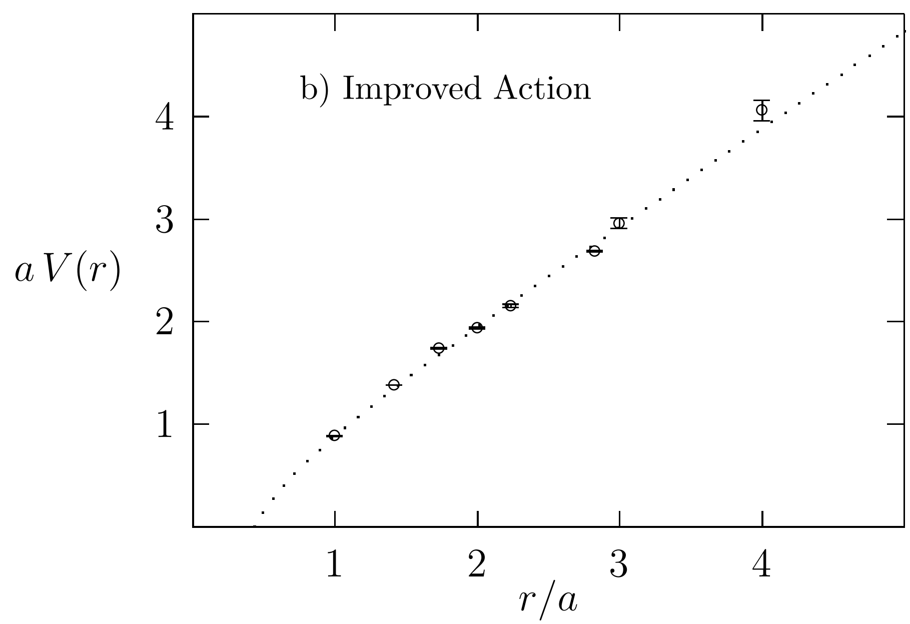
Gluons

Original discretization of the gluon action (Wilson, 1974) has $\mathcal{O}(a^2)$ errors:

$$\mathcal{L}_{\rm Wil} \approx \sum_{\mu,\nu} \left\{ \frac{1}{2}\,\mathrm{Tr}\,F_{\mu\nu}^2 + \frac{a^2}{24}\,\mathrm{Tr}\,F_{\mu\nu}(\mathrm{D}_\mu^2 + \mathrm{D}_\nu^2)F_{\mu\nu} \cdots \right\}.$$

$\mathcal{O}(a^2)$ error violates rotation/Poincaré invariance (due to lattice); removed by adding correction terms.


a) Wilson Action
4
3
2
1
$a\,V(r)$
1
2
3
4
r/a


b) Improved Action
4
3
2
1
$a\,V(r)$
1
2
3
4
r/a

Quarks

The standard discretization of the quark action has $\mathcal{O}(a^2)$ errors:

$$\mathcal{L}_{\rm lat} \approx \overline{\psi}(D\cdot\gamma + m)\psi + \frac{a^2}{6}\sum_\mu \overline{\psi} D_\mu^3 \gamma^\mu \psi + \cdots .$$

$\mathcal{O}(a^2)$ error violates rotation/Poincaré invariance; removed by adding correction term.

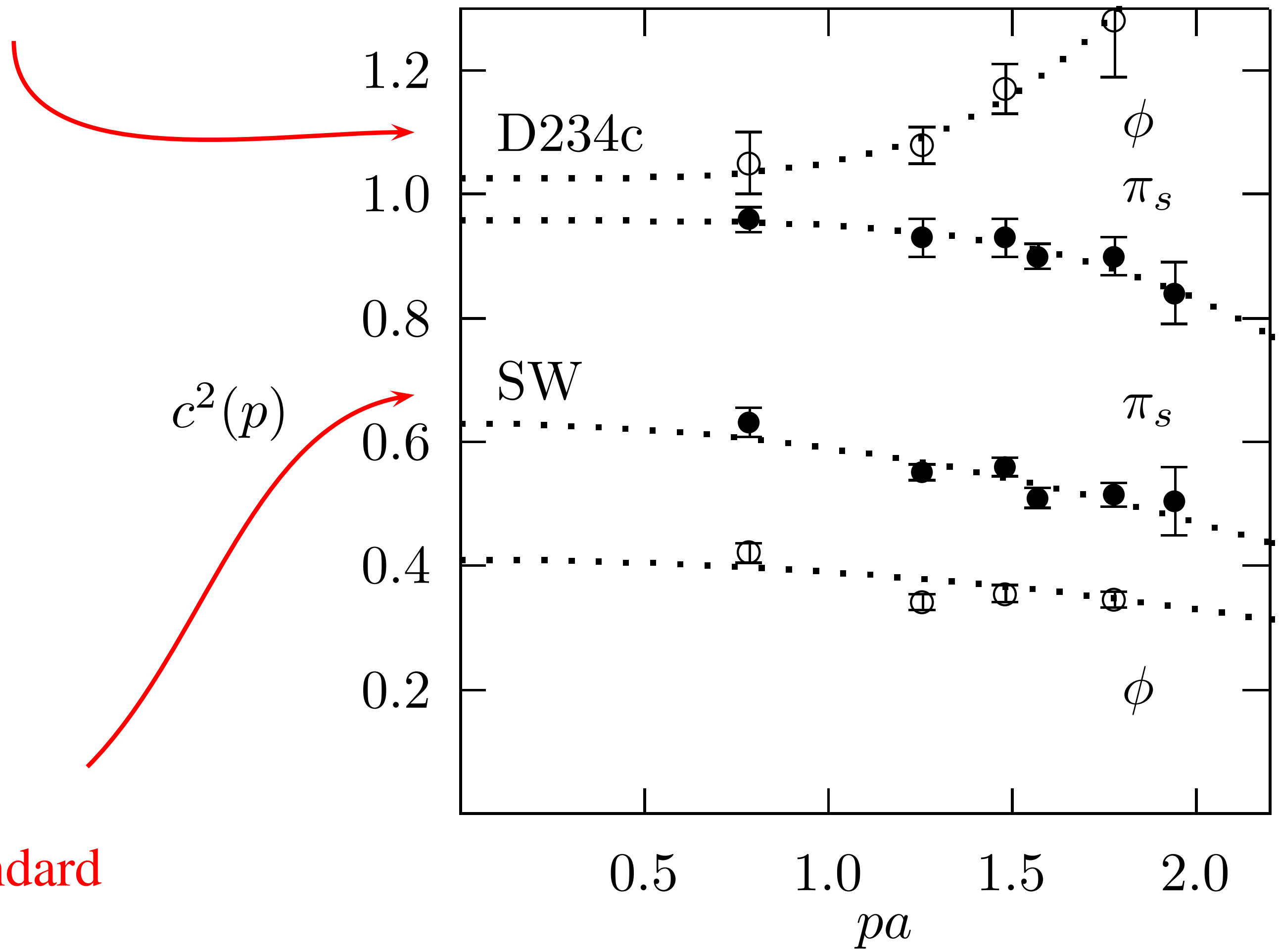
Test by computing

$$c^2(\mathbf{p}) \equiv \frac{E^2(\mathbf{p}) - m^2}{\mathbf{p}^2};$$

Lorentz invariance implies:

$$c^2(\mathbf{p}) = 1 \qquad \forall\, \mathbf{p}.$$

Improved



Standard

Alford et al (1997).

Heavy Quarks

Lattice errors $\propto (a\,E)^n$, $(a\,p)^n$

$\Rightarrow$ need $a \ll 1/M$ where M = hadron mass.

$B, \Upsilon \ldots \quad \Rightarrow \quad$ Need $a \to a/10$

$\Rightarrow \quad$ Cost $\to\ 10^6$ cost!

$\Rightarrow \quad$ Impossible?

No! b quark is nonrelativistic:

$$\frac{v^2}{2} \approx \frac{\Delta M}{M} \approx \frac{0.5\,\mathrm{GeV}}{10\,\mathrm{GeV}}$$

$\Rightarrow$ $v^2 \approx 0.1$;

$\Rightarrow$ don't use Dirac; use effective field theory.

$$\begin{aligned} \text{Schrödinger} \quad &+ \quad \mathcal{O}(a, a^2)\ \text{corrections} \\ &+ \quad \mathcal{O}(v^2, v^4)\ \text{corrections} \\ &+ \quad \cdots. \end{aligned}$$

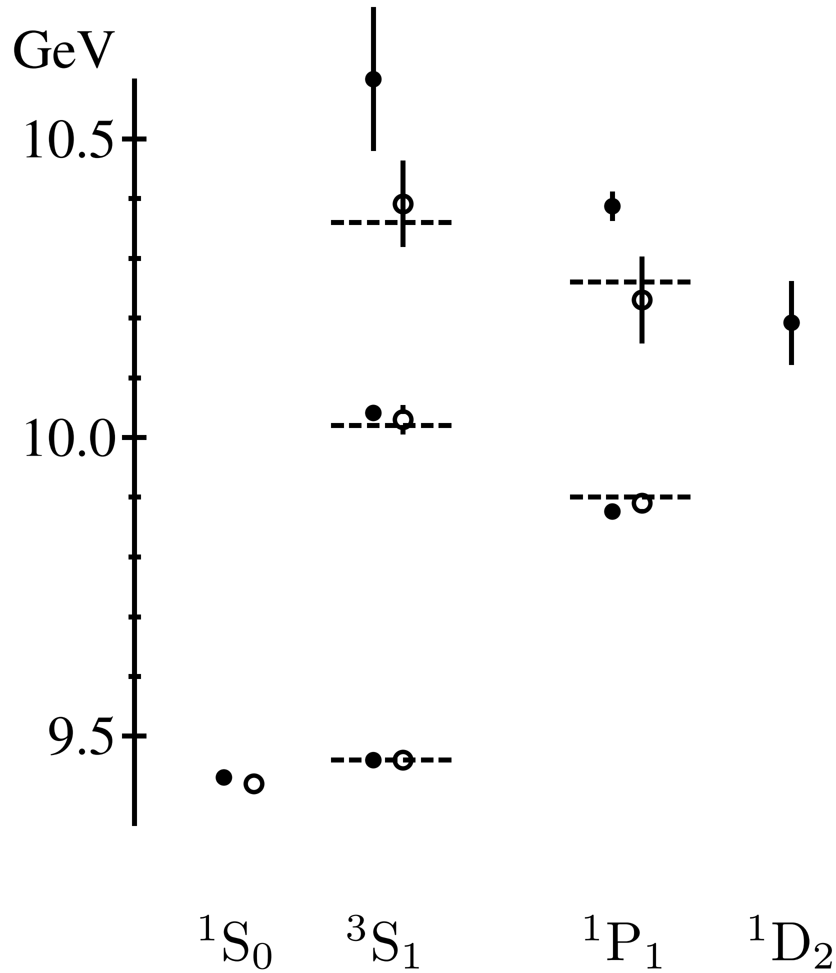
Lattice NRQCD:

Schrödinger:
$$H_0 \sim -\frac{\mathbf{D}^2}{2M_0} + ig\,A_0,$$

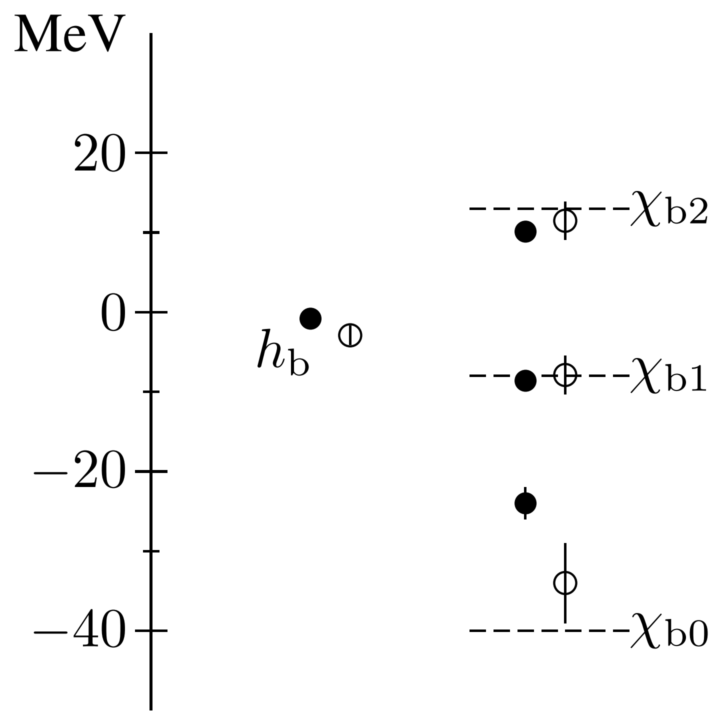
Corrections:
$$\begin{aligned}\delta H \quad \sim \quad & -c_1 \frac{(\mathbf{D}^2)^2}{8M_0^3}\left(1 + \frac{aM_0}{2n}\right) + c_2 \frac{a^2 \sum_i \mathrm{D}_i^4}{24M_0} \\ & -c_3 \frac{g}{2M_0}\,\sigma\cdot\mathbf{B} + c_4 \frac{ig}{8M_0^2}\left(\mathbf{D}\cdot\mathbf{E} - \mathbf{E}\cdot\mathbf{D}\right) \\ & -c_5 \frac{g}{8M_0^2}\sigma\cdot\left(\mathbf{D}\times\mathbf{E} - \mathbf{E}\times\mathbf{D}\right).\end{aligned}$$

where perturbation theory $\Rightarrow c_i = 1 + c_{i1}\alpha_s(\pi/a) + \cdots;$

$\Rightarrow$ only two parameters: α_s and M_0.



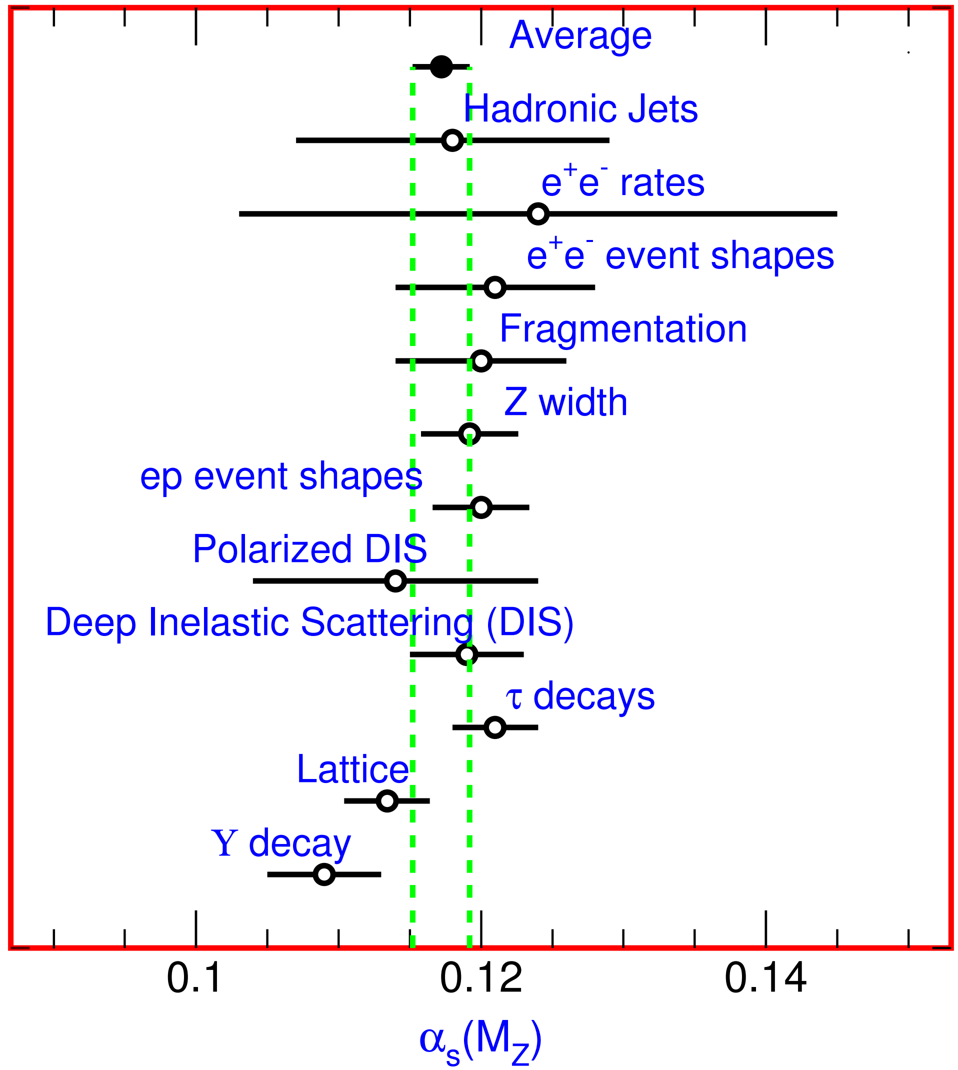
Davies et al. (1997).



Davies et al. (1997).

Tune two parameters to reproduce experimental data

$\Rightarrow$ few % accurate results for M_b and $\alpha_{\overline{\mathrm{MS}}}(M_Z)$.



Particle Data Group (2001).

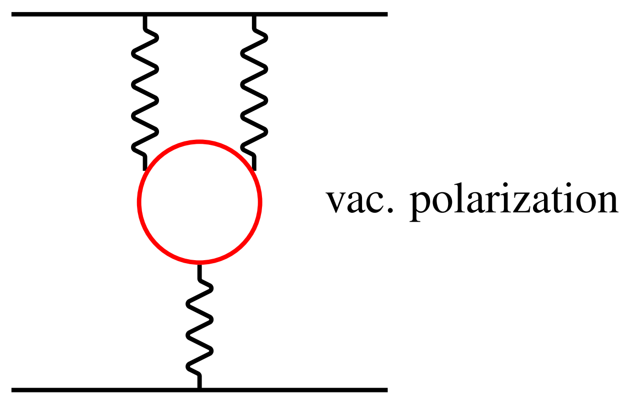
“Unquenching”

Unquenched LQCD

“Quenched” QCD $\equiv$ QCD without quark vacuum polarization.

$\Rightarrow$ 15–20% errors in most calculations;

$\Rightarrow$ *the* major limitation of LQCD *until 2000*.



Naive/staggered quarks + improved discretization

$\Rightarrow$ 10–100 times faster
& smallest finite-a errors
& best behavior in chiral limit!

$\Rightarrow$ high-precision (few %) LQCD possible *now!*

$\Rightarrow$ MILC collaboration has already produced thousands of configurations:

- $n_f = 3$;
- smallest $(m_u = m_d)$ ever: $m_s \ldots m_s/5$, $m_s/7$;
- small as: $1/8$ fm, $1/11$ fm;
- large Ls: 2.5 fm, 3.0 fm.

Naive/Staggered Quarks

Simplest discretization of light quarks,

$$\mathcal{L} = \overline{\psi}(x)(\Delta \cdot \gamma + m)\psi(x)$$

$\Rightarrow$ an exact "doubling" symmetry:

$$\psi(x) \quad \rightarrow \quad \tilde{\psi}(x) \equiv i\gamma_5\gamma_\rho\,(-1)^{x_\rho/a}\,\psi(x)$$
$$= i\gamma_5\gamma_\rho \exp(i\,x_\rho\pi/a)\,\psi(x).$$

$\Rightarrow$ Any low-energy mode ψ $\equiv$ Another mode, $\tilde{\psi}$, with $p_\rho \approx \pi/a$.

max. p on lattice

General case:

$$\psi(x) \;\rightarrow\; \mathcal{B}_\zeta(x)\,\psi(x)$$

where

$$\mathcal{B}_\zeta(x) \;\equiv\; \prod_\rho \,(i\gamma_5\gamma_\rho)^{\zeta_\rho}\;\; \exp(i\,x\cdot\zeta\,\pi/a)$$

$\zeta = (1,0,0,0),\,(0,1,0,0)\ldots\,(1,1,0,0)\ldots,$ 15 in all.

$\Rightarrow$ 1 field $\psi(x)$ creates 16 *different* but *exactly equivalent* flavors of quark ($p \approx \zeta\pi/a$)!

16 flavors is bad!

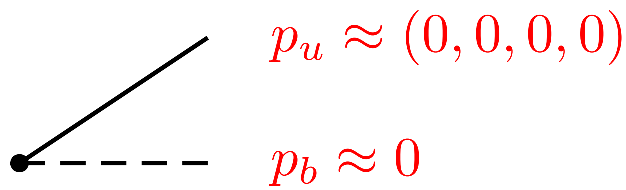
Two traditional options:

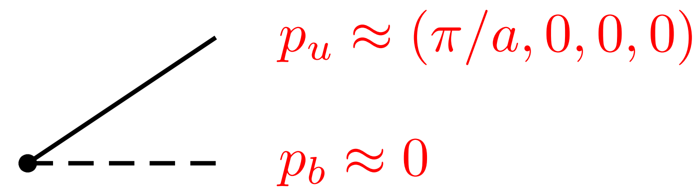
1. (Wilson, SW ...) Break doubling symmtry by adding $-a\overline{\psi}(D\cdot\gamma)^2\psi/2$ to $\mathcal{L}$; destroys all chiral symmetry $\Rightarrow$ small $m_{u,d}$ very difficult.

2. (Kogut-Susskind ...) Live with the 16 flavors by inserting factors of $1/16$ in strategic places. Preserves a chiral symmetry $\Rightarrow$ small masses relatively efficient.

Follow option 2!

E.g., 16 equivalent B mesons constructed from a b quark and 16 flavors of u antiquark: e.g.,





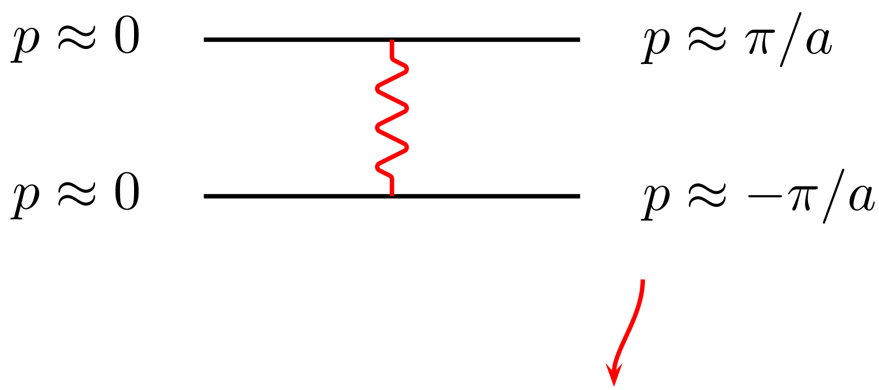
Ignore all but first (equivalent) by limiting total momentum:

$$P_B \equiv p_b + p_u < \frac{\pi}{2a}.$$

$\Rightarrow P_B$ distinguishes between flavors.

Light hadrons harder.

Bad News: Flavor-changing strong interactions —



Quarks on-shell, but different flavor

Good News: The gluon carries the largest lattice momentum, π/a;

$\Rightarrow$ highly virtual and perturbative;

$\Rightarrow$ can remove by (local) perturbative modifications to $\mathcal{L}$ to any order in $\alpha_s(\pi/a)$.

The new idea!

$\Rightarrow$ Four quark operators $+ a^2\Delta^3$ correction as before.

$\Rightarrow$ Most accurate discretization.

See Lepage (1998); MILC (1999).

An amazing fact:

$$\Omega(x) \equiv \prod_{\mu} (\gamma_\mu)^{x_\mu/a}$$

$$\Rightarrow \qquad S_F(x,y;A_\mu) = g(x,y;A_\mu)\,\Omega(x)\,\Omega^\dagger(y)$$

Propagator.

Dirac scalar for any gluon field A_μ.

Dirac structure is completely independent of A_μ!

$\Rightarrow$ 10–100 times faster than all other alternatives!

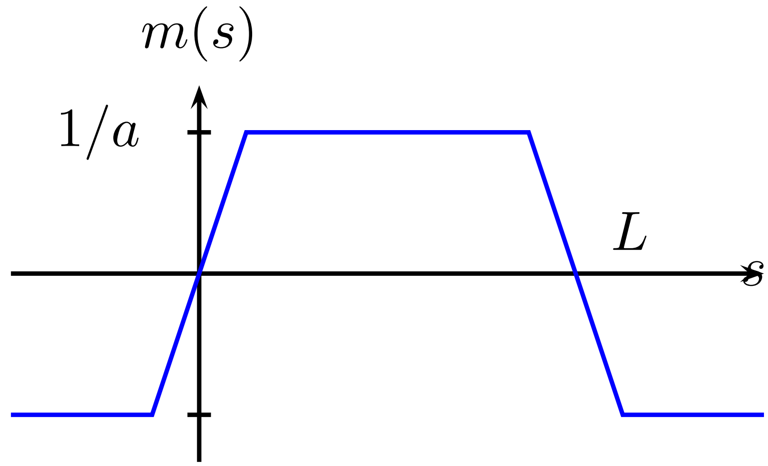
Domain-Wall Fermions

An approach for chiral fermions?

- Embed 4-D space-time in 5-D: (x^μ, s).
- Use Wilson lattice Dirac equation in 5-D,

$$\left(D\cdot\gamma + D_s\gamma_5 - \frac{a}{2}(D^2 + D_s^2) + m(s)\right)\,\Psi = 0,$$

but with

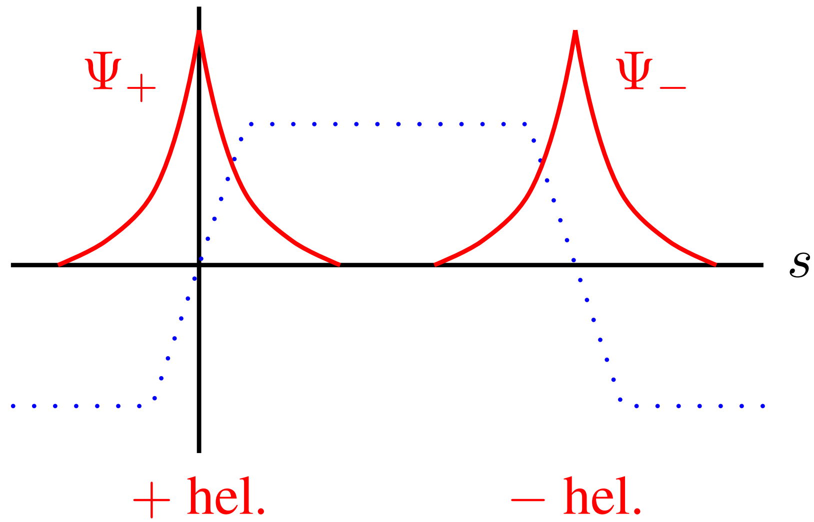


See Kaplan (1992).

- Separable $\Rightarrow$ chiral $\Psi_\pm$, such that

$$D \cdot \gamma \, \Psi_\pm = 0,$$

localized where $m(s) = 0$ — domain walls:



- "Keep" only $s = 0$ or L solution $\Rightarrow$ chiral theory $\Rightarrow$ wide range of new applications?
- Key issue: Do the two solutions communicate?

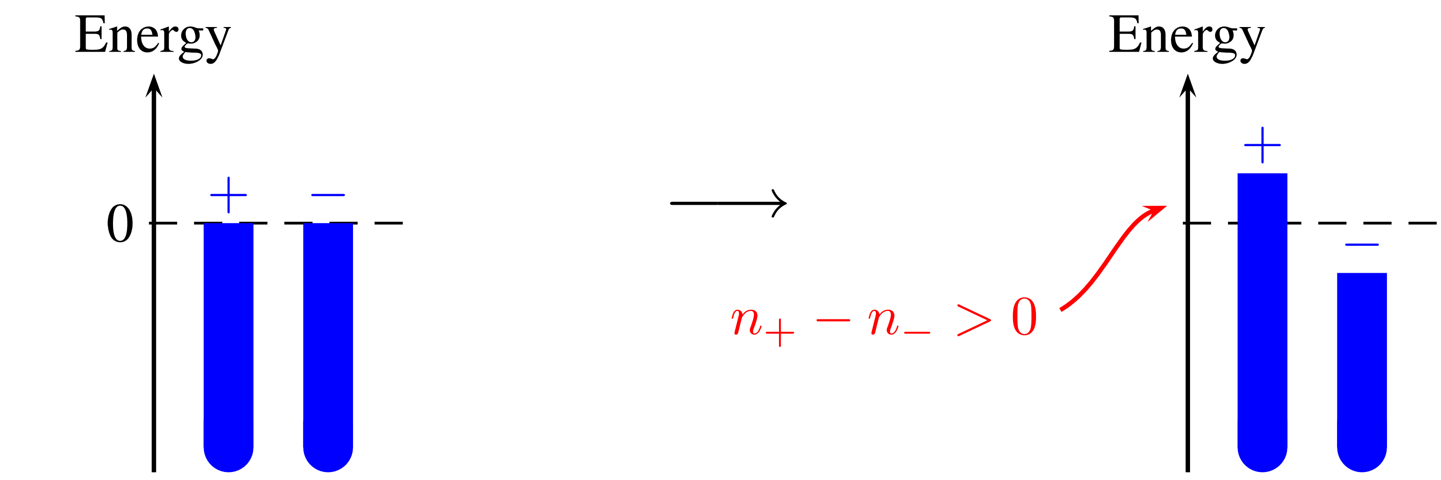
Why are Quarks so Hard?

Anomalies!? Chiral symm. $+$ massless quarks $\Rightarrow$

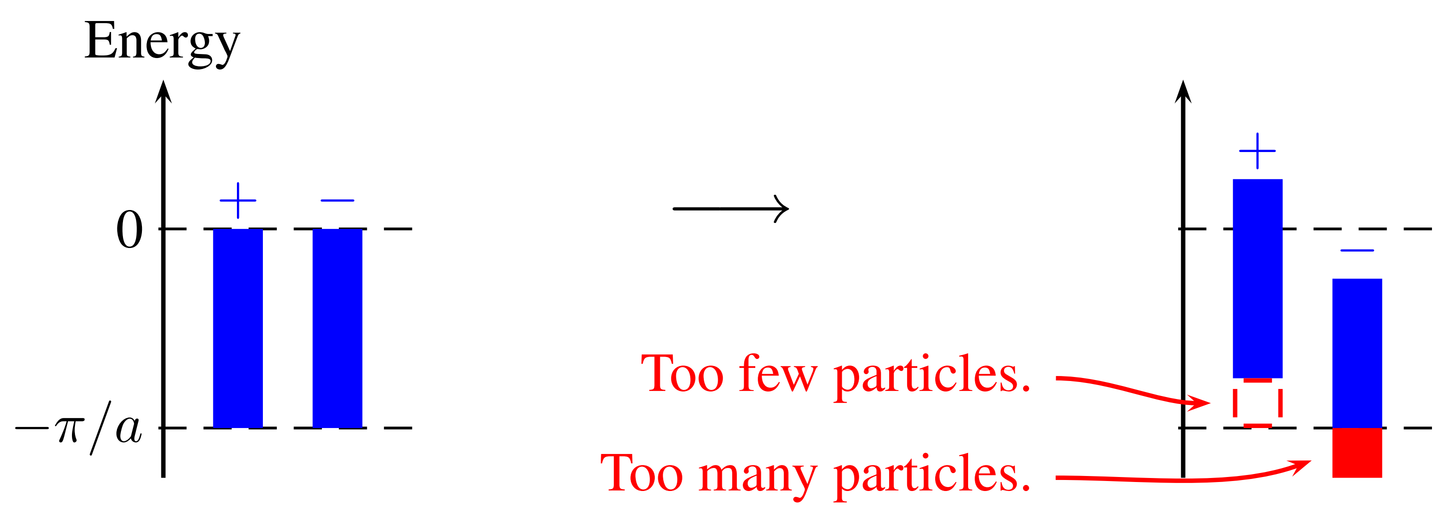
- Quark helicity $(\pm)$ is Lorentz invariant.
- $\partial^\mu j_{5\mu} = 0$ (classically)
  $\Rightarrow$ $n_+ - n_-$ conserved.
  $\Rightarrow$ No interaction in $\mathcal{L}$ changes $-$ to $+$.
- Quantum effects (anomaly)
  $\Rightarrow$ $\partial \cdot j_5 \propto \mathbf{E} \cdot \mathbf{B}$
  $\Rightarrow$ $n_+ - n_-$ can change!

E.g., vacuum Fermi levels shift as $\mathbf{E} \cdot \mathbf{B}$ varies:

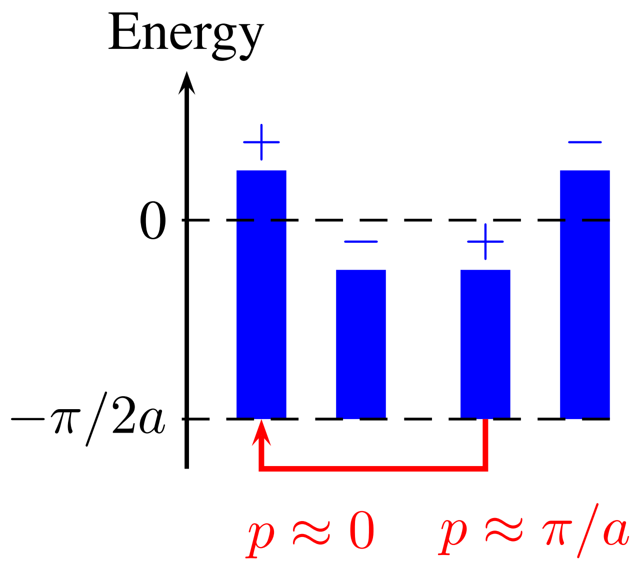
Continuum:



Lattice??

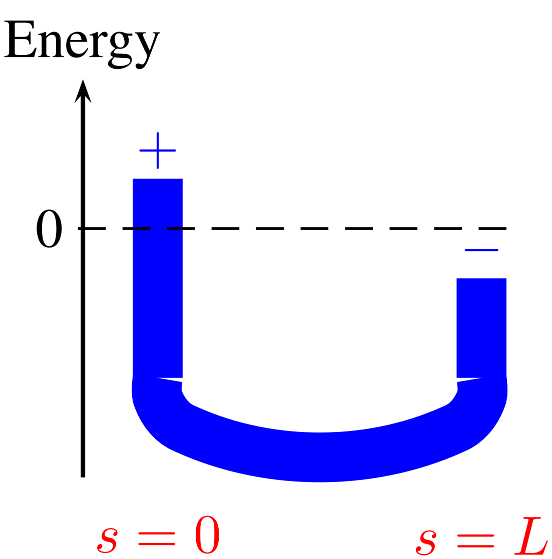


Naive/Staggered Quarks:



$n_+ - n_- = 0$, still;
$\Rightarrow$ anomalies cancel between flavors.

Domain Wall Quarks:



Particles move from $-$ to $+$ via 5th dimension.

Near Future

Algorithmic Advances in 1990's

1992 LQCD perturbation theory.

1992 Effective field theory (e.g., NRQCD, Fermilab).

1995 Improved discretizations $\Rightarrow$ larger a.

1999 Improved staggered/naive quarks:

- $\mathrm{Det}(D \cdot \gamma + m) > 0$ as in continuum (small m okay).
- Flavor-changing perturbative $\Rightarrow$ remove systematically.
- Naive quarks $\Rightarrow$ simple analyses, operators.
- No $\mathcal{O}(a)$ errors *and* much faster unquenching!

$\Rightarrow$ Lattice QCD is in revolution:

- 100–∞% errors in 1990.
- 10–20% errors today for wide range of masses, form factors . . .
- 1–3% possible now and in next few years.

High-Precision Possible Now

Few % accuracy for "gold-plated" calculations:
(Cornell Workshops, 2001 and 2002)

- Masses, decay constants, semileptonic form factors, and mixing amplitudes for D, D_s, D^*, D_s^*, B, B_s, B^*, B_s^*, and baryons.
- Masses, leptonic widths, electromagnetic form factors, and mixing amplitudes for any meson in ψ/Υ families below D/B threshold.
- Masses, decay constants, electroweak form factors, charge radii, magnetic moments and mixing for low-lying light-quark hadrons.
- High-precision $\Rightarrow$ masses and amplitudes with at most one hadron in the initial and/or final state, for stable or nearly stable hadrons ($\Gamma < 10$–$20\,\text{MeV}$).

- New collaboration (HPQCD):

  M. Alford, C. Davies (Glasgow)
  A. El-Khadra (Illinois)
  S. Gottlieb (Indiana)
  R. Horgan (Cambridge)
  K. Hornbostel (SMU)
  A. Kronfeld, P. Mackenzie, J. Simone (Fermilab)
  P. Lepage (Cornell)
  J. Shigemitsu (OSU)
  H. Trottier (SFU)
  R. Woloshyn (TRIUMF)
  + working closely with MILC Collaboration

Uses current techniques.

- Progress driven by improved methods.
- Future pace will be much faster than pace of hardware evolution.

HPQCD Plan

Compute dozens (?) of gold-plated quantities to few percent over next few years.

- Unquenched $n_f = 6$ with improved staggered quarks.
- NRQCD and Fermilab actions for c, b quarks through v^6.
- Actions, operators corrected through order a^2, $1/M^2$.
- One and two-loop perturbation theory (automated).
- PC cluster is optimal (simulations, pert'n theory . . .).

HPQCD Plan

Focus on B and D physics $\Rightarrow$ maximum impact on experimental community (BaBar, Belle, CLEO . . .).

- Gold-plated quantities for every off-diagonal CKM element.

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \pi \to l\nu & K \to \pi l\nu & B \to \pi l\nu \\ V_{cd} & V_{cs} & V_{cb} \\ D \to l\nu & D_s \to l\nu & B \to Dl\nu \\ D \to \pi l\nu & D \to Kl\nu & \\ V_{td} & V_{ts} & V_{tb} \\ \langle B_d|\overline{B}_d\rangle & \langle B_s|\overline{B}_s\rangle & \end{pmatrix}$$

- Extensive cross-checks for error calibration: Υ, ψ, B, D, K, π . . . .

HPQCD So far . . .

- Automated one-loop perturbation theory.
  - $\Lambda_V/\Lambda_{\mathrm{latt}}$ for improved gluons.
  - f_π and f_K, B_K, and m_s.
  - Flavor-changing in naive/staggered quarks.
  - Anisotropy and "speed of light" on anisotropic lattices.
- Tree-level spin-independent NRQCD through v^6 and spin-dependent through v^8; one-loop soon.
- Operator design with naive quarks.
- High-β techniques for nonperturbative perturbation theory.
- Constrained/Bayesian curve fitting.

- Very preliminary tunings/results:
  - HPQCD+MILC collaborations.
  - $n_f = 3$, $a = 1/8\,\mathrm{fm}$.
  - Tune $m_u = m_d$, m_s, m_c, m_b and α_s using m_π, m_K, m_ψ, m_Υ, and $\Delta E_\Upsilon(1P-1S)$.

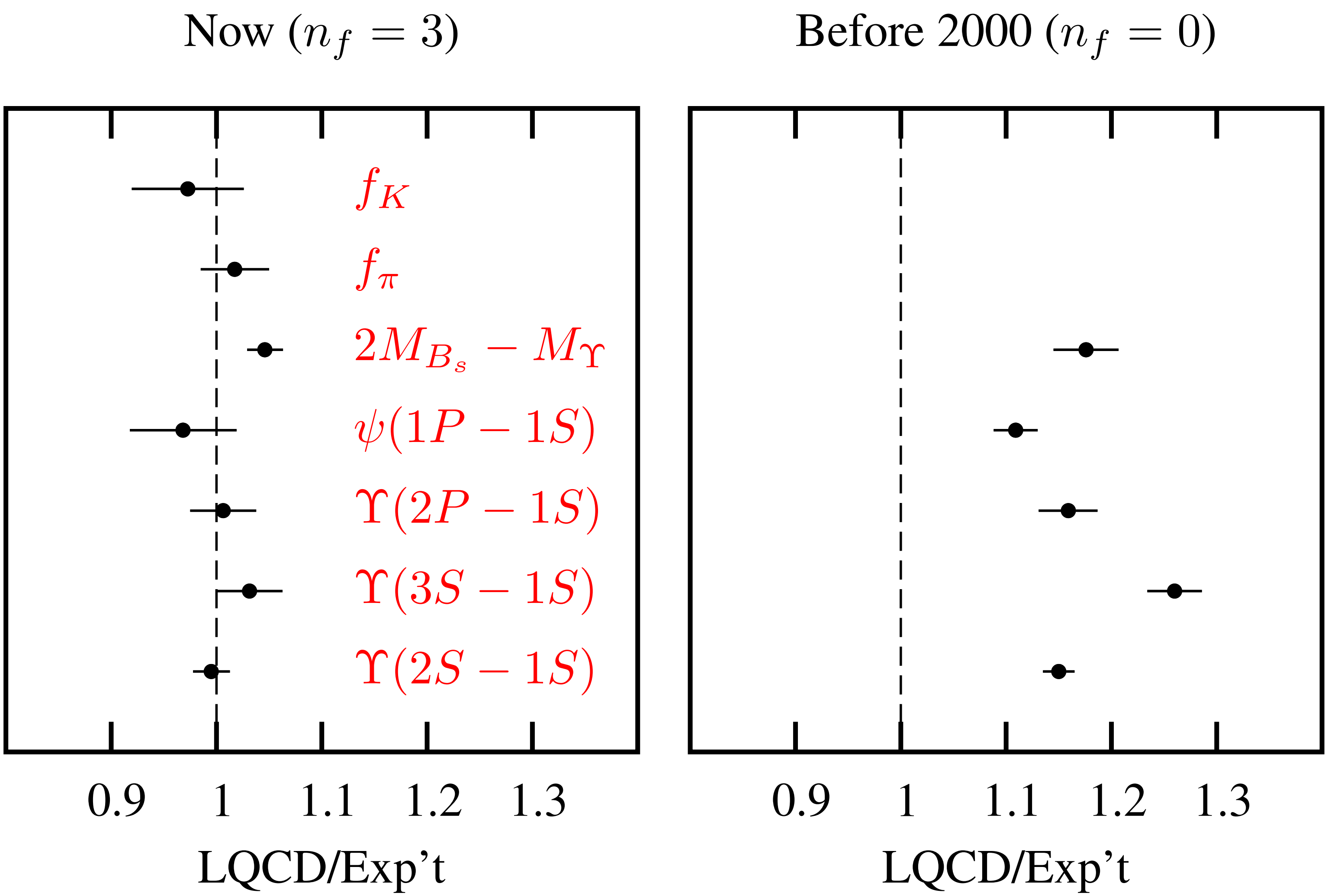
$\Rightarrow$ $\alpha_{\overline{MS}}(M_Z) = 0.116\,(4)$

$m_s(2\,\mathrm{GeV}) = 80\,(20)\,\mathrm{MeV}$

$\ldots$

Errors reduced 3–4$\times$ by Fall ‘02.

⇒ New results: (lattice QCD)/(experiment)



HPQCD+MILC: Very Preliminary

HPQCD case study: f_D errors

Perturbation theory:

- Connects lattice to continuum (fills in gaps): for f_D use

$$J_{\text{cont}} = Z\, J_{\text{latt}} + a\, \Delta J$$

where

$$Z = 1 + c_1 \alpha_s(\mu) + c_2 \alpha_s^2 + \cdots$$

and $\mu \approx 2/a$ is typical (for α_V and $n_f = 3$).

- Current work uses 1^{st}-order results; relative error is $\mathcal{O}(\alpha_s^2) \approx 7\%$ for $a = 0.1\,\text{fm}$. This is the dominant error.

- Next generation will use 2^{nd}-order results, giving relative errors of $\mathcal{O}(\alpha_s^3) \approx 1.6\%$ at $a = 0.1\,\text{fm}$.

HPQCD case study: f_D errors

Finite-lattice spacing errors:

- E.g., on lattice

$$p^2 \rightarrow (\sin(pa)/a)^2 = p^2 \left(1 - (pa)^2/3 + \cdots\right).$$

- Remove a, $a^2 \ldots$ errors (improved discretizations).
- $(pa)^2/3 \approx 0.7\%$ for $p \approx 300\,\mathrm{MeV}$ and $a = 0.1\,\mathrm{fm}$.
- $\mathcal{O}(a^2)$ improvement crucial for high-momentum form factors, and for suppressing flavor-changing interactions.

HPQCD case study: f_D errors

$1/M$ errors:

- Effective field theory (e.g., NRQCD) essential for heavy quarks $\Rightarrow 1/M$ expansion.
- Current work accurate through $\mathcal{O}(1/M)$ errors:
  - $\mathcal{O}(1/M^2) \approx 2\%$ or less for f_D;
  - $\mathcal{O}(\alpha_s/M) \approx 3.6\%$ for f_D;
  - 3–10 times smaller for B mesons.
- Future work accurate through $\mathcal{O}(\alpha_s/M, 1/M^2)$; relative error is $\mathcal{O}(\alpha_s^2/M) \approx 0.9\%$.

HPQCD case study: f_D errors

Unquenching:

- Most past work is quenched: $m_{u,d,s} \to \infty$ for sea quarks (i.e., no vacuum polarization) $\Rightarrow$ errors of 10–20%.
- Current simulations use realistic m_s and $m_{u,d} = m_s/5, m_s/7 \ldots$.
  - Chiral perturbation theory $\Rightarrow$ error estimates/corrections.
  - Relative errors $= \mathcal{O}(15\% \times (m_{u,d}/m_s)^2) \approx 1\%$ for $m_{u,d} = m_s/5$.
  - Complicated by flavor-changing interactions ($\approx$ couple %?).
- Simulations with $a = 0.1\,\mathrm{fm}$, $n_f = 6$, $m_{u,d} = m_s/4$ require $\approx$ 3 months on 200-node PC cluster for 1% statistical errors.
  - Use improved staggered quarks.
  - Lots of $n_f = 3$ gluon configurations already (MILC).
  - No more quenched analyses!

HPQCD case study: $B \to \pi l \nu$

$B \to \pi l \nu \Rightarrow V_{ub}$ but ...

$$(p_l + p_\nu)^2 \to 0 \quad \Rightarrow \quad p_\pi \to 2.5\,\text{GeV}$$

$\Rightarrow$ need

$$a \ll \frac{1}{2.5\,\text{GeV}} = 0.08\,\text{fm}$$

$\Rightarrow$ computers $3^7\times$ larger.

Or use mNRQCD (moving NRQCD):

a) Choose frame where B moves $\Rightarrow$ share momentum between B and π.

b) Parameterize b-quark's momentum

$$p_b^\mu \equiv M_b\, u_B^\mu + k^\mu$$

4-velocity of B meson.

$k \approx (\Lambda/M_b)\, p_b \ll p_b$
where $\Lambda \approx 0.5\,\mathrm{GeV}$.

$\Rightarrow$ **discretize k but treat $M_b\, u_B^\mu$ exactly.**

N.B. Best frame has $p_B \approx 8\,\mathrm{GeV}$ and

$p_\pi \approx k \approx \sqrt{\Lambda M_b}/2 \approx 0.8\,\mathrm{GeV}$

mNRQCD lagrangian:

$$\mathcal{L}_{\mathrm{mNRQCD}} = \chi^\dagger \left(iD_t + i\mathbf{v}\cdot\mathbf{D} + \frac{1}{2m\gamma}\left(\mathbf{D}^2 - (\mathbf{v}\cdot\mathbf{D})^2 + \sigma\cdot\mathbf{B}'\right) + \cdots \right)\chi$$

where

$$\mathbf{B}' = \gamma\left(\mathbf{B} - \mathbf{v}\times\mathbf{E} - \frac{\gamma}{\gamma+1}\,\mathbf{v}\,\mathbf{v}\cdot\mathbf{B}\right).$$

Hashimoto and Matsufuru (1996); Sloan (1998); Foley et al (2002); c.f. HQET.

Challenge for lattice QCD

Demonstrate reliability at the level of 1–3% errors, given past history of 10–20% errors.

- Requires comparison with wide variety of highly accurate experimental data.
  - High precision $\Rightarrow$ differentiate QCD from models.
  - Wide variety $\Rightarrow$ independent tests of all components.
- Must test:
  - Heavy-quark actions (NRQCD, Fermilab, etc.).
  - Light-quark actions (improved stagg. quarks).
  - Gluon action.
  - High-order perturbation theory.
  - Techniques for computing spectra, form factors . . . .

A problem for lattice QCD

There is very little high-precision QCD data from experiment.

Solution: new CLEO-c experiment!

CLEO-c: High-Precision QCD

2002 Υ family:

- Masses, spin splittings, widths, form factors, mixing . . . to few %.
- Richest testing ground for heavy-quark actions ⇒ *independent* calibration/test of b-quark action used in B simulations.
- Overconstrain b-quark action.

2004/5 D, D_s mesons:

- Leptonic, semileptonic widths and form factors to few %.
- Calibrate LQCD on analogues of crucial B processes.
- V_{cd} and V_{cs} to few %, new unitarity triangles, new physics (?).

2006 ψ/J family, glueball spectrum.

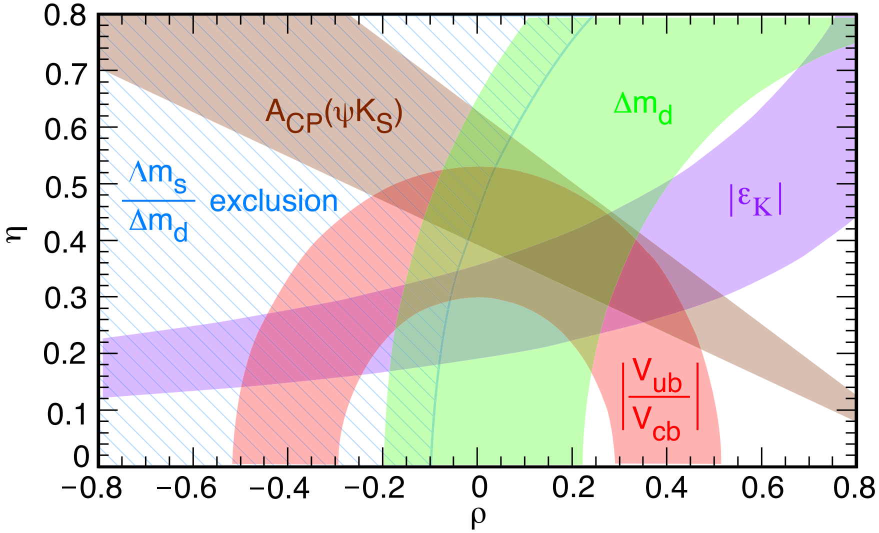
⇒ LQCD in race to *predict* CLEO-c results!

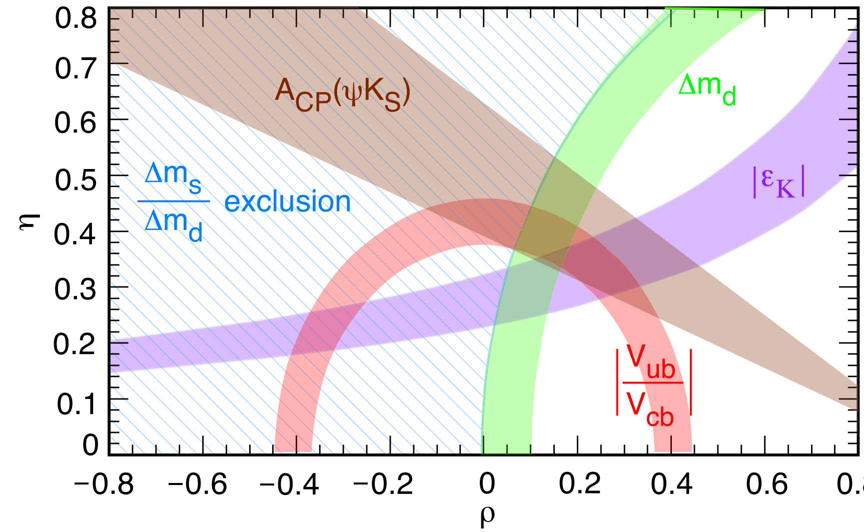
Conclusion

Superb opportunity for LQCD to have an impact on particle physics.

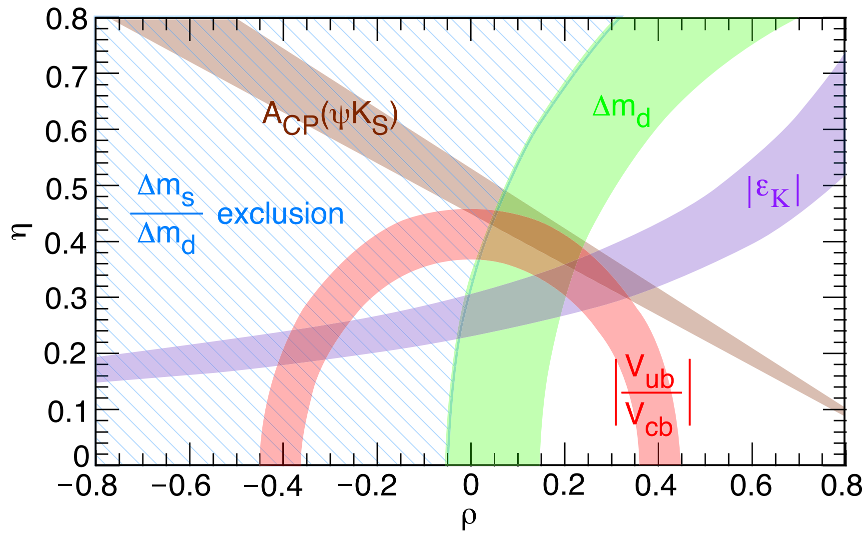
- LQCD essential to high-precision B/D physics at BaBar, Belle, CLEO-c ...
- *Predicting* CLEO-c, BaBar/Belle results $\Rightarrow$ much-needed credibility for LQCD.
- Landmark in history of quantum field theory: quantitative verification of nonperturbative technology (c.f., 1950's).
- Ready for beyond the Standard Model!

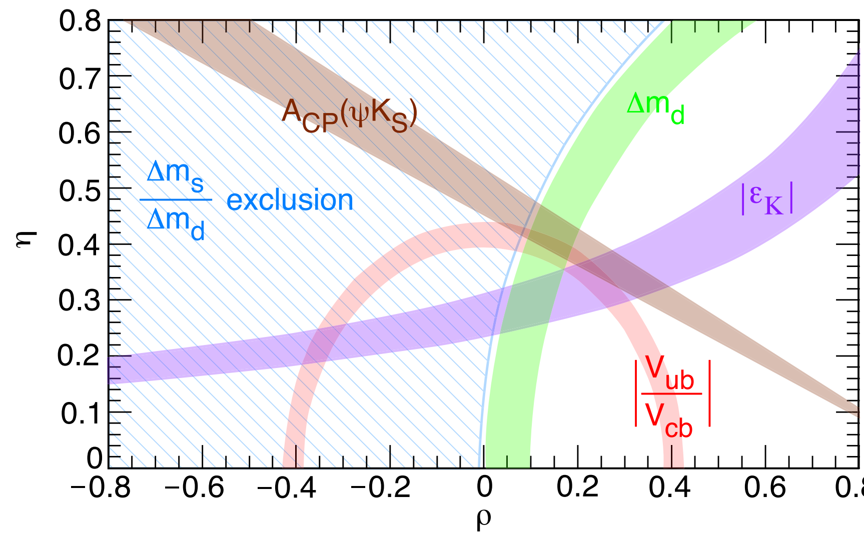
CKM today . . .   . . . and with 2–3% theory errors.





And with B Factories . . .





95% confidence levels; CLEO-c (2001).