Second Lecture

- Heavy quark symmetry

  ... Spectroscopy with HQS

- Exclusive semileptonic decays

  ... $B \to D^{(*)}\ell\nu$ decays and $|V_{cb}|$

  ... Heavy to light decays

- Inclusive semileptonic decays

  ... $B \to X_c\ell\bar{\nu}$ and $|V_{cb}|$

  ... Inclusive $|V_{ub}|$ measurements and rare decays

- Summary

- Additional topics

  ... B decays to excited D mesons; exclusive & inclusive rare decays

Preliminaries

- Theoretical tools to analyze semileptonic and rare decays are similar

  Allow measurements of CKM elements and are sensitive to new physics

  Improved understanding of hadronic physics and accuracy of theoretical predictions affects sensitivity to new physics

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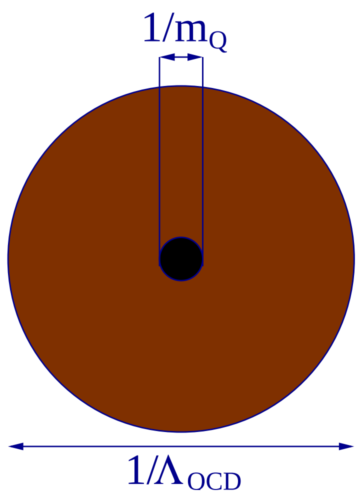
- For the purposes of this and tomorrow's talks, [strong interaction] model independent $\equiv$ theoretical uncertainty suppressed by small parameters

  Most of the recent progress comes from expanding in powers of Λ/m_Q, $\alpha_s(m_Q)$
  ... a priori not known whether $\Lambda \sim 200\mathrm{MeV}$ or $\sim 2\mathrm{GeV}$ $(f_\pi, m_\rho, m_K^2/m_s)$
  ... need experimental guidance to see which cases work how well

Heavy quark symmetry

$Q\,\overline{Q}$: positronium-type bound state, perturbative in $m_Q \gg \Lambda_{\rm QCD}$ limit

$Q\,\overline{q}$: wave function of the light degrees of freedom
("brown muck") insensitive to spin and flavor of Q



B meson is a lot more complicated than just a $b\,\overline{q}$ pair

In the $m_Q \to \infty$ limit, the heavy quark acts as a static color source with fixed four-velocity v^μ

$\Rightarrow SU(2n)$ heavy quark spin-flavor symmetry at fixed v^μ

Similar to atomic physics ($m_e \ll m_N$):

1. Flavor symmetry $\sim$ isotopes have similar chemistry [Ψ_e independent of m_N]
2. Spin symmetry $\sim$ hyperfine levels almost degenerate [$\vec{s}_e - \vec{s}_N$ interaction $\to 0$]

Spectroscopy of heavy-light mesons

- In $m_Q \to \infty$ limit, spin of the heavy quark is a good quantum number, and so is the spin of the light d.o.f., since $\vec{J} = \vec{s}_Q + \vec{s}_l$ and

$$\left.\begin{array}{r} \text{angular momentum conservation: } [\vec{J}, \mathcal{H}] = 0 \\ \text{heavy quark symmetry: } [\vec{s}_Q, \mathcal{H}] = 0 \end{array}\right\} \Rightarrow [\vec{s}_l, \mathcal{H}] = 0$$

For a given s_l, two degenerate states:

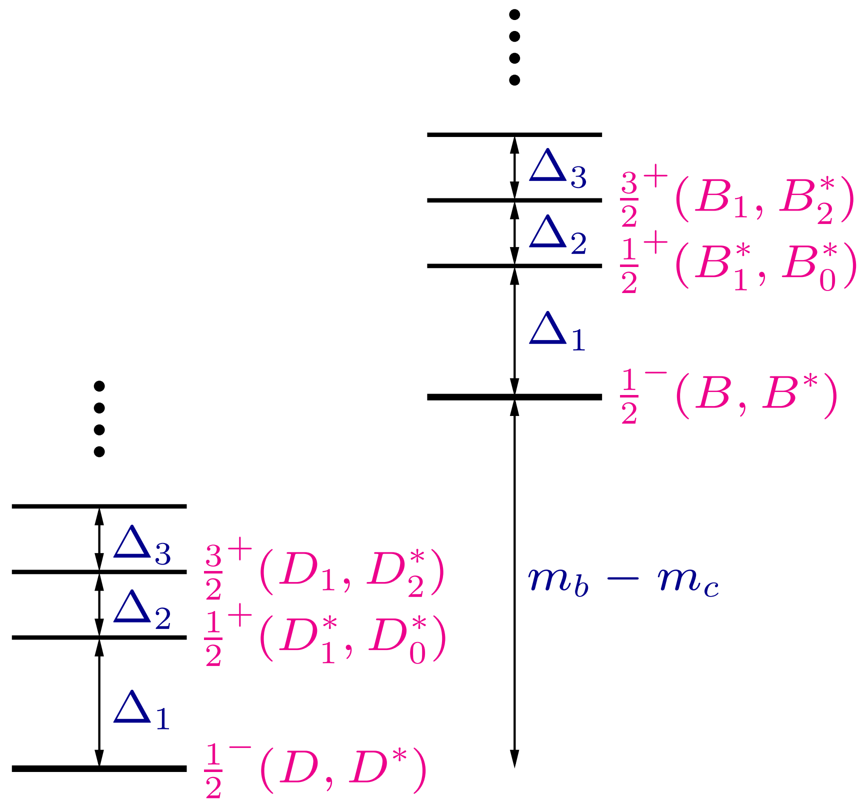
$$J_\pm = s_l \pm \tfrac{1}{2}$$

$\Rightarrow \Delta_i = \mathcal{O}(\Lambda_{\rm QCD})$ — same in B and D sector

Doublets are split by order $\Lambda^2_{\rm QCD}/m_Q$, e.g.:

$m_{D^*} - m_D \simeq 140$ MeV

$m_{B^*} - m_B \simeq 45$ MeV



Aside: a puzzle

Since vector–pseudoscalar mass splitting $\propto 1/m_Q$, expect: $m_V^2 - m_P^2 =$ const.

This argument relies on $m_Q \gg \Lambda_{\rm QCD}$

Experimentally:

$$m_{B^*}^2 - m_B^2 = 0.49\,\text{GeV}^2$$

$$m_{B_s^*}^2 - m_{B_s}^2 = 0.50\,\text{GeV}^2$$

$$m_{D^*}^2 - m_D^2 = 0.54\,\text{GeV}^2$$

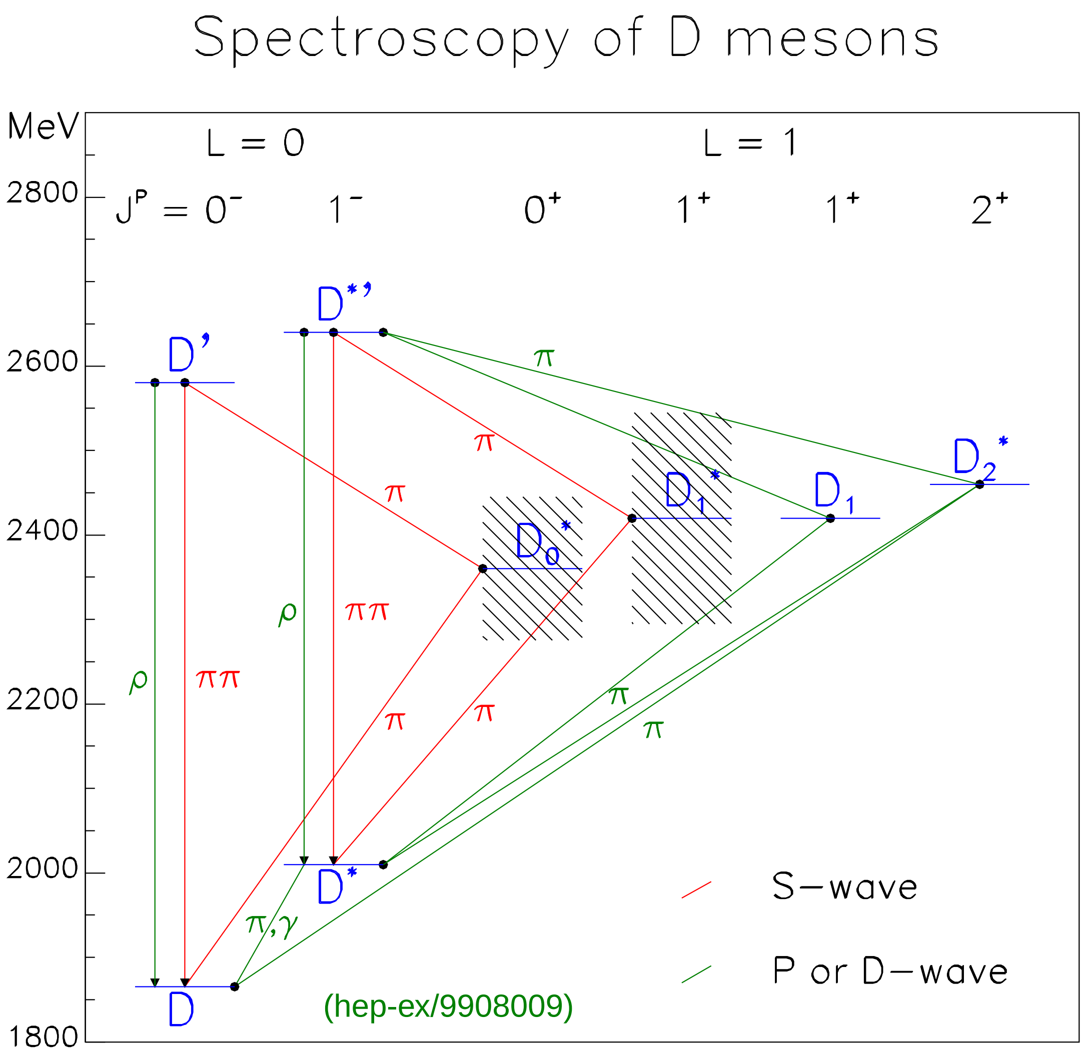
$$m_{D_s^*}^2 - m_{D_s}^2 = 0.58\,\text{GeV}^2$$

$$m_\rho^2 - m_\pi^2 = 0.57\,\text{GeV}^2$$

$$m_{K^*}^2 - m_K^2 = 0.55\,\text{GeV}^2$$

Not understood... there is something more going on than just HQS!

Charmed meson spectrum



"Successes:"

D_1 is narrow: S-wave $D_1 \to D^*\pi$ amplitude allowed by angular momentum conservation, but forbidden in the $m_Q \to \infty$ limit by heavy quark spin symmetry

Mass splittings of orbitally excited states is small:
$m_{D_2^*} - m_{D_1} = 37\,\text{MeV} \ll m_{D^*} - m_D$
vanishes in the quark model, since it arise from $\langle \vec{s}_Q \cdot \vec{s}_{\bar{q}}\, \delta^3(\vec{r}) \rangle$

Aside: strong decays of D_1 and D_2^*

- The strong interaction Hamiltonian conserves the spin of the heavy quark and the light degrees of freedom separately

$(D_1, D_2^*) \to (D, D^*)\pi$ — four amplitudes related by heavy quark spin symmetry

$$\Gamma(j \to j'\,\pi) \propto (2s_l + 1)(2j' + 1) \left| \begin{Bmatrix} L & s_l' & s_l \\ \frac{1}{2} & j & j' \end{Bmatrix} \right|^2$$

Multiplets have opposite parity $\Rightarrow$ π must be in $L = 2$ partial wave

$\Gamma(D_1 \to D\pi)$	:	$\Gamma(D_1 \to D^*\pi)$	:	$\Gamma(D_2^* \to D\pi)$	:	$\Gamma(D_2^* \to D^*\pi)$
0	:	1	:	$\frac{2}{5}$	:	$\frac{3}{5}$
0	:	1	:	2.3	:	0.92

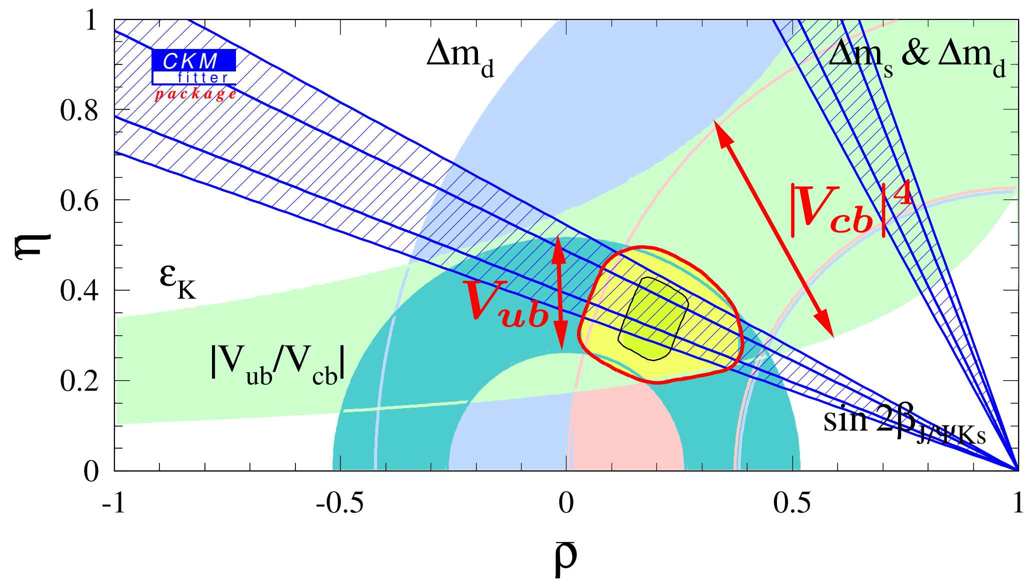
- Last line includes large $|p_\pi|^5$ HQS violation from phase space, which changes $\Gamma(D_2^* \to D\pi)/\Gamma(D_2^* \to D^*\pi)$ from $2/3$ to 2.5 (data: 2.3 ± 0.6)

[Note: prediction for ratio of D_1 and D_2^* total widths works less well (Falk & Mehen)]

Semileptonic and rare B decays

$|V_{ub}|$ is the dominant uncertainty of the side of the UT opposite to $\beta = \phi_1$

Error of $|V_{cb}|$ is a large part of the uncertainty in the ϵ_K constraint, and in $K \to \pi\nu\bar{\nu}$ when it's measured



Rare decays mediated by $b \to s\gamma$, $b \to s\,\ell^+\ell^-$, and $b \to s\,\nu\bar{\nu}$ transitions are sensitive probes of the Standard Model

Exclusive $B \to D^{(*)}\ell\bar{\nu}$ decay

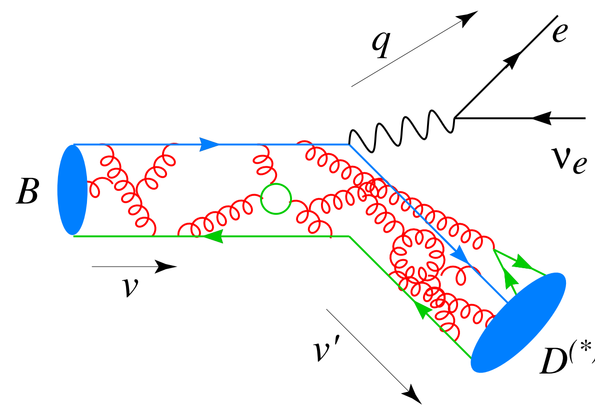
- In the $m_{b,c} \to \infty$ limit, configuration of brown muck only depends on the four-velocity of the heavy quark, but not on its mass and spin

  Weak current changes $b \to c$, i.e.:

  $\vec{p}_b \to \vec{p}_c$ and possibly flips $\vec{s}_Q$, on a time scale $\ll \Lambda_{\rm QCD}^{-1}$

  In $m_{b,c} \gg \Lambda_{\rm QCD}$ limit brown muck only feels $v_b \to v_c$

  Form factors independent of Dirac structure of weak current $\Rightarrow$ all form factors related to a single function of $w = v \cdot v'$, the Isgur-Wise function, $\xi(w)$



  $\Uparrow$

  Contains all nonperturbative low-energy hadronic physics

- $\xi(1) = 1$, because at "zero recoil" configuration of brown muck not changed at all

$B \to D^{(*)}\ell\bar{\nu}$ form factors

- Lorentz invariance $\Rightarrow$ 6 form factors

$$\langle D(v')|V_\nu|B(v)\rangle = \sqrt{m_B m_D}\,\left[h_+\,(v+v')_\nu + h_-\,(v-v')_\nu\right]$$

$$\langle D^*(v')|V_\nu|B(v)\rangle = i\sqrt{m_B m_{D^*}}\,h_V\,\epsilon_{\nu\alpha\beta\gamma}\epsilon^{*\alpha}v'^\beta v^\gamma$$

$$\langle D(v')|A_\nu|B(v)\rangle = 0$$

$$\langle D^*(v')|A_\nu|B(v)\rangle = \sqrt{m_B m_{D^*}}\,\left[h_{A_1}\,(w+1)\epsilon^*_\nu - h_{A_2}\,(\epsilon^*\cdot v)v_\nu - h_{A_3}\,(\epsilon^*\cdot v)v'_\nu\right]$$

$V_\nu = \bar{c}\gamma_\nu b,\quad A_\nu = \bar{c}\gamma_\nu\gamma_5 b,\quad w \equiv v\cdot v' = \dfrac{m_B^2+m_D^2-q^2}{2m_B m_D},$ and $h_i = h_i(w,\mu)$

- In $m_Q \to \infty$ limit, up to corrections suppressed by α_s and $\Lambda_{\rm QCD}/m_{c,b}$

$$h_- = h_{A_2} = 0\,, \qquad h_+ = h_V = h_{A_1} = h_{A_3} = \xi(w)$$

α_s corrections calculable

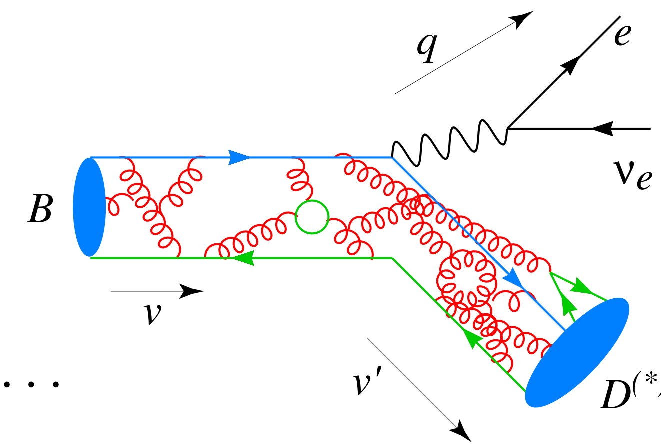
$\Lambda_{\rm QCD}/m_{c,b}$ corrections is where model dependence enters

$|V_{cb}|$ from $B \to D^{(*)}\ell\bar\nu$

- Extract $|V_{cb}|$ from $w \equiv v \cdot v' = \frac{m_B^2+m_D^2-q^2}{2m_B m_D} = 1$ limit of $B \to D^{(*)}\ell\bar\nu$ rate

$$\frac{\mathrm{d}\Gamma(B \to D^{(*)}\ell\bar\nu)}{\mathrm{d}w} = \text{(known factors)}\ |V_{cb}|^2\, \mathcal{F}^2_{(*)}(w)$$

$$\mathcal{F}_{(*)}(w) = \text{Isgur-Wise function} + \mathcal{O}(\alpha_s, \Lambda_{\text{QCD}}/m_{c,b})$$



$$\mathcal{F}(1) = 1_{\text{Isgur−Wise}} + 0.02_{\alpha_s,\alpha_s^2} + \frac{\text{(lattice or models)}}{m_{c,b}} + \dots$$

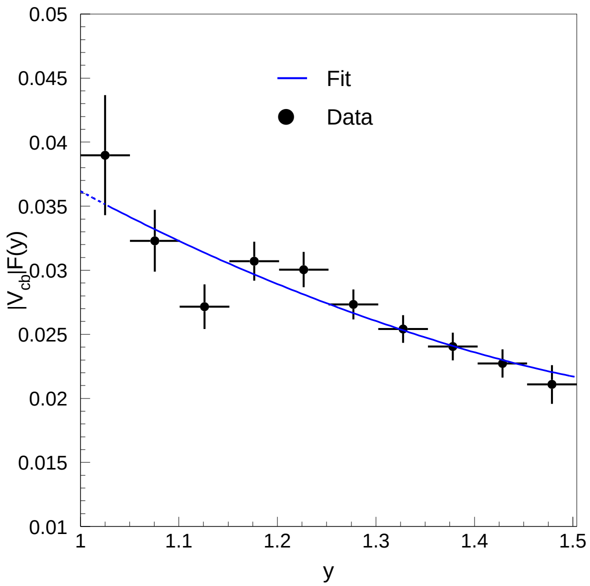
$$\mathcal{F}_*(1) = 1_{\text{Isgur−Wise}} - 0.04_{\alpha_s,\alpha_s^2} + \frac{0_{\text{Luke}}}{m_{c,b}} + \frac{\text{(lattice or models)}}{m_{c,b}^2} + \dots$$

$\Rightarrow$ theorists argue about small corrections

Near zero recoil: $\mathrm{d}\Gamma/\mathrm{d}w \propto \begin{cases} \sqrt{w^2-1} & \text{for } B \to D^* \\ (w^2-1)^{3/2} & \text{for } B \to D \ \text{(helicity!)} \end{cases}$

$B \to D^*$ preferred both experimentally and theoretically (except lattice QCD)

Experimental status of $|V_{cb}|_{\text{exclusive}}$



Functional form used to extrapolate to zero recoil is very important — shape related to $B \to D^{**}\ell\bar{\nu}$ decay

Experiments measure: $|V_{cb}|\, \mathcal{F}_*(1)$

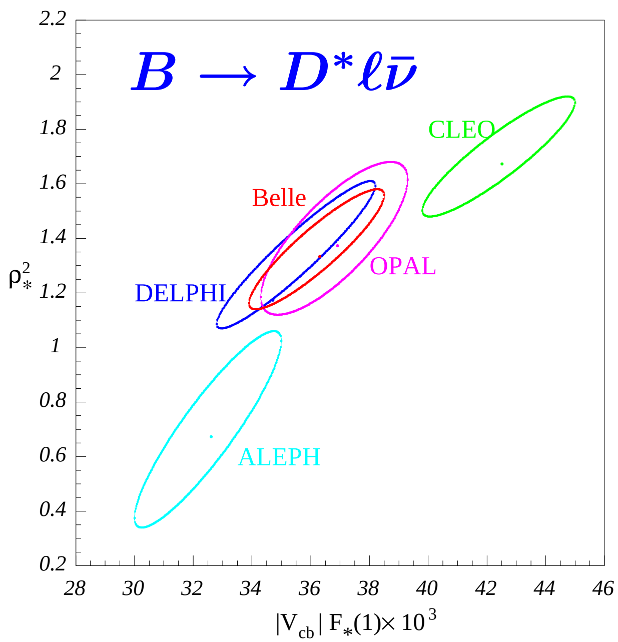
Theory predicts: $\mathcal{F}_*(1) = 0.91 \pm 0.04$

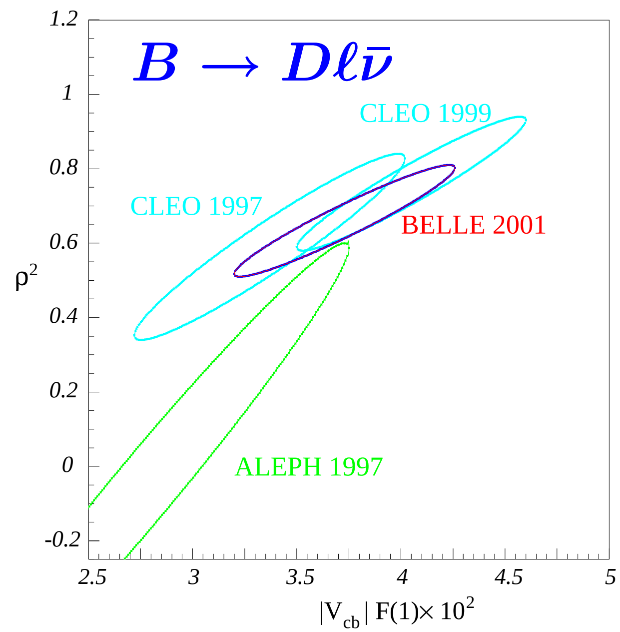
$\Rightarrow |V_{cb}| = (41.9 \pm 1.1 \pm 1.9) \times 10^{-3}$ (Battaglia @ ICHEP)

$B \to D\ell\bar{\nu}$ may be important:

Difference of slopes is an order $\Lambda_{\text{QCD}}/m_{c,b}$ effect...

Corellation between slope and $|V_{cb}|$ very large





Uncertainties in $|V_{cb}|_{\rm exclusive}$

- Nonperturbative correction at zero recoil
  - Bounds from sum rules or models[1]
  - Lattice QCD: Calculate $\mathcal{F}_{(*)} - 1$ from a double ratio of correlation functions

    $\mathcal{F}(1) = 1.06 \pm 0.02\,,\ \mathcal{F}_*(1) = 0.91 \pm 0.03,$ D not harder than D^* (FNAL, quenched)

  Checks: consistency between $B \to D^*$ and D, and the form factor ratios ($R_{1,2}$)

- Extrapolation to zero recoil
  - Unitarity constraints: strong correlation between slope & curvature of $\mathcal{F}_{(*)}(w)$

    (Boyd, Grinstein, Lebed; Caprini, Lellouch, Neubert)
  - Constrain slopes by studying decays to excited D^{**}, $B \to D^{**}\ell\bar{\nu}$, near $w = 1$

[1] "When you have to descend into the brown muck, you abandon all pretense of doing elegant, pristine, first-principles calculations. You have to get your hands dirty with uncontrolled approximations and models. When you are finished with the brown muck you should wash your hands." (H. Georgi, TASI' 1991)

$B \longrightarrow$ light form factors

- Limited use of HQS: relate $B \to \rho\ell\bar\nu,\ K^*\ell^+\ell^-,\ K^*\gamma$ form factors in large q^2 region, but HQS neither reduces number of form factors, nor determines their normalization at any value of q^2

$$\begin{array}{ccccccc} & \bar B & \xrightarrow{\bar u \Gamma b\, V_{ub}} & \rho\,\ell\bar\nu & & & \\ \begin{matrix}\text{flavor}\\ SU(2)\end{matrix} & \updownarrow & & \updownarrow & \begin{matrix}\text{chiral}\\ SU(3)\end{matrix} & \Rightarrow & \text{relations at same } v\cdot v' \\ & D & \xrightarrow{\bar d \Gamma c\, V_{cs}} & K^*\ell\bar\nu & & & \end{array}$$

Can predict $B \to \rho\ell\bar\nu$ rate from measured $D \to K^*\ell\bar\nu$ form factors

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- Corrections to heavy quark symmetry and chiral symmetry could be $\sim 20\%$ each (First order corrections can be eliminated — complicated)

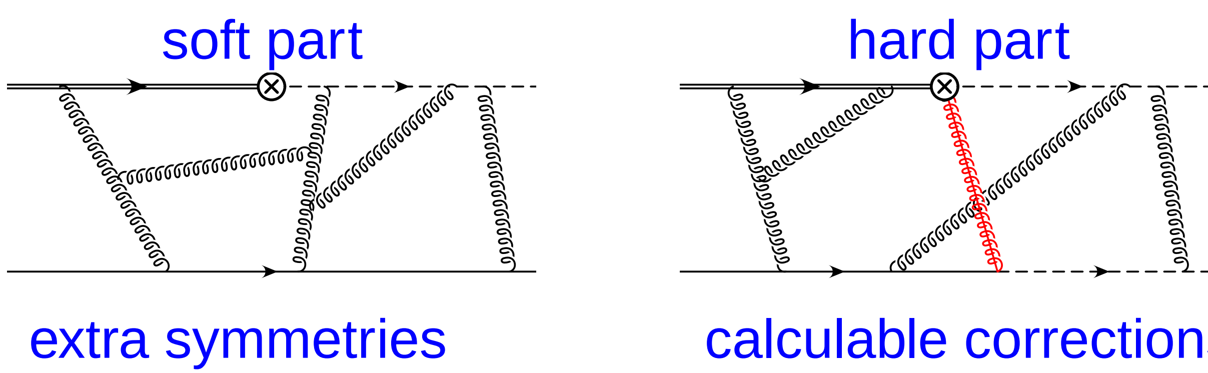
Large q^2 region is also what's most accessible to lattice QCD

Soft-collinear effective theory

- Recently proposed: for $q^2 \ll m_B^2$, 7 vector meson form factors (V, A, T currents) related to 2 functions; 3 pseudoscalar form factors related to just 1 (Charles *et al.*)

  SCET: a new effective field theory for energetic particles (simplify power counting, helps to make all-order proofs, etc.) (Bauer, Fleming, Luke, Pirjol, Stewart)

  Systematic framework to describe form factors when light hadron is very energetic



  Consistency of separation only proven to 1-loop yet (Beneke & Feldman)

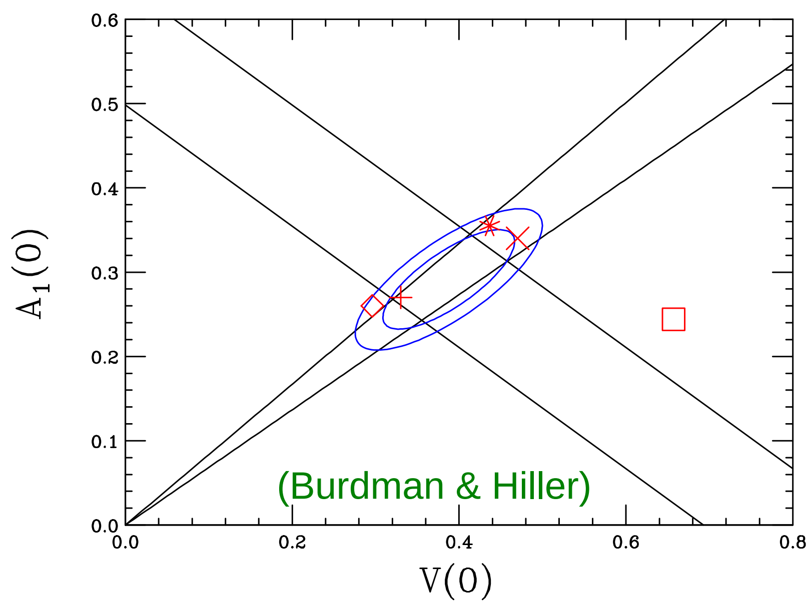
  (In $B \to D^{(*)} \ell \bar{\nu}$, nonperturbative part is in Isgur-Wise function to all orders)

  ... Expect progress!

Aside: an application

- The hope is to use some measurements in a theoretically controlled way to predict other decay rates; e.g., use $B \to K^* \gamma$ data to reduce uncertainty of $B \to K^* \ell^+ \ell^-$ and $B \to \rho \ell \bar{\nu}$ predictions, and also constrain models

Perturbative order α_s corrections have been computed (Beneke, Feldman, Seidel)



Crucial questions: all orders proof and understand power suppressed corrections

Exclusive decays — Summary

- Heavy quark symmetry provides many model independent predictions, similar to chiral symmetry

  Spectroscopy, strong and weak decays much better understood

- $B \to D^{(*)}\ell\bar{\nu}$: six semileptonic form factors depend on a single Isgur-Wise function in the $m_Q \to \infty$ limit; at zero recoil $\xi(1) = 1$, sometimes no $\Lambda_{\rm QCD}/m_Q$ corrections

  $|V_{cb}|$ known at $\sim 5\%$ level from exclusive decays (improvements will rely on lattice)

- Progress to understand exclusive heavy $\to$ light semileptonic and rare decays for small q^2; SCET might lead to rigorously proving

  Form factor relations between $B \to \pi\ell\bar{\nu}$, $B \to \rho\ell\bar{\nu}$, $B \to K^*\gamma$, $B \to K^{(*)}\ell^+\ell^-$

  – increase sensitivity to new physics

  – tests some assumptions for factorization in charmless decays (more tomorrow)

Inclusive decays

Operator product expansion

- Consider semileptonic $b \to c$ decay: $O_{bc} = -\frac{4G_F}{\sqrt{2}} V_{cb} \underbrace{(\bar{c}\,\gamma^\mu P_L\, b)}_{J^\mu_{bc}} \underbrace{(\bar{\ell}\,\gamma_\mu P_L\,\nu)}_{J_{\ell\mu}}$

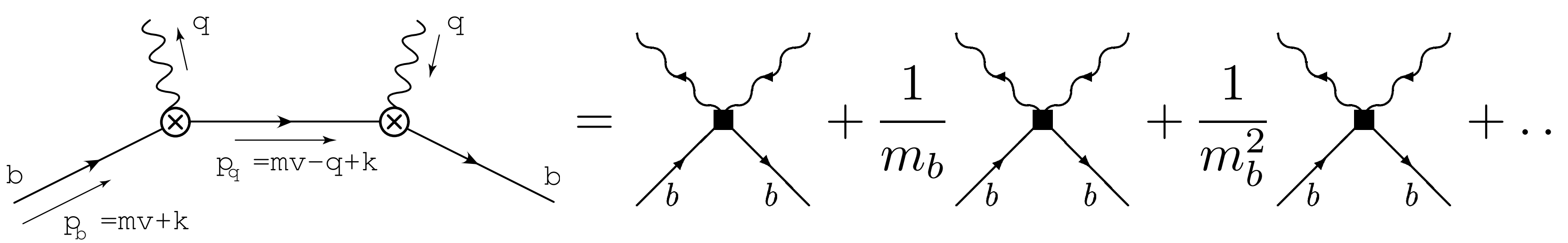
Decay rate: $\Gamma(B \to X_c \ell \bar{\nu}) \sim \sum_{X_c} \int \mathrm{d}[\mathrm{PS}]\, \left| \langle X_c \ell \bar{\nu} | O_{bc} | B \rangle \right|^2$

Factor to: $B \to X_c W^*$ and $W^* \to \ell\bar{\nu}$, concentrate on hadronic part

$$W^{\mu\nu} \sim \sum_{X_c} \delta^4(p_B - q - p_X) \left| \langle B | J^{\mu\dagger}_{bc} | X_c \rangle \, \langle X_c | J^\nu_{bc} | B \rangle \right|^2$$

(optical theorem) $\quad \sim \mathrm{Im} \int \mathrm{d}x\, e^{-iq\cdot x} \langle B | \, T\{ J^{\mu\dagger}_{bc}(x)\, J^\nu_{bc}(0) \} \, | B \rangle$

In $m_b \gg \Lambda_{\mathrm{QCD}}$ limit, time ordered product dominated by $x \ll \Lambda^{-1}_{\mathrm{QCD}}$



OPE (cont.)

- The $m_b \to \infty$ limit is given by free quark decay

  No $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$ corrections

  Order $\Lambda_{\rm QCD}^2/m_b^2$ corrections depend on two hadronic matrix elements

$$\lambda_1 = \frac{1}{2m_B}\,\langle B|\,\bar b\,(iD)^2\,b\,|B\rangle \qquad \lambda_2 = \frac{1}{6m_B}\,\langle B|\,\bar b\,\frac{g}{2}\,\sigma_{\mu\nu}\,G^{\mu\nu}\,b\,|B\rangle$$

not well-known $\qquad$ $\lambda_2 = (m_{B^*}^2 - m_B^2)/4$

- OPE predicts decay rates in an expansion in $\Lambda_{\rm QCD}/m_b$ and $\alpha_s(m_b)$

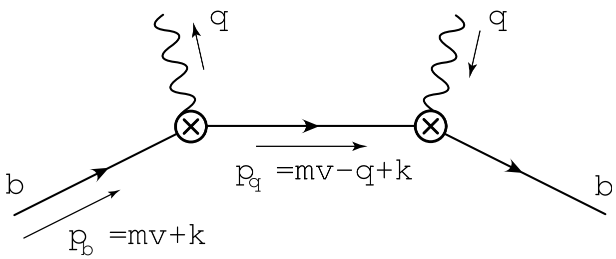
$$\mathrm{d}\Gamma = \begin{pmatrix} b \text{ quark} \\ \text{decay} \end{pmatrix} \times \left\{ 1 + \frac{0}{m_b} + \frac{f(\lambda_1, \lambda_2)}{m_b^2} + \ldots + \alpha_s(\ldots) + \alpha_s^2(\ldots) + \ldots \right\}$$

  Interesting quantities computed to order α_s, $\alpha_s^2\beta_0$, and $1/m^3$

  When can we trust the result?

Inclusive decay rates

- In which regions of phase space can we expect the OPE to converge?



Can think of the OPE as an expansion in $k \sim \Lambda_{\rm QCD}$

$$[(m_b v + k - q)^2 - m_q^2]^{-1}$$
$$= [(m_b v - q)^2 - m_q^2 + 2k\cdot(m_b v - q) + k^2]^{-1}$$

Need to allow: $m_X^2 - m_q^2 \gg E_X \Lambda_{\rm QCD} \gg \Lambda_{\rm QCD}^2$

Implicit assumption: "quark-hadron duality" valid once $m_X \gg m_q$ allowed

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- Good news: Total rates calculable at few $(\lesssim 5)$ percent level (duality...) $\Rightarrow$ $|V_{cb}|$

Need to know m_b (or $\bar\Lambda = m_B - m_b$) and λ_1

$$|V_{cb}| \sim \big[42 \pm (\text{error mostly in } m_b\ \&\lambda_1)\big] \times 10^{-3} \left(\frac{\mathcal{B}(B\to X_c\ell\bar\nu)}{0.105}\,\frac{1.6\,\text{ps}}{\tau_B}\right)^{1/2}$$

- Bad news: In certain restricted regions of phase space the OPE breaks down

To determine $|V_{ub}|$, cuts required to eliminate ~ 100 times larger $b\to c$ background

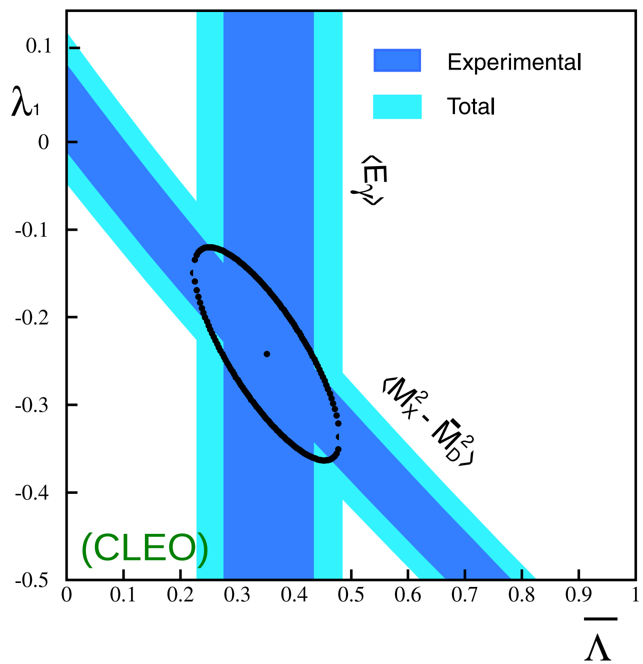
Determination of m_b & $\lambda_1 \Rightarrow |V_{cb}|$

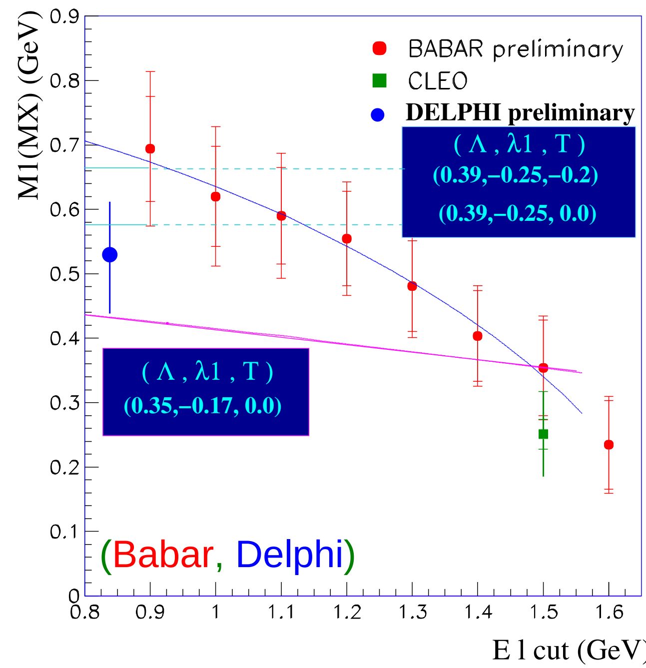
- Progress likely to come from determining m_b and λ_1 from "shape variables" in inclusive B decays $\sim \langle E_\gamma^n \rangle$ in $B \to X_s\gamma$, $\langle E_\ell^n \rangle$ and $\langle m_{X_c}^n \rangle$ in $B \to X_c \ell \bar{\nu}$

These have been computed to $\alpha_s^2\beta_0$ and $(\Lambda_{\rm QCD}/m_Q)^3$

CLEO:

$$\bar{\Lambda} = 0.35 \pm 0.13\,\text{GeV}$$
$$\lambda_1 = -0.24 \pm 0.11\,\text{GeV}^2$$
$$\Downarrow$$
$$|V_{cb}| \sim (40.4 \pm 1.6) \times 10^{-3}$$

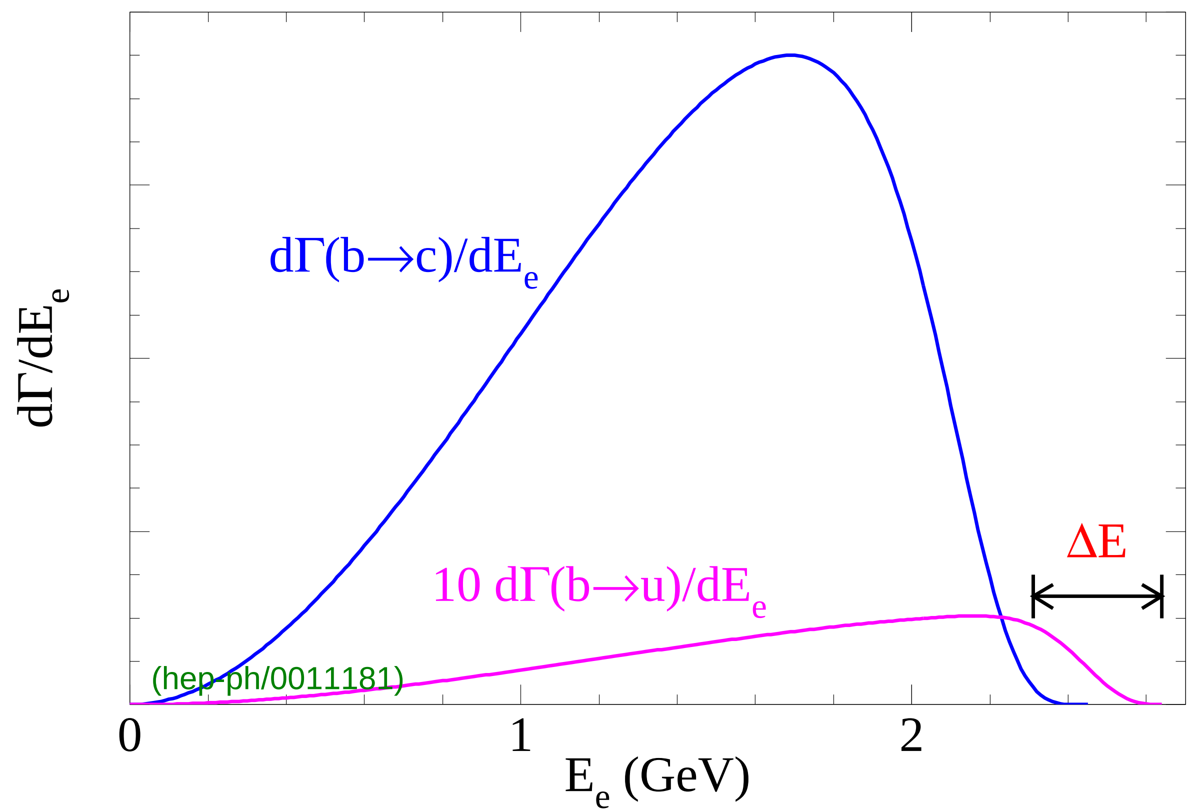




Level of (in)consistency will test accuracy of OPE and quark-hadron duality

$\Rightarrow$ May lead to $\sigma(V_{cb}) \sim 2-3\%$ if all works out

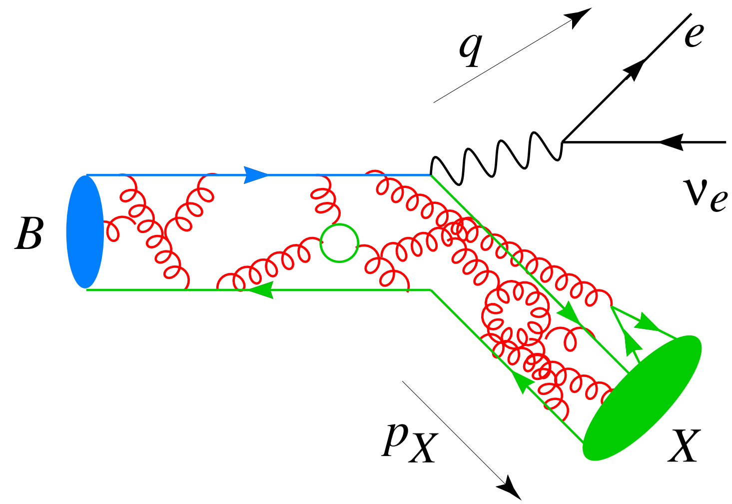
Inclusive $b \to u$: the problem



$$|V_{ub}| \sim \frac{1}{10} |V_{cb}| \quad \Rightarrow \text{cuts}$$

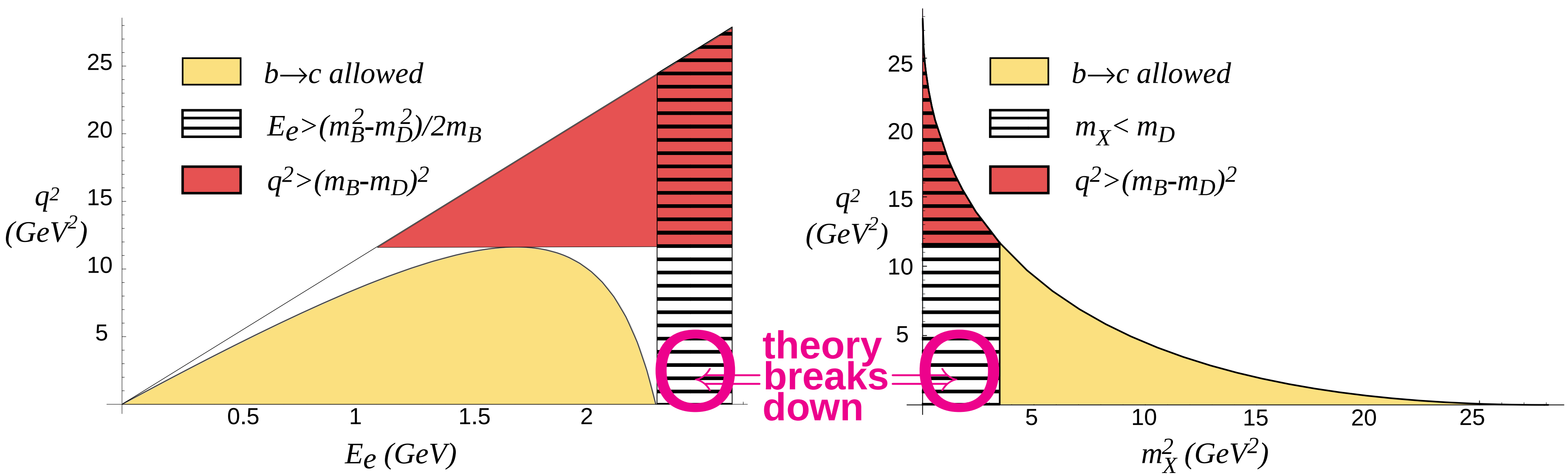
... and the troubles begin

Inclusive $B \to X_u \ell \bar{\nu}$ decay and $|V_{ub}|$



Proposals to measure $|V_{ub}|$:

- Lepton spectrum: $E_\ell > (m_B^2 - m_D^2)/2m_B$
- Hadronic mass spectrum: $m_X < m_D$
- Dilepton mass spectrum: $q^2 > (m_B - m_D)^2$



$B \to X_u \ell \bar{\nu}$ spectra

- Three qualitatively different regions of phase space:

1) $m_X^2 \gg E_X \Lambda_{\rm QCD} \gg \Lambda_{\rm QCD}^2$: the OPE converges, first few terms can be trusted
2) $m_X^2 \sim E_X \Lambda_{\rm QCD} \gg \Lambda_{\rm QCD}^2$: infinite set of terms in the OPE equally important
3) $m_X \sim \Lambda_{\rm QCD}$: resonance region — cannot compute reliably

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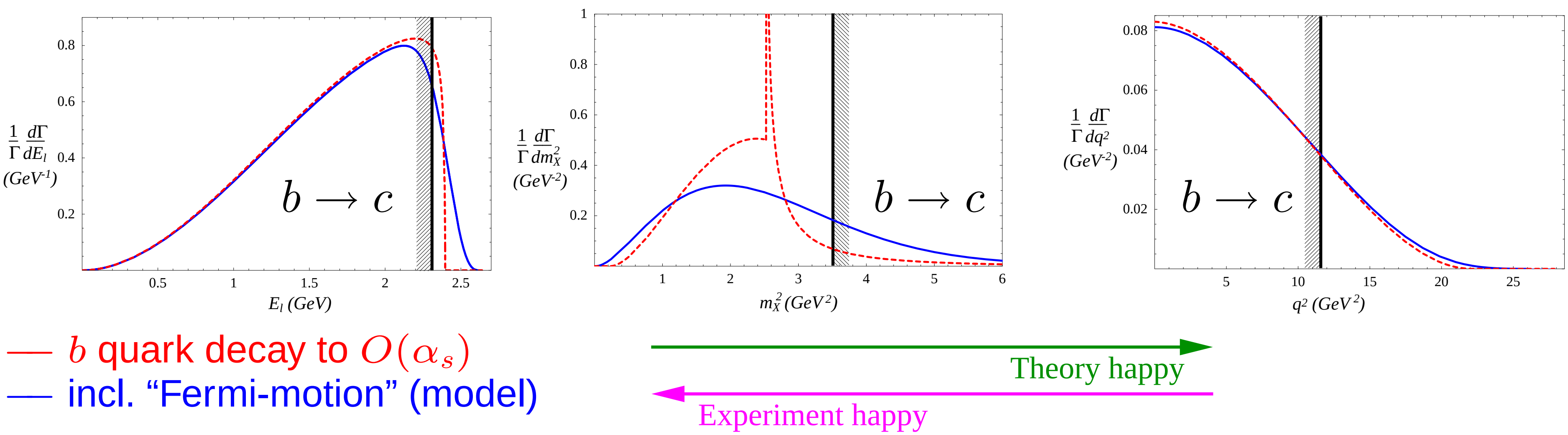
- Problem: $E_\ell > (m_B^2 - m_D^2)/2m_B$ and $m_X < m_D$ are in (2) since $m_B \Lambda_{\rm QCD} \sim m_D^2$



— b quark decay to $O(\alpha_s)$
— incl. "Fermi-motion" (model)

V_{ub}: lepton endpoint region

- Bad: an infinite set of terms in the OPE are equally important

  Good: it is related to $B \to X_s\gamma$ photon spectrum (Neubert; Bigi, Shifman, Uraltsev, Vainshtein)

  Recently: Perturbative corrections worked out to higher order (Leibovich, Low, Rothstein)

  Terms in the OPE not related to $B \to X_s\gamma$ are also significant (Leibovich, ZL, Wise; Bauer, Luke, Mannel)

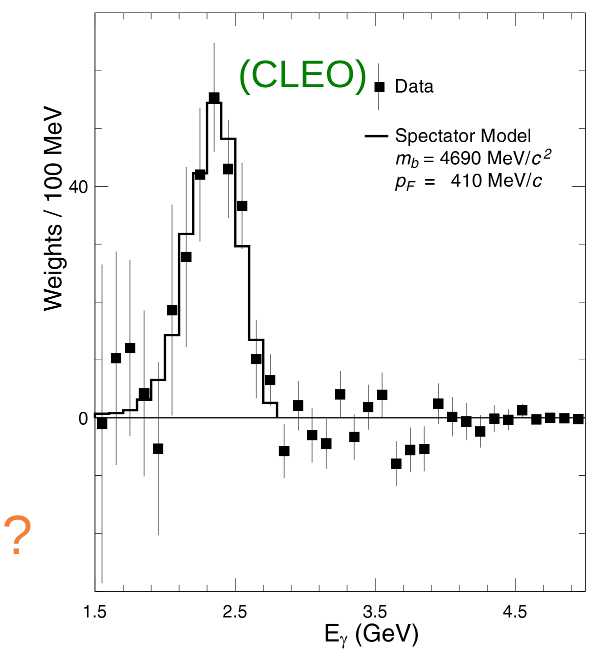
CLEO used the $B \to X_s\gamma$ photon spectrum as an input to determine $|V_{ub}|$

... measures the “Fermi-motion” of the b quark

$$|V_{ub}| = (4.08 \pm 0.63) \times 10^{-3}$$

Limiting uncertainties: subleading corrections

quark-hadron duality applicable?



V_{ub}: q^2 spectrum

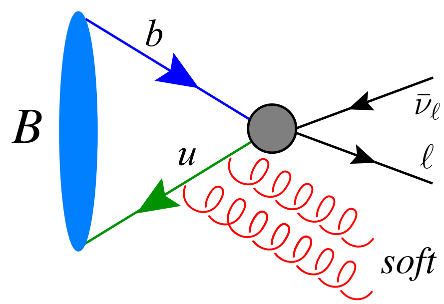
- In large q^2 region, first few terms in OPE can be trusted (Bauer, ZL, Luke)

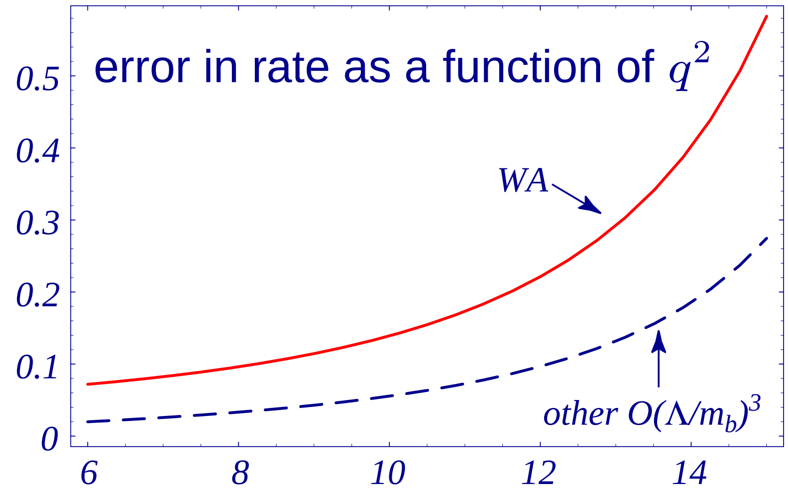
Reason: $q^2 > (m_B - m_D)^2$ cut implies $E_X < m_D$, therefore $m_X^2 \gg E_X \Lambda_{\rm QCD}$

Some nonperturbative corrections are $(\Lambda_{\rm QCD}/m_c)^3$, and not $(\Lambda_{\rm QCD}/m_b)^3$ (Neubert)

Possibly sizable corrections at $\mathcal{O}(\Lambda_{\rm QCD}/m_b)^3$ from weak annihilation (Voloshin)

Guesstimate: $\sim$ 2–3% of $b \to u$ semileptonic rate; delta-function at maximal q^2 and maximal E_ℓ





Comparing D^0 vs. D_s SL widths, or V_{ub} from $B^\pm$ vs. B^0 decay can constrain WA

V_{ub}: combine q^2 & m_X cuts

- Can get $|V_{ub}|$ with theoretical uncertainty at the 5–10% level, from up to $\sim 45\%$ of the events (Bauer, ZL, Luke)

Such precision can be achieved even with cuts away from the $b \to c$ threshold

Cuts on $(q^2,\, m_X)$	included fraction of $b \to u\ell\bar{\nu}$ rate	error of $\lvert V_{ub}\rvert$ $\delta m_b = 80/30\,\text{MeV}$
$6\,\text{GeV}^2,\ m_D$	46%	$8\%/5\%$
$8\,\text{GeV}^2,\ 1.7\,\text{GeV}$	33%	$9\%/6\%$
$(m_B - m_D)^2, m_D$	17%	$15\%/12\%$

Strategy: (i) reconstruct q^2 and m_X; make cut on m_X as large as possible

(ii) for a given m_X cut, reduce q^2 cut to minimize overall uncertainty

... Would significantly reduce the uncertainty of a side of the unitarity triangle

Semileptonic & rare decays — Summary

- $|V_{cb}|$ is known at the $\sim 5\%$ level; error may become half of this in the next few years using both inclusive and exclusive determinations (latter will rely on lattice)
- Situation for $|V_{ub}|$ may become similar to present $|V_{cb}|$; for precise inclusive determination the neutrino reconstruction seems crucial; the exclusive will use lattice
- For both $|V_{cb}|$ and $|V_{ub}|$ it is important to pursue both inclusive and exclusive
- Progress in understanding exclusive rare decays for $q^2 \ll m_B^2$ (expect more!) $B \to K^{(*)}\gamma$ and $B \to K^{(*)}\ell^+\ell^-$ below the $\psi \Rightarrow$ increase sensitivity to new physics

  Related to some issues in factorization in charmless decays (tomorrow)

Additional Topics

- B decays to excited D mesons
- Exclusive rare decays
- Inclusive rare decays

Decays to excited states: $B \to D^{**}\ell\bar\nu$

- HQS $\Rightarrow$ matrix elements of weak currents vanish at zero recoil for excited states
  Become non-zero at $\mathcal{O}(\Lambda_{\rm QCD}/m_Q)$ — most of the phase space is near zero recoil

$m_Q \to \infty$: for each doublet, all form factors are related to an Isgur-Wise function

$\mathcal{O}(\Lambda_{\rm QCD}/m_Q)$: in $B \to (D_1, D_2^*)\ell\bar\nu$, 8 subleading I-W fn's, but only 2 independent

$$\frac{{\rm d}\Gamma(B\to D_1\ell\bar\nu)}{{\rm d}w} \propto \sqrt{w^2-1}\,[\tau(1)]^2 \bigg\{ 0 + 0\,(w-1) + (\ldots)(w-1)^2 + \ldots$$

$$+\frac{\Lambda_{\rm QCD}}{m_Q}\big[\,0 + ({\rm almost\ calculable})(w-1) + \ldots\big]$$

$$w \equiv \frac{m_B^2+m_{D_1}^2-q^2}{2m_B\,m_{D_1}} \in (1, 1.3)$$

$$+\frac{\Lambda_{\rm QCD}^2}{m_Q^2}\big[\,({\rm calculable!}) + \ldots\big] + \ldots \bigg\}$$

In $B \to$ (orbitally excited D) decays, the zero recoil matrix element at $\mathcal{O}(1/m_Q)$ is given by mass splittings and the $m_Q \to \infty$ Isgur-Wise fn. (Leibovich, ZL, Stewart, Wise)

More $B \to D^{**}\ell\bar\nu$

- Bjorken sum rule for the slope of Isgur-Wise function ($\exists$ many more sum rules):

$$\rho^2 = \frac{1}{4} + \sum_m \frac{|\zeta^{(m)}(1)|^2}{4} + 2\sum_p \frac{|\tau^{(p)}(1)|^2}{3} + \text{nonresonant}$$

$\zeta^{(m)}$ and $\tau^{(p)}$ are Isgur-Wise fn's for the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ states

$B \to D_1\ell\bar\nu$ rate is enhanced at order $\Lambda_{\rm QCD}/m_Q$ by much more than $D_2^*\ell\bar\nu$

The present world average is about 0.4 ± 0.15

Approximation	$\Gamma_{D_2^*}/\Gamma_{D_1}$
$m_Q \to \infty$	1.65
Finite m_Q $\{$ B_1	0.52
Finite m_Q $\{$ B_2	0.67

- To compare $B \to (D_1, D_2^*)$ with (D_0^*, D_1^*), need to know the Isgur-Wise functions

Quark models (ISGW, etc.) and QCD sum rules predict that the Isgur-Wise function for the broad doublet is not larger than for the narrow doublet

If you buy these arguments, then the large $B \to (D_0^*, D_1^*)\ell\bar\nu$ rate is a puzzle

$B \to D^{**}\pi$ decays

- Factorization is expected to work as well as in $B \to D^{(*)}\pi$

$$\Gamma_\pi = \frac{3\pi^2\,|V_{ud}|^2\,C^2\,f_\pi^2}{m_B^2\,r} \times \left(\frac{\mathrm{d}\Gamma_{\rm sl}}{\mathrm{d}w}\right)_{w_{\max}}$$

$$r = m_{D^{**}}/m_B\,, \quad w_{\max} = (1+r^2)/(2r) \simeq 1.3\,, \quad f_\pi \simeq 132\,\text{MeV}\,, \quad C\,|V_{ud}| \simeq 1$$

- An interesting ratio from which Isgur-Wise function cancels out:

$$\frac{\mathcal{B}(B^- \to D_2^{*0}\pi^-)}{\mathcal{B}(B^- \to D_1^0\pi^-)} = 0.89 \pm 0.14 \qquad \text{(BELLE @ ICHEP)}$$

This looks OK and can teach us about $1/m$ corrections (in '97 ratio was 1.8 ± 0.9, theory could not accommodate such a large central value) (Leibovich, ZL, Stewart, Wise)

Sorting out these semileptonic and nonleptonic decays to excited D's will be important for HQET, factorization, and will impact $|V_{cb}|$ determinations

Exclusive rare decays

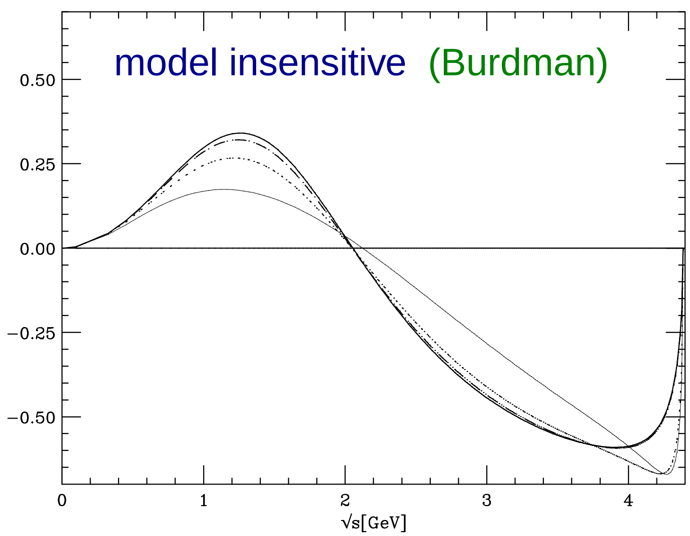
- Important probes of NP — measurements of $|V_{ij}|$

  Exclusive decays are experimentally easier — need to understand form factors

  – $B \to K^*\gamma$ or $B \to X_s\gamma$: best $m_{H^\pm}$ limits in 2HDM — in SUSY many param's

  – $B \to K^{(*)}\ell^+\ell^-$ or $B \to X\,\ell^+\ell^-$: bsZ penguins, SUSY, right handed couplings

- There is an observable insensitive to the precise values of the form factors:



Forward-backward asymmetry in $B \to K^*\ell^+\ell^-$ changes sign:

$$C_9^{\rm eff}(s_0) = -\frac{2m_B m_b}{s_0}\, C_7^{\rm eff} \times [\,1 + O(\alpha_s, \Lambda_{\rm QCD}/m_b)]$$

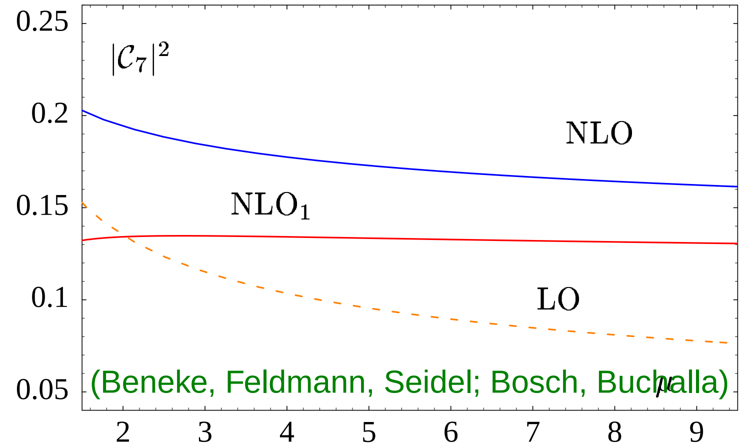
$\mathcal{O}(\alpha_s)$ corrections computed (Beneke, Feldman, Seidel)

May give clean measurement of C_9 (sensitive to NP)

$B \to K^*\gamma$ briefly

Large ($\sim 80\%$) enhancement of $B \to K^*\gamma$ decay rate found at NLO

$\Rightarrow 1/m$ correction large or/and form factors significantly different from model predictions



Form factors also enter predictions for isospin splitting — power suppressed correction, but claimed to be calculable

$$\Delta_{0-} = \frac{\Gamma(\overline{B}^0 \to \overline{K}^{*0}\gamma) - \Gamma(B^- \to K^{*-}\gamma)}{\Gamma(\overline{B}^0 \to \overline{K}^{*0}\gamma) + \Gamma(B^- \to K^{*-}\gamma)} = 0.02 \pm 0.07 \quad \text{(data)}$$

$$\Delta_{0-} = (0.08^{+2.1}_{-3.2})\% \times \frac{0.3}{T_1^{B\to K^*}} \quad \text{(Kagan \& Neubert)}$$

Testing these predictions may be important for understanding various approaches to factorization in charmless decays

Inclusive rare B decays

- Important probes of new physics — measurements of CKM elements
  - $B \to K^* \gamma$ or $X_s \gamma$: Best $m_{H^\pm}$ limits in 2HDM — in SUSY many param's
  - $B \to K^{(*)} \ell^+ \ell^-$ or $X_s \ell^+ \ell^-$: bsZ penguins, SUSY, right handed couplings

A crude guide... ($\ell = e$ or μ)

Decay	$\sim$SM rate	physics examples
$B \to s\gamma$	3×10^{-4}	$\lvert V_{ts}\rvert$, $H^\pm$, SUSY
$B \to s\nu\nu$	4×10^{-5}	new physics
$B \to \tau\nu$	4×10^{-5}	$f_B \lvert V_{ub}\rvert$, $H^\pm$
$B \to s\ell^+\ell^-$	7×10^{-6}	new physics
$B_s \to \tau^+\tau^-$	1×10^{-6}	
$B \to s\tau^+\tau^-$	5×10^{-7}	⋮
$B \to \mu\nu$	3×10^{-7}	
$B_s \to \mu^+\mu^-$	4×10^{-9}	
$B \to \mu^+\mu^-$	1×10^{-10}	

Replacing $b \to s$ by $b \to d$ costs factor ~ 20 (in SM)

In $B \to q\, l_1\, l_2$ decays expect ~ 10–20% K^*/ρ, and ~ 5–10% K/π (model dependent)

So far the $b \to s\ell^+\ell^-$ data agrees with the SM expectation within the still sizable errors

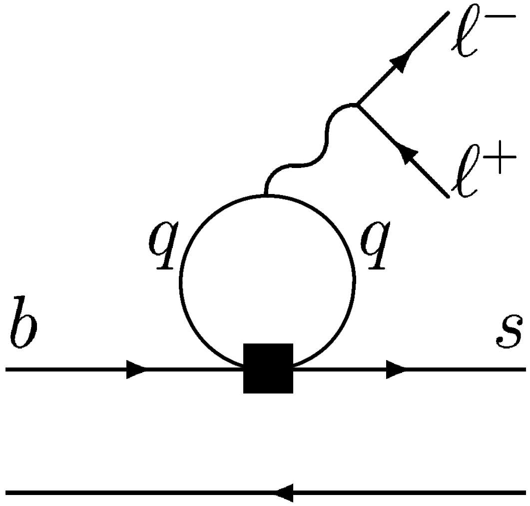
Something to worry about?

$$\mathcal{B}(B \to \psi X_s) \sim 4 \times 10^{-3}$$
$$\downarrow$$
$$\mathcal{B}(\psi \to \ell^+\ell^-) \sim 6 \times 10^{-2}$$

So this "long distance" contribution is:

$$\mathcal{B}(B \to X_s \ell^+\ell^-) \sim 2 \times 10^{-4}$$

This is ~ 30 times the short distance contribution!



Averaged over a large region of invariant masses (and $0 < q^2 < m_B^2$ should be large enough), the $c\bar{c}$ loop expected to be dual to $\psi + \psi' + \ldots$ This is what happens in $e^+e^- \to$ hadrons, in τ decay, etc., but NOT here

Is it consistent to "cut out" the ψ and ψ' regions and then compare data with the short distance calculation? (Maybe..., but understanding is unsatisfactory)