Theory of muon $g-2$

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Outline

1. Introduction
   - What is $\mathbf{g_\mu - 2}$ and why it is interesting
   - Current status
2. Standard Model prediction for $\mathbf{g_\mu - 2}$
   - QED and electroweak contributions
   - Hadronic vacuum polarization
   - Hadronic light-by-light scattering
3. Conclusions and prospects

1. Introduction

- Definitions:

$$\mu = g\frac{e}{2m}s, \qquad a_\mu = \frac{g_\mu - 2}{2}.$$

  $g = 2$ from the Dirac equation.

- $g_e \neq 2$ was first discovered in atomic experiments and then derived from QED.

  Foley, Kush, Schwinger

- What makes the muon so heavy?

  Precision of 1961 CERN experiment: 2% ;
  Precision of 2001 BNL experiment 1 ppm .

- Muons are good for the precise a_μ measurement because:
  - pions decay to polarized muons;
  - electron from $\mu \to e\nu_e\nu_\mu$ follows the muon's spin.

- $\mathbf{g_\mu - 2}$ is a good observable in terms of precision/New Physics discovery potential:

$$\boldsymbol{\Delta a_\mu^{\mathrm{NewPhysics}} \approx \left(\frac{\alpha}{\pi}\right) \frac{m_\mu^2}{\Lambda_{\mathrm{NP}}^2}}.$$

  Many New Physics models "predict"

$$\boldsymbol{\Delta a_\mu \sim \mathrm{few} \cdot 10^{-9}} :$$

  - muon substructure;
  - anomalous W boson magnetic moment;
  - supersymmetry;
  - two Higgs doublet models;
  - extra dimensions;
  - lepton mixing.

  There is a chance that manifestation of the New Physics can be seen with E821 ultimate precision of about $\mathbf{(40-60) \cdot 10^{-11}}$!

- Muon's only competitor is the electron.

$$\boldsymbol{\Delta a_\mu^{\mathrm{NewPhysics}} \sim \left(\frac{m_\mu}{m_e}\right)^2 \Delta a_e^{\mathrm{NewPhysics}}}.$$

$$\boldsymbol{\delta a_\mu \sim 100 \cdot 10^{-11}, \quad \delta a_e \sim 1 \cdot 10^{-11}}.$$

- Summary of the results:

  The new result for a_μ (E821, 2002) :

$$a_\mu^{\mathrm{exp}} = 116\ 592\ 040(80) \cdot 10^{-11}.$$

  The year 2001 result for a_μ :

$$a_\mu^{\mathrm{exp}} = 116\ 592\ 020(150) \cdot 10^{-11}.$$

  The updated SM prediction:

$$a_\mu^{\mathrm{th}} = 116\ 591\ 672(113) \cdot 10^{-11},$$

$$\left[a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{th}}\right] \cdot 10^{11} = 368 \pm 80|_{\mathrm{exp}} \pm 113|_{\mathrm{th}};$$

- The $g_\mu - 2$ theory will be reviewed.

  Diverse physics:

  - precision QED and electroweak physics;
  - fine details of QCD at low energies;
  - τ physics.

  Focus on the hadronic light-by-light scattering.

2. SM prediction for a_μ

$$a_\mu^{\text{th}} = a_\mu^{\text{QED}} + a_\mu^{\text{weak}} + a_\mu^{\text{hadr}}.$$

- QED:

$$a_\mu^{\text{QED}} = 116\ 584\ 721(3) \cdot 10^{-11}.$$

  Major changes unlikely.

- The weak corrections:

$$a_\mu^{\text{weak}} \approx 150 \cdot 10^{-11}.$$

  Small contribution; changes unlikely.

- The hadronic contribution:

$$a_\mu^{\text{hadr}} = \left\{ \begin{array}{c} 7032(100) \\ 6774(100) \end{array} \right. \cdot 10^{-11}.$$

  Large contribution, extraordinary precision.

  Has changed by $(200^{+120}_{-185}) \cdot 10^{-11}$recently.

- Details of the QED contribution

Kinoshita, Nizic, Okamoto

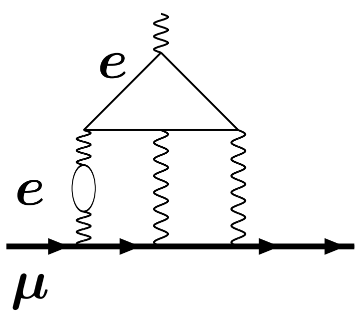
$\mathcal{O}(\alpha^4)$ computed (single calculation);

$\mathcal{O}(\alpha^5)$ estimated (not yet relevant).

$$a_\mu^{\mathrm{QED}} = 116\ 584\ 721(3) \cdot 10^{-11},$$

$\mathcal{O}(\alpha^4) \sim 360 \cdot 10^{-11}$, 469 diagrams.

The largest contribution comes from:



$$= 337 \cdot 10^{-11}$$

Kinoshita
Chlouber, Samuel

The remainder fluctuates (numerics):

Kinoshita

$$70(17) \cdot 10^{-11} \to 31(1) \cdot 10^{-11} \to 46 \cdot 10^{-11}$$

It is a challenge for QED theorists to verify this result by an independent calculation.

- No sensitivity to last digits in α_{QED} .

- Electroweak contribution:

$$a_\mu^{\mathrm{EW}} = \frac{5}{24}\frac{G_\mu m_\mu^2}{\sqrt{2}\pi^2}\left[1 + \frac{1}{5}\left(1 - 4\sin^2\theta_W\right)\right]$$

The two-loop calculation yields:

$$a_\mu^{\mathrm{EW}} \cdot 10^{11} = (195 - 43(4)) = 152(4).$$

Czarnecki, Krause, Marciano
Kuhto, Kuraev, Silagadze, Schiller

The second order correction is large since

$$L_f = \ln\frac{M_z}{m_f} \approx 7 \gg 1\ .$$

$$\frac{\delta a_\mu^{\mathrm{EW}}}{a_\mu^{\mathrm{EW}}} = \frac{\alpha}{\pi}\left[-\frac{43}{3}L_\mu + \frac{36}{5}\sum_f N_f Q_f^2 T_f L_f\right],$$

The L_f-enhanced terms can be computed using RG techniques (similar to $b \to s\gamma$):

Degrassi, Giudice

$$\mathcal{L}_{\mathrm{eff}} = \sum C_f(\mu)\mathcal{O}_f(\mu),$$

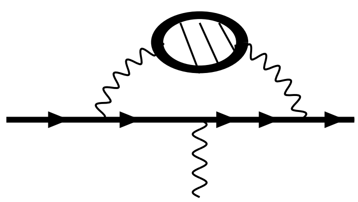
$$\mathcal{O}_1 \approx \bar{\mu}\sigma_{\alpha\beta}\mu F^{\alpha\beta}$$

$$\mathcal{O}_i \approx \bar{\mu}\gamma_\mu(\gamma_5)\mu\ \bar{f}\gamma_\mu(\gamma_5)f.$$

Hadronic contributions:

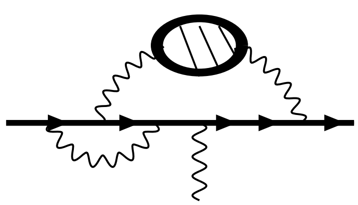
$$a_\mu^{\text{had}} = \left\{ \begin{array}{l} 7032(100) \\ 6774(100) \end{array} \right. \cdot 10^{-11}.$$

Firmly establishing a_μ^{had} with the 1% precision is the key to the successful $g_\mu - 2$ physics program.



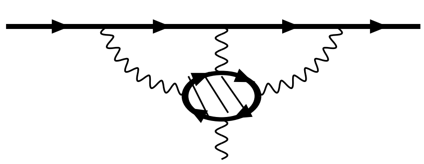
$= 6924(62) \cdot 10^{-11}$

Davier, Hocker(1998)



$= -100(6) \cdot 10^{-11}$

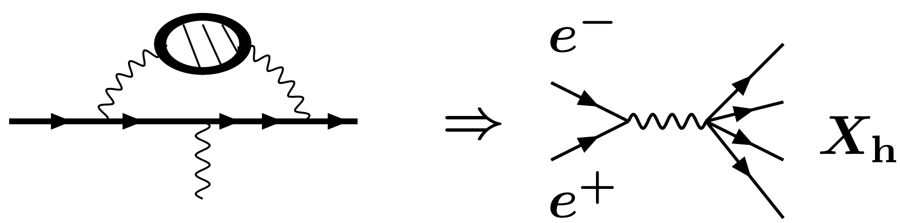
Krause



$= 85(30) \cdot 10^{-11}$

Knecht, Nyffeler
Kinoshita, Hayakawa
Bijnens,Prades,Pallante
Blokland, Czarnecki, K.M.

LO hadronic vacuum polarization



$$a_\mu^\mathrm{vp} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} \mathrm{d}s \; K(s) \; \sigma_\mathrm{h}(s),$$

$$K(s) \sim \frac{m_\mu^2}{s} \text{ for } s \gg m_\mu^2.$$

- a_μ^vp receives the major contribution from the lightest hadronic states:

  a) $\mathbf{72\%}$ from $\pi^+\pi^-$;

  b) $\mathbf{92\%}$ from $\sqrt{s} < 2$ GeV.

- Recent evaluations of a_μ^{vp}:

author	$a_\mu^{\mathrm{vp}} \cdot 10^{11}$	year	method
Davier et al.	**6924(62)**	**1998**	τ
Davier et al.	**7047(69)**	**2002**	τ
Jegerlehner	**6974(105)**	**2001**	e^+e^-
Hagiwara et al.	**6865(60)**	**2002**	e^+e^-
Davier et al.	**6789(70)**	**2002**	e^+e^-

- Both the τ and the e^+e^-data-based results have shifted significantly from their year 2001 values!

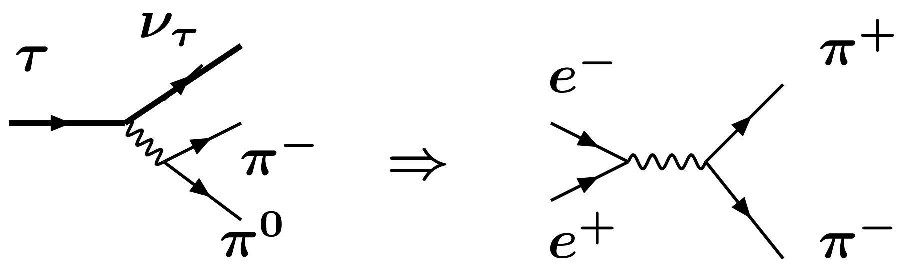
- Changes come from:
  - new e^+e^-data (VEPP2-M, BEPC)
  - the re-analysis of the ALEPH data on τ

- New values unambiguously establish large differences between $\tau \to \nu_\tau \pi \pi_0$ and $e^+e^- \to \pi^+\pi^-$.

How does the τ data fit in?

- The isospin symmetry: **up** $\Leftrightarrow$ **down**

$$\Gamma(\tau \to \nu_\tau \pi^- \pi^0) \approx \sigma(e^+e^- \to \pi^+\pi^-).$$



- Problem: isospin violations:
  - $m_u \neq m_d$
  - QED corrections
- The isospin violating effects in a_μ^{vp} can be

$$1\% \Rightarrow \pm 50 \cdot 10^{-11}.$$

- With the current precision, such corrections can not be neglected.

- Attempts to compute the isospin breaking corrections (empirical approach, χPT).

Davier and Hocker
Ecker, Cirigliano, Neufeld

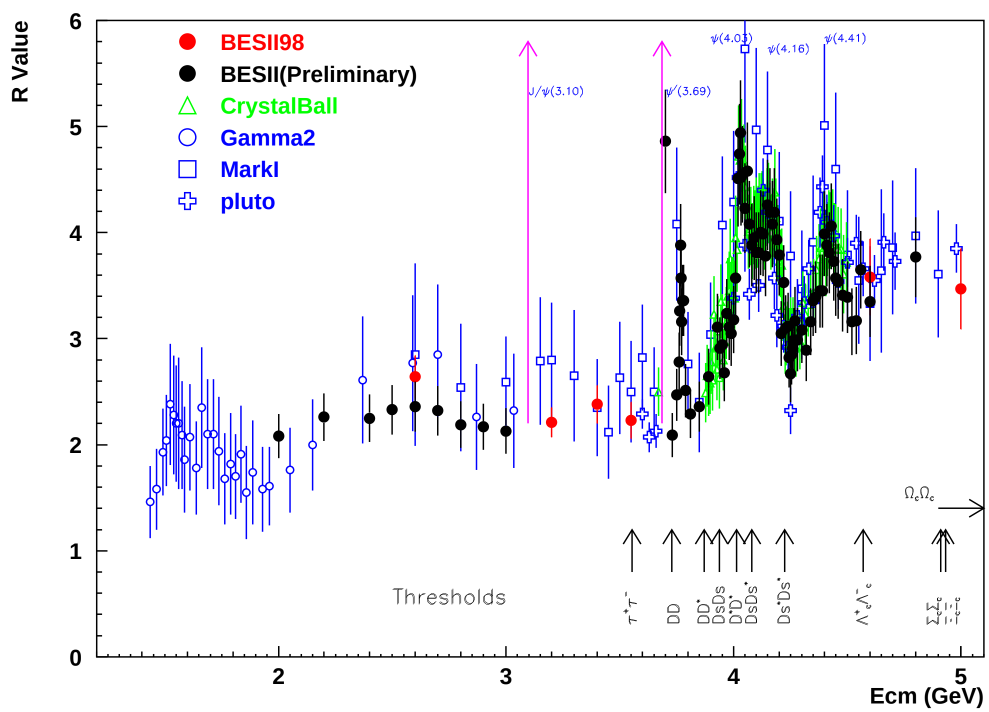
- What has been considered:
  - The QED Wilson coefficient for the four quark operator S_{ew}: $-97 \cdot 10^{-11}$
  - $m_\pi^\pm \neq m_\pi^0$: $-75 \cdot 10^{-11}$
  - the $\rho - \omega$ mixing: $40 \cdot 10^{-11}$
  - $\Gamma_\rho^0 \neq \Gamma_\rho^\pm$: $20 \cdot 10^{-11}$
  - Photon bremsstrahlung in $\tau \to \nu_\tau \pi^- \pi^0 +$ virtual QED corrections in χPT: $(-10 \div 16) \cdot 10^{-11}$
- An apparent problem:

Br	exp	CVC
$\tau \to 2\pi + \nu_\tau$	25.46 ± 0.12	$23.97 \pm 0.24 \pm 0.21$
$\tau \to 4\pi + \nu_\tau$	4.54 ± 0.13	$3.68 \pm 0.19 \pm 0.09$

Davier, Eidelman and Hocker

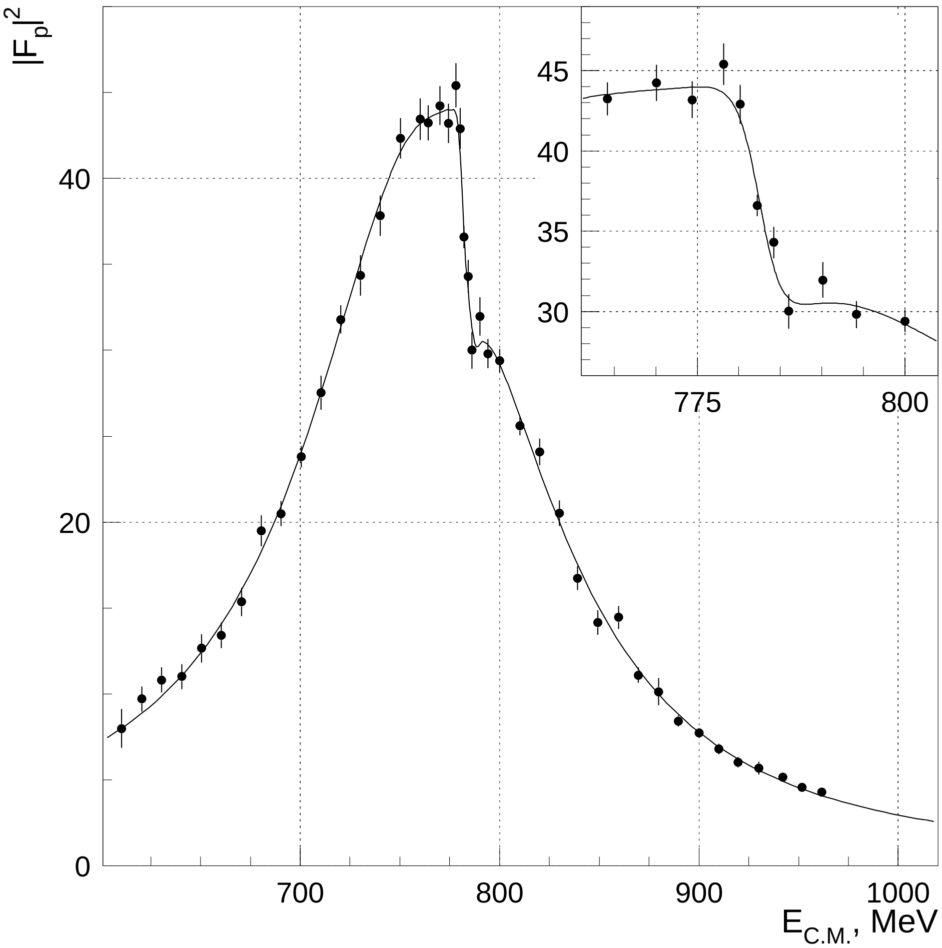
Progress in low-energy e^+e^- annihilation experiments makes the use of the τ data unnecessary (but still useful for cross-checks at the few per cent level).

- BEPC data: $\mathbf{2\ GeV} \leq \sqrt{\mathbf{s}} \leq \mathbf{3\ GeV}$.



- Data somewhat (but not too significantly) higher than pQCD.

- VEPP-2M data: $\boldsymbol{\rho}$ meson region.



$$a_\mu^{\mathbf{vp}}|_\rho = (\mathbf{3748 \pm 41 \pm 85}) \cdot \mathbf{10^{-11}} \quad [\mathbf{old}],$$
$$a_\mu^{\mathbf{vp}}|_\rho = (\mathbf{3681 \pm 26 \pm 22}) \cdot \mathbf{10^{-11}} \quad [\mathbf{new}].$$

The precision $\mathbf{0.6\%}$ is outstanding.

Systematic error is dominated by the radiative corrections.

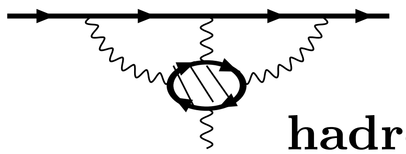
- Important issue: how to "prove" that this measurement is correct?

- To make a convincing case for the $\mathbf{1}$ per cent measurement of $\boldsymbol{\sigma}(\boldsymbol{e}^+\boldsymbol{e}^- \longrightarrow \boldsymbol{\pi}^+\boldsymbol{\pi}^-)$ it is important to:
  - Separate the notion of $\boldsymbol{F_\pi}(\boldsymbol{s})$ and $\gamma^* \longrightarrow \boldsymbol{\pi}^+\boldsymbol{\pi}^-$.
  - Remove all the QED corrections, specific to the ISR and the vacuum polarization from $\boldsymbol{\sigma}(\boldsymbol{e}^+\boldsymbol{e}^- \longrightarrow \boldsymbol{\pi}^+\boldsymbol{\pi}^-)$.
  - Measure separately, or include in some approximation, the part of $\gamma^* \longrightarrow \boldsymbol{\pi}^+\boldsymbol{\pi}^-\gamma$ that is affected by cuts.
  - Measure forward-backward asymmetry in $\boldsymbol{\pi}^+\boldsymbol{\pi}^-\gamma$ to test the point-like pion approximation (Novosibirsk, 1991; DAPHNE).

To reiterate:

- the 1% uncertainty on $a_\mu^{\rm vp}$ is crucial for the required precision on $g_\mu - 2$;
- The use of the τ decay data can bring in uncontrollable effects due to the isospin violation;
- the e^+e^- data has become a viable alternative because of Novosibirsk and Beijing results.
- $a_\mu^{\rm vp}$ from τ decays and $a_\mu^{\rm vp}$ from e^+e^- are not consistent (3σ or 4%).
- Compared to the year 2000 values,
  - the τ result has shifted up, by 1.98σ (re-analysis);
  - the e^+e^- result has shifted down, by 1.8σ (new data).
- The 4% difference due to an unaccounted isospin is hard to believe in – very likely either the e^+e^-or the τ data is wrong.

Hadronic light-by-light



The current world average estimate:

$$a_\mu^{\rm lbl} = 85(30) \cdot 10^{-11}.$$

Knecht, Nyffeler
Kinoshita, Hayakawa
Bijnens,Prades,Pallante
Blokland, Czarnecki, K.M.

- pQCD is not applicable;
- No information on $\gamma^*\gamma^* \to \gamma\gamma^*$;
- No simple dispersion representation for $\gamma^*\gamma^* \to \gamma\gamma^*$;
- Models are used; the precision is uncertain.

Interesting history; recent change of the sign:

The sign error was of arithmetic origin.

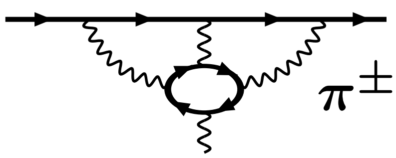
The sign issue is not to be confused with the question if the physics beyond that results is sound!

- What are the degrees of freedom to be used in the calculation?
  - Quark and gluons: this does not quite work since $m_\mu \leq \Lambda_{\mathbf{QCD}}$.
  - Hadrons: this is hopeless, if no small parameter can be found.
- Since $m_\mu \ll m_\rho$, can it play a role of a small parameter?
- If momentum scales are small, we expect that:
  - heavy hadrons are not important;
  - the interactions between pions are small.
- This makes the problem manageable.

$$\mathcal{L}_{\mathrm{eff}} = |D_\mu \pi|^2 - m_\pi^2 \pi^2 + \mathcal{O}\left(\frac{m_\pi}{4\pi f_\pi}\right)$$

$$D_\mu = \partial_\mu + ieA_\mu$$

- Neglecting the power-suppressed terms, perform the three loop calculation in scalar QED:



plus 7 other diagrams

- For $m_\mu = m_\pi$, we obtain

$$\delta a_\mu^{\pi-\mathrm{box}} = \left(\frac{\alpha}{\pi}\right)^3 \left[-\frac{11}{72} - \frac{16}{3} a_4 - \frac{\zeta_3}{6} + \frac{11\pi^2}{36}\zeta_3 - \frac{5\zeta_5}{4} + \frac{31\pi^4}{540} + \frac{2\pi^2}{9}\ln^2 2 - \frac{1925\pi^2}{216} + 12\pi^2 \ln 2 - \frac{2}{9}\ln^4 2 \right].$$

- For $m_\mu \neq m_\pi$, we derive:

$$\delta a_\mu^{\pi-\mathrm{box}} = -0.035 \left(\frac{\alpha}{\pi}\right)^3 \approx -43.5 \cdot 10^{-11}.$$

- How natural is this value?

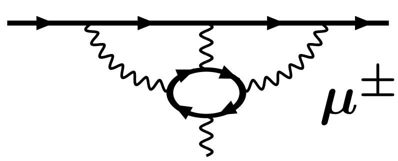
- What is "natural"? The intermediate state of the mass M should contribute:

$$\delta a_\mu \approx \frac{m_\mu^2}{M^2}\left(\frac{\alpha}{\pi}\right)^3$$

For $M \sim 2m_\pi \sim 2m_\mu$, one expects:

$$\delta a_\mu \approx 0.25\left(\frac{\alpha}{\pi}\right)^3$$

- The muon contribution confirms that this is a reasonable estimate:



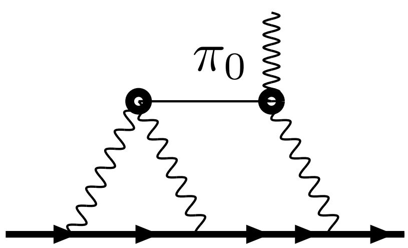
$$a_\mu^{\mu-\mathrm{box}} = 0.37\left(\frac{\alpha}{\pi}\right)^3$$

- The π-box contribution is one tenth of its "natural" value. This makes the subleading terms important.

- Can the subleading contribution be computed?
- Consider one of the $\mathcal{O}(m_\pi/m_\rho)$ suppressed terms in the Lagrangian:

$$\mathcal{L}_{\mathrm{WZW}} = \frac{\alpha N_c}{12\pi f_\pi} F_{\mu\nu}\tilde{F}^{\mu\nu}\pi_0$$

- The corresponding contribution is:



$$\sim \frac{\alpha^3 m^2}{(4\pi f_\pi)^2}\int_{m_\pi}^{\Lambda}\frac{\mathrm{d}k}{k}\ln\frac{\Lambda}{k}.$$

- The infinity is removed by adding the counter-term to the effective Lagrangian:

$$\delta\mathcal{L} = C\bar{\psi}\sigma_{\alpha\beta}\psi F^{\alpha\beta}.$$

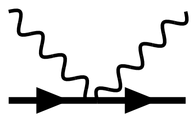
- But, this operator is the anomalous magnetic moment itself!

- Since the counter-term in EFT is the anomalous magnetic moment itself, the predictive power of the model-independent approach is very limited.
- The other possibility is to resort to a model.
- The model:
  - Scalar QED with power-suppressed corrections through the pion form-factor;
  - Large N_c approximation to reduce the number of power-suppressed operators – only WZW term remains;
  - Quark model to estimate the counter-term.
- Hardly consistent...

- The pion form factor is introduced through:

$$= \frac{-i}{q^2}\frac{M^2}{M^2-q^2}, \quad M \approx 770 \text{ MeV}.$$

- The $\pi^2 A_\mu^2$ vertex is modified:



$$= \delta_{\mu\nu}\left(1 - \frac{p_1^2 p_2^2}{M^4}\right).$$

- The calculation of the π-box contribution involves three different scales. Can be done as an expansion in
$m_\pi - m_\mu \ll m_\pi \ll M$.

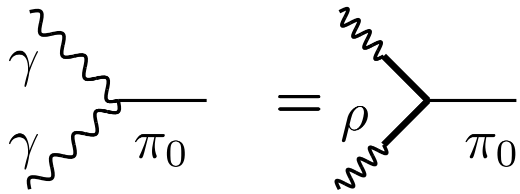
$$\delta a_\mu^{\pi-\mathrm{box}} = \frac{m^2}{M^2}\left[\frac{3}{2}L^2 + \left(\frac{13}{4} - \frac{2\pi^2}{3}\right)L + ..\right]$$

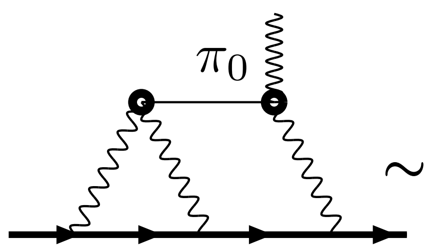
where $L = \ln M/m \approx 1.7$.

- The final result is:

$$a_\mu^{\pi-\mathrm{box}} = -0.003\left(\frac{\alpha}{\pi}\right)^3 = -4.4 \cdot 10^{-11}$$

- The contribution of the WZW term is computed by introducing the pion transition form factor through VMD:





$$\sim \left(\frac{\alpha}{\pi}\right)^3 \frac{m_\mu^2}{(\pi f_\pi)^2}\left[\frac{3}{16}\ln^2\frac{M}{m_\mu}\right]$$

- The full result is then:

$$a_\mu^{\pi_0} = 56 \cdot 10^{-11}.$$

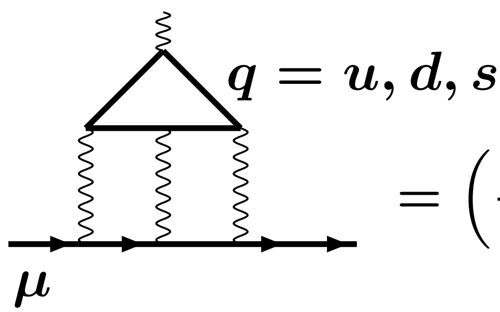
- The large N_c argument does not seem to work well.

- Heavy pseudoscalar mesons η, η' are not included on purpose.

- The counter-term:

$$\delta\mathcal{L} = C\bar{\psi}\sigma_{\alpha\beta}\psi F^{\alpha\beta}.$$

- Use the quark model for the estimate. The quark mass is a free parameter.



$$= \left(\frac{\alpha}{\pi}\right)^3 \frac{m_\mu^2}{M_Q^2}\left(\zeta_3 - \frac{19}{24}\right)$$

- As a "reasonable estimate" for the quark masses, take the range

$$M_Q = 250 \div 400 \text{ MeV}.$$

- Then:

$$a_\mu^{\text{quark}} = (35 \div 90) \cdot 10^{-11}.$$

- Hadronic light-by-light summary:

Contribution	$\mathcal{O}(1)$	$\mathcal{O}(m^2/M^2)$
$\pi^\pm$ loop	-43.4	39
π_0	0	56
counterterm	0	$35-90$

- "Duality": the result is stable if M_Q and M_ρ are increased(decreased) simultaneously.
- The final result is the sum of all the entries in the table:

$$a_\mu^{\rm lbl} = 110(30) \cdot 10^{-11}$$

- The uncertainty of this contribution is entirely subjective.
- How to match the quark model for the counterterm and the hadronic calculations for the matrix elements in a more rigorous way.

3. Conclusions and prospects

- The final estimate for a_μ:

contr.	old	new	change
QED	116 584 706(3)	116 584 721(3)	numerics
l.o. vp	6924(62)	6789(70)	e^+e^-
nlo. vp	$-100(6)$	$-100(6)$	
had. lbl.	$-85(25)$	110(30)	error
EW	152(4)	152(4)	
result	116 591 597(67)	116 591 672(110)	

- Using the e^+e^- data:

$$\left[a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{th}}\right] \cdot 10^{11} = 368 \pm 80|_{\mathrm{exp}} \pm 113|_{\mathrm{th}},$$

- Few standard deviations; any definite conclusion is difficult but the situation is uncomfortable.

- For comparison: using the τ data:

$$\left[a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{th}}\right] \cdot 10^{11} = 110 \pm 80|_{\mathrm{exp}} \pm 113|_{\mathrm{th}},$$

- In the future, $\mathbf{80}|_{\mathbf{exp}} \longrightarrow \mathbf{40}|_{\mathbf{exp}}$. Are we able make use out of it?
- It seems, we are at the bottom line of the possible confusion...
- There is a clear disagreement between the e^+e^- and the τ data; it is unlikely that the current difference between the two will be accommodated by the isospin violation effects.
- To resolve the situation, new measurements are needed.
- Additional studies at e^+e^- machines:
  a) further data analysis (VEPP2-M, BEPC);
  b) Radiative return measurements at existing facilities (DAPHNE, BaBar, CESR).
- Further checks on various contributions to $a_\mu^{\rm SM}$.
- The major conceptual problem is the hadronic light-by-light where we are bound to rely on the theoretical models.

An exciting year for $g - 2$:

- "We are now 99 percent sure that the present Standard Model calculations cannot describe our data" (2001)
- "There are three possibilities for the interpretation of this result. Firstly, new physics beyond the Standard Model... Thirdly, although unlikely,... there is always the possibility of mistakes in experiments and theories" (2001)
- "The observed change in frequency fits supersymmetry like a glove" (2001)
- "We are telling them (*theorists*), " Look, you guys, get the damn number on the table" (2002)
- "Obviously, this is all work in progress" (2002)