Neutrinos in Astrophysics

SLAC Summer Institute

August 4, 2004
Stanford Linear Accelerator Center

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Core Collapse Supernovae as the Ultimate Neutrino Physics Laboratories

Exploring/constraining the singlet neutrino mass/mixing spectrum with X-ray Observatories, nucleosynthesis, and other cosmological considerations.

Neutrinos Likely "Responsible" for Core Collapse Supernova Explosions/Nucleosynthesis

- We do not understand the origin of the viable shocks and associated explosions in core collapse events (Type II, Ib, Ic supernovae).

- However, given that a huge reservoir of energy (~10% of the rest mass of the core) is in the neutrino seas trapped in the core, it seems likely that neutrinos play an important and perhaps dominant role. Attempts to understand the explosions have centered on neutrino heating of material above and below the shock and neutrino-driven convective/hydro transport of this energy.

- Since neutrino flavors *may* have quite different energy spectra, active-active matter-enhanced conversion of neutrino flavors can have important effects on supernova dynamics/nucleosynthesis.

- Likewise, active-sterile matter-enhanced neutrino conversion also can affect core/explosion dynamics/nucleosynthesis.

- All channels of neutrino flavor conversion can affect the detected neutrino burst signal from a distant core collapse event.

Core Collapse Supernovae (Types II, Ib, Ic)

I. Collapse and Bounce Epoch

- Massive star (>10 solar masses) evolves in millions of years
- Forms “Fe”-core of 1.4 to 1.6 solar masses
- Core goes dynamically unstable
- Collapse duration of order 1 sec
- Entropy-per-baryon S/k of order 1 (really “Cold”)
- Shock generated at core bounce (at edge of homologous core)
- Shock energy subsequently degraded by photo-dissociation of nuclei

II. Shock Re-Heating Epoch

- Time “post-bounce,” $t_{pb,}$ from 0.1 s to 0.6 s
- Neutrino processes re-energize shock, drive convection
- Entropy-per-baryon S/k of order 40

III. Hot Bubble/r-Process Epoch

- t_{pb} from 1 s to 20 s
- Entropy-per-baryon S/k of order 70 to 500
- Neutrino-driven “wind”

Nuclear Burning Stages of a 25 M_{sun} Star

Burning Stage	Temperature	Density	Time Scale
Hydrogen	5 keV	5 g cm^{-3}	7 X 10^{6} years
Helium	20 keV	700 g cm^{-3}	5 X 10^{5} years
Carbon	80 keV	2 X 10^{5} g cm^{-3}	600 years
Neon	150 keV	4 X 10^{6} g cm^{-3}	1 year
Oxygen	200 keV	10^{7} g cm^{-3}	6 months
Silicon	350 keV	3 X 10^{7} g cm^{-3}	1 day
Core Collapse	700 keV ↓	4 X 10^{9} g cm^{-3} ↓	~ seconds of order the free fall time
"Bounce"	~ 2 MeV	~10^{15} g cm^{-3}	~milli-seconds
Neutron Star	< 70 MeV initial ~ keV "cold"	~10^{15} g cm^{-3}	initial cooling ~ 15-20 seconds ~ thousands of years

Massive Stars are Giant Refrigerators

From core carbon/oxygen burning onward
the neutrino luminosity exceeds the photon luminosity.

Neutrinos carry energy/entropy away from the core!

Core goes from $S/k \sim 10$ on the Main Sequence (hydrogen burning)
to a thermodynamically cold $S/k \sim 1$ at the onset of collapse!

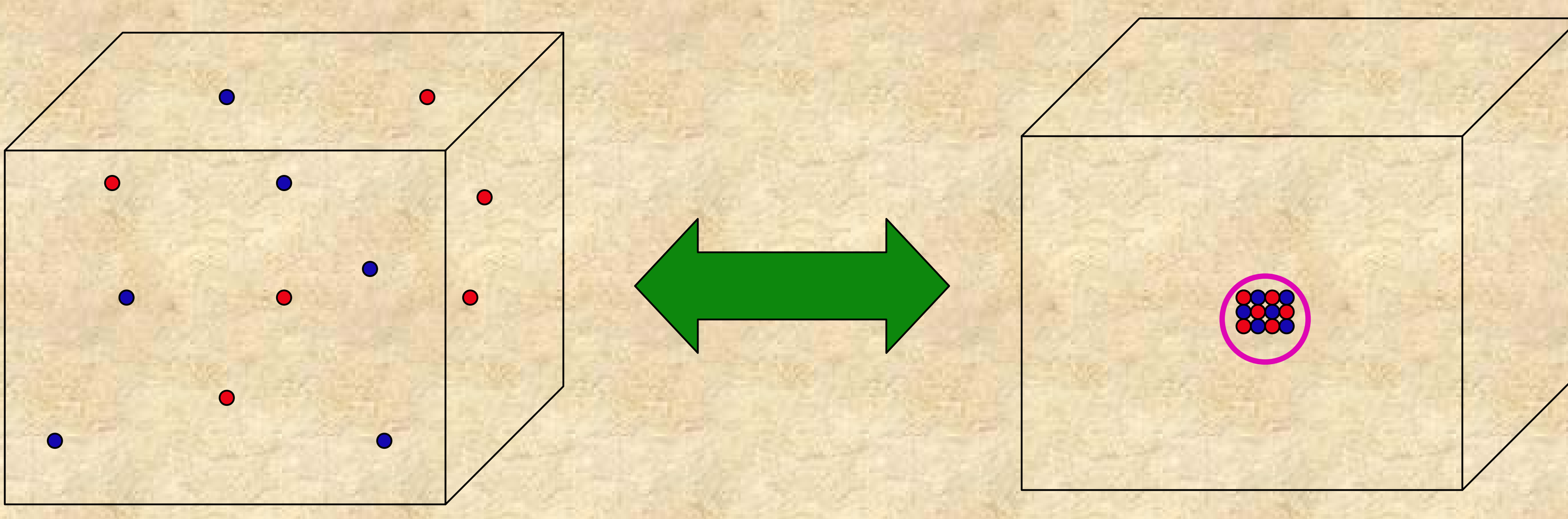
e.g., the collapsing core of a supernova can be a
frozen (Coulomb) crystalline solid with a
temperature ~1 MeV!

Entropy

$$S = k \log \Gamma$$

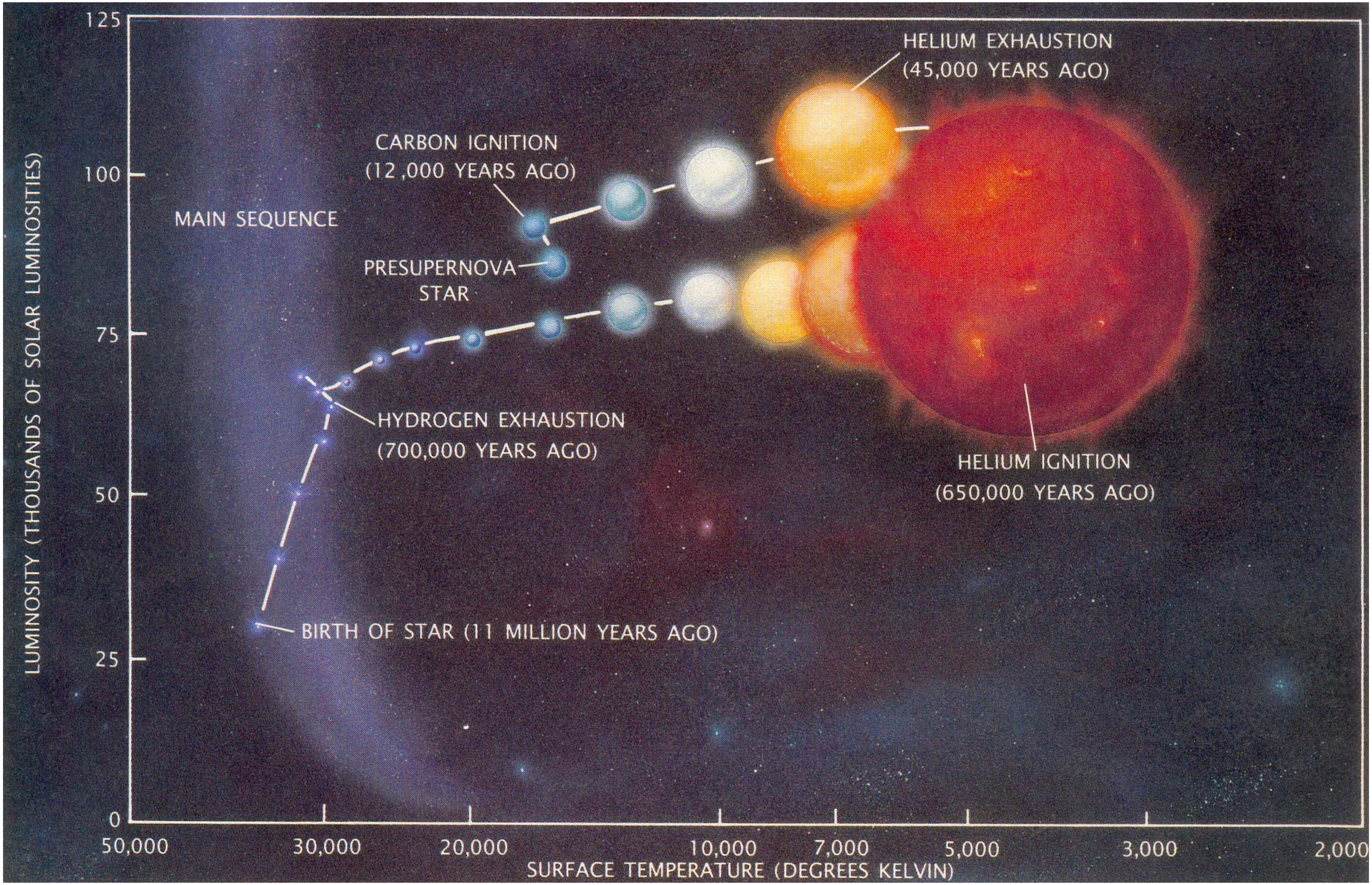
a measure of a system's disorder/order

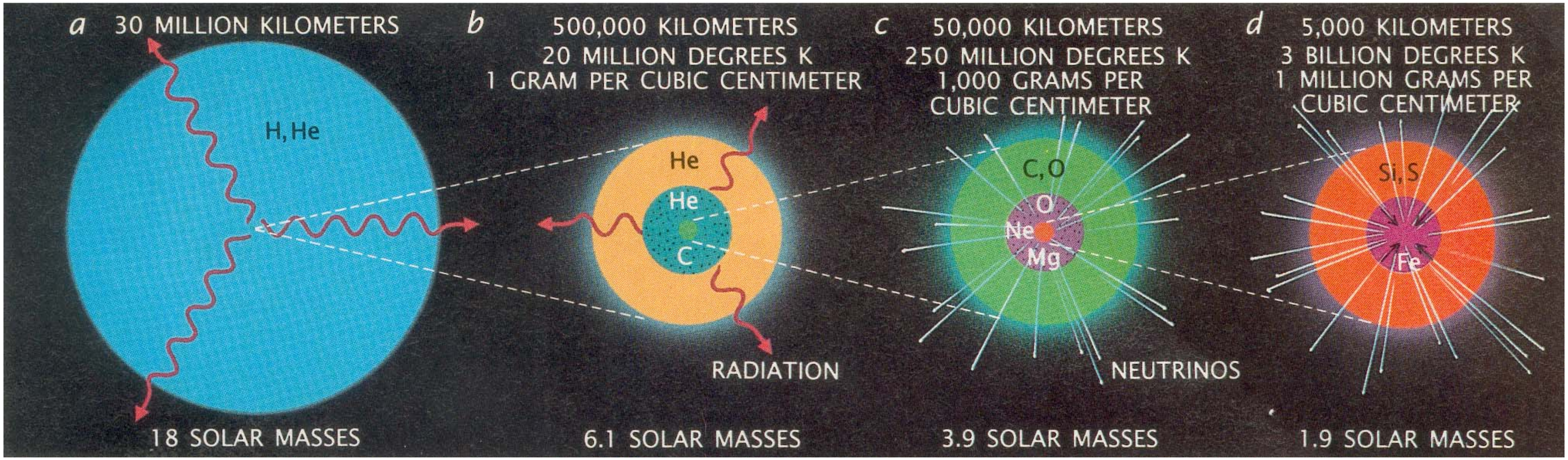
High Entropy **Low Entropy**



6 free nucleons **^{12}C nucleus**

Weaver & Woosley, ***Sci Am***, **1987**

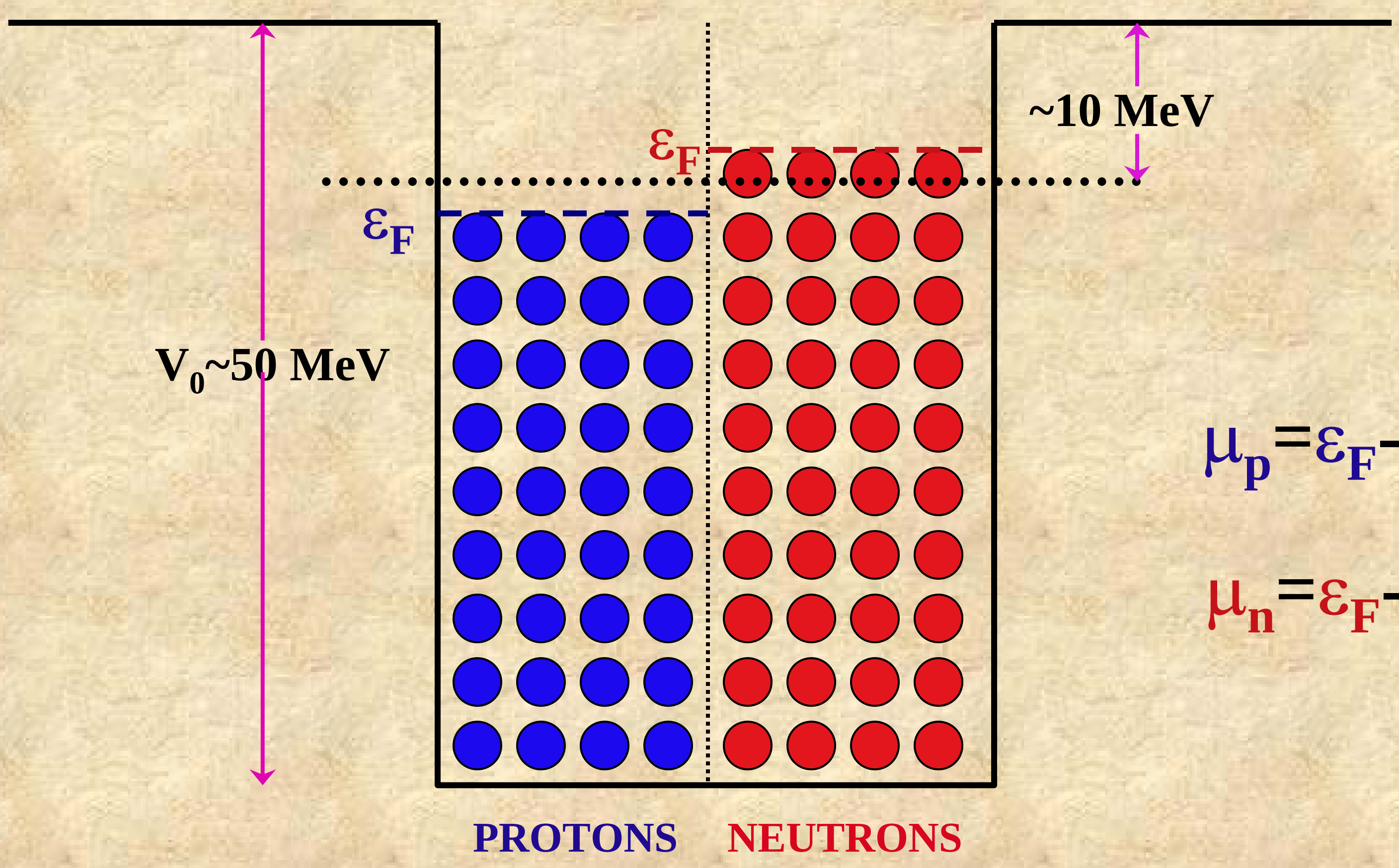




Weaver & Woosley, *Sci Am*, 1987

Schematic "Nucleus"

(ignore Coulomb potential for protons)



$$\mu_p = \varepsilon_F - V_0$$

$$\mu_n = \varepsilon_F - V_0$$

Total entropy for *N* Maxwell-Boltzmann particles in volume *V*

$$S \approx \tfrac{5}{2}N + \tfrac{3}{2}N\left[\ln(kT) + \ln\left(\frac{mc^2}{2\pi\hbar^2}\right) + \tfrac{2}{3}\ln\left(\frac{V}{N}\right)\right]$$

Entropy per unit volume for nucleons inside nuclei with Fermi level ε_F (*nonrelativistically degenerate*).

$$S \approx \frac{\pi^2}{2}\frac{kT}{\varepsilon_F}$$

Entropy per unit volume for electrons with chemical potential μ_e (*relativistically degenerate*).

$$S \approx \pi^2 \frac{kT}{\mu_e}$$

e.g., at the onset of core collapse, $\rho_{10}=\rho/10^{10}$g cm^{-3}=0.37, T=0.66 MeV, Y_e=0.42

→ **S/k (per baryon) = 0.91**

Infall Epoch

Electron capture proceeds on free protons and nuclei. Eventually neutrinos become trapped in the core and begin to thermalize. Reverse process of neutrino capture becomes significant and system approaches beta equilibrium.

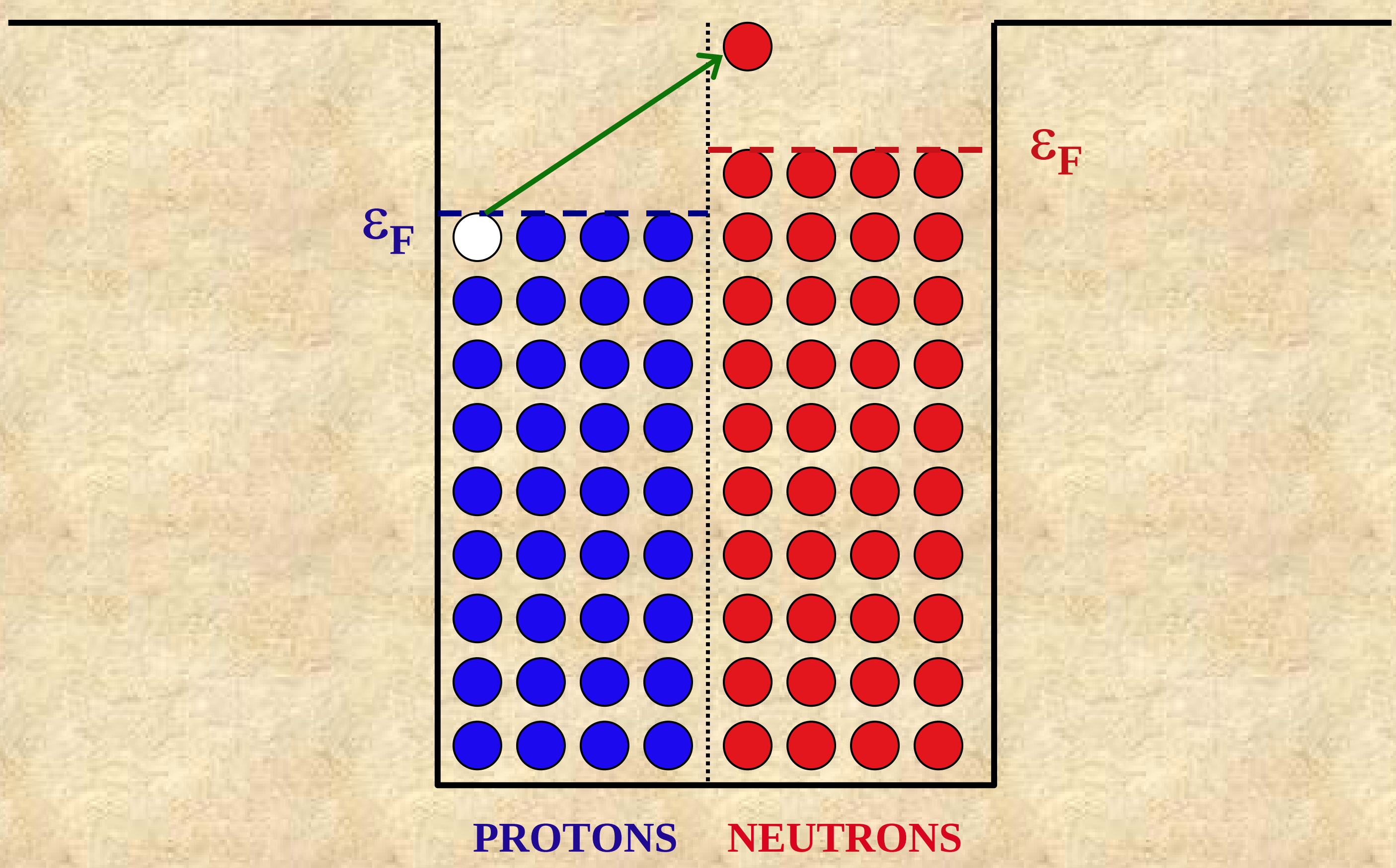
$$e^- + p \Longleftrightarrow n + \nu_e \longrightarrow \mu_e - \mu_{\nu_e} = \mu_n - \mu_p + \delta m_{np}$$

$$\mu_e \sim 11.1\ \mathrm{MeV}\ (\rho_{10} Y_e)^{1/3}$$

System is extremely sensitive to lepton number violating processes in the neutrino sector.

$$\left.\begin{array}{l} \nu_e \longrightarrow \nu_{\mu,\tau} \quad \textbf{or} \quad \nu_e \longrightarrow \nu_s \\ \textbf{leads to entropy generation/loss} \end{array}\right\} \begin{array}{l} \Delta S/k > 3 \\ \textbf{melts nuclei} \end{array}$$

Electron Capture on Nuclei Heats the System



The Dynamics of the Collapsing Stellar Core can be Exquisitely sensitive to lepton number violating processes.

Low entropy, highly ordered system with huge electron and significant electron neutrino degeneracy (high Fermi energies).
(Few muon/tau neutrinos/antineutrinos.)

Pressure dominated by degenerate electrons:
reducing number of electrons by opening holes in electron neutrino distribution allows further electron capture and lowers homologous core mass, increasing nuclear photo-disintegration burden on shock.

For example: **probe of R-parity violating supersymmetry interactions with collapse/shock dynamics or neutrino signal.**

P. Amanik, G. M. Fuller, B. Grinstein, hep-ph/0407130

(see poster by Phil Amanik at this SSI)

Neutrinos Dominate the Energetics of Core Collapse Supernovae

Explosion
only ~1% of
neutrino energy

- Total optical + kinetic energy, 10^{51} ergs
- Total energy released in Neutrinos, 10^{53} ergs

10 % of star's rest mass!

$$E_{\mathrm{GRAV}} \approx \frac{3}{5}\frac{G\,M_{\mathrm{NS}}^2}{R_{\mathrm{NS}}} \approx 3\times 10^{53}\,\mathrm{ergs}\left[\frac{M_{\mathrm{NS}}}{1.4\mathrm{M}_{\mathrm{sun}}}\right]^2\left[\frac{10\ \mathrm{km}}{R_{\mathrm{NS}}}\right]$$

- Neutrino diffusion time, $\tau_\nu \approx 2\ \mathrm{s}$

$$\boxed{L_\nu \approx \frac{1}{6}\frac{GM_{\mathrm{NS}}^2}{R_{\mathrm{NS}}}\frac{1}{\tau_\nu} \approx 4\times 10^{51}\,\mathrm{ergs\ s^{-1}}}$$

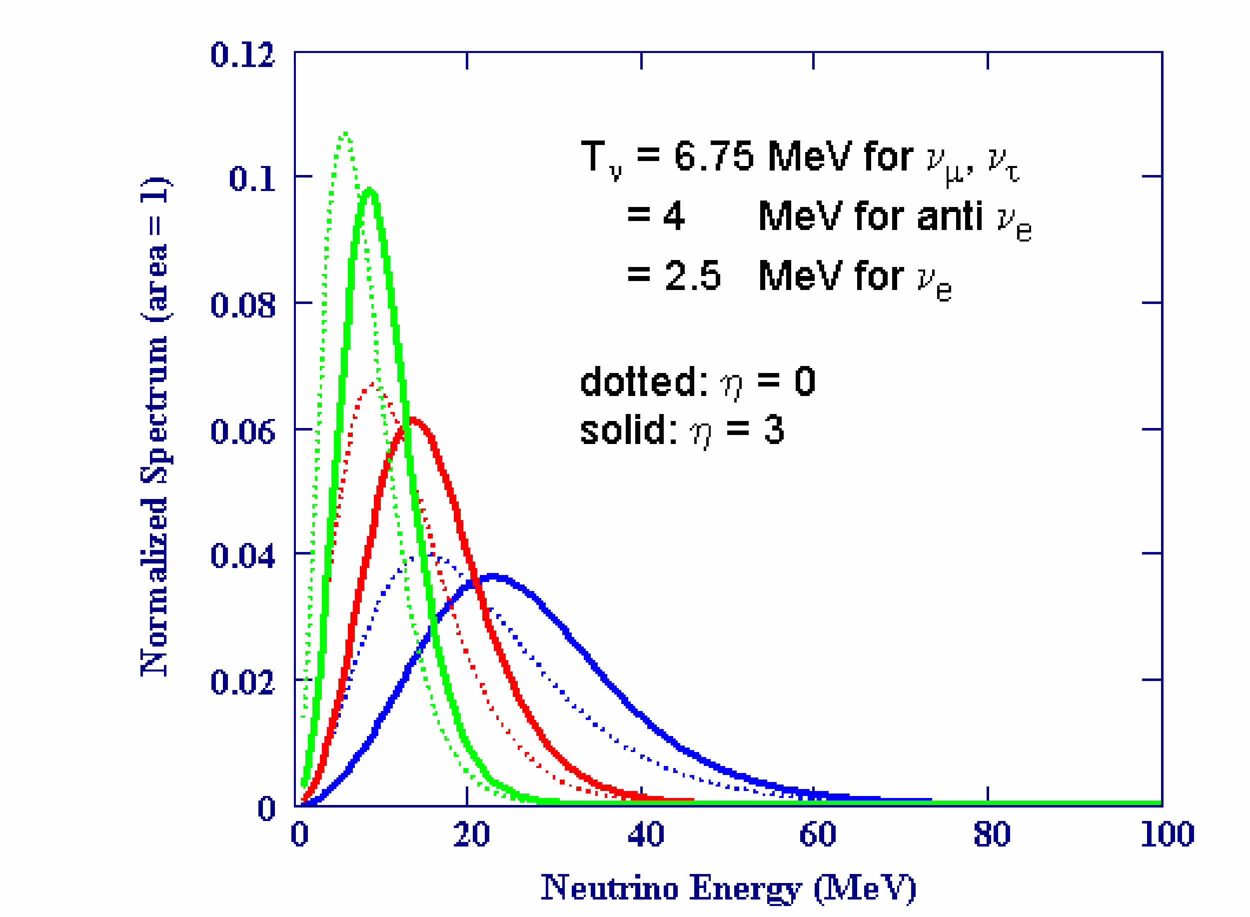
Two lessons can be drawn:

- Even a small change in the way neutrinos couple to matter can result in significant energy and isopspin (neutron/proton ratio) consequences at a variety of epochs and locations.

- The explosion is a sideshow with a very small fraction of the energy budget! Therefore, a small change in neutrino interactions can have leverage in explosion/nucleosynthesis. (Same reason sub-dominant B-fields, rotation could be important.)

Neutrino Energy Spectra

- at "Neutrino Sphere"
- Near Fermi-Dirac energy distribution



$$\langle E_{\nu_e} \rangle \sim 10\ \mathrm{MeV}$$

$$\langle E_{\bar{\nu}_e} \rangle \sim 16\ \mathrm{MeV}$$

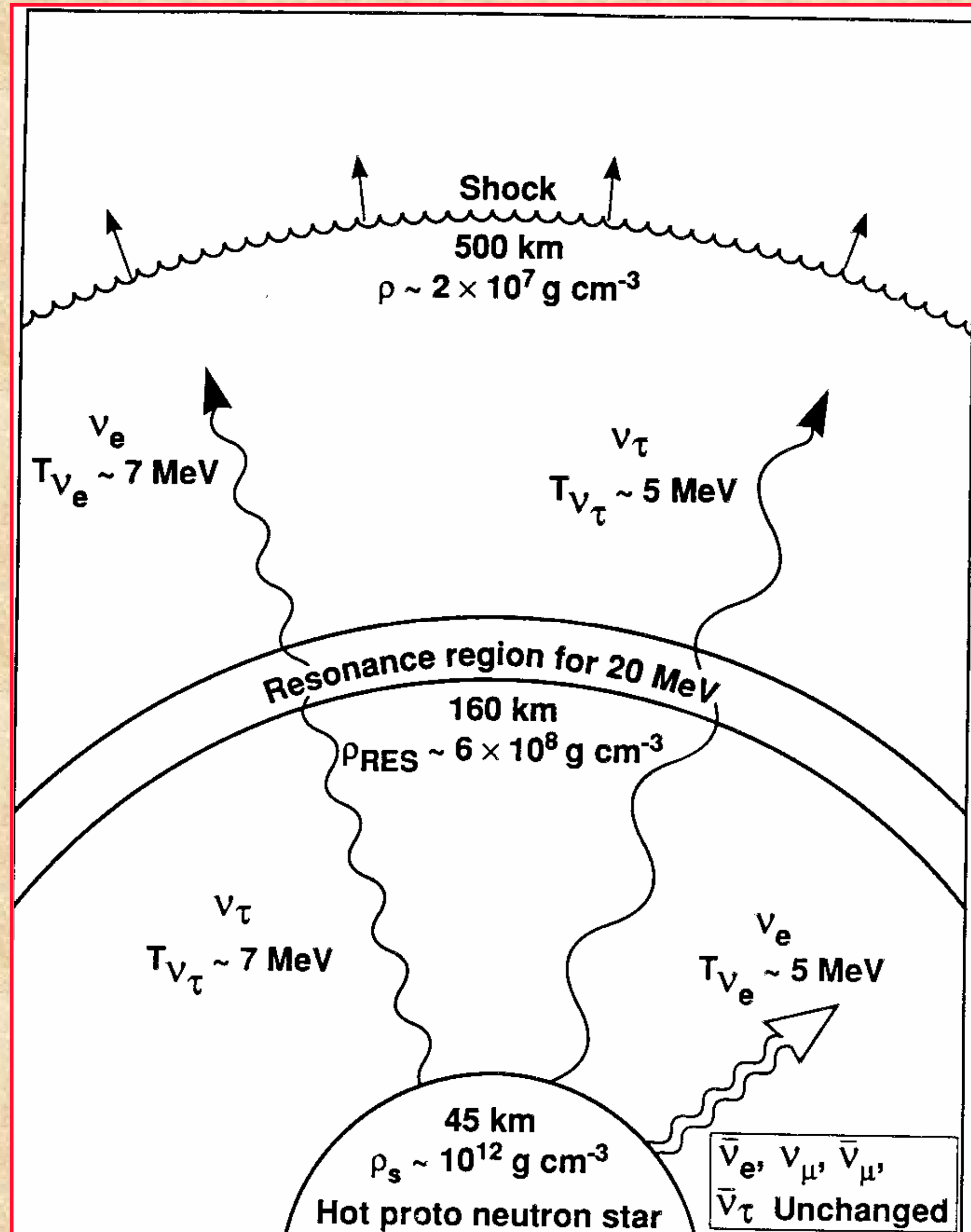
$$\langle E_{\nu_{\mu,\tau}} \rangle \sim 27\ \mathrm{MeV}$$

We really do not know what the spectra are but they are likely to be different For different flavors at *some* times of interest.

Shock likely re-energized principally by neutrino processes occurring underneath shock:

$$\nu_e + n \rightarrow p + e^-$$
$$\bar{\nu}_e + p \rightarrow n + e^+$$

Neutrino-nucleus processes, both charged and neutral current, may also be important in "pre-heating" of material ahead of shock. Can this alleviate the **nuclear photo-dissociation problem**?



The flux of neutrinos in a pencil of directions and energies is

$$d\varphi_\nu \approx \frac{L_\nu}{\pi R_\nu^2}\left(\frac{d\Omega_\nu}{4\pi}\right)\frac{1}{\langle E_\nu \rangle} f(E_\nu) dE_\nu$$

The (black body) neutrino distribution function is

$$f(E_\nu) = \frac{1}{T_\nu^3 F_3(\eta_\nu)} \frac{E_\nu^2}{e^{E_\nu/T_\nu - \eta_\nu} + 1}$$

$$F_k(\eta_\nu) = \int_0^\infty \frac{x^k dx}{e^{x-\eta_\nu} + 1}$$

Neutrino Charged Current Capture Cross Section

$$\sigma(E_\nu) \approx \langle G \rangle \frac{\ln 2}{\langle ft \rangle} \left(\frac{2\pi^2 (\hbar c)^3}{c} \right) (m_e c^2)^{-5} (E_\nu + Q_n)^2$$

For neutrino capture on neutrons or protons the appropriate Q-values are

$$Q_n = \pm(m_n - m_p) \approx \pm 1.293 MeV$$

While the Coulomb wave correction factors in this case are $\langle G \rangle \approx 1$

And the appropriate matrix element/*ft*-values are $\langle ft \rangle \approx 10^{3.035} s$

The mass absorption coefficient (opacity) for neutrino capture on nuclear species *i* is

$$K_i = N_a Y_i \langle \sigma(E_\nu) \rangle$$

Bethe & Wilson (1985) have shown that the rate of Net energy deposition from neutrino charged current capture is

$$\dot{E} \approx K(T_\nu)\left[\frac{L_\nu}{4\pi r_m^2} - \left(\frac{T_m}{T_\nu}\right)^2 acT_m^4\right]$$

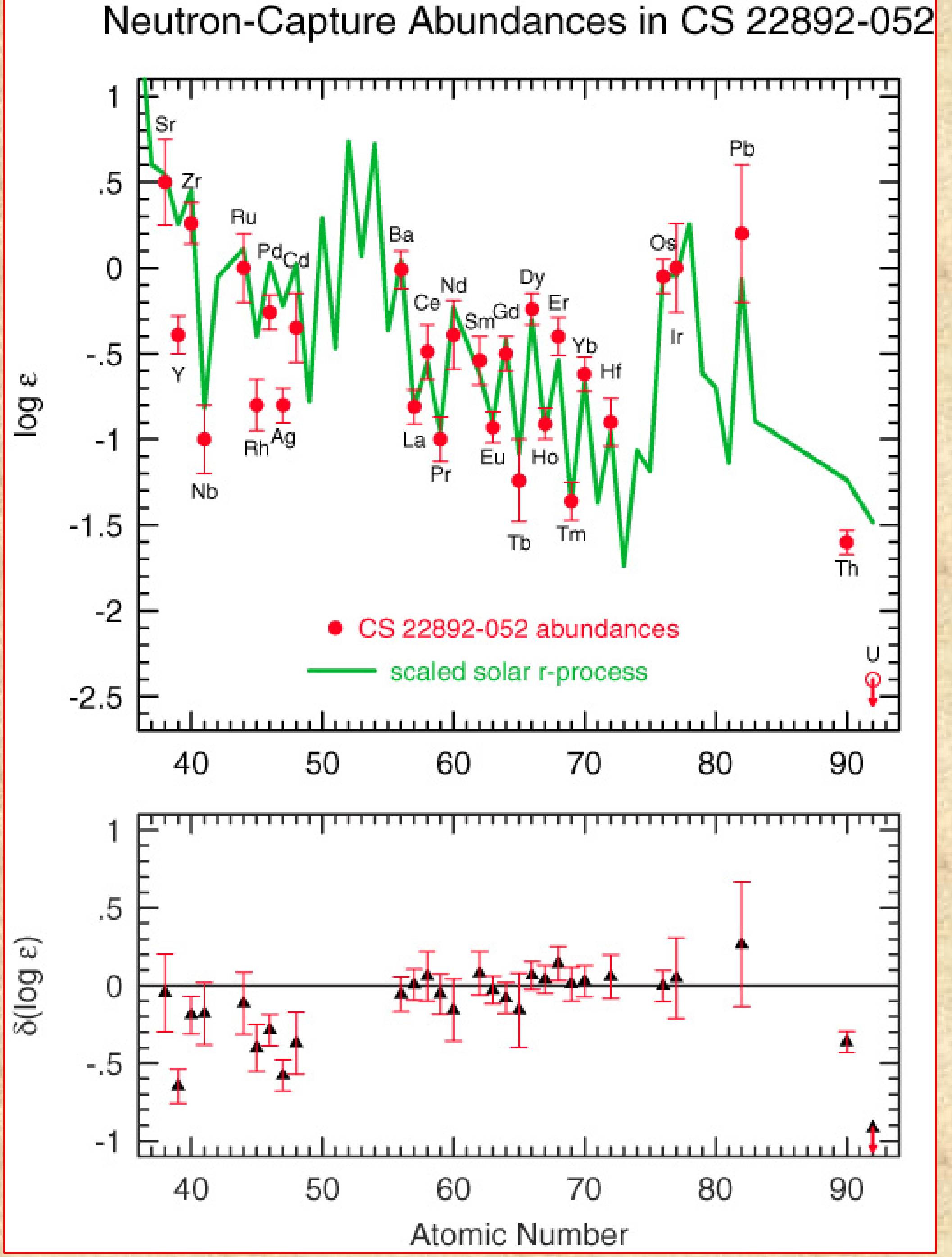
Where r_m is the radius of a matter element and the matter at that point has tempertaure T_m, which will generally be less than the temperature characterizing the neutrino distribution function, T_ν

Sneden, *et al.*, Astrophys. J., 533, L139 (2000).

Heavy element abundance determinations in Ultra Metal-Poor halo star CS 22892-052

$$[Fe/H] \approx -3.1$$

(Includes New Keck Data)



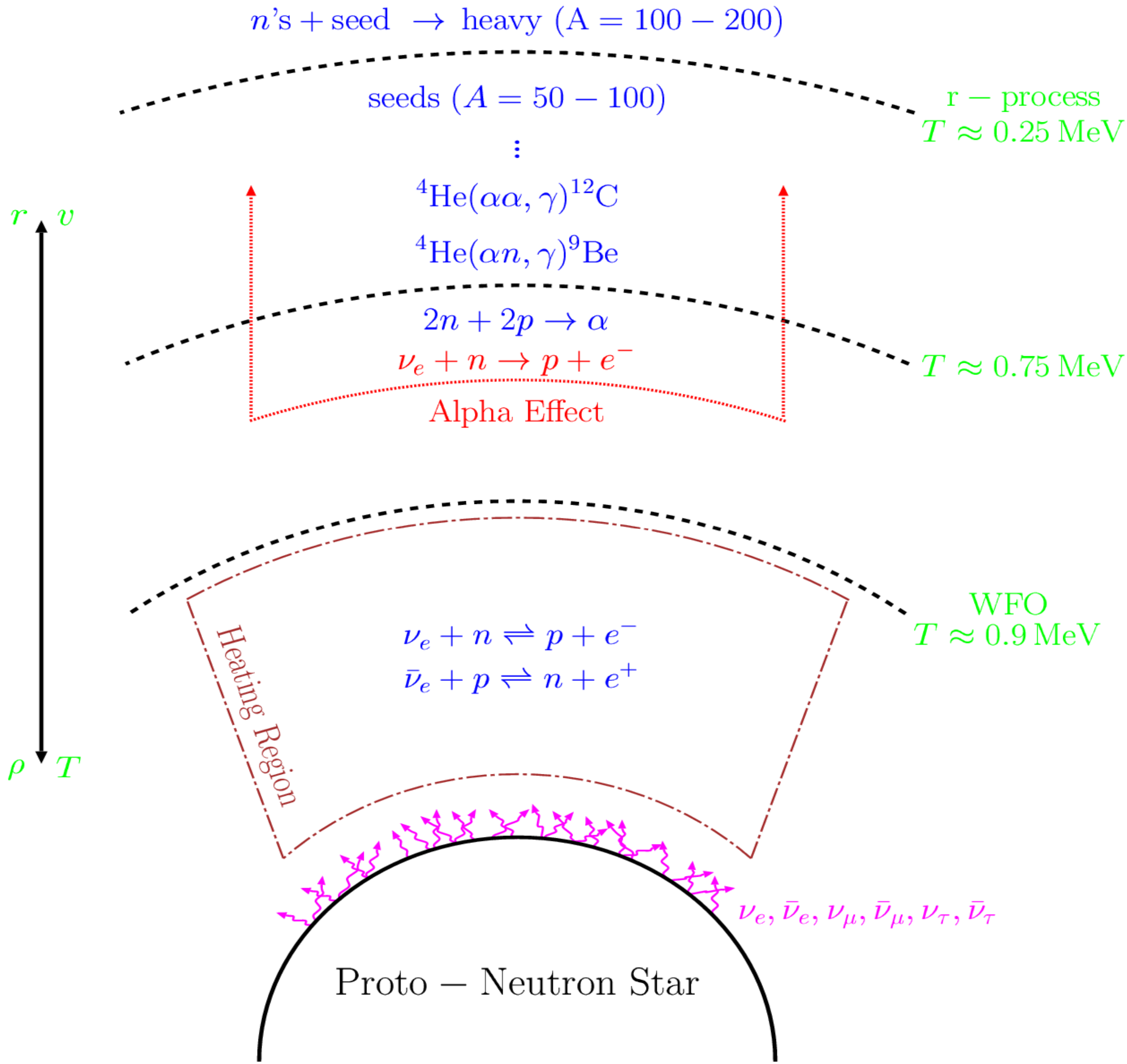
Mass A>100 abundance pattern fits that of solar system, lower nuclear mass material has an abundance pattern which does not, in general, fit the solar pattern. This trend is evident in other Ultra Metal-Poor Halo stars as well.

universal abundance pattern?

A = 130 & 195 peak have comparable abundances

Shock Propagation

R. Schirato & G. Fuller, astro-ph/0205390



n's + seed $\rightarrow$ heavy (A = 100 − 200)
seeds (A = 50 − 100)
r − process
$T \approx 0.25$ MeV
$^4\mathrm{He}(\alpha\alpha,\gamma)^{12}\mathrm{C}$
$^4\mathrm{He}(\alpha n,\gamma)^{9}\mathrm{Be}$
r
v
$2n + 2p \rightarrow \alpha$
$\nu_e + n \rightarrow p + e^-$
$T \approx 0.75$ MeV
Alpha Effect
WFO
$T \approx 0.9$ MeV
$\nu_e + n \rightleftharpoons p + e^-$
$\bar{\nu}_e + p \rightleftharpoons n + e^+$
Heating Region
ρ
T
$\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau$
Proto − Neutron Star

Neutrinos set Y_e

$\nu_e + n \rightarrow p + e^-$ $\bar{\nu}_e + p \rightarrow n + e^+$

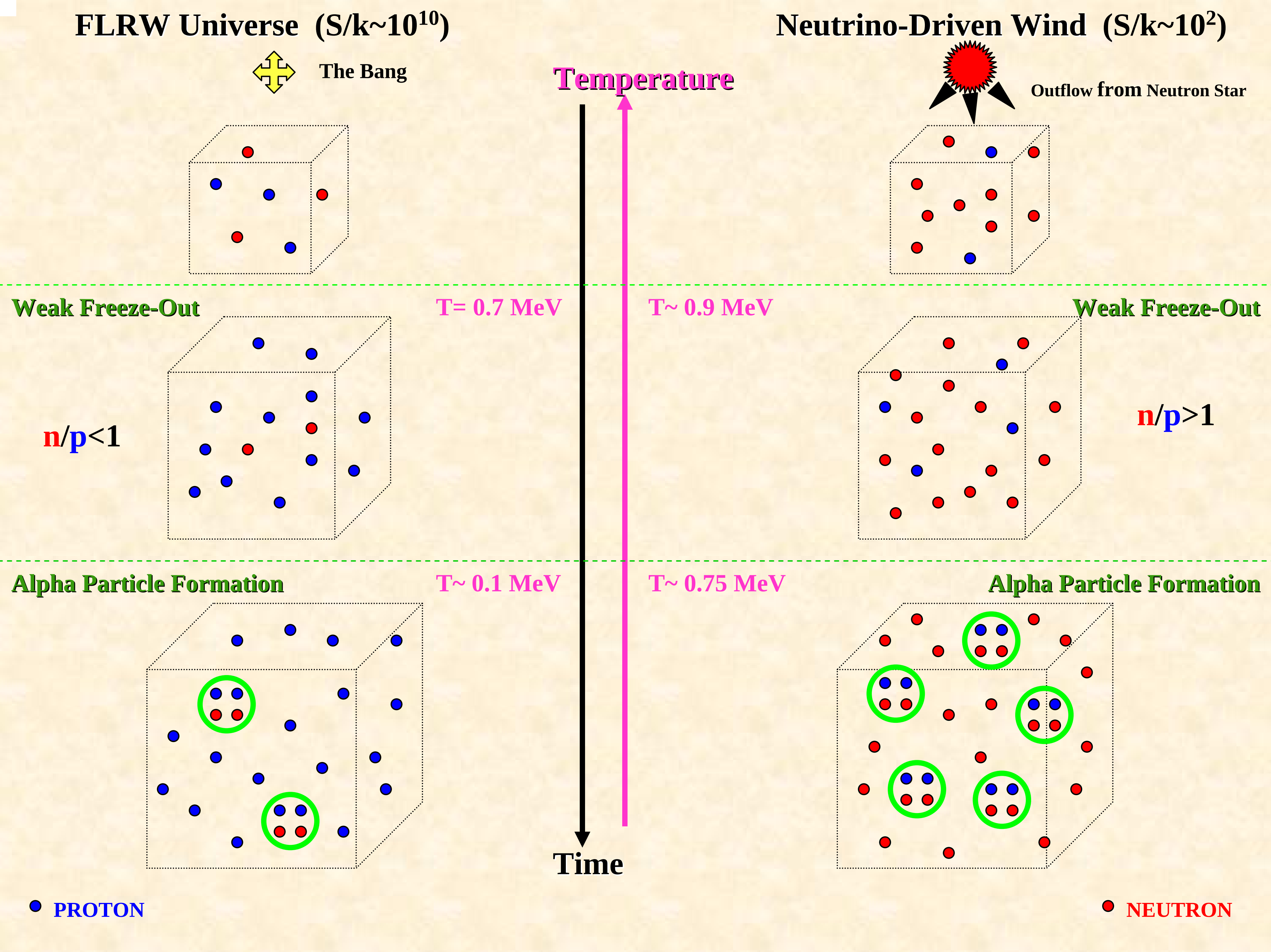
Cross sections $\sigma \sim G_F^2 \ E_\nu^2$

Rates: λ = (flux)(cross section) $\sim (L_\nu / \langle E_\nu \rangle) \langle E_\nu^2 \rangle$
$\sim L_\nu \langle E_\nu \rangle$

Integrate rate equations to find that

$Y_e = \lambda_{\nu en} / (\lambda_{\nu en} + \lambda_{\bar{\nu} ep}) \sim (1 + \langle E_{\bar{\nu}_e} \rangle / \langle E_{\nu_e} \rangle)^{-1}$

$n/p = \lambda_{\bar{\nu} ep} / \lambda_{\nu en}$


FLRW Universe (S/k~10^10)
Neutrino-Driven Wind (S/k~10^2)
The Bang
Temperature
Outflow from Neutron Star
Weak Freeze-Out
T= 0.7 MeV
T~ 0.9 MeV
Weak Freeze-Out
n/p<1
n/p>1
Alpha Particle Formation
T~ 0.1 MeV
T~ 0.75 MeV
Alpha Particle Formation
Time
PROTON
NEUTRON

Freeze-Out from **Nuclear Statistical Equilibrium** (NSE)

In NSE the reactions which build up and tear down nuclei have equal rates, and these rates are large compared to the local material expansion rate.

$$\underbrace{Z\,\mathbf{p} + N\,\mathbf{n} \longleftrightarrow A(Z,N) + \gamma}_{\text{nuclear mass A is the sum of protons and neutrons A=Z+N}}$$

$$Z\,\mu_p + N\,\mu_n = \mu_A + Q_A$$

Saha Equation

abundance of nucleus A(Z,N) with binding energy Q_A

$$Y_{A(Z,N)} \propto \left[S^{1-A}\right] e^{Q_A/T}$$

The Alpha Effect

The paradox of neutrino-heated *r*-Process nucleosynthesis

Require neutrino interactions on free nucleons to give enough energy to each baryon to overcome the gravitational binding energy near the neutron star (~100 MeV per baryon). Since the average energies of neutrinos are ~ 10 MeV, we need some ~10 neutrino and antineutrino captures per nucleon to ensure ejection of the material.

However, formation of alpha particles incorporates all protons thereby isolating some free neutrons. These can capture electron neutrinos to become protons, which are immediately incorporated into alpha particles. Each reaction $\nu_e + n \rightarrow p + e^-$ takes out two neutrons!

In short order there are not enough neutrons to make the *r*-Process

Matter-Enhanced Neutrino Flavor Transformation can affect supernova explosion physics/nucleosynthesis

active-active :

G. M. Fuller, R. Mayle, J. R. Wison, and D. N. Schramm, Astrophys. J., 322, 795 (1987).
G. M. Fuller, R. Mayle, B. S. Meyer, and J. R. Wilson, Astrophys. J., 389, 517 (1992).
Y.-Z. Qian, G. M. Fuller, G. J. Mathews, R. Mayle, J. R. Wilson, S. E. Woosley, Phys. Rev. Lett. 71, 1965 (1993).
Y.-Z. Qian & G. M. Fuller, Phys. Rev. D51, 1479 (1995).
S. Bruenn & A. Mezzacappa, in “Sources and Detection of Dark Matter,” World Scientific (1998).
S.Pastor & G. Raffelt, preprint (2002); Sigl & Raffelt, Nucl. Phys. B406, 423 (1993).

active-sterile:

J. T. Peltoniemi hep-ph/9511323; H. Nunokawa, J. T. Peltoniemi, A. Rossi, J. Valle, Phys. Rev. D56, 1204 (1997).
G. C. McLaughlin, J. M. Fetter, A. B. Balantekin, G. M. Fuller, Phys. Rev. C59, 2873 (1999).
D. O. Caldwell, G. M. Fuller, and Y.-Z. Qian, Phys. Rev. D61, 123005 (2000).
K. Abazajian, G. M. Fuller, M. Patel, Phys. Rev. D64, 023501 (2001).

RSFP:

E. Akhmedov, A. Lanza, S. T. Petcov, D. Sciama, E. Akhmedov, Z. Berezhiani, Nuclear Physics, B373, 479 (1992).
H. Nunokawa, Y.-Z. Qian, G. M. Fuller, Phys. Rev. D55, 3265 (1997).

The weak interaction, or flavor basis is not coincident with the energy eigenstate, or mass basis.

These bases are related through a unitary transformation,

$$\left|\nu_\alpha\right\rangle = \sum_i U^*_{\alpha i}\left|\nu_i\right\rangle$$

where the flavors are $\alpha = e, \mu, \tau, s, s', \ldots$

and where the mass states are $i = 1, 2, 3, 4, \ldots$

$U_{\alpha i}$ **is parameterized by vacuum mixing angles and CP-violating phases, in general.**

If we consider only two-by-two neutrino mixing then the unitary transformation is parameterized by a single vacuum mixing angle:

$$|\nu_\alpha\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$

$$|\nu_\beta\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$$

Difference of the squares of the neutrino mass eigenvalues:

$$\delta m^2 = m_2^2 - m_1^2$$

Ignore LSND…

Experiment/Observation now has given us almost everything!

$\nu_\mu/\nu_\tau/\nu_e$

ν_3

$\delta m^2 \approx 3\times 10^{-3} eV^2$

ν_2

ν_1

$\delta m^2 \approx 7\times 10^{-5} eV^2$

(near) maximal mixing between ν_μ/ν_τ

also between $\nu_\mu/\nu_\tau/\nu_e$

only θ_{13} and CP-violating phase
and the absolute masses remain to be determined in this case

Atmospheric Neutrinos

$$\delta m_{23}^2 \approx 2.5 \times 10^{-3}\ \text{eV}^2$$
$$\sin^2 2\theta_{23} \approx 1.0$$

"Solar"/KamLaND Neutrinos

$$\delta m_{\text{sol}}^2 \approx 7 \times 10^{-5}\ \text{eV}^2$$
$$\tan^2 \theta_{12} \approx 0.42 \ \leftrightarrow \ 0.45$$

Chooz/KamLaND

Chooz limit on $\theta_{13} \Rightarrow$

$\left|U_{e3}\right|^2 < 2.5\%$ or $\sin^2 2\theta_{13} < 0.1$ $(\theta_{13} < \frac{\pi}{20} \approx 9^\circ)$

plus KamLaND $\Rightarrow$

$\sin^2 2\theta_{13} < 6.65 \times 10^{-2}$ $(< 0.2 \text{ at } 3\sigma)$

Coherent Neutrino Flavor Evolution

above the Neutron Star Surface (neutrino sphere)

Active-Active Neutrino Flavor Transformation

Consider active-active neutrino mixing:

in vacuum

$$|\nu_\alpha\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$
$$|\nu_\beta\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$$

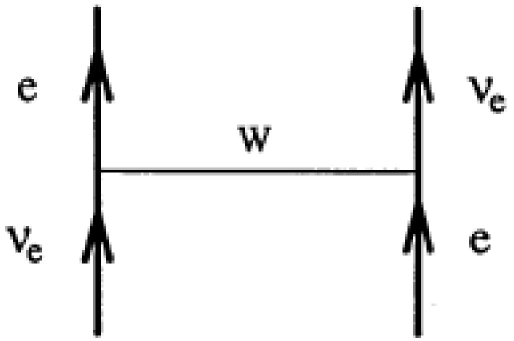
here α, β **=** **e,**μ, τ

in "medium," in the supernova core or envelope

$$|\nu_\alpha\rangle = \cos\theta_M|\nu_1\rangle + \sin\theta_M|\nu_2\rangle$$
$$|\nu_\beta\rangle = -\sin\theta_M|\nu_1\rangle + \cos\theta_M|\nu_2\rangle$$

For the ultra-high density core/neutron star limit see for example Abazajian, Fuller, Patel, Phys. Rev. D64, 023501 (2001).

The **A** potential arises from charged current forward exchange



A schematic view of neutrino effective masses

The current-current Lagrangian for neutrino-electron scattering

$$\begin{cases} L_{total} = \overline{\Psi}_\nu (i\partial\!\!\!/ - m_\nu)\Psi_\nu + \overline{\Psi}_e (i\partial\!\!\!/ - m_e)\Psi_e \\ -\frac{G_F}{\sqrt{2}}(\overline{\Psi}_\nu \gamma^\mu (1-\gamma_5)\Psi_\nu)(\overline{\Psi}_e \gamma_\mu (1-\gamma_5)\Psi_e) \end{cases}$$

From this we can define a potential stemming from the electron background:

$$A^\mu \equiv \frac{G_F}{\sqrt{2}}\left[\overline{\Psi}_e \gamma^\mu (1-\gamma_5)\Psi_e\right] = (\varphi, \vec{A})$$

The neutrino Lagrangian is then:

$$L_\nu = \overline{\Psi}_\nu \left(i\partial\!\!\!/ - A\!\!\!/(1-\gamma_5) - m_\nu\right)\Psi_\nu$$

The equation of motion (Dirac equation) corresponding to this is . . .

$$\left(i\frac{\partial}{\partial t} - \varphi\right)\Psi_\nu = \left[\vec{\alpha}\bullet(\tfrac{1}{i}\vec{\nabla} - \vec{A}) + \beta m_\nu\right]\Psi_\nu$$

and the dispersion relation is . . .

$$(E_\nu - \varphi)^2 = \left(\vec{p}_\nu - \vec{A}\right)^2 + m_\nu^2$$

$$E_\nu^2 = \vec{p}_\nu^{\,2} + \left[m_\nu^2 + 2E_\nu\varphi - 2\vec{p}_\nu \bullet \vec{A} + \left(\vec{A}^2 - \varphi^2\right)\right]$$

$$2E_\nu \frac{G_F}{\sqrt{2}}\left(\overline{\Psi}_e \gamma^0 (1-\gamma_5)\Psi_e\right) \approx 1.5\times 10^{-7} eV^2 \left(\frac{\rho N_a Y_e}{1 g cm^{-3}}\right)\left(\frac{E_\nu}{MeV}\right)$$

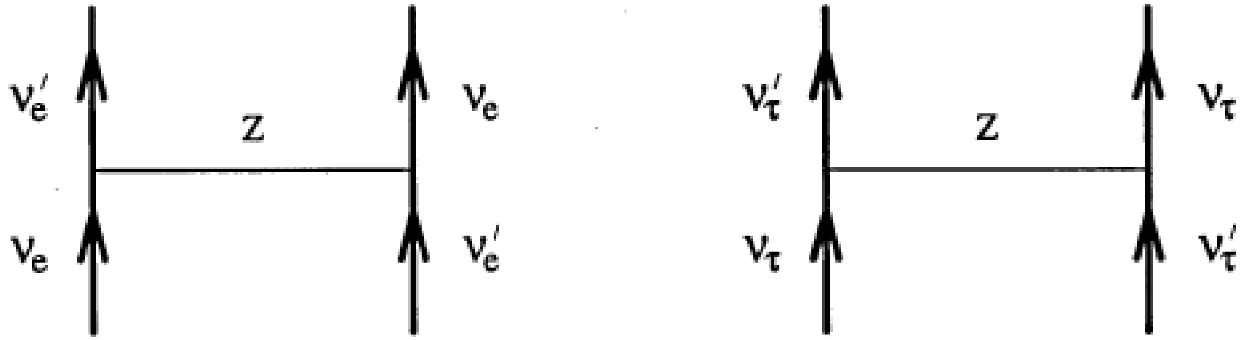
$$E_\nu^2 = \vec{p}_\nu^{\,2} + \left[m_\nu^2 + 2E_\nu\langle\varphi\rangle - 2\left\langle \vec{p}_\nu \bullet \vec{A}\right\rangle + \left(\left\langle \vec{A}^2\right\rangle - \left\langle \varphi^2\right\rangle\right)\right]$$

$$\sim G_F^2$$

$$2\left\langle p_\nu A\cos\theta\right\rangle \approx 2E_\nu\langle\varphi\rangle\langle\cos\theta\rangle$$

zero if electron distribution is isotropic

The neutrino “background” potential B arises from neutral current forward exchange



Low-Temperature Neutrino Forward Scattering Potentials

$$H(\nu_s) \approx 0$$

$$H(\nu_e) = \sqrt{2}G_F\left(n_e - \tfrac{1}{2}n_n\right) + \sqrt{2}G_F\left[2\left(n_{\nu_e} - n_{\bar{\nu}_e}\right) + \left(n_{\nu_\mu} - n_{\bar{\nu}_\mu}\right) + \left(n_{\nu_\tau} - n_{\bar{\nu}_\tau}\right)\right]$$

$$H(\nu_\mu) = \sqrt{2}G_F\left(-\tfrac{1}{2}n_n\right) + \sqrt{2}G_F\left[\left(n_{\nu_e} - n_{\bar{\nu}_e}\right) + 2\left(n_{\nu_\mu} - n_{\bar{\nu}_\mu}\right) + \left(n_{\nu_\tau} - n_{\bar{\nu}_\tau}\right)\right]$$

$$H(\nu_\tau) = \sqrt{2}G_F\left(-\tfrac{1}{2}n_n\right) + \sqrt{2}G_F\left[\left(n_{\nu_e} - n_{\bar{\nu}_e}\right) + \left(n_{\nu_\mu} - n_{\bar{\nu}_\mu}\right) + 2\left(n_{\nu_\tau} - n_{\bar{\nu}_\tau}\right)\right]$$

Matter-Enhanced Neutrino Flavor Transformation

example : $\nu_e \leftrightarrow \nu_\tau$

$$A = \sqrt{2} G_F \left(n_{e^-} - n_{e^+} \right)$$

$$B = \sqrt{2} G_F \int \left(1 - \cos\theta_q\right) \left\{ \left(\rho_q - \overline{\rho_q}\right)_{ee} - \left(\rho_q - \overline{\rho_q}\right)_{\tau\tau} \right\} d^3 q$$

$$B_{e\tau} = \sqrt{2} G_F \int \left(1 - \cos\theta_q\right) \left\{ \left(\rho_q - \overline{\rho_q}\right)_{e\tau} \right\} d^3 q$$

Neutrino density operator: $$\boxed{\rho_q d^3 q = \sum_\alpha dn_{\nu_\alpha} \left| \psi_{\nu_\alpha} \right\rangle \left\langle \psi_{\nu_\alpha} \right|}$$

Flavor amplitude evolution: $$\boxed{i \frac{\partial \Psi_f}{\partial r} = \left[H_e + H_{\nu\nu} \right] \Psi_f}$$

$$\Delta \equiv \delta m^2 / 2E_\nu$$

$$H_e = \frac{1}{2} \begin{pmatrix} A - \Delta\cos 2\theta & \Delta \sin 2\theta \\ \Delta \sin 2\theta & \Delta \cos 2\theta - A \end{pmatrix}$$

$$H_{\nu\nu} = \frac{1}{2} \begin{pmatrix} B & B_{e\tau} \\ B_{\tau e} & -B \end{pmatrix}$$

The neutrino background term B renders the neutrino flavor transformation problem very nonlinear

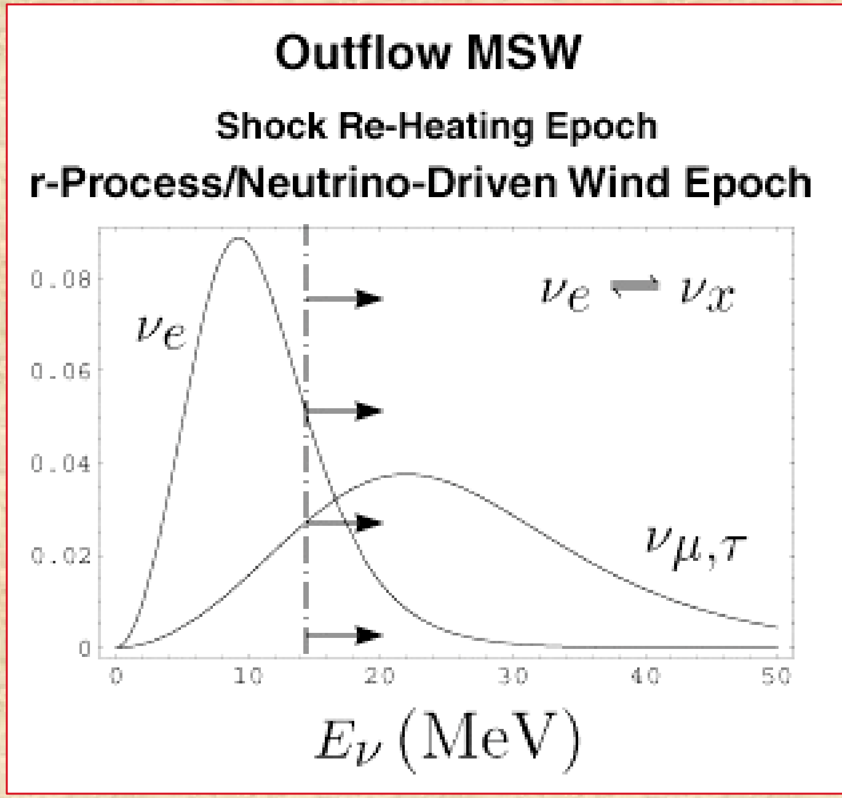
The neutrino background term B can be driven negative, which can cause antineutrino flavor transformation.
For example, note that

$$B \propto \left(n_{\nu_e} - n_{\bar{\nu}_e}\right) - \left(n_{\nu_\tau} - n_{\bar{\nu}_\tau}\right)$$

As a fluid element moves away from the neutron star the resonance energy will sweep from low to high neutrino energy:

$$E_\nu^{RES} \propto \frac{\delta m^2 S^4}{A+B}$$

$A+B$ evolves toward zero via conventional MSW.



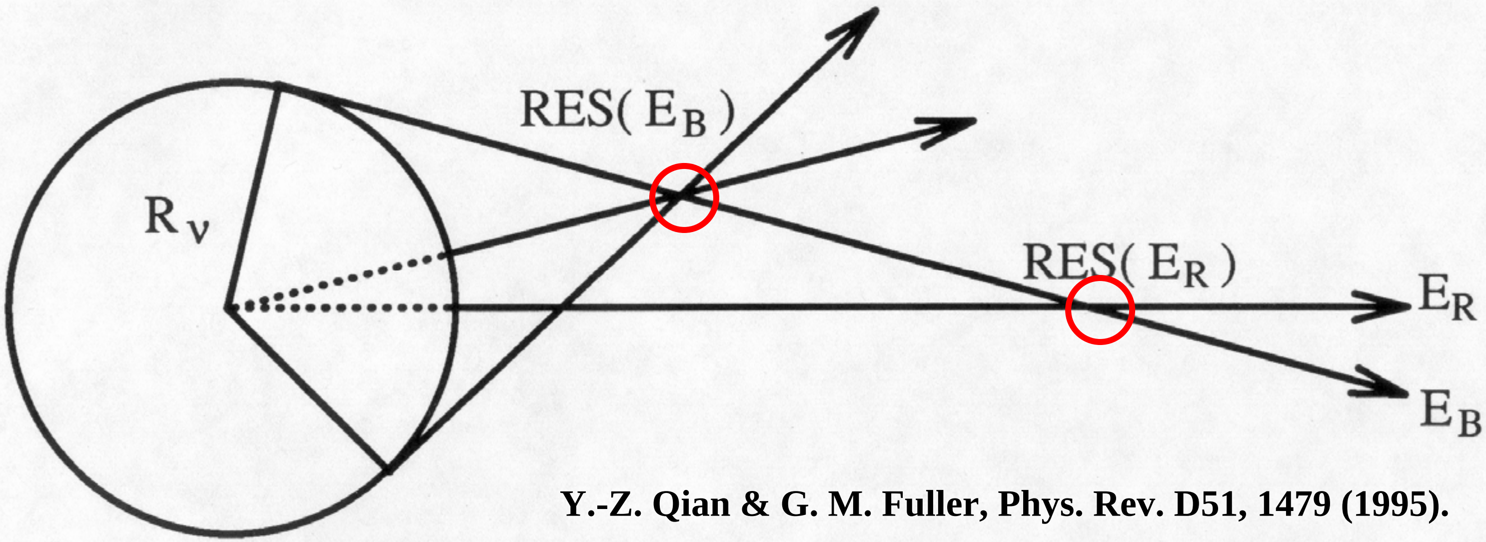
An entropy-per-baryon in excess of S/k~100, plus *A+B*~ 0, can cause neutrino flavor transformation in the hot bubble/r-process environment, possibly even for the atmospheric neutrino mass-squared difference ($3\ 10^{-3}$ eV^2).

$$T_9^{\mathrm{RES}} \approx 1.3 \left(\frac{20\ \mathrm{MeV}}{E_\nu}\right)^{1/3} \left(\frac{0.42}{Y_\mathrm{e}+Y_\nu}\right)^{1/3} \left(\frac{\delta m^2 \cos 2\theta}{3\times10^{-3}\ \mathrm{eV}^2}\right)^{1/3} S_{100}^{1/3}$$

Fuller & Haxton 2004

But what happens when we put in the flavor basis off-diagonal potential ?

The flavor amplitude evolution history of a given neutrino depends on the prior amplitude evolution histories of the background neutrinos which intersect its world line.



Y.-Z. Qian & G. M. Fuller, Phys. Rev. D51, 1479 (1995).

So, this cannot be followed with the simple *one dimensional* mean-field Schroedinger equation shown earlier.
Instead we must face a *computationally daunting problem*, one where the geometric entangling of histories is treated adequately.

Each forward scattering event results in a quantum mechanical "entanglement" of the flavor histories of the two neutrinos!

In light of this, one could legitimately ask about the efficacy of a mean field Schroedinger equation treatment for this problem.

A. Friedland & C. Lunardini PRD 68, 013007 (2003);
JHEP 43, 0310 (2003).

N. Bell, A. Rawlinson, & R. F. Sawyer Phys. Lett. B573, 86 (2003).

J. Hidaka & G.M.F.

Active-Sterile Neutrino Flavor Transformation

and

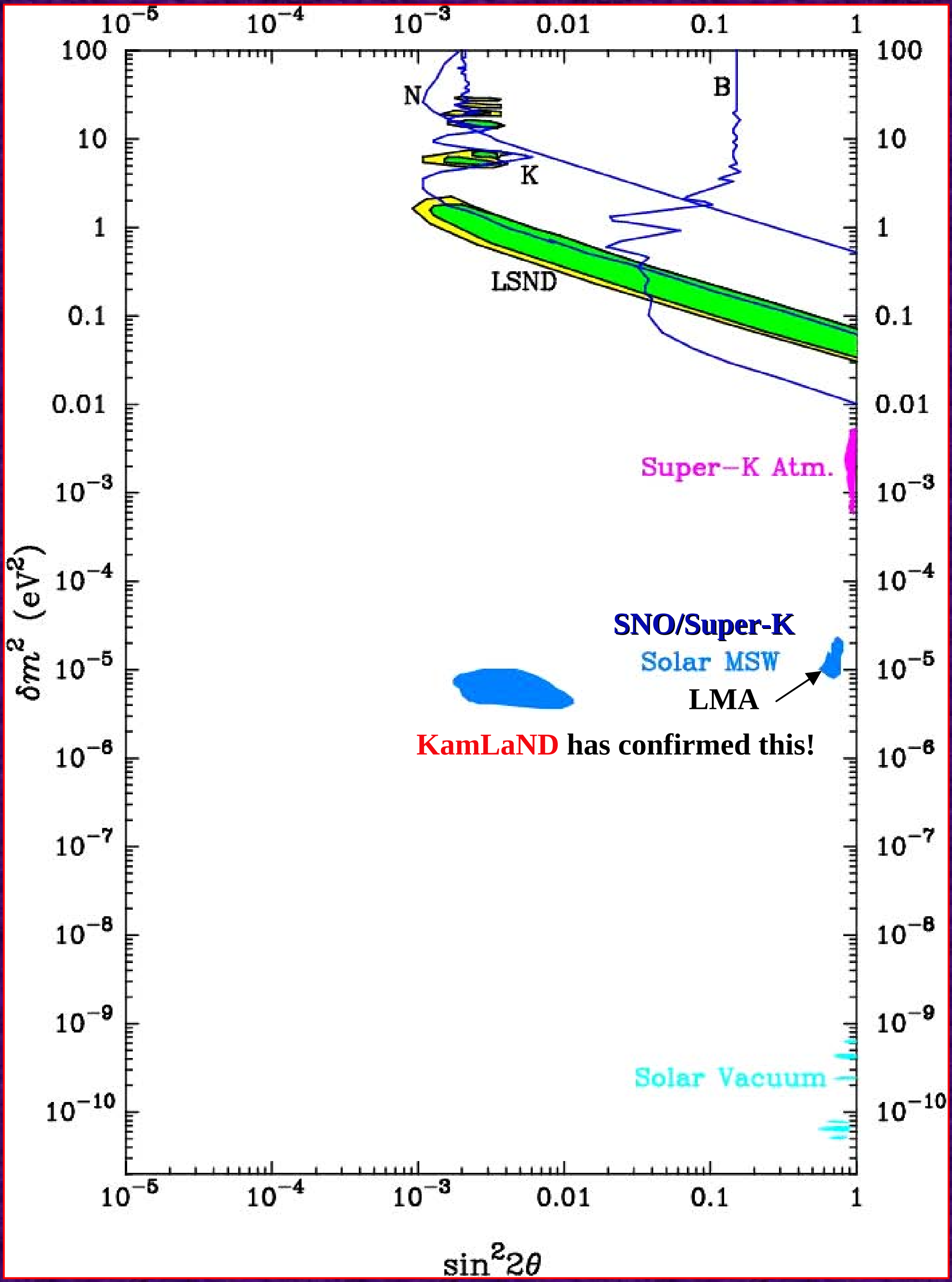
decoherence at high density in both the active-active and active-sterile channels

The Experimental Neutrino Mass/Mixing Plot

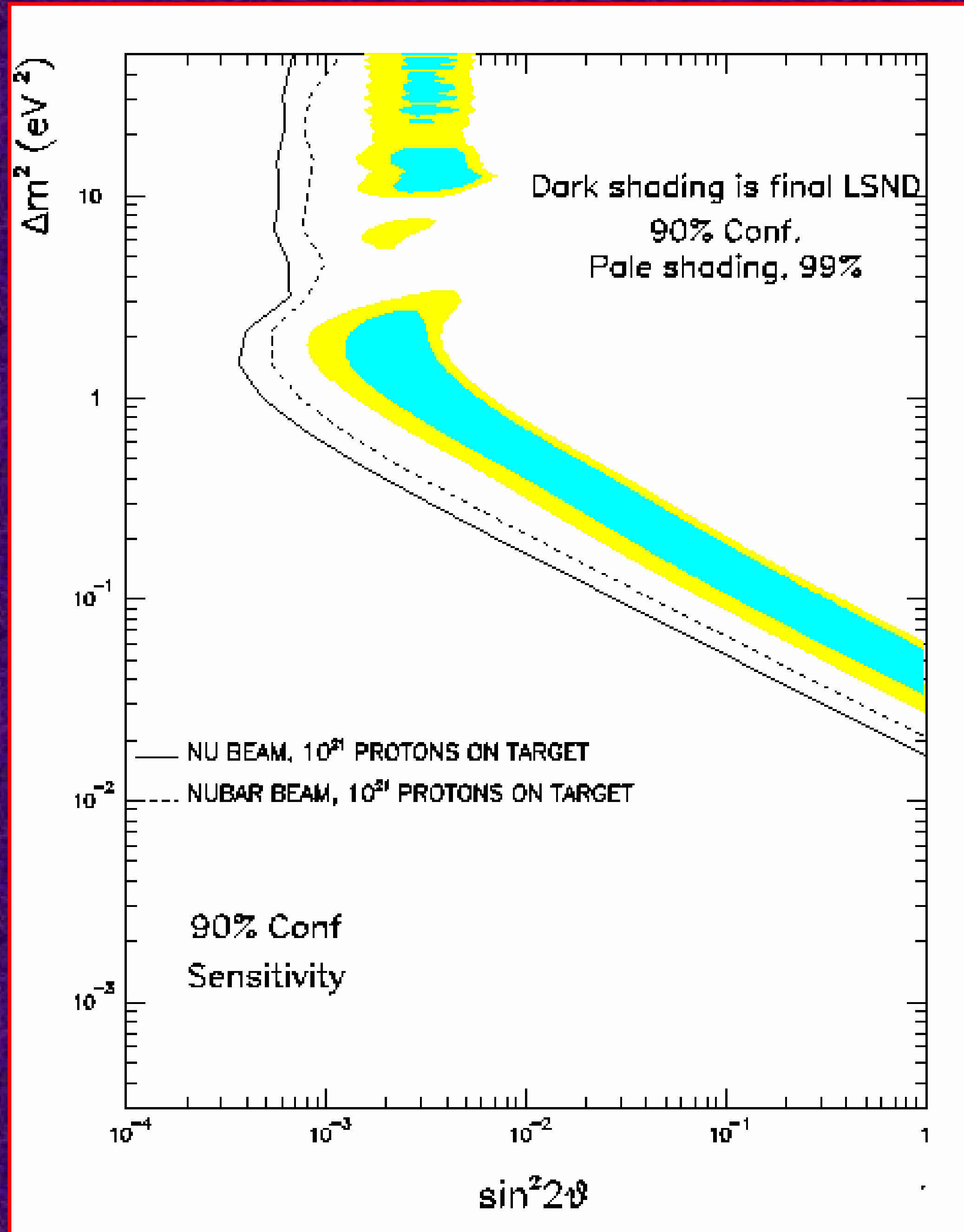
LSND $\nu_\mu \Leftrightarrow \nu_e$

Atmospheric $\nu_\mu \Leftrightarrow \nu_{\tau,s?}$

Solar $\nu_e \Leftrightarrow \nu_{\mu,\tau,s\,?}$



Mini-BooNE explores astrophysically important mixing parameter space well beyond that probed by LSND.



Why Are Neutrinos So Light?

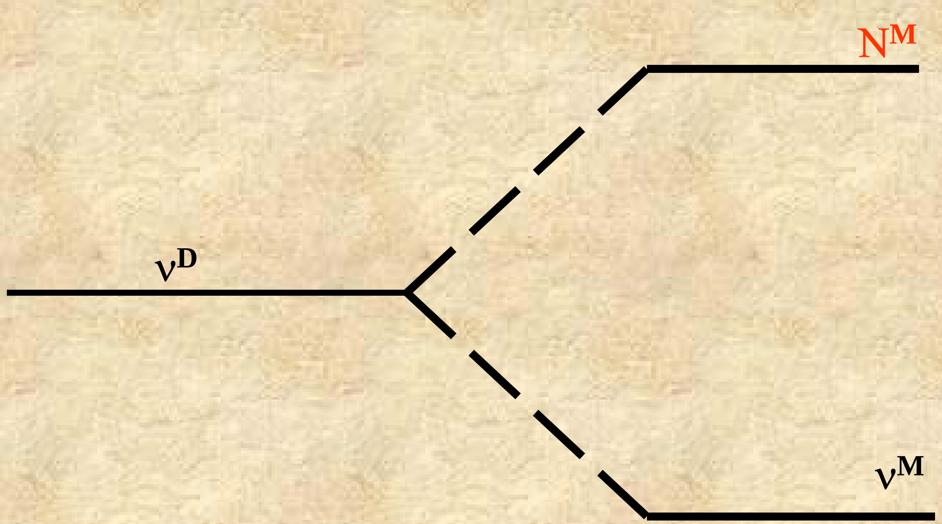
Dirac Neutrinos $\nu \neq \bar{\nu}$ $\nu + \bar{\nu} = 4$ **states**

Majorana Neutrinos $\nu = \bar{\nu}$ $\nu + \bar{\nu} = 2$ **states**

4 states

ν_L, ν_R

$\bar{\nu}_R, \bar{\nu}_L$



2 states ("sterile" Majorana)

$\nu_R, \bar{\nu}_L$

2 states (active Majorana)

$\nu_L, \bar{\nu}_R$

See-Saw Relation for the Product of Neutrino Masses: (m_N)(m_ν) ~ (Really Big Mass Scale)^2

Unification Scale?

Gell-Mann, Ramond, Slansky; Yanagida; Mohapatra & Senjanovic

(after a slide by Boris Kayser)

Sorel, Conrad, Shaevitz hep-ph/0305255 argue that a "3+2" fit is better.

ν_5 $\quad m_2' \approx 4.6\ \text{eV}$

$$U_{e5} \approx 0.07$$
$$\delta m_{51}^2 \approx 21.5\ \text{eV}^2$$

ν_4 $\quad m_1' \sim 1\ eV$

$\nu_1/\nu_2/\nu_3$

"LSND"

$$U_{e4} \approx 0.12$$
$$\delta m_{41}^2 \approx 0.91\ \text{eV}^2$$

A cynic would say that more parameters always make for a better fit, but if there is one light sterile why wouldn't there be others?

A possible neutrino physics solution to the alpha effect problem:

Matter-enhanced active-sterile transformation in the 3+1 scheme coupled to hydrodynamics and weak rates (feedback):

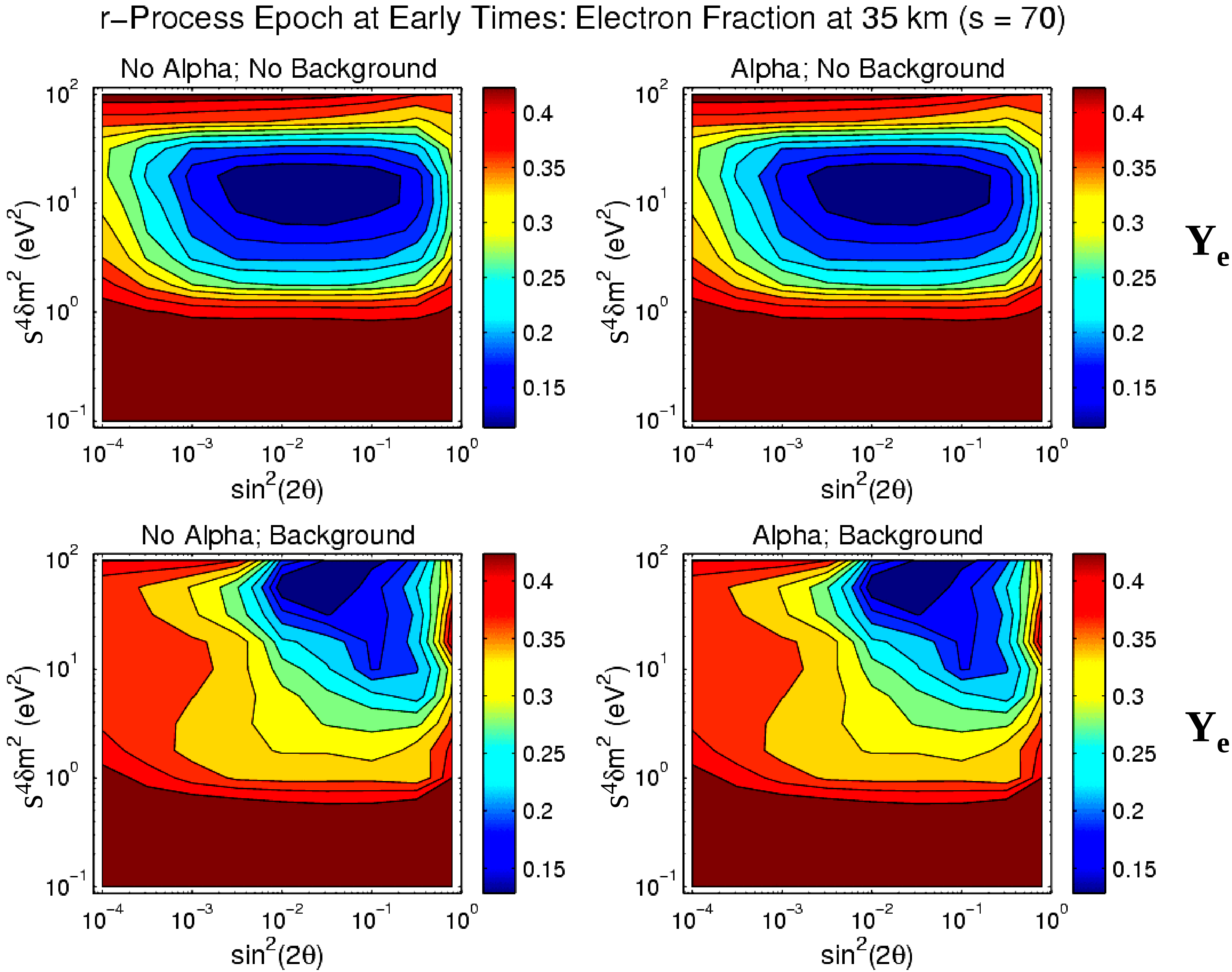
$$\nu_e \leftrightarrow \nu_s \quad \textbf{and} \quad \bar{\nu}_e \leftrightarrow \bar{\nu}_s$$

$$A \propto \left(Y_e - \frac{1}{3} \right)$$

➡ **no alpha effect, extreme neutron excess, fission cycling in r-process nicely tie together abundances of 130 and 195 peaks (a fundamental feature of the observations).**

McLaughlin, Fetter, Balantekin, Fuller, Phys. Rev. C39, 2873 (1999).
Fetter, Mclaughlin, Balantekin, Fuller PRD (2002).

Patel (unpublished, 2001) has considered neutrino background effects.
See also Caldwell, Fuller, & Qian, Phys. Rev. D61, 123005 (2000) for similar “2+2” scheme.



r−Process Epoch at Early Times: Electron Fraction at 35 km (s = 70)
No Alpha; No Background
Alpha; No Background
No Alpha; Background
Alpha; Background
$s^4\delta m^2$ (eV^2)
$\sin^2(2\theta)$
10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0}
10^{-1} 10^{0} 10^{1} 10^{2}
0.4 0.35 0.3 0.25 0.2 0.15
Y_e

Neutrino Burst Signal

- **Detectors: (1.) SuperK/HyperK**
  ? **(2.) SNO**
  (3.) OMNIS

The Figure of Merit:

Supernova rate: 1 per ~30 years

- **Neutrino Oscillation Effects on Burst Signal:**

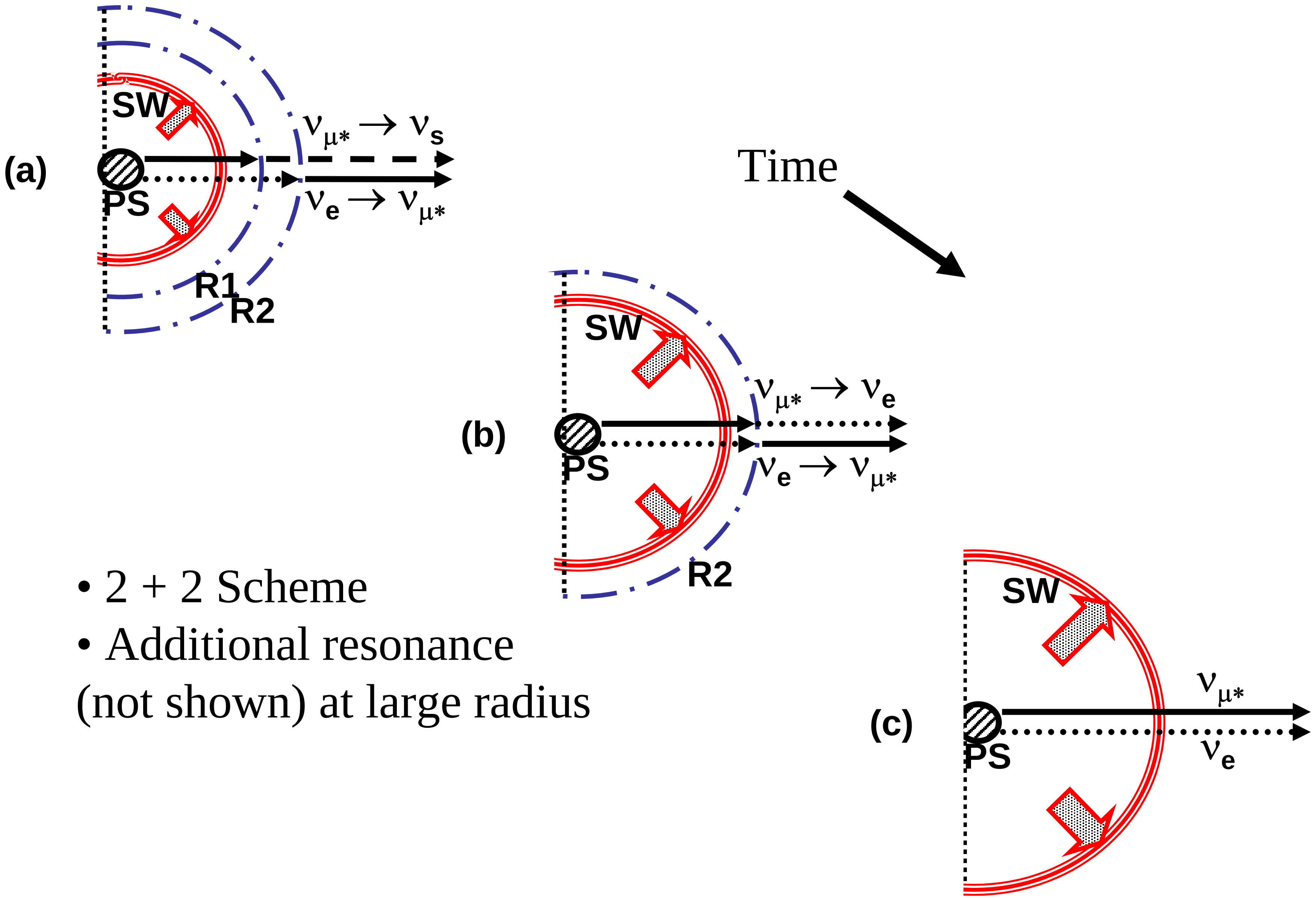
  Dighe & Smirnov 2000; Peres & Smirnov 2001; Dutta et al. 2000;
  Barger, Marfatia, and Wood 2001; Kachelriess, Strumia, Valle 2002;
  Fuller, Haxton, McLaughlin 1999.

- **Effects of the Shock on Neutrino Flavor Conversion and Signal:**

  R. C. Schirato & G. M. Fuller, astro-ph/0205390.

  C. Lunardini & A. Yu. Smirnov, hep-ph/0302033;
  K. Takahashi, K. Sato, H. E. Dalhed, J. R Wilson, astro-ph/0212195

Shock Propagation across Resonances



- 2 + 2 Scheme
- Additional resonance
(not shown) at large radius

MSW Resonances in the Region Above the Neutron Star

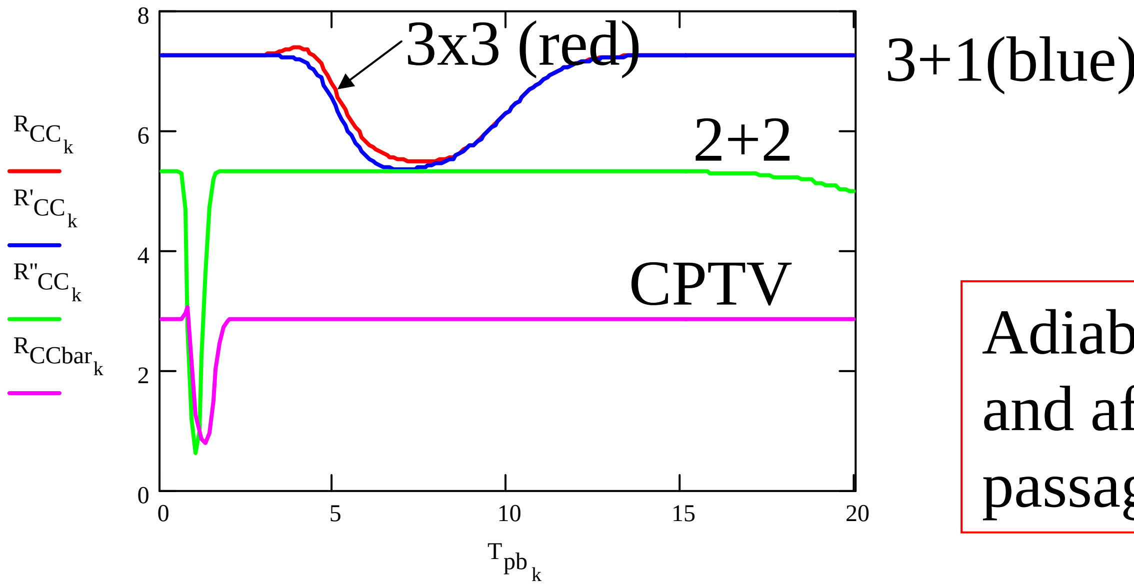
ρ

$\sin^2 2\theta_M$

R. Schirato & G. M. Fuller 2003

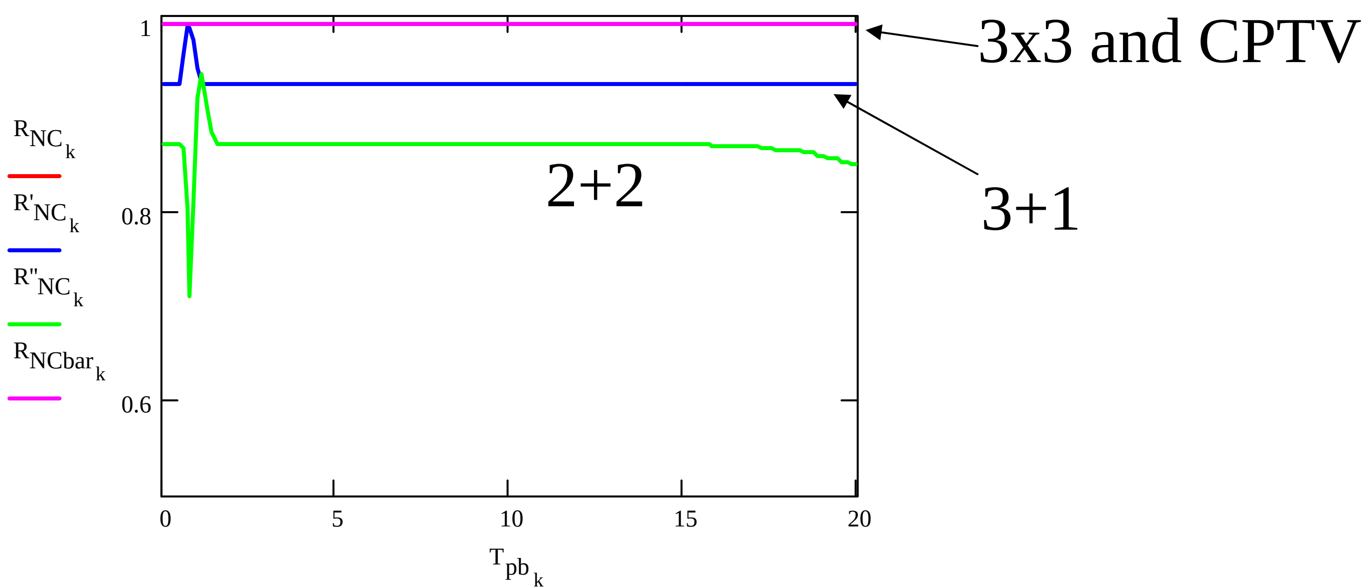
Neutrino Burst Signals

Charged Current Ratios



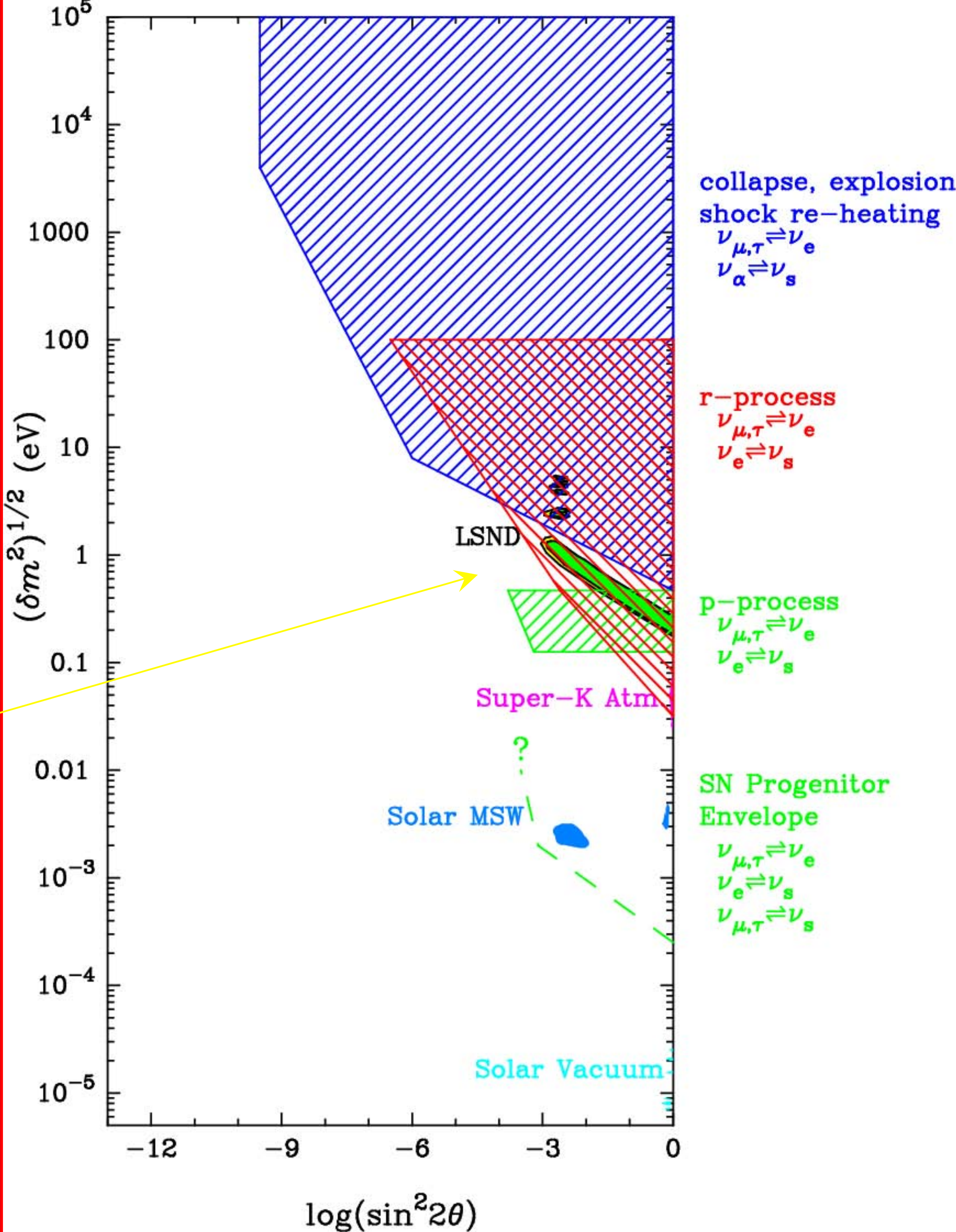
Adiabatic before and after shock passage

Neutral Current Ratios



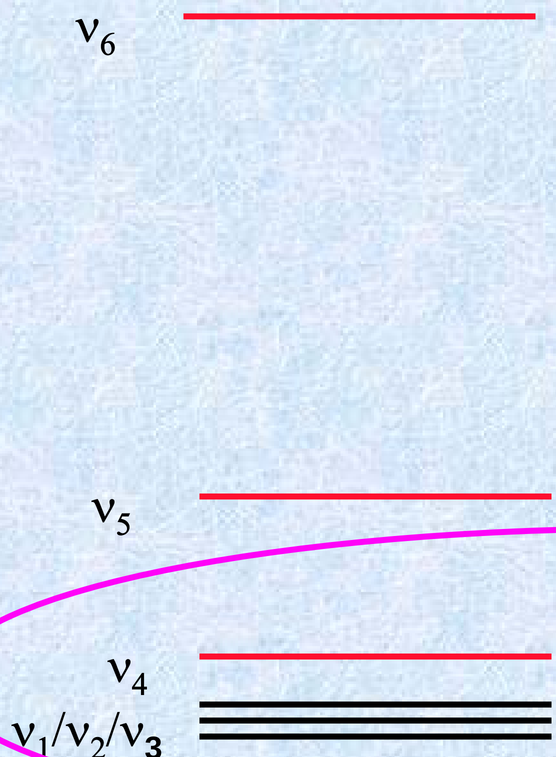
Neutrino Mass/Mixing Parameters which imply *CONSEQUENCES* For Supernova Physics

Fixed Point (active-active)



Abazajian & Fuller 2004

What if there is a "light" sterile? Are there others?



$m_3' \approx 32keV \quad \sin^2 2\theta \approx 10^{-12}$

$m_2' \approx 4keV \quad \sin^2 2\theta \approx 10^{-10}$

$m_1' \approx 1eV$ } **"LSND"** $\sin^2 2\theta \approx 10^{-3}$

A guess at masses for lightest neutrinos (mostly active)

$$\left\{\begin{array}{l} m_1 \approx 2\times 10^{-6} eV \\ m_2 \approx 7.7\times 10^{-3} eV \\ m_3 \approx 6.3\times 10^{-2} eV \end{array}\right.$$

?

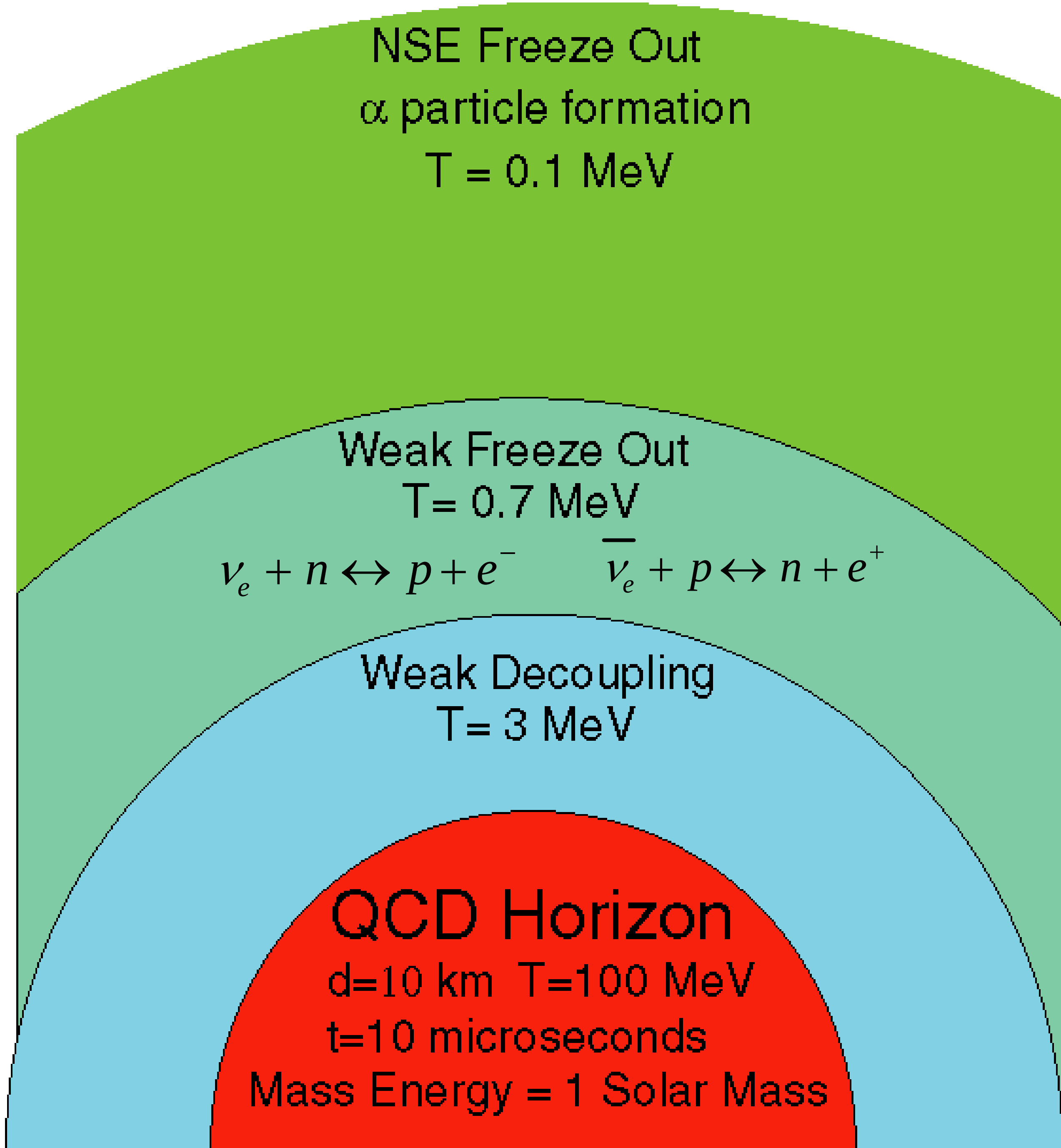
In both Supernova Cores and the Early Universe a key problem is how a system of active and/or sterile neutrinos coupled only via vacuum mass terms (vacuum mixing) evolves.

Follow the Boltzmann evolution of a gas of initially all active neutrinos whose effective masses and couplings with steriles are determined by scattering processes. A neutrino will propagate coherently (matter-affected neutrino oscillations with a sterile species) until a scattering event. The scattering process is like a "measurement," collapsing the neutrino's wave function, wherein there is a small probability that a sterile neutrino results.

- **Population of singlet ('sterile') sea depends on quantum decoherence processes which are complicated. (for recent work see Bell, Sawyer, Volkas, quant-phys/0106082)**
- **Wide ranges of singlet masses affect the physics of supernova cores and the early universe (not just "LSND"-inspired masses).**
- **See recent work by Friedland, Lunardini, Sawyer**

The History of
The Early Universe:

(shown are a succession of temperature and causal horizon scales)



The QCD horizon is essentially an ultra-high entropy Neutron Star

Consider active-sterile neutrino mixing:

in vacuum

$$|\nu_\alpha\rangle = \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle$$
$$|\nu_s\rangle = -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle$$

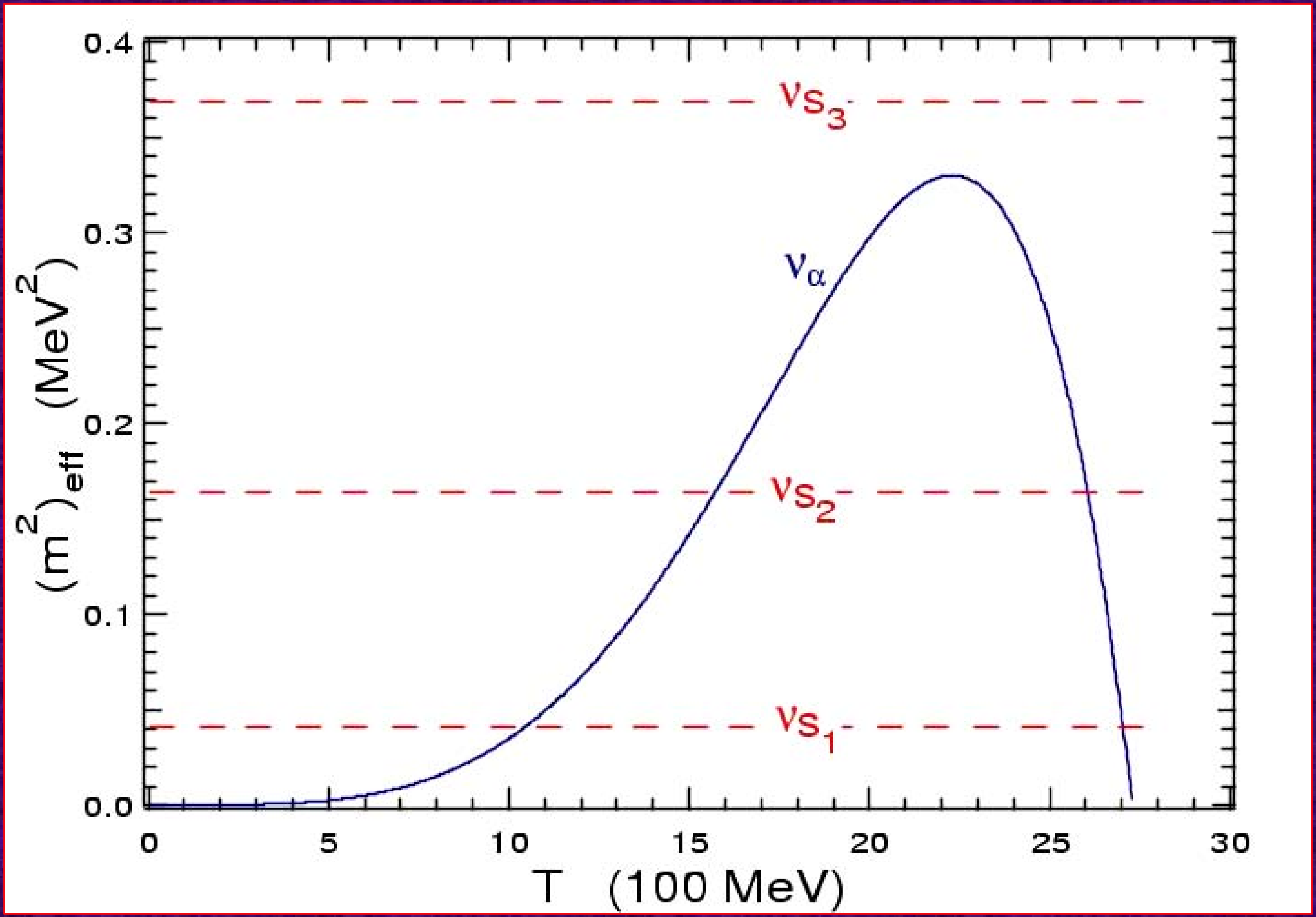
here α = e,μ,τ

in "medium," in the neutron star or early universe

$$|\nu_\alpha\rangle = \cos\theta_M|\nu_1\rangle + \sin\theta_M|\nu_2\rangle$$
$$|\nu_s\rangle = -\sin\theta_M|\nu_1\rangle + \cos\theta_M|\nu_2\rangle$$

See for example
Abazajian, Fuller, Patel, Phys. Rev. D64, 023501 (2001).

Active neutrinos ν_α could have two *level crossings* with **singlet states (sterile neutrinos) ν_s** in the early universe.



Effective "matter" mixing angle

$$\sin^2 2\theta_M \approx \frac{\tan^2 2\theta}{\left(1 - E_\nu / E_R\right)^2 + \tan^2 2\theta}$$

Where the resonance energy E_R is related to the forward scattering potential V through

$$E_R = \frac{\delta m^2 \cos 2\theta}{2V}$$

Evolution of the "sterile" and active neutrino distribution functions given by the Boltzmann equation:

$\alpha = e, \mu, \tau$

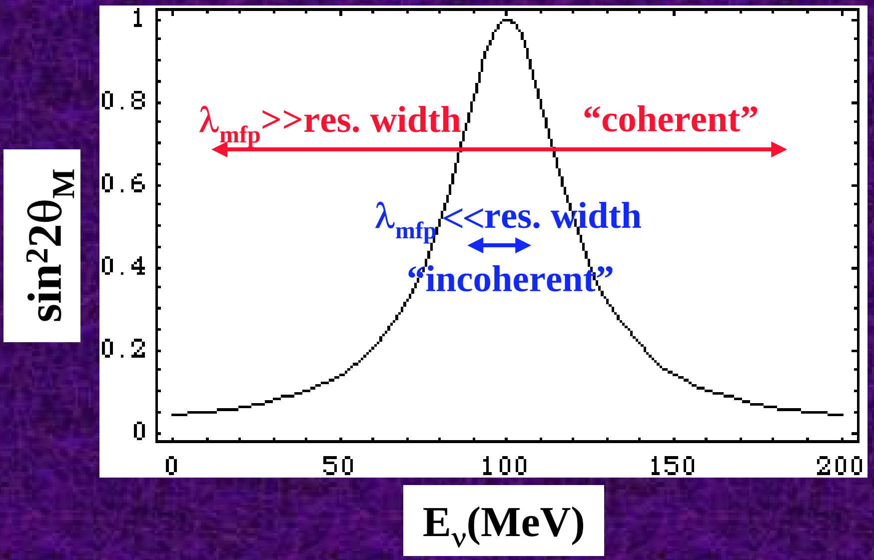
$$\frac{\partial}{\partial t} f_s(p,t) - Hp\frac{\partial}{\partial p} f_s(p,t) \approx \Gamma(\nu_\alpha \to \nu_s; p,t)\left[f_\alpha(p,t) - f_s(p,t)\right]$$

active neutrino scattering rate

Sterile neutrino production rate

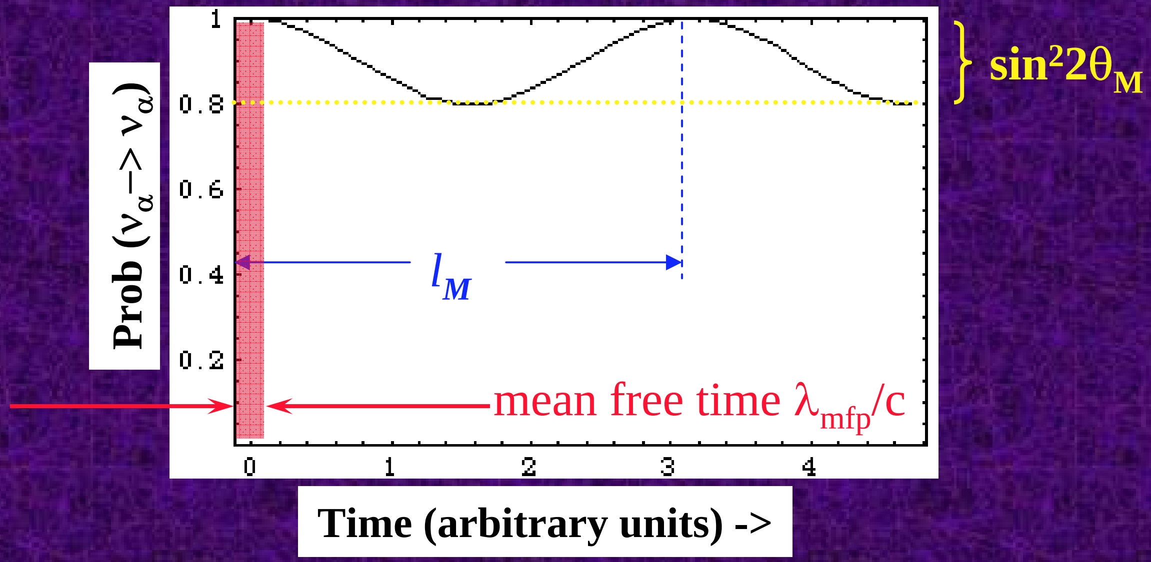
$$\approx \tfrac{1}{2}\Gamma_\alpha(p)\sin^2 2\theta_M \left[1 + \left(\tfrac{1}{2}\Gamma_\alpha(p)\, l_M\right)^2\right]^{-1}$$

$$\left|\nu_\alpha\right\rangle = \cos\theta_M \left|\nu_1\right\rangle + \sin\theta_M \left|\nu_2\right\rangle$$
$$\left|\nu_s\right\rangle = -\sin\theta_M \left|\nu_1\right\rangle + \cos\theta_M \left|\nu_2\right\rangle$$

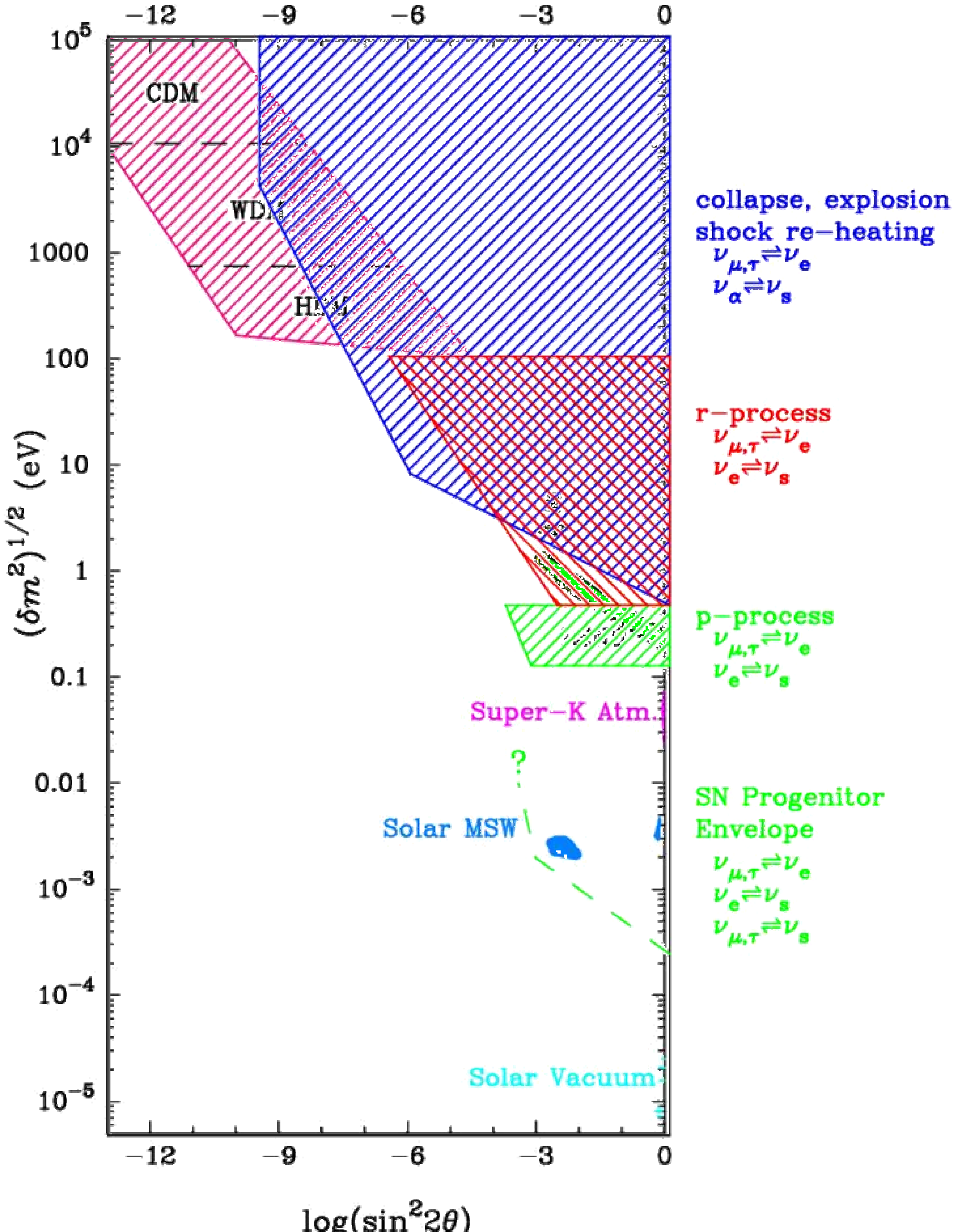


Quantum Zeno Effect

Phase in the neutrino oscillation begins to develop after a collision, but is reset at the next collision. If the mean free path λ_{mfp} is short compared to the oscillation length l_M, the probability of sterile neutrino production is suppressed.



Dark Matter $\nu_\alpha \rightleftharpoons \nu_s$ (Abazajian, Fuller, & Patel 2001)



What if there are heavier "sterile" (singlet) states which have very small vacuum mixings with active species?

$$\left|\nu_\alpha\right\rangle = \cos\theta\left|\nu_1\right\rangle + \sin\theta\left|\nu_2\right\rangle$$

$$\left|\nu_s\right\rangle = -\sin\theta\left|\nu_1\right\rangle + \cos\theta\left|\nu_2\right\rangle$$

where, for example, m_2 = 1 keV to 100 keV
and where $\sin^2 2\theta \le 10^{-10}$

Every time an active neutrino scatters in the early universe there is a very very small probability that the neutrino "scatters" into a "sterile" state. This process can be matter-enhanced as well.
Abazajian, Fuller, Patel, Phys. Rev. D64, 023501 (2001) follow the Boltzmann evolution of a system of active neutrinos to calculate the relic singlet neutrino density.

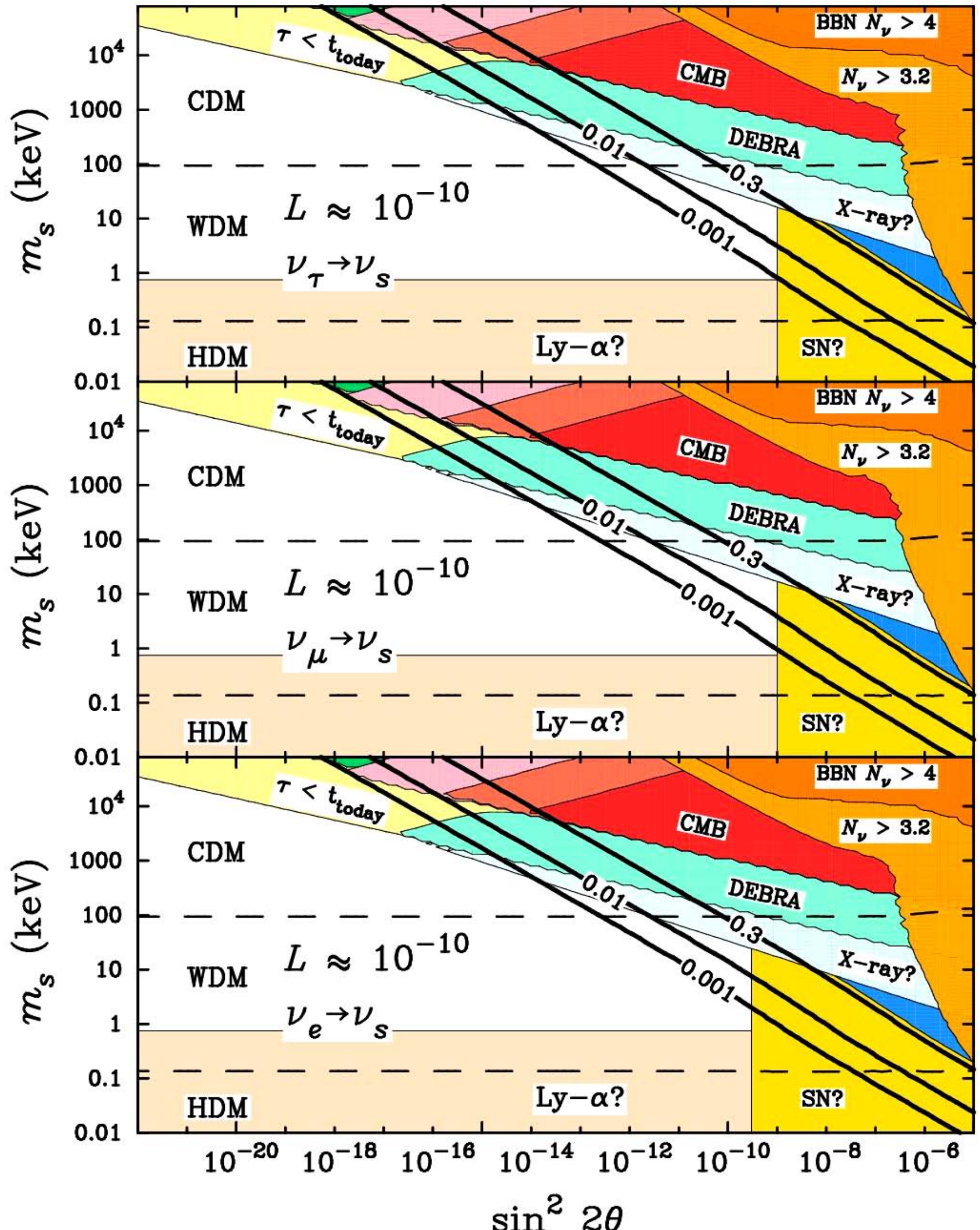
Singlet “Sterile” Neutrino Dark Matter

- **Scattering-dominated, matter-suppressed production**
  S. Dodelson & L. M. Widrow, Phys. Rev. Lett. 72, 17 (1994).

- **Matter-enhanced (resonant) production**
  X. Shi & G. M. Fuller, Phys. Rev. Lett. 82, 2832 (1999).

- **Re-look at matter-suppressed production**
  A. D. Dolgov and S. Hansen, Astropart. Phys. 16, 339 (2002).
  hep-ph/0009083.

- **Matter-enhanced (resonant) plus matter-suppressed production**
  K. Abazajian, G. M. Fuller, M. Patel, Phys. Rev. D64, 023501 (2001).
  astro-ph/0101524



BBN $N_\nu > 4$
$N_\nu > 3.2$
$\tau < t_{today}$
CMB
DEBRA
X-ray?
CDM
WDM
HDM
$L \approx 10^{-10}$
$\nu_\tau \to \nu_s$
$\nu_\mu \to \nu_s$
$\nu_e \to \nu_s$
0.01
0.3
0.001
Ly−α?
SN?
m_s (keV)
$\sin^2 2\theta$

Radiative decay graphs for heavy singlets.
The final state neutrino and the photon
equally share the rest mass energy of the singlet.

Singlet Neutrino Radiative Decay Rate

$$\Gamma_\gamma \approx \frac{\alpha G_F^2}{64\pi^4} m_2^5 \left[\sum_\beta U_{1\beta} U_{2\beta} F(r_\beta)\right]^2$$

$$\approx 6.8 \times 10^{-33} s \left(\frac{\sin^2 2\theta}{10^{-10}}\right)\left(\frac{m_s}{keV}\right)^5$$

no GIM suppression for sterile neutrinos

$$F(r_\beta) \approx -\tfrac{3}{2} + \tfrac{3}{4} r_\beta$$

$$r_\beta = \left(M_\beta^{lep} / M_W\right)^2$$

Abazajian, Fuller, & Tucker, astro-ph/0106002 considered the radiative decays of heavy singlets and possible x-ray constraints.

XMM-Newton and Chandra have greatest sensitivity for photons with energies between about 1 keV to 10 keV, serendipitously coincident with the expected photon energies from decaying WDM/CDM singlets.

Typical singlet lifetimes against radiative decay are some $\sim 10^{16}$ Hubble times! However, if singlets are the dark matter, then in a typical cluster of galaxies there could be $\sim 10^{79}$ of these particles.

This could allow x-ray observatories to probe physics at interaction strengths some 10-14 orders of magnitude smaller than the Weak Interaction.

X-Ray Constraints on Decaying Singlet Neutrinos

- K. Abazajian, G. M. Fuller, W. H. Tucker, astro-ph/0106002
  "Direct Detection of Warm Dark Matter in the X-Ray"
  Astrophys. J., 562, 593-604 (2001).

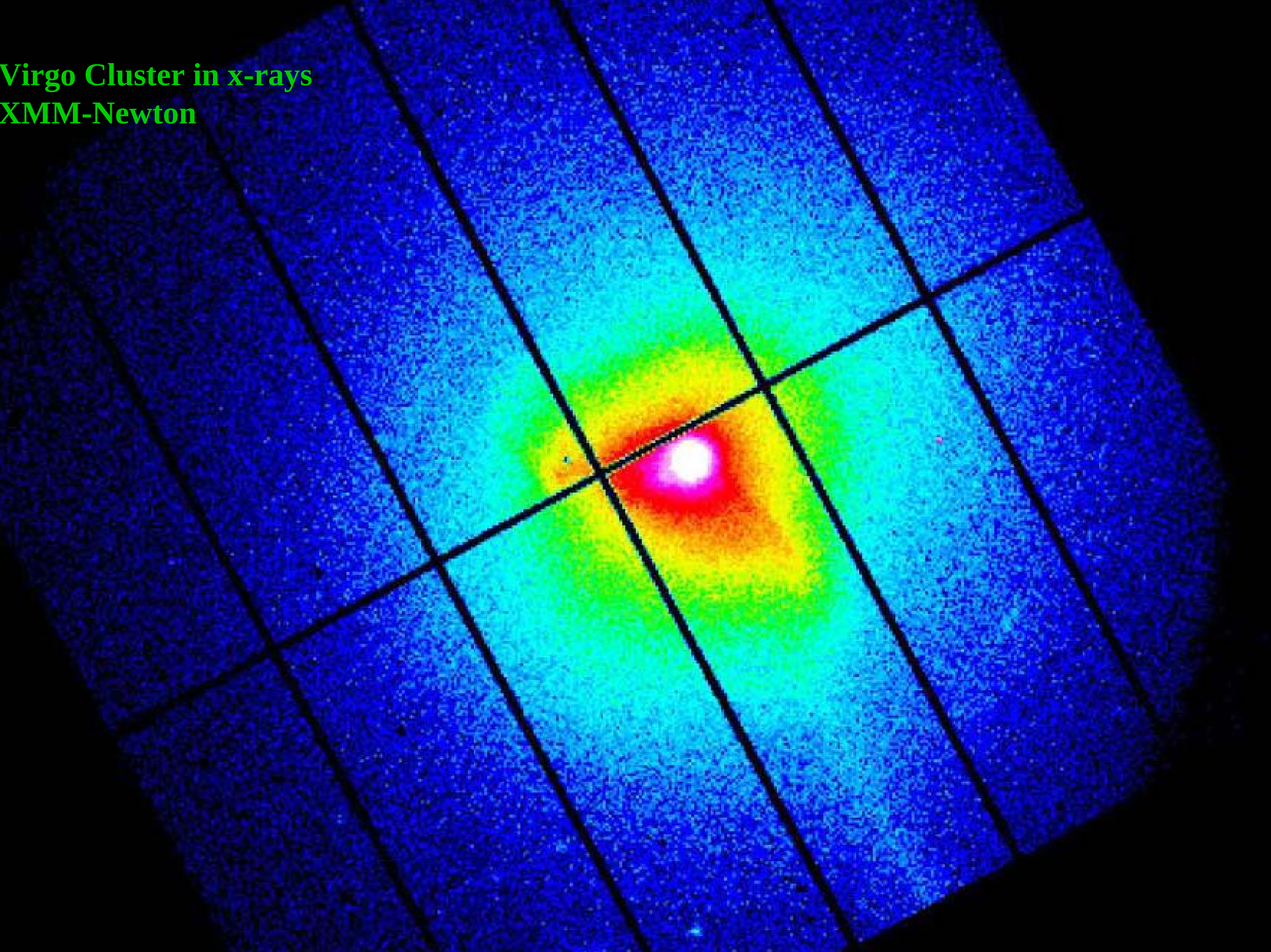
  pointed out serendipitous coincidence
  between x-ray detector technology
  and Dark Matter particle mass;
  suggested looking in clusters of galaxies, field galaxies

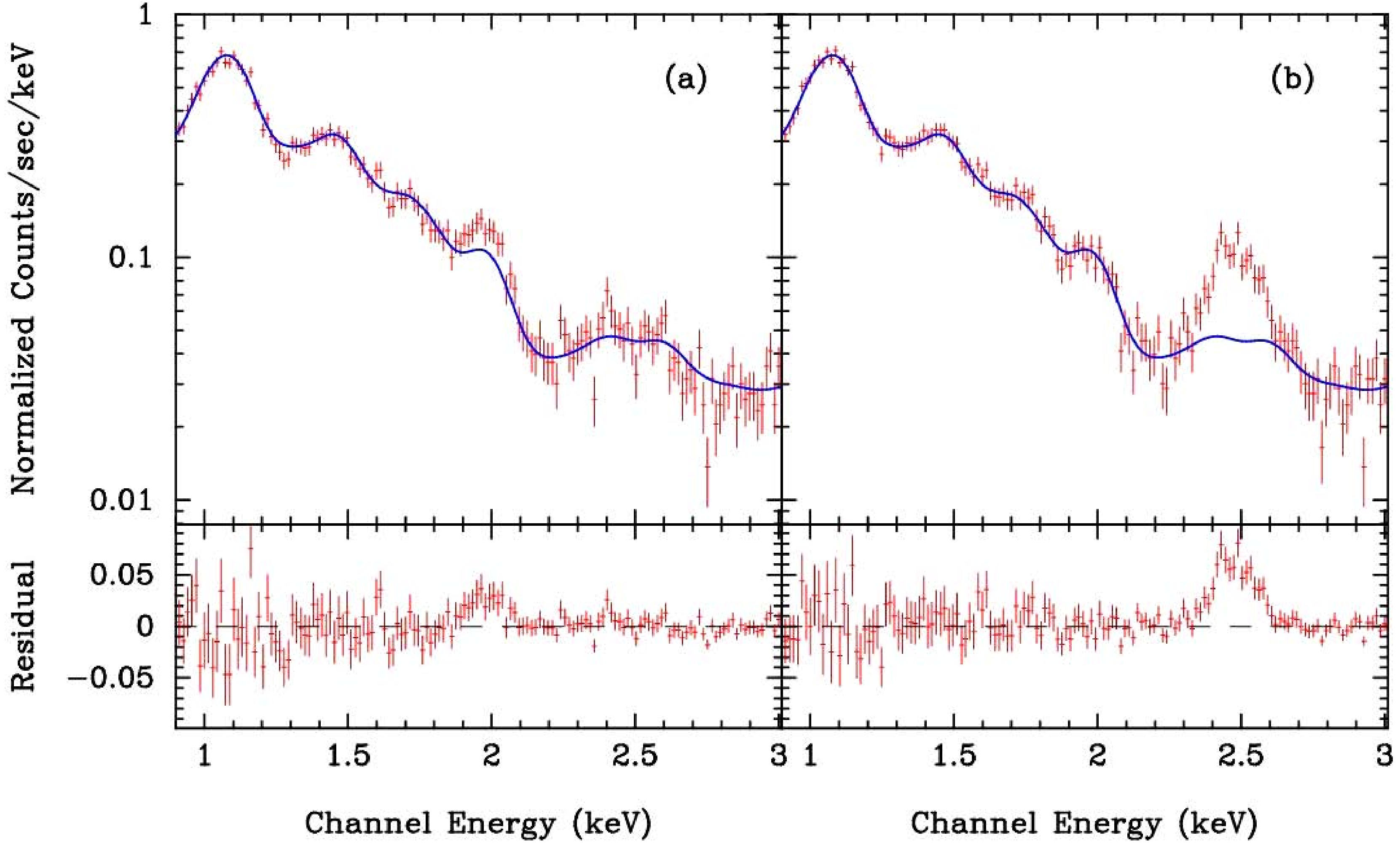
- S. Hansen, J. Lesgourgues, S. Pastor, J. Silk, astro-ph/0106108
  "Constraining the Window on Sterile Neutrinos as Warm Dark Matter"
  MNRAS 333, 544 (2002).

  suggested looking in Dark Matter "blobs,"
  refined limits with Colombi et al. energy spectra considerations

Chandra X-Ray Observatory

Virgo Cluster in x-rays
XMM-Newton

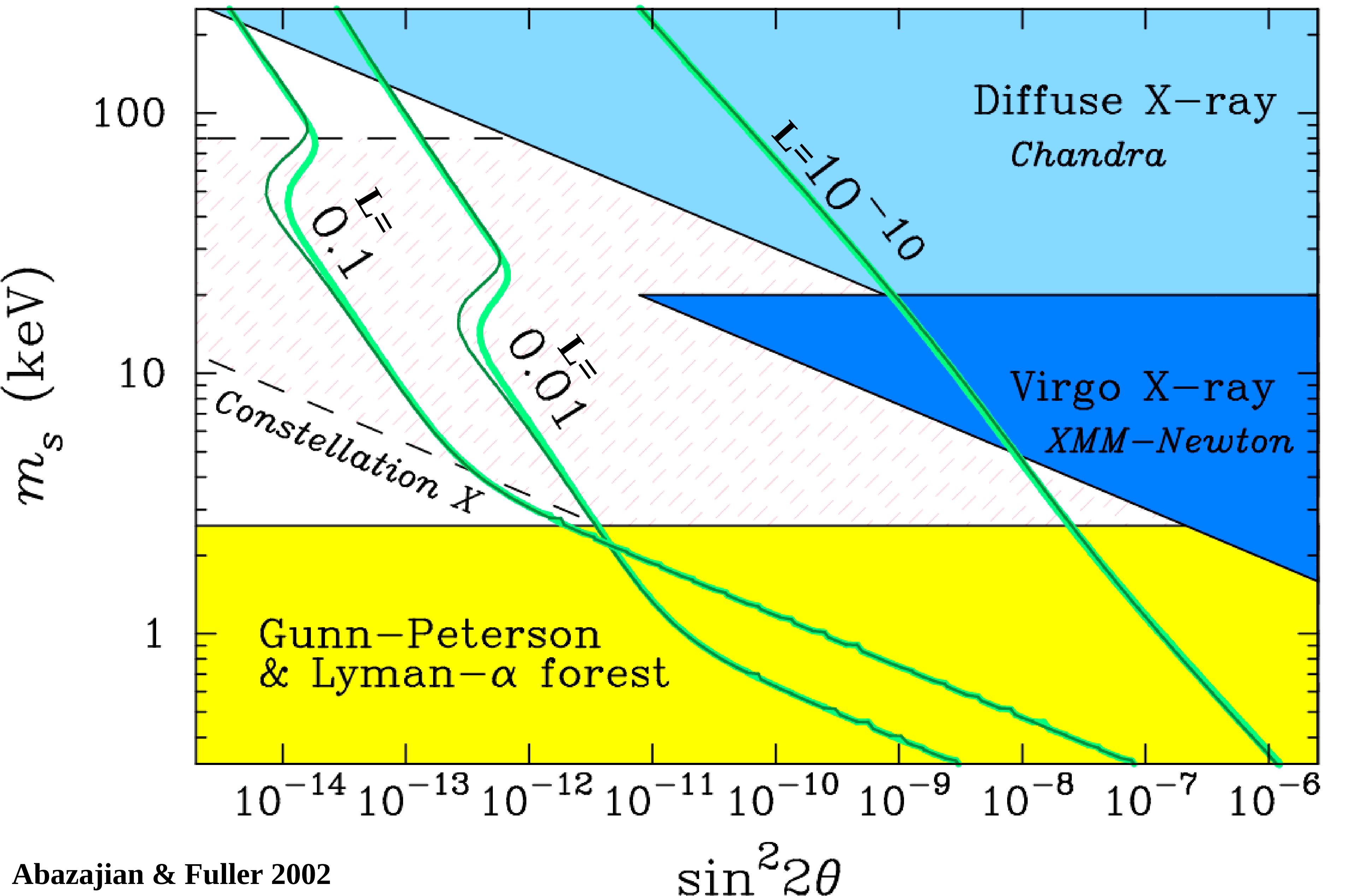




Synthetic spectra for the Virgo cluster when the Dark Matter is composed of singlets with rest mass (b) m_s=5 keV and (a) m_s=4 keV.

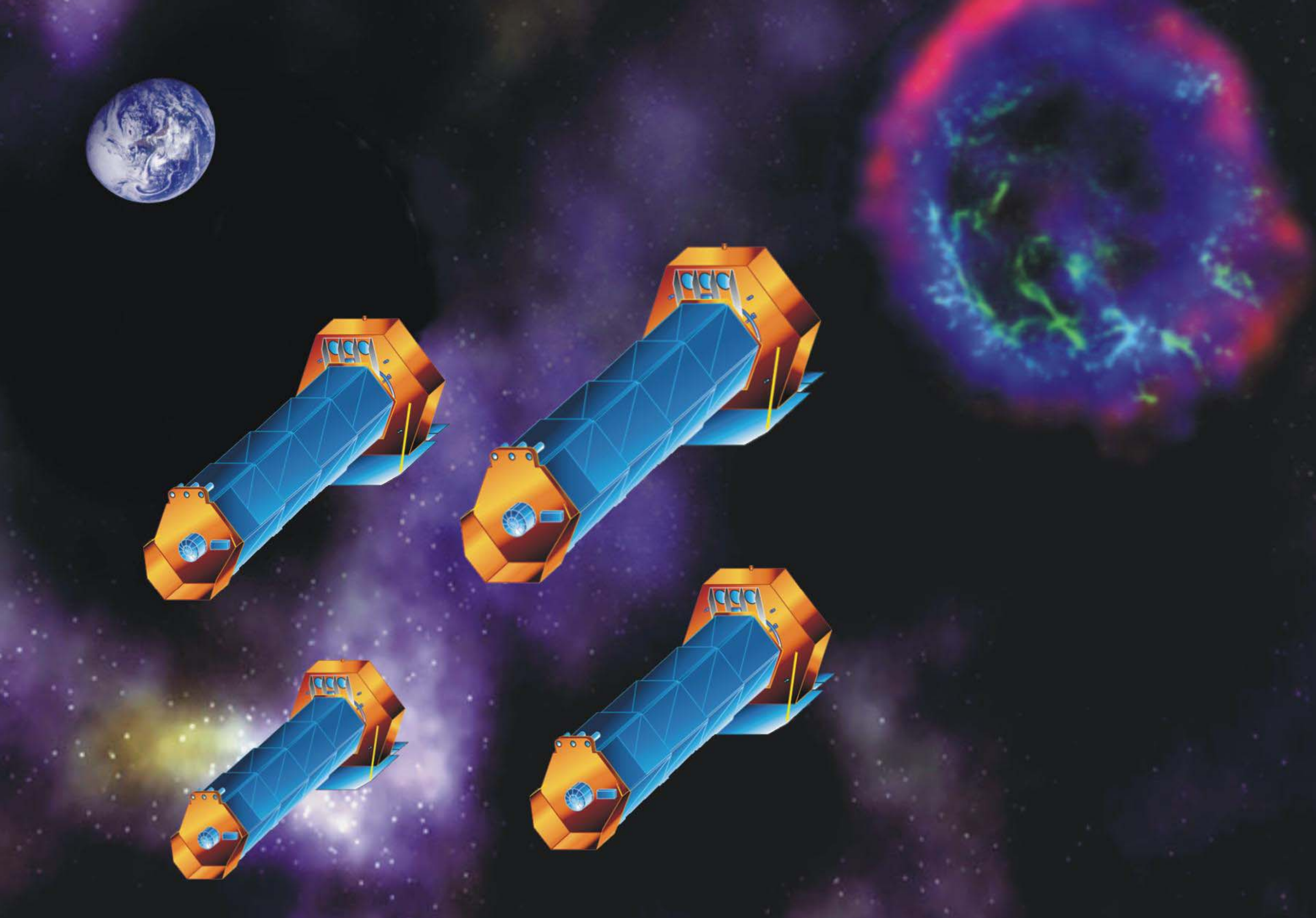
Abazajian, Fuller, and Tucker 2001

Contours of Singlet Neutrino Relic densities giving Ω_s=0.3 for various Lepton Numbers L



Abazajian & Fuller 2002

Constellation X



These Dark Matter candidate sterile neutrinos would be produced in the core of the supernova and could impact neutrino energy transport physics in the core.

Fuller, Kusenko, Mocioiu, Pascoli PRD 68, 103002 (2004)
Have shown that these neutrinos can produce
large kicks, and can give neutron star velocities
~ 1000 km s^{-1}

Conclusions

Core Collapse Events represent the ultimate neutrino physics "laboratories."

. . . If only we could decipher the nucleosynthesis fossil record
and/or record a detailed neutrino burst signal . . .

- Better understanding of neutrino mass/mixing physics may help shed light on r-process nucleosynthesis and the mechanism of shock revival and, just possibly, *vice versa*.

- We need to incorporate known active-active neutrino mixing physics in core collapse/explosion simulations.

  A positive result in the mini-BooNE experiment would call into question much of what we hold dear in our model of the neutrino/energy transport in core collapse supernovae.

Conclusions continued . . .

- A positive signal in the mini-BooNE experiment has profound implications for the origin of neutrino mass and primordial lepton number

- X-Ray Observatories are probably our best (only?) probes of the singlet neutrino mass/mixing spectrum if singlet masses are in the 1 keV to 1 MeV range and vacuum mixings with active neutrinos are in the range of one part in $\sim 10^{12}$ or lower.