

Gravity and Strings

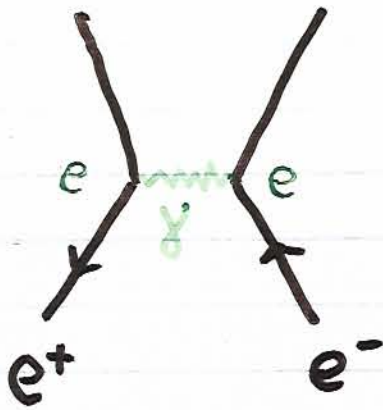
String theory = 2-3 quarters at UKSE

⇒ impressionistic view of some feature
introducing important ideas

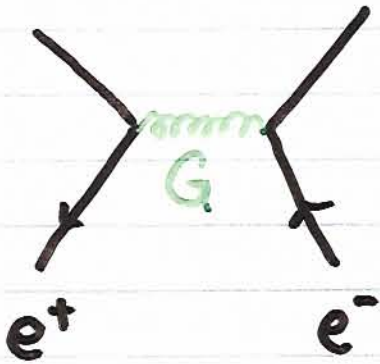
Question of the day:

Why is gravity so weak?

e.g.



$\propto e^2 = \alpha$ ($\hbar = c = 1$)

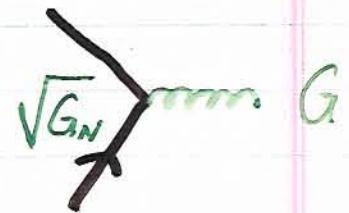


(*)

$$\int d^4x \sqrt{-g} \left[\frac{R}{G_N} + \mathcal{L}(\psi_e) \right]$$

expand: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
metric

$\leadsto \int d^4x \left(h^{\mu\nu} \square_{\kappa} h_{\mu\nu} + \sqrt{G_N} h^{\mu\nu} T_{\mu\nu} + \dots \right)$
h³, etc.



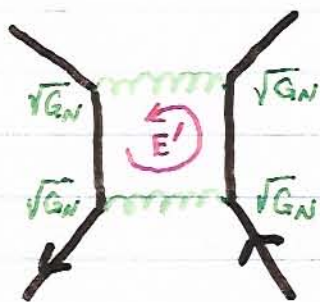
$[G_N] = M^{-2} \leadsto G_N = M_{\text{Planck}}^{-2}$ (M_p^{-2+n} w/extra dim)

$\Rightarrow (*) \propto G_N E^2$; small if $E \ll M_p \sim 10^{19} \text{ Ge}$

Why is energy so low?

Quantum corrections

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$$G_N^2 \int dE' E'^3 = \infty$$

dim analysis

More G_N 's \Rightarrow worse divergences

"Non-renormalizable" (non-predictive)

$$\leftrightarrow [G_N] = M^p \quad p < 0$$

Compare 4-fermi:

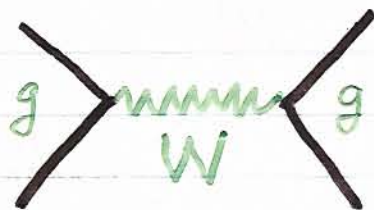


$$\mathcal{L}_4 \sim G_F J_W^\mu J_{W\mu} ; [G_F] = M^{-2}$$

But:

$$\mathcal{L}_4 \sim \frac{g^2}{M_W^2} J_W^\mu J_{W\mu}$$

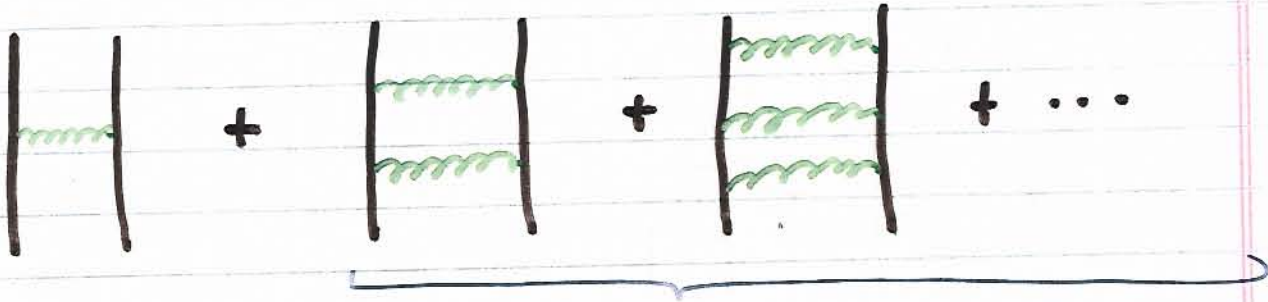
from



Renormalizable, predictive

Why do we care about predictivity? 4

1. High energy scattering



Important at $E \gtrsim M_p$

→ **Black hole formation**

(maybe $M_p \sim \text{TeV} ???$ Large/warped dims)

2. Cosmology: early on, $E_{\text{typical}} \gtrsim M_p$

⇒ need for initial conditions

3. Black holes: what happens

- inside

- when Hawking evaporate

4. Dark energy ??

5. Boundary condition for lower energy physics

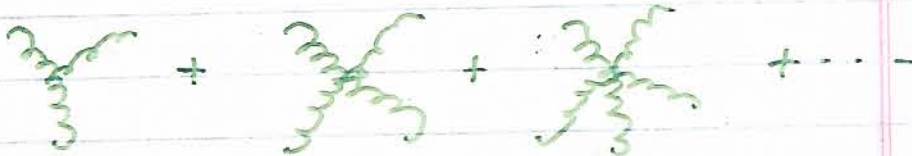
→ **Standard Model**

+ more ...

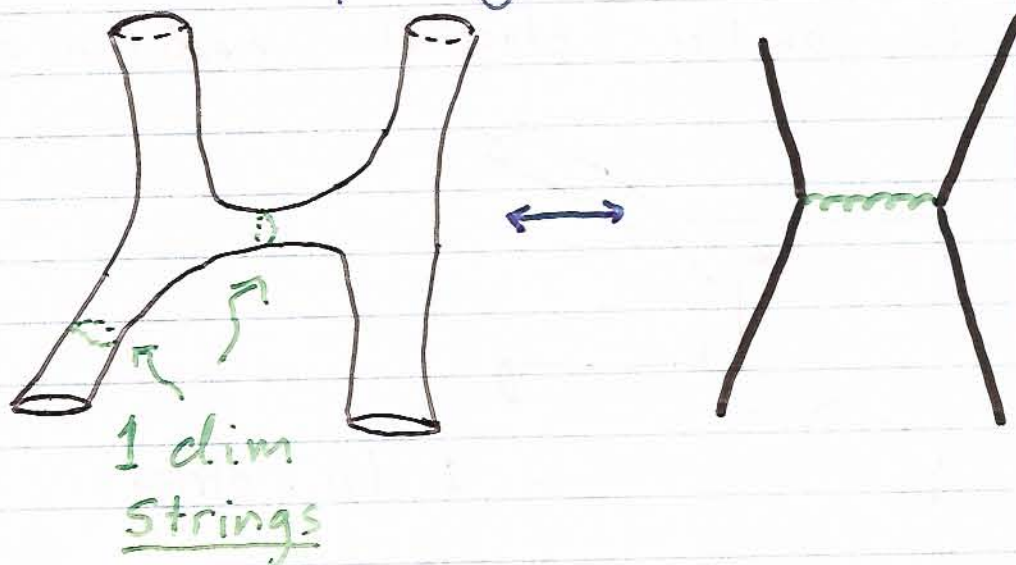
Gravity?

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No convincing underlying field theory ...

note 

But a more surprising classical modification:

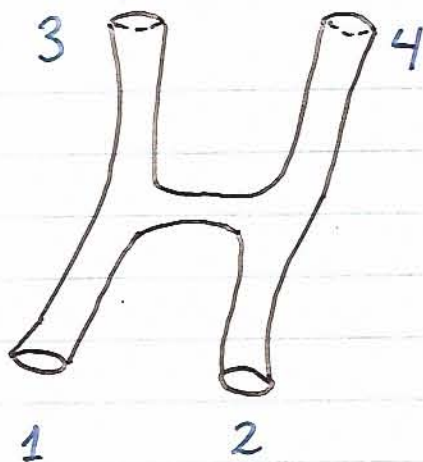


$$\text{Amplitude} \sim \int \mathcal{D}(\text{worldsheets}) e^{i \text{Area}}$$

(analogous to QFT via Schwinger)

How do we know this gives gravity?

Compute:



$$A_{4\pi} = \int \mathcal{G}[WS] e^{i \text{Area}}$$

$$= \frac{\Gamma\left(-\frac{\alpha' s}{4} - 1\right) \Gamma\left(-\frac{\alpha' t}{4} - 1\right) \Gamma\left(-\frac{\alpha' u}{4} - 1\right)}{\Gamma\left(-\frac{\alpha'(s+t)}{4} - 2\right) \Gamma\left(-\frac{\alpha'(t+u)}{4} - 2\right) \Gamma\left(-\frac{\alpha'(s+u)}{4} - 2\right)}$$

"Virasoro - Shapiro"

$$s = -(p_1 + p_2)^2 \quad t = -(p_1 + p_3)^2 \quad u = -(p_1 + p_4)^2$$

$$[\alpha'] = M^{-2} \quad \alpha' = M_{\text{string}}^{-2}$$

resonances (poles) at :

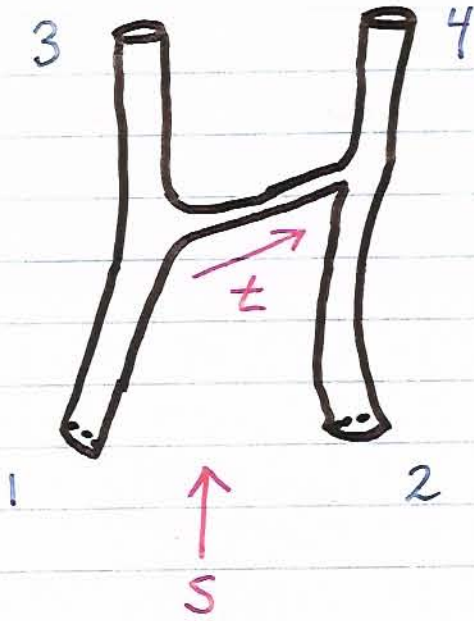
$$s = \frac{4}{\alpha'}, 0, \frac{4}{\alpha'}, \dots$$

(exercise)
eliminated in SUSY version

$$s = 0 \leftrightarrow \underline{\text{massless}} \quad m^2 = 0$$

spin ??

Another limit: $s \rightarrow \infty$, t fixed ("Regge")¹



$$A_{4T} \sim s^{2 + \frac{\alpha' t}{2}} f(t) \quad (\text{exercise})$$

\uparrow
 J

$$t=0 \Rightarrow J=2$$



General result (Feynman, ...):

interacting spin 2 massless particle \Rightarrow

gravity

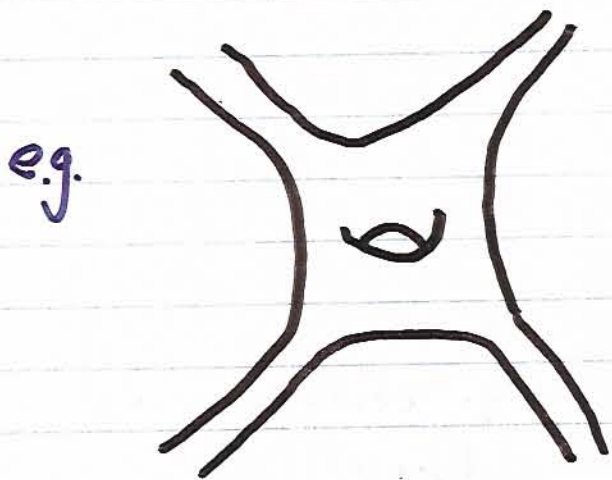
Some catches (or, bonuses)

* Only works in $D=26$ dimensions

* Really only sensible theory if SUSY
($\Rightarrow D=10$)

(In fact, SUSY first discovered in string theory)

But, then, nonrenormalizability cured:



no
high-energy divergence
UV finite!

Reason: In position space, singularity from coincident interaction points



Strings: no special interaction points

(deeper reason: duality...)

\Rightarrow Strings UV finite order-by-order in perturbation theory.

What about gauge theories?

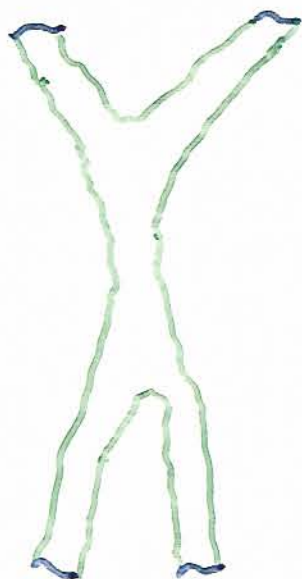


closed



open

← Charges,
eg. N of $SU(N)$



$$A_{4T} = \int \mathcal{D}[ws] e^{i \text{Area}} = \frac{\Gamma(\alpha's-1) \Gamma(\alpha't-1)}{\Gamma(\alpha's-\alpha't-2)}$$

$\rightarrow m^2 = 0, J = 1$

Non-abelian gauge bosons

Many possibilities?

No: tight structure

Open + closed : $SO(32)$

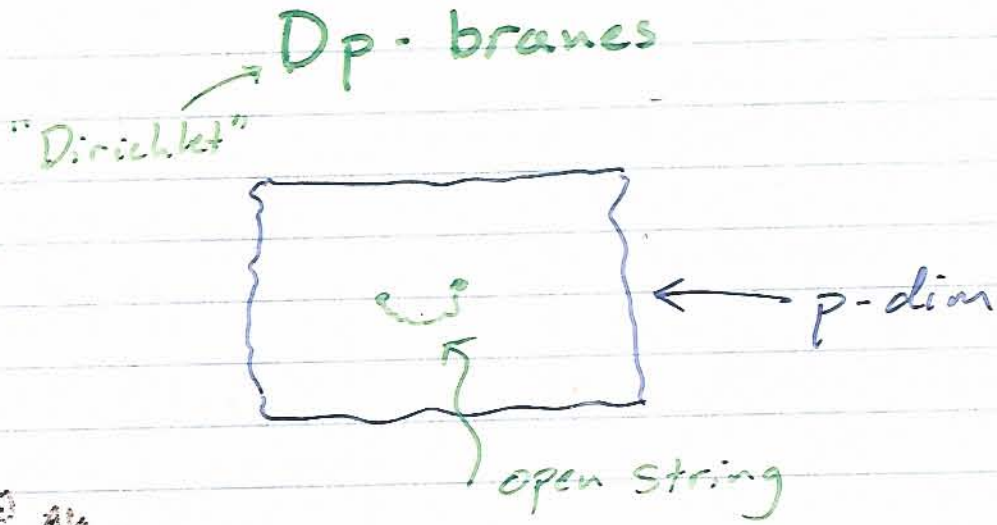


Closed :

IIA } No gauge
 IIB } group
 (a priori)

Het $E_8 \times E_8$ } differ
 Het $SO(32)$ } mechani

The theory also predicts



~> other gauge groups ...

(localized ... not Poinc. invt.)

Naively ~> more possible constructions,
 but at same time allow one to show
 all S theories are different
 versions of the same theory

("duality symmetries")

Taking stock so far:

Gravity ✓

Fermionic matter ✓ (Bonus #1, from SUSY)

Gauge symmetry ✓

But

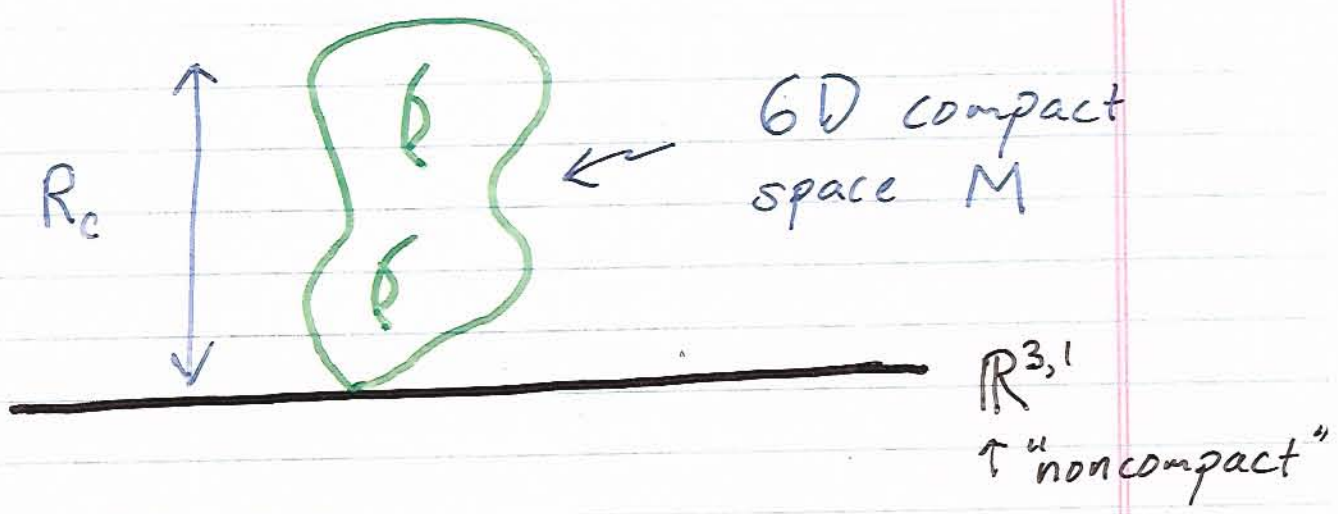
$D = 10$

Wrong gauge group ?

Generations ?

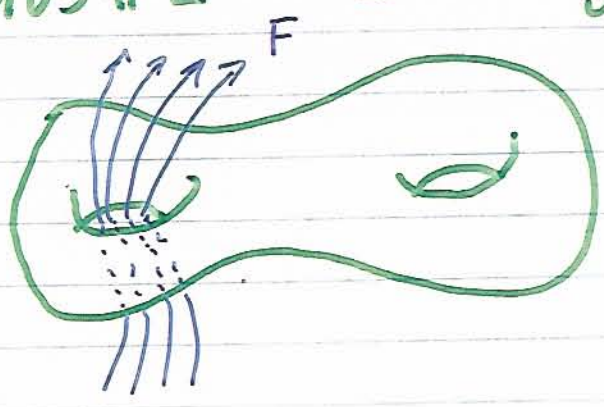
... Solve one problem - $D = 10$ -
and get mechanisms to solve others

Idea: 9+1 dim configured as



If $R_c \lesssim \frac{1}{\text{TeV}}$, no reason to have noticed. (Or even bigger, w/ brane worlds)

Bonus #2: Break gauge symmetry



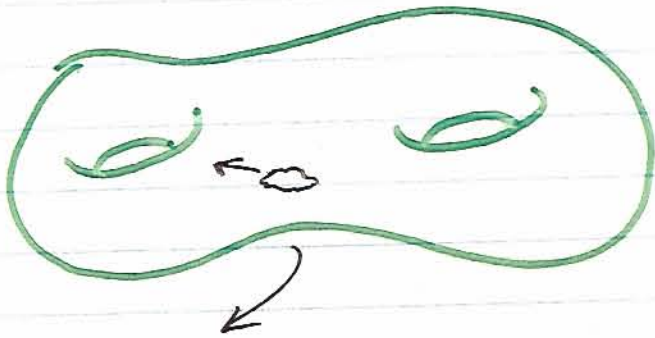
Gauge field configuration ~ trapped flux ~ e.g

$$E_8 \rightarrow E_6 \rightarrow SO(10) \rightarrow SU(3) \times SU(2) \times U(1)$$

\uparrow
 good grand
 unification group

(or can get Standard model from intersecting branes...)

Bonus #3: Generations



String configuration described by wavefunction $\Psi(x, y)$ (like for QM particle, e.g. on circle)

roughly, $\not{D}_{10} \Psi = \not{D}_4 \Psi + \not{D}_6 \Psi = 0$

\uparrow Dirac operator \uparrow lightest oscill. state

$\not{D}_6 \Psi = m \Psi$ "normal modes"

\nwarrow 6D Evalue \leftrightarrow 4D mass

Can have several mode wavefunctions w/
same charge \rightsquigarrow

$$\begin{pmatrix} \nu_e \\ e \\ u \\ d \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \\ c \\ s \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \\ t \\ b \end{pmatrix}$$

... easy to replicate.

Summary, so far.

Assume strings.

- Get :
- gravity - UV finite
 - gauge theories (→ Standard model)
 - fermions
 - generations
 - scalars (≈ Higgs - from compact.)

So, some say

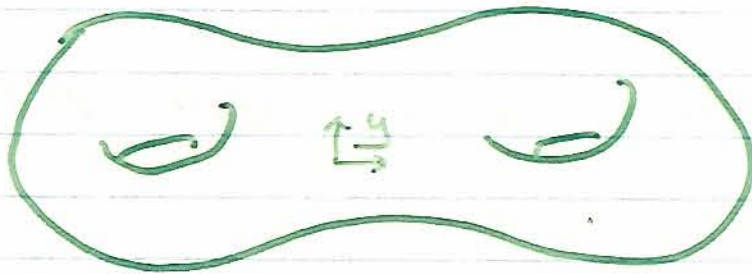
"Theory of everything" (TOE)

(I prefer "Theory of all physics,"
TOP)

Before we get too elated ...

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What fixes compact space M ??



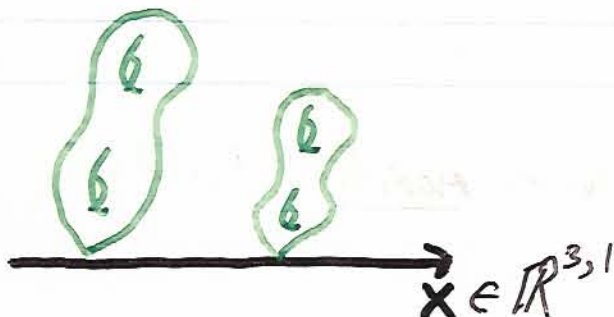
The equations of motion of string theory?

To a good approximation:

$$\begin{array}{l} R_{mn}(y) = 0 \quad (\text{vacuum Einstein eqs}) \\ \uparrow \\ \text{Ricci} \end{array} \rightsquigarrow \text{"Calabi-Yau manifold"}$$

Many solutions

- Many topologies (though connected in S.T.)
- Many brane configurations
- Continuous families
... size, shape



e.g. $R(x)$

Massless moduli fields

All apparently bad news.

- loss of predictivity
- massless scalars, gravitationally coupled

→ 5th forces

time dep. couplings

extra matter

} Not seen
experimentally

But ...

there are newer ideas how to solve, with more complicated configurations ...

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First, one more ingredient: fluxes

E.M.: A_μ ; $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
V.
antisymmetric

$$S_{\text{Maxwell}} = \int F^2$$

q-form flux:

$$A_{\mu_1 \dots \mu_{q-1}}$$

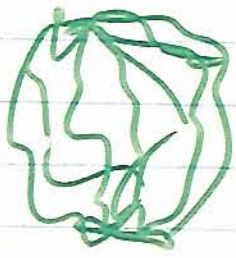
$$F_{\mu_1 \dots \mu_q} = \partial_{\mu_1} A_{\mu_2 \dots \mu_q} \pm \text{perms}$$

$$S_q = \int F_q^2$$

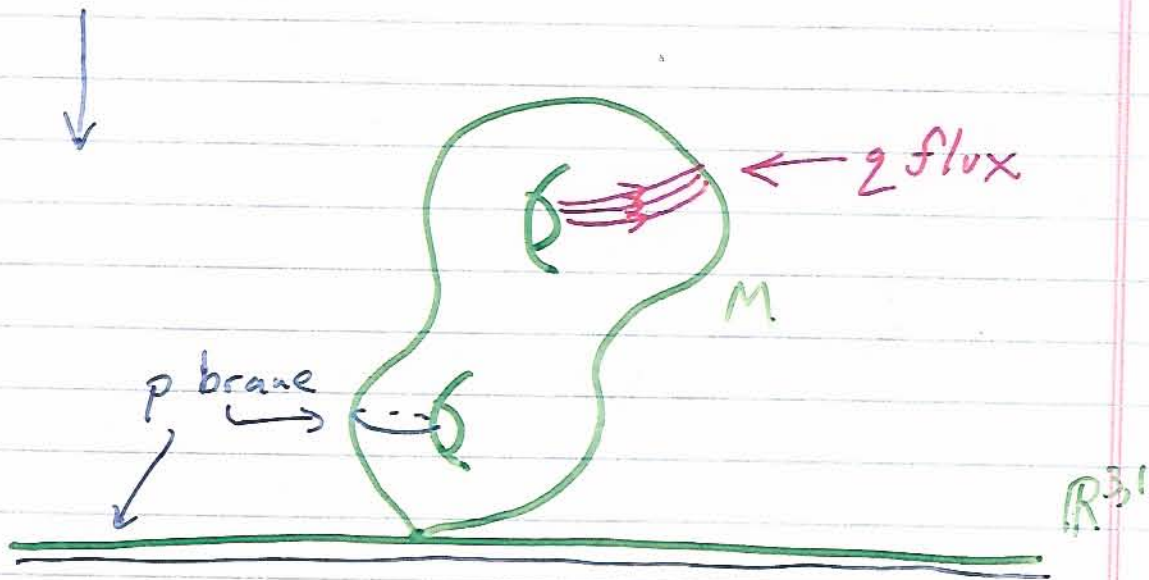
... also present in string theory

Cartoon of cosmological evolution:

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"Quantum geometry"



Can "freeze in":

p-branes - spacefilling

q-fluxes

Their energies depend on sizes/shapes

$$E_p \propto R^{p-3}; \quad \int F_q = \text{fixed} \Rightarrow E_q = \int F_q^2 \propto \frac{1}{R^{2q}}$$

$$E_{\text{tot}}(R) = E_p(R) + E_2(R) + \dots = V(R)$$

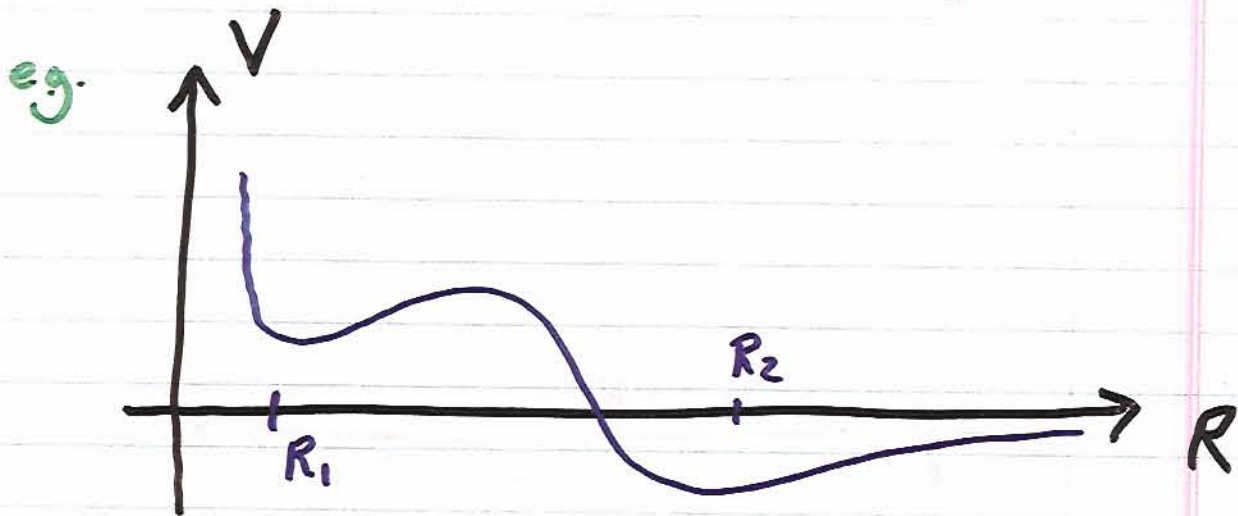
... potential for $R(x)$

4 dim "effective theory"

$$S = \int d^4x \sqrt{-g} \left(\mathcal{R} - (\nabla R)^2 + V(R) + \dots \right)$$

⇒

Combination of effects can fix moduli



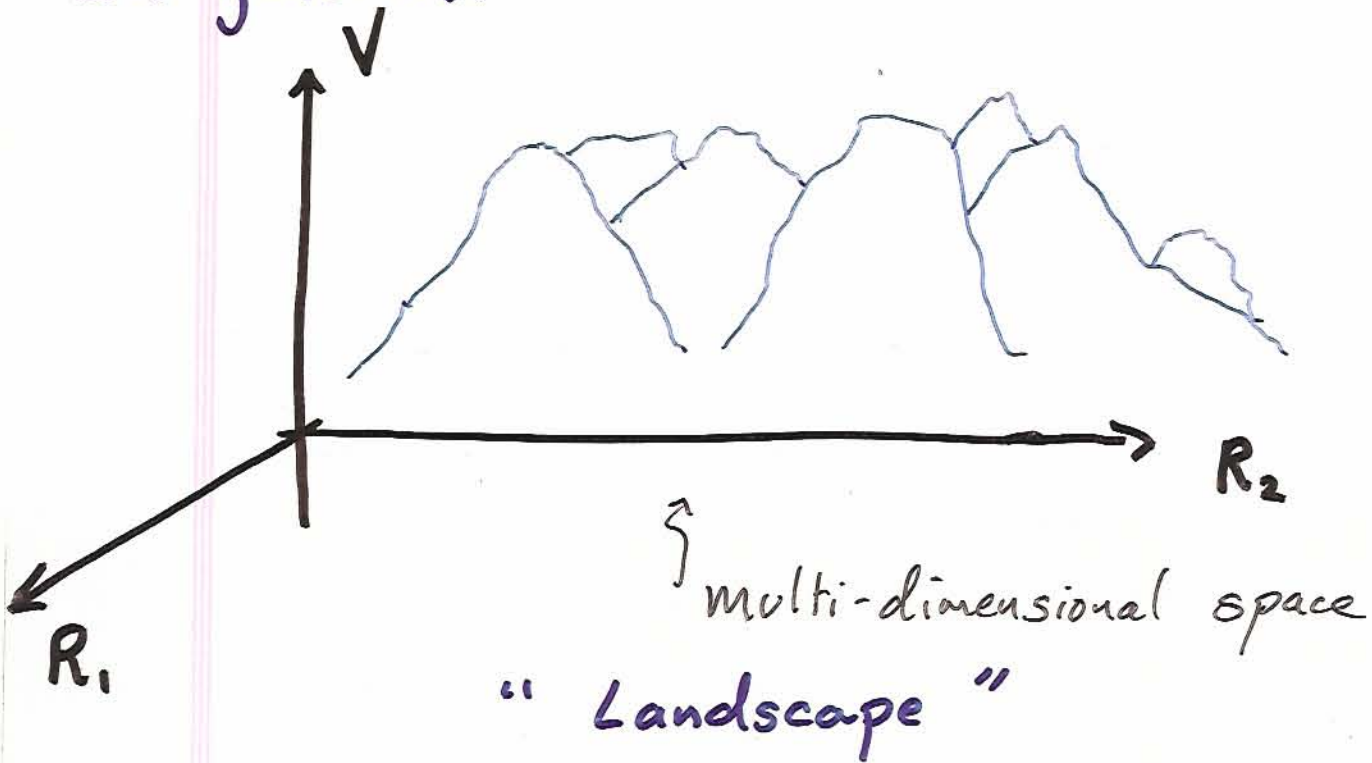
Moreover:

$$V(R_1) = \Lambda_{\text{eff}} > 0 \quad \text{de Sitter}$$

$$V(R_2) = \Lambda_{\text{eff}} < 0 \quad \text{anti de Sitter}$$

So: energy from fluxes, branes, ... trapped²¹
in extra dimensions may explain the
presence of dark energy (Λ)

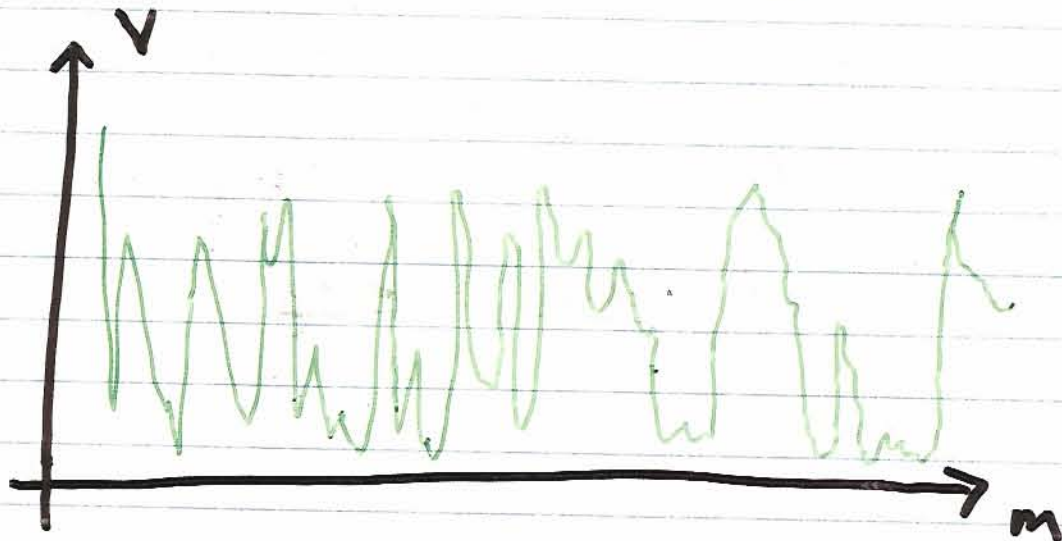
In general:



Another problem: typically $V(R_i) \sim M_p^4 \sim 10^{120} \Lambda_d$

But ...

Many vacua randomly distributed



\Rightarrow if enough (yes), can
have vacua w/

$$\Lambda \sim \Lambda_{\text{obs}}$$

Since Λ_{obs} is \sim the maximum allowed for galaxy formation (\rightarrow life) perhaps the entire landscape is populated (many different regions), and we evolved where life is possible, i.e. $\Lambda \sim \Lambda_{\text{obs}}$

"Anthropic principle"

This picture is new, under test, but plausible.

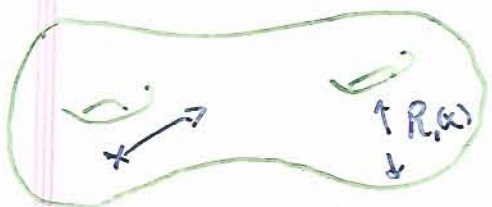
Potential issue - does it really emerge from a complete string theory analysis ... ?

approximations ...

time dependence ...

Other aspects of the "landscape" ²⁴

* Inflation candidates: eg



D3 brane
 $y(x)$

moduli, brane motion

(brane collisions ... ?)

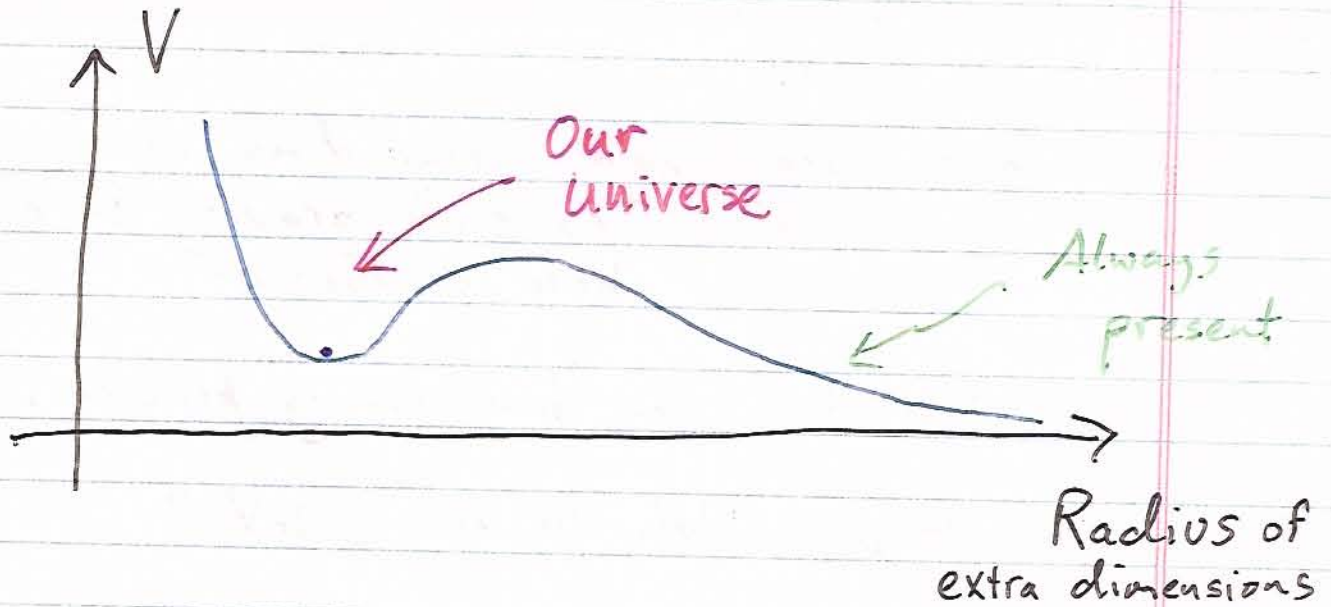
* Some points may have large dims.
or large "warping" \leadsto TeV scale
gravity and black hole
production at accelerators.

* Have eliminated one "fine-tuning"
problem (\wedge).

Is m_{SUSY} likely to be \sim TeV?

debate continues ...

* Ultimate fate of the universe.



⇒ our universe is unstable

extra dimensions will grow

"Spontaneous decompactification"

The end of the Universe is a good place²⁶ to end this survey.

Summary

- * String theory resolves the nonrenormalizability problem of perturbative gravity.
- * It predicts: SUSY, fermions, gauge theories, generations, ...
- * Candidate "TOP"
- * Moduli (size/shape) problem is critical
- * One resolution via frozen fluxes/branes
- * \leadsto dark energy
- * landscape - likely, under investigation
- * inflation candidates? TeV scale gravity?
high M_{SUSY} ??
- * Spontaneous decompactification