Other issues:

1. Anomaly cancellation?
   OK for SM.
   Since we have not added any new exotic chiral fermions when embedding into $\bar{5} \oplus 10$
   \[ \Rightarrow \text{anomaly cancellation is still OK} \]
   \[ \text{[anomaly contribution from 5 exactly cancels anomaly contribution from 10]} \]

2. Overall hypercharge $Y$ normalization finally fixed
   because it is now joined with $SU(2), SU(3)$ non-abelian generators to fit into a single $SU(5)$ non-abelian group!

What rescaling is needed in order to accomplish this unification?
Our method

Think of the

$\text{SU}(2) \otimes \text{U}(1)_Y \subset \text{SU}(3)$

analogy we discussed earlier

\[
\begin{array}{c}
\text{\text{SU}(2)} \\
\end{array}
\]

only when $\text{U}(1)_Y$ scale for vertical axis
is correct relative to $\text{SU}(2)$ scale

do we get a proper $\text{SU}(3)$ representation
(equilateral triangle)!

With wrong normalization for $\text{U}(1)_Y$ vertical axis,
we would obtain, e.g.,

\[
\begin{array}{c}
\text{SU}(2) \\
\end{array}
\]

full $\text{SU}(3)$ symmetry would be lost!

MUST CHOOSE HYPERCHARGE NORMALIZATION
RELATIVE TO $\text{SU}(2)$, $\text{SU}(3)$ normalizations
SO THAT ALL GAUGE BOSON ROOTS HAVE
SAME LENGTH AS IN $\text{SU}(5)$!
Recall normalizations in SM:

\[
\begin{align*}
SU(3): \quad \mathcal{D}_\mu &= \partial_\mu - ig_3 \left( \frac{a^a}{2} \right) A^a_\mu \\
SU(2): \quad \mathcal{D}_\mu &= \partial_\mu - ig_2 \left( \mathbf{T}_2 \right) A^a_\mu \\
U(1)_Y: \quad \mathcal{D}_\mu &= \partial_\mu - ig \left( \frac{Y}{2} \right) B_\mu
\end{align*}
\]

Thus, in SM, \( \left( \frac{Y}{2} \right) \) is like \( \left( \frac{a^a}{2} \right) \).

Look at \( SU(2) \) generators:

\[
\left[ \mathbf{T}_i, \mathbf{T}_j \right] = 2i \epsilon_{ijk} \mathbf{T}^k \quad \Rightarrow \quad \left[ \mathbf{J}^i, \mathbf{J}^j \right] = i \epsilon_{ijk} \mathbf{J}^k
\]

where \( \mathbf{J}^i = \frac{\mathbf{T}^i}{2} \)

\[
\Rightarrow \quad \left[ \mathbf{J}^\pm, \mathbf{J}^\mp \right] = \pm \mathbf{J}^\pm
\]

so \( \mathbf{J}^\pm \) raises/lowers by one unit.

\( \Rightarrow \) \( SU(2) \) roots have length = 1 in SM!

SAME for \( SU(3) \): these roots also have length = 1 in SM!

We therefore need to choose our normalization for \( U(1)_Y \)

such that all* of the \( SU(5) \) gauge bosons correspond to roots of length = 1.

\( * \) not including the diagonal generators, of course.
Using the SM normalization for $Y_{SM}$, we already saw

$$2^4 \rightarrow (8, 1)_0 + (1, 3)_0 + (4, 1)_0 + (3, 2)^5_3 + (\bar{3}, 2)^5_{\frac{5}{3}}$$

SM gauge bosons: each has length = 1

These states "point" non-trivially in color, weak, and hypercharge directions simultaneously.
These directions are orthogonal.

Thus

$$(\text{total length})^2 = (\text{length of SU(3) triplet})^2 + (\text{length of SU(2) doublet})^2 + c^2 \left( \frac{5}{6} \right)^2 = 1$$

$\frac{3}{2}$ is generator

Rescaling factor

Thus

$$\frac{1}{3} + \frac{1}{4} + c^2 \left( \frac{5}{6} \right)^2 = 1 \implies c = \sqrt{\frac{3}{5}}$$

required rescaling factor for U(1) $Y$!
Thus \[ Y_{su(5)} = \sqrt{\frac{3}{5}} Y_{SM} \]

However, since we must preserve strength of observed interactions

\[
[D\mu = \partial\mu + ig_Y Y B_\mu] \Rightarrow \text{product} (g_Y Y) \text{ must be preserved!}
\]

\[
\begin{align*}
su(5) & \quad \frac{su(5)}{g_Y} = \sqrt{\frac{5}{3}} g_Y \\
\frac{su(5)}{\alpha_Y} & = \frac{5}{3} \alpha_Y
\end{align*}
\]

In general, \( \frac{su(5)}{g_Y} \) is renamed \( g_1 \),

\[
\begin{align*}
\frac{su(5)}{\alpha_Y} & \quad \text{is renamed } g_1
\end{align*}
\]

So GUT unification into a single GUT group such as \( su(5) \) requires all generators to act with a common coupling:

\[
\begin{align*}
& \quad g_5 = (g_3 = g_2 = g_1 = \sqrt{\frac{5}{3}} g_Y) \\
\text{or} \quad \alpha_5 = (\alpha_3 = \alpha_2 = \alpha_1 = \frac{5}{3} \alpha_Y)
\end{align*}
\]
A priori, this does not fix the overall value of the coupling. But it does fix the ratios between couplings!

\[ \frac{g_3}{g_2} = 1 \quad \text{and} \quad \sin^2 \theta_W = \frac{g_2^2}{g_2^2 + g_Y^2} = \frac{3}{8} \approx 0.375 \]

But recall:

- \( g_2^{-1} \approx 8.5 \)
- \( g_Y^{-1} \approx 28.1 \)
- \( g_Y^{-1} \approx 98.3 \Rightarrow g_1^{-1} \approx 59.0 \)

at Z scale

\( \{ \text{couplings are not equal!} \} \)

\( \sin^2 \theta_W \approx 0.23 \) not \( \frac{3}{8} \)!

Uh, oh! Trouble for GUT's?

No...
Unification of gauge couplings...
Thus, the natural energy scale for grand unification is

\[ M_{\text{GUT}} \approx 2 \times 10^{16} \text{ GeV} \]

where the gauge couplings unify and where we have the potential to realize all gauge forces as emerging from a single GUT group [e.g., SU(5)] with a single coupling constant \( g_5 \)

GUTs are high-scale physics!

\[ \sim 10^2 \text{ GeV} \]

\[ \sim 10^{16} \text{ GeV} \]

\[ \text{SM or MSSM} \]

\[ \text{GUT} \]

\[ \text{SU}(5), \text{SO}(10), \text{etc.} \]

\[ \text{Electroweak Phase Transition} \]

\[ \text{GUT phase transition} \]
But how does the GUT break at $M_GUT$?

Just as in SM, one idea is to use the Higgs mechanism.

Need a Higgs $H$ such that

$$SU(5) \rightarrow SU(3) \otimes SU(2) \otimes U(1) \rightarrow SU(3) \times U(1)_{EM}$$

Can we use $H = \Phi_3$?

- The "colored" Higgs which was joined with the EW Higgs $\Phi$ to form the 5 of $SU(5)$?

No! Recall $\Phi_3 : (3,1)_{-2/3} \leftrightarrow \Phi_{EM} = -1/3$ for each component of the triplet.

Thus, if $<\Phi_3> \neq 0$

$$\Rightarrow SU(3) \text{ and } U(1)_{EM} \text{ will be broken - EXACTLY THE OPPOSITE OF WHAT WE WANT!}$$

Indeed, to preserve $SU(3) \otimes SU(2) \otimes U(1)$ subgroup, need a NEW HIGGS REP of $SU(5)$ containing a component $(1,1,1)_0 = \text{neutral under SM}$!
Turns out that the smallest SU(5) rep that can do this is the $24$ rep = adjoint

Fairly big!
But OK, since we are going to need lots of Goldstone bosons to be "eaten" by the twelve $(X,Y)$ gauge bosons to make them massive and thereby do the breaking!

This is a general property for all simple GUT groups \([SU(5), SO(10), E_6, \text{ etc.}]\)

\[ \Rightarrow \text{no GUT Higgs for reps smaller than the adjoint!} \]

Recall for SU(5),

$24 \rightarrow (8,1) \oplus (1,3) \oplus (1,1)$.

\[ \oplus (3,2)_{-\frac{5}{3}} \oplus (\overline{3},2)_{\frac{5}{3}} \]
In matrix notation, $24 = \text{traceless } 5 \times 5 \text{ matrix singlet } (1,1)$. A representative should look like identity for both $SU(3)$ and $SU(2)$ components.

$\Rightarrow$ we want 

$$
\langle H \rangle = \nu_{{}_{\text{ew}}} \begin{bmatrix} 2 & 2 & 2 \\
2 & 2 & 3 \\
-3 & -3 & -3 
\end{bmatrix}
$$

(this is the analogue of demanding $\langle \phi \rangle = (\frac{\psi}{\sqrt{2}}) \text{ for EW}$)

**WHAT POTENTIAL WOULD HAVE THIS MINIMUM?**

Try something familiar:

$$
V(H,\phi) = -m_1^2 \text{ tr } H^2 + A_1 (\text{tr } H^2)^2 + A_2 (\text{tr } H^4) - m_2^2 (\phi^+ \phi) + A_3 (\phi^+ \phi)^2 + A_4 (\text{tr } H^2)(\phi^+ \phi) + A_5 (\phi^+ H^2 \phi)
$$

This is a general 4th-order $SU(5)$-invariant polynomial where we imposed discrete ($H \rightarrow -H$) symmetries to eliminate cubic terms... (two Mexican hats & interaction terms)
Find: for $A_2 > 0$, $A_1 > -(\frac{7}{30})A_2$

$\Rightarrow$ reproduces desired minimum

with $V_{\text{GUT}}^2 = \frac{m_1^2}{60A_1 + 14A_2}$

We then find:

\[ H = \]

\[ m^2 + V_{\text{GUT}}^2 \]

\[ m^2 + m^2 \]

massless Goldstone bosons

massless Goldstone bosons

\( (X,Y) \) gauge bosons then "eat" these massless Goldstone Higgs fields, get masses

\[ m_{X,Y} \sim g_{\text{GUT}} V_{\text{GUT}} \]

while remaining massive Higgs fields survive as physical Higgses, but at the GUT scale.
However, the presence of the cross-couplings $(\bar{A}_4, A_5)$ between $H$ and $\phi$ in the potential implies that our $\phi$ fields also get masses!

\[
\begin{align*}
\text{colored triplet } & \phi_3 \quad \text{got masses} \\
\text{usual Higgs doublet } & \phi
\end{align*}
\]

Top result for $\phi_3$: \[\Box\text{OK}\], triplet Higgs also will have GUT-scale mass \[\checkmark\]

But bottom result for EW Higgs $\phi$

is problematic!

$\phi$ needs to be light $(m_\phi \lesssim 6\text{ GeV})$ so that it can survive down to EW scale and trigger EW symmetry breaking!

$\Rightarrow$ MUST BE A VERY PRECISE CANCELLATION

over $12\to14$ orders of magnitude

between $(A_4, A_5)$ and $m_2$!
Two questions:

- Where does this remarkable fine-tuned cancellation come from?

- Why/how is it stable against radiative corrections?

These are two parts of the so-called gauge hierarchy problem.

→ solution is unknown!

- susy?
- extra dimensions?
- string landscape?

{lots of ideas, no experimental evidence yet.}

*Upshot*

Need a lot of fine-tuning in the Higgs sector because we want to maintain two widely separated scales of gauge symmetry breaking within one UV theory emerging from one potential $V(H, \phi)$ with GUT-scale parameters.
The Higgs sector may also need to be extended for other, more phenomenological reasons.

Consider fermion masses:

SU(5) structure of Yukawa couplings can be shown to require certain mass relations to hold at the GUT scale:

\[ m_e = m_d \quad \Rightarrow \quad m_\mu = m_s \quad \Rightarrow \quad m_\tau = m_\nu. \]

When extrapolated down to EW scale using RG equations, do these equations hold even approximately?

For third generation, we find \( m_b \approx 3 m_\tau \), which is reasonably successful.

But even with RG equation evolution, it is easy to see that the ratios

\[ \frac{m_e}{m_\tau} = \frac{m_d}{m_s} \], etc. must remain invariant.

Same quantum numbers must behave the same!

This is obviously violated!
Are there solutions to this problem?

Yes, mostly by expanding the Higgs sector!

e.g., if we introduce a $\bar{45}$ representation as another new Higgs $h$

then one can show that

\[
\frac{m_e}{m_{\mu}} = \frac{m_d}{m_{\nu}} \quad \text{(becomes)} \quad \frac{m_e}{m_{\mu}} = \frac{1}{9} \frac{m_d}{m_{\nu}}
\]

This is satisfied rather well by exp., but obviously requires a non-minimal SU(5) Higgs structure!

Other similar solutions are also possible...
But the classic signature of GUT's is proton decay!

Since GUT's necessarily break

\[ \begin{align*}
B &= \text{baryon} \\
L &= \text{lepton}
\end{align*} \]

the lightest baryon (= proton) is no longer stable!

⇒ NEW DECAY PROCESSES ARE MEDIATED, PRIMARILY BY \((X, Y)\) GAUGE BOSONS!

Recall the relevant diagrams:

\[ \begin{align*}
\Delta B &= \frac{1}{3} \\
\text{LEPTOQUARK CHANNEL} \\
\Delta B &= -\frac{2}{3} \\
\text{DIQUARK CHANNEL}
\end{align*} \]

⇒ Therefore, through \((X, Y)\) exchange, can build a \(\Delta B=1\) process to allow the proton to decay!
e.g.,

\[ p \rightarrow \pi^0 e^+ \] as dominant mode

in this minimal SU(5) GUT

\[ M(p \rightarrow \pi^0 e^+) \approx \frac{9 G_{\text{U}}}{M_{X,Y}^2} \]

from (xy propagator)

Thus, for \( M_{X,Y} \gg 10^{15} \) GeV

\[ \Rightarrow \text{proton lifetime} \gg 10^{34} \text{ years} \]

In general, heavy xy gauge boson masses are crucial for suppressing proton decay in GUTs!
Important observation:

Even though $B$ and $L$ are broken by the $(X,Y)$ bosons in the SU(5) model, the difference $B-L$ is conserved!

$\Rightarrow \Delta B = \Delta L$

eg $\mu \rightarrow \tau \gamma$ (lose a baryon, lose a lepton)

This is an "accidental" global symmetry in SU(5).

 Miracle? Not really - it can be shown that any four-fermion (dim=6) effective operators in $L$ which break $B$ yet preserve $SU(3)_C \otimes SU(2)_W \otimes U(1)_Y$ will also preserve $B-L$!

Thus B-L conservation is independent of the particular unification scheme!

(\Rightarrow to break B-L, require operators of dim > 6 !)
Aside: The breaking of $B$ has very important COSMOLOGICAL IMPLICATIONS!

One major problem is the baryon-number asymmetry in the Universe:

$$\delta = \frac{N_\text{b} - N_{\bar{\text{b}}}}{N_\text{b} + N_{\bar{\text{b}}}} \approx 10^{-9} \quad \text{(not zero!)}$$

Where does this baryon/anti-baryon asymmetry come from?

- Initial condition? (Landscape?)
- Symmetric initial condition, with subsequent dynamics generating the asymmetry?

Would require: (Sakharov)

- Violate C and CP conservation
- Violate B conservation
- Have these properties occur in processes which are out of thermal equilibrium

(So reverse process does not occur with equal probability!)
GUT's have all three off
the required conditions!

Processes which violate $B, C$, and $CP$
are those which are mediated by
$(X,Y)$ gauge bosons & color Higgs triplets $\phi_3$

$\implies$ These are forced out of thermal
equilibrium by cosmological expansion ...

Thus, GUT's may provide an explanation
of the primordial baryon/antibaryon
asymmetry of the Universe!
Beyond the minimal $SU(5)$ GUT:

1. Add $N=1$ SUSY:
   - improves gauge coupling unification
   - helps to stabilize (and perhaps even dynamically produce) the gauge hierarchy
   - may help keep proton-decay rates within increasingly tight bounds

2. Add more Higgs fields, other matter:
   - helps to obtain better fermion mass relations

3. Enlarge the gauge group!

   $SO(10)$?
   $E_6$? ...

   Why do this?
Recall the SM particle content:

\[ d^c : (\overline{3}, 1)_{2/3} \quad \Rightarrow \quad \overline{5} \text{ of } SU(5) \]
\[ L : (1, 2)_{-1} \quad \Rightarrow \quad 10 \text{ of } SU(5) \]
\[ Q : (3, 2)_{1/3} \]
\[ u^c : (\overline{3}, 1)_{-1/3} \quad \Rightarrow \quad 10 \text{ of } SU(5) \]
\[ e^c : (1, 1)_{2} \]
\[ \gamma^c : (1, 1) \quad \Rightarrow \quad \text{if it exists...?} \]

Two unsolved issues:

1. \( \overline{5} \) and \( 10 \) are still two different reps of \( SU(5) \) \( \Rightarrow \) they themselves are not unified!

2. We now know neutrinos have mass!

\[ \gamma^c \] needed for Dirac mass through Yukawa couplings!

and needed for Majorana mass through Seesaw mechanism!
Thus it seems we need a third representation so that all 16 particles in each SM generation would be realized as

\[ 5 + 10 + 1 \]

\[ \text{singlet } \nu^c. \]

This is hardly a "unification"!

But if we consider the next largest GUT group \( SO(10) \rightarrow SU(5) \otimes U(1) \)

we find that it has representations

\[ 1 \]

\[ 16 \rightarrow (\bar{5})_3 + (10)^{-1} + (1)^{-5} \]

\[ 45 \]

\[ 54 \]

\[ 120 \]

\[ 126 \ldots \text{ etc.} \]

Just what we need, all bundled together into the fundamental \((\text{spinor})\) representation of \(SO(10)\)!
• Thus, using $SO(10)$, one entire SM generation—including the RH neutrino—can be realized as coming from a single, unified, fundamental, anomaly-free, 16-dimensional spinor representation.

  ALL QUANTUM NUMBERS WORK PERFECTLY!

• Moreover, the extra $U(1)$ is $B-L$!

  Thus, in $SO(10)$ unification, $B-L$ conservation is no longer "accidental" but arises naturally from a gauge symmetry!

  Prevents processes like:

  $n \rightarrow e^- \pi^+$, $n \rightarrow \gamma \pi^0$

  $p \rightarrow \gamma \pi^+$

  $n \rightarrow \bar{n}$

  forbidden to all orders...

• Other cosmological benefits:

  e.g., parities now part of the gauge symmetries of $SO(10)$ [unlike $SU(5)$]

  $\Rightarrow$ avoids the cosmological domain wall problem normally associated with parity-breaking!
Can also extend to even larger

GUT groups:

- $E_6$, etc $\to$ natural "string-Theory" candidate

- $G_{\text{GUT}} \times G_{\text{flavors}}$

\[ \uparrow \quad \text{to incorporate family (flavors) symmetries across three generations} \]

... Lots of possibilities exist

The field of GUT model-building is huge & well-explored!

$\rightarrow$ see original literature.
The Problem of Gauge Coupling Unification

Recall situation in field theory:

Experimental Data —
- Standard Model, no SUSY: no unification of couplings
- MSSM, $N = 1$ SUSY: unification occurs
  Unification scale $M_{\text{MSSM}} \approx 2 \times 10^{16}$ GeV.

$\implies$ MSSM very successful!
- one of the few surviving extensions to SM which is in agreement with all experimental data!
- SUSY also provides elegant solutions for:
  finiteness
gauge hierarchy problem...

Therefore, current field-theoretic scenario:
- Grand Unified Group above $M_{\text{MSSM}}$
- MSSM gauge group and spectrum between $M_{\text{MSSM}}$ and $M_{\text{SUSY}}$
- then .... (soft?, spontaneous?) SUSY-breaking
  — constrained to control Higgs mass, solve gauge hierarchy problem
  — requires $\text{Str}(M^2) < $ some number.
- SM gauge group and spectrum below $M_{\text{SUSY}}$
Compelling picture except for various problems:

- $M_{\text{MSSM}}$ close to Planck scale $\implies$ what about gravity?

- Why the spectrum of SM and MSSM with all of these arbitrary parameters? Need to explain:
  - three generations
  - fermion mass matrices...

- If GUT theory above $M_{\text{MSSM}}$, what about proton lifetime? Require some sort of doublet-triplet splitting mech.

- Why require a GUT at all?
  - Theoretical prejudice...
  - Not required for consistency of model...
String theory can solve these problems.

- Naturally incorporates quantized gravity
  — spin-two massless particle (graviton) always appears in spectrum.

- \( N = 1 \) SUSY field theories with non-abelian gauge groups appear as low-energy limits.

- May provide uniform framework for understanding
  - three generations
  - fermion mass matrices
  - doublet-triplet splitting mechanism, etc...
  — in principle, no free parameters!

- Natural unification of couplings:
  - gauge and gravitational couplings automatically unify to form one coupling constant \( g_{\text{string}} \):

\[
8\pi \frac{G_N}{\alpha'} = g_i^2 k_i = g_{\text{string}}^2
\]

where \( G_N \equiv \) gravitational (Newton) coupling
\( \alpha' \equiv \) Regge slope
\( k_i \equiv \) affine level of group factor \( G_i \):

\[
J^a(z)J^b(w) \sim \frac{i f^{abc}}{z-w} J^c(w) + k \frac{\delta^{ab}}{(z-w)^2} + \ldots
\]

"Schwinger term"
Field Theory vs. String Theory: Crucial Differences

- **String theory is finite** $\Rightarrow$ **running** of gauge couplings is within framework of low-energy *effective* theory only.

- In string theory, **all couplings are dynamical variables**, related to expectation values of *moduli fields*.

- Dependence on KM level $k_i$.
  
  - *Essentially a normalization*: analogous to hypercharge normalization [e.g., $k_Y = 5/3$ for $SU(5)$ or $SO(10)$], but now also appears for non-abelian gauge factors!
  
  - Most easily constructed string models have $k_i = 1$. 

Unfortunately, this is not the unification scale expected in heterotic string theory!

- At tree level, \( M_{\text{string}} = \frac{1}{\sqrt{\alpha'}} = g_{\text{string}} M_{\text{Planck}} \).

- At one loop,

\[
M_{\text{string}} \approx g_{\text{string}} \times 5 \times 10^{17} \text{ GeV}
\]

in \( \overline{\text{DR}} \) scheme.

Assuming \( g_{\text{string}} \approx \mathcal{O}(1) \),

\[
M_{\text{string}} \approx 5 \times 10^{17} \text{ GeV}
\]

\[\implies\] factor of \( \approx 20 \) discrepancy!!

---

**Is this a major problem?**

- only 10\% effect in logarithms of mass scales, but

- leads to **wildly incorrect** values for \( \sin^2 \theta_W \) and \( \alpha_{\text{strong}} \):

\[
\frac{16\pi^2}{g_i^2(\mu)} = k_i \frac{16\pi^2}{g_{\text{string}}^2} + b_i \ln \frac{M_{\text{string}}^2}{\mu^2}
\]

\[\implies\] Major problem for string phenomenology!
The Fundamental Problem...

\[ G_N^{-1} \equiv \mu^2 G_N^{-1} \]

\[ \alpha^{-1} \]

\[ M_1 \quad M_2 \]

\[ M_{\text{MSSM}} = 2 \times 10^{16} \text{ GeV} \]

\[ M_{\text{string}} = 5 \times 10^{17} \text{ GeV} \]
Path #1: String GUT's

Intermediate-scale gauge group:

\[ SU(3) \times SU(2) \times U(1)_Y \subset G \]

For example, \( G = SU(5), SO(10), \) or \( E_6. \)
Path #2: Non-Standard Levels \((k_Y, k_2, k_3)\)

In MSSM, we have \((k_Y, k_2, k_3) = (5/3, 1, 1)\).

In string theory, however, this is not necessary! Other values are allowed.

This would be a stringy effect...

\[
\begin{align*}
\cdot \text{ KRD, A.E. Faraggi, \\ & J. March-Russell.} \text{ Nucl. Phys. B467 (1996) 44}
\end{align*}
\]
Path #3: Heavy String Thresholds

Heavy string threshold corrections ($\Delta_Y, \Delta_2, \Delta_3$) from infinite towers of massive (Planck-scale) states. Also a purely stringy effect...

\[ \alpha^{-1} \]

\[ G_N^{-1} \]

\[ M_1 \quad M_2 \]

Path #4: Intermediate-Scale Corrections

- **Light SUSY Thresholds**
  - from breaking SUSY at intermediate scales
  - analyzed in effective field theory

- **Intermediate Gauge Structure**
  - *e.g.*, Pati-Salam $SU(4) \times SU(2)_L \times SU(2)_R$
    - flipped $SU(5) \times U(1)$
  - also analyzed in effective field theory

\[ \alpha^{-1} \]

\[ G_N^{-1} \]

\[ (d) \]

\[ M_1 \quad M_2 \]

Path #5: Extra Matter Beyond the MSSM

Such matter *ad hoc* from field-theory perspective, but *required for self-consistency* in many string models!

\[ \alpha^{-1} \]

\[ G_N^{-1} \]

\[ M_i \quad M_i' \quad M_1 \quad M_2 \]

or even...

two-loop effects

$\alpha^{-1}$

$G_N^{-1}$

$M_i\quad M_1\quad M_2$
Path #6: Strings without SUSY

In field theory, unification impossible without SUSY. In string theory, however, may be possible!

\[
\begin{align*}
\alpha_1^{-1}(\mu) & \quad (k_Y = \frac{13}{10}) \\
\alpha_2^{-1}(\mu) & \\
\alpha_3^{-1}(\mu) & \\
\end{align*}
\]

\[
\log_{10} (\mu/\text{GeV})
\]

Non-SUSY Standard Model

\begin{itemize}
\item KRD, *Nucl. Phys. B429* (1994) 533
\end{itemize}
Path #7: Strong-Coupling Effects

If underlying string coupling is strong,  
\[\Rightarrow\] non-perturbative relations to new theories  
e.g., M-theory or open string theory.

These theories have weaker unification relations.  
Thus, original mismatch may not be problematic.
Thus, one over-riding question:

Which path(s) to unification
does string theory actually take?

...or equivalently:

Can realistic string models be constructed
which exploit these possibilities?

There has been significant recent progress
in understanding/utilizing each of these paths...

- Review Article: KRD, hep-th/9602045
  (Physics Reports 287 (1997) 447)
Phenomenological Lessons from String GUT's


New Rules for GUT Model-Building —

- **No 120 or 144** representations of $SO(10)$
  — typically used for
    - fixing light quark/lepton mass ratios
    - inducing GUT symmetry breaking
  — ruled out in $SO(10)$ string models!

- **No 126** representation of $SO(10)$
  — typically used for
    - heavy Majorana right-handed neutrino mass
    - GJ factor of 3 in light quark/lepton mass ratio
  — ruled out in $SO(10)$ string models!

- **ALL** larger representations of $SO(10)$ ruled out!

- All **54** representations of $SO(10)$ must transform as **singlets** under all gauge symmetries beyond $SO(10)$.
  — hard to build models with multiple 54 reps

- **New restrictions on allowed couplings** — *e.g.,*
  — no couplings of form $X \cdot 54 \cdot 54$ where $X = \text{singlet}$
  — no couplings of form $X \cdot 54 \cdot 54'$ where $X = \text{singlet}$
  — no explicit mass terms in superpotential
  — no cubic level-terms in superpotential
• All 45 representations of SO(10) must transform as singlets under all gauge symmetries beyond SO(10)!
  — can only have $\mathbb{Z}_3$ or $\mathbb{Z}_6$ discrete quantum numbers
  — hard to build models with multiple 45 reps
  — hard to implement doublet/triplet splitting
  — hard to implement "fake" $126 \subset 45 \times 45' \times 10$

• All 78 representations of $E_6$ must transform as singlets under all gauge symmetries beyond $E_6$!

• ALL larger representations of $E_6$ ruled out!

• All 24 representations of $SU(5)$ must transform as singlets under all gauge symmetries beyond $SU(5)$!

etc...

Guided by these results, there is much current activity —

• Field Theory: new efforts to build low-energy GUT models satisfying these constraints

• String Theory: new mechanisms for building string GUT models
Another modern topic, potentially even more exciting...
GUT's at a TeV ??
One of the most compelling proposals for physics beyond the Standard Model is the emergence of a **SUSY GUT**

**SUSY GUT**'s would have many benefits —

- **Unify strong, weak, and hypercharge forces**
  - ⇒ explain relative gauge couplings
  - ⇒ explain quantization of electric charge

- **Unify diverse particle representations**
  - ⇒ explain fermion quantum numbers
  - ⇒ explain relative fermion masses

- **Predict baryon-number violation**
  - ⇒ explain cosmological baryon/anti-baryon asymmetry
Unfortunately, the GUT scale is very remote!

implies hard to probe GUT physics directly
implies must look for very rare processes, e.g., proton decay

All GUT physics is suppressed by heavy scale!

Is there a way to bring GUT physics down to accessible energy scales?

Is it possible to probe GUT physics directly?
Hardly seems possible...

GUT's require *unified* gauge couplings
- would need to somehow achieve gauge coupling unification at lower energy scales!

*But once again, this hardly seems possible*...
- only possibility is to add extra matter
but this generally *increases* unification scale,
also pushes unification towards strong coupling.

**Is there a way around this?**

---

**Why is unification scale so high?**

Gauge couplings run only *logarithmically* versus energy scale $\mu$
(or *linearly* versus $\log \mu$)
- must extrapolate over many orders of magnitude before different low-energy gauge couplings can be reconciled!

---

**How can we make the gauge couplings run faster?**

**How can we change *linear running* into *exponential running*?**
Extra spacetime dimensions!

- Naturally predicted in string theory
- Radii generally unfixed by string dynamics
- Large radii play an important role in string duality

\[ \Rightarrow \] Such scenarios should be easy to realize within string theory!

Nevertheless, extra dimensions can be discussed in purely field-theoretic terms...
Consider running of gauge couplings...

Recall: Without extra dimensions, gauge couplings have usual logarithmic running:

\[ \alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi} \ln \frac{\mu}{M_Z} \]

...result of evaluating the vacuum polarization diagram:
Now, imagine $\delta \equiv D - 4$ extra dimensions at scale $R^{-1}$. Must also include Kaluza-Klein states in loop!

$\Rightarrow$ Below $R^{-1}$, no appreciable effect. Above $R^{-1}$, couplings now evolve according to:

$$
\alpha_i^{-1}(\mu) \approx \alpha_i^{-1}(R^{-1}) - \frac{b_i - \tilde{b}_i}{2\pi} \ln (R\mu) - \frac{\tilde{b}_i X_\delta}{2\pi\delta} \left[(R\mu)^\delta - 1\right]
$$

where

- $$(b_1, b_2, b_3) \equiv (33/5, 1, -3)$$
- $$(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3) \equiv (3/5, -3, -6)$$
- $$X_\delta \equiv \frac{2\pi^{\delta/2}}{\Gamma(\delta/2)}$$

Power-law behavior is the consequence of extra dimensions! But how does this affect gauge coupling unification?
Let us choose:

- $R^{-1} = 1 \text{ TeV}$ (the most extreme case)
- $\delta = 1$ (one extra dimension)

We then find...

- Evolution is dramatically altered,
  \textit{but gauge couplings still unify!}
- Potential new scale for grand unification!
How does this depend on the chosen radius?

Unification is always preserved!
Extra Dimensions and Proton Decay

If we imagine a GUT theory emerging at such low energy scales, we immediately face the problem of proton decay!

Can be mediated by X-bosons, colored Higgs triplets, etc...

Usually, proton decay is suppressed by large GUT mass scale!
In our case, however, situation is drastically altered:

- Unified coupling is weaker than usual GUT value
- Unification scale is lower than usual GUT value

$\quad \Rightarrow$ Proton-decay amplitude is apparently increased by factor

\[
\left( \frac{\alpha'_{\text{GUT}}}{\alpha_{\text{GUT}}} \right) \left( \frac{M_{\text{GUT}}}{M'_{\text{GUT}}} \right)^2 \gg 1.
\]

Seems to be trouble!
Is there another solution?

One idea:

This is actually a GUT in higher dimensions!

Might there be a higher-dimensional solution to the proton-decay problem?
Yes!

Extra GUT states beyond MSSM should not be observable below unification scale!

$\Rightarrow$ Such states should not have zero-modes!

$\Rightarrow$ Thus, wavefunctions must be chosen odd under $y \rightarrow -y$:

$$
\Phi^{-} = \sum_{n=1}^{\infty} [\Phi^{(n)}(x) - \Phi^{(-n)}(x)] \sin \left( \frac{ny}{R} \right)
$$

But recall that in the “minimal” scenario with $\eta = 0$, the MSSM fermions are restricted to the orbifold fixed points

$$
y = 0 \quad \text{and} \quad y = \pi R.
$$

Thus, all GUT states beyond the MSSM have wavefunctions which vanish at the higher-dimensional fermion locations!

These states do not couple to the MSSM fermions!
Not only for the lowest modes of $X$-bosons and Higgs triplets...

but also for their infinite towers of KK excitations...
Thus, such proton-decay diagrams vanish to all orders in perturbation theory!

- This is an intrinsically higher-dimensional solution to the proton-decay problem.
- Valid for all radii $R$ and all spacetime dimensions $D$.
- Makes use of a higher-dimensional (orbifold) symmetry.
- No analogue in usual four-dimensional GUT’s.

Are there other contributions to proton decay?

Generally, yes:

- Non-perturbative effects leading to proton decay (e.g., instantons $\sim \exp(-1/g^2)$, hence small!)
- Additional, non-renormalizable interactions (typical in string theory)

However, it may be possible to cancel these by making use of additional discrete symmetries (also typical in string theory).

This then becomes a model-dependent question...
Proton Stability

- C.P. Burgess, L.E. Ibáñez, F. Quevedo, hep-ph/9810536
- Z. Kakushadze, hep-th/9812163

How to avoid rapid proton decay without heavy mass scales?
Two sets of ideas...

(1) higher-dimensional discrete orbifold symmetries
   - suppress vertices of baryon number-violating processes
     rather than propagators
   - many leading diagrams can be cancelled using only MSSM symmetries
   - generalizations exist to prevent dangerous operators to all orders, assuming various kinds of extra matter
   - intermediate-scale unification may not require all-order discrete symmetries
   - task remains to realize such mechanisms within Type I string models

(2) approximate symmetries broken on "distant" branes
   - may also be able to preserve baryon/lepton-number symmetries to desired accuracy

(3) "split fermions" inside a "fat brane"
   - spatial separation between quarks & leptons in extra dimension
   - prevents quark/lepton vertices leading to rapid proton decay