Tests of Alternative Theories of Gravity

Gilles Esposito-Farese (GReCO, IAP, France)

■ General relativity passes all tests with flying colors

■ ⇒ WHY considering alternative theories?

- ∃ theoretical motivations for alternatives to G.R.:
  Quantization of gravity & unification with other forces
  [strings] predict the existence of **PARTNERS to graviton**

- Useful to **contrast** their predictions with G.R.:
  – What theoretical information can we extract from experimental data?
  – What can be further tested?

- ∃ some puzzling exprimental issues:
  Dark energy (72%), **dark** matter (24%), Pioneer anomaly

■ GENERAL RELATIVITY

$$S_{\text{action}} = \underbrace{\frac{c^3}{16\pi G}\int d^4x\sqrt{-g}R}_{\text{dynamics of gravity}} + \underbrace{S_{\substack{\text{standard}\\\text{model}}}\left[{}^{\text{all matter}}_{\text{fields}}, g_{\mu\nu}\right]}_{\text{coupling of matter to gravity}}$$

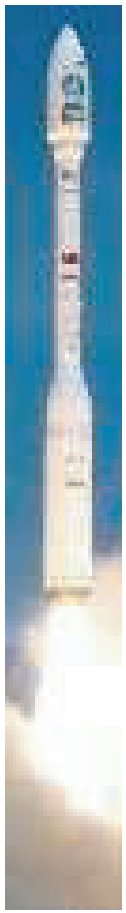
SPIN 2 FIELD

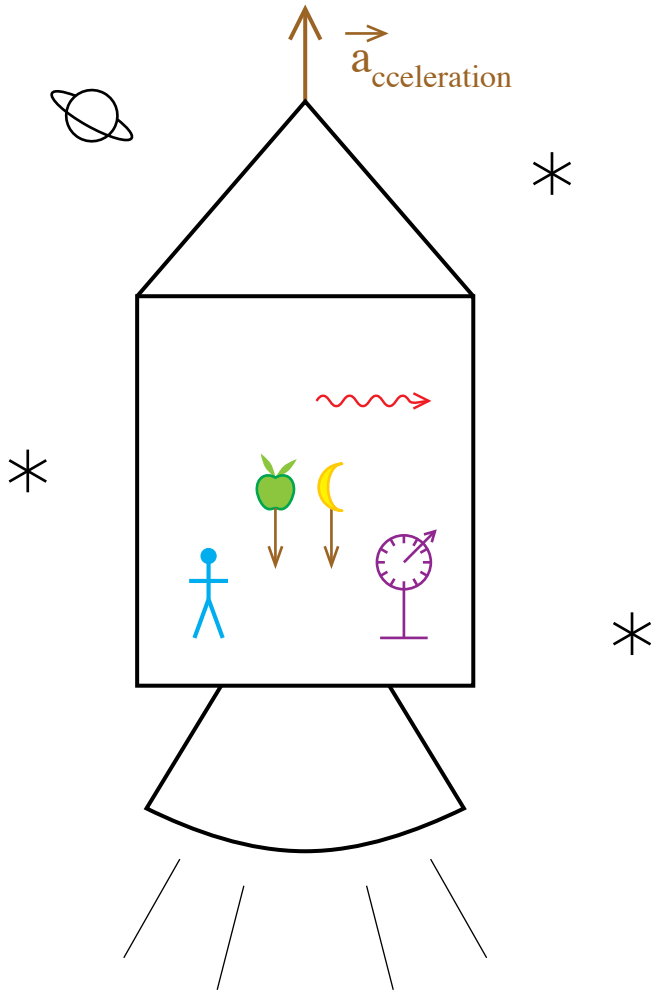
MINIMAL COUPLING TO $g_{\mu\nu}$

MATTER–GRAVITY COUPLING

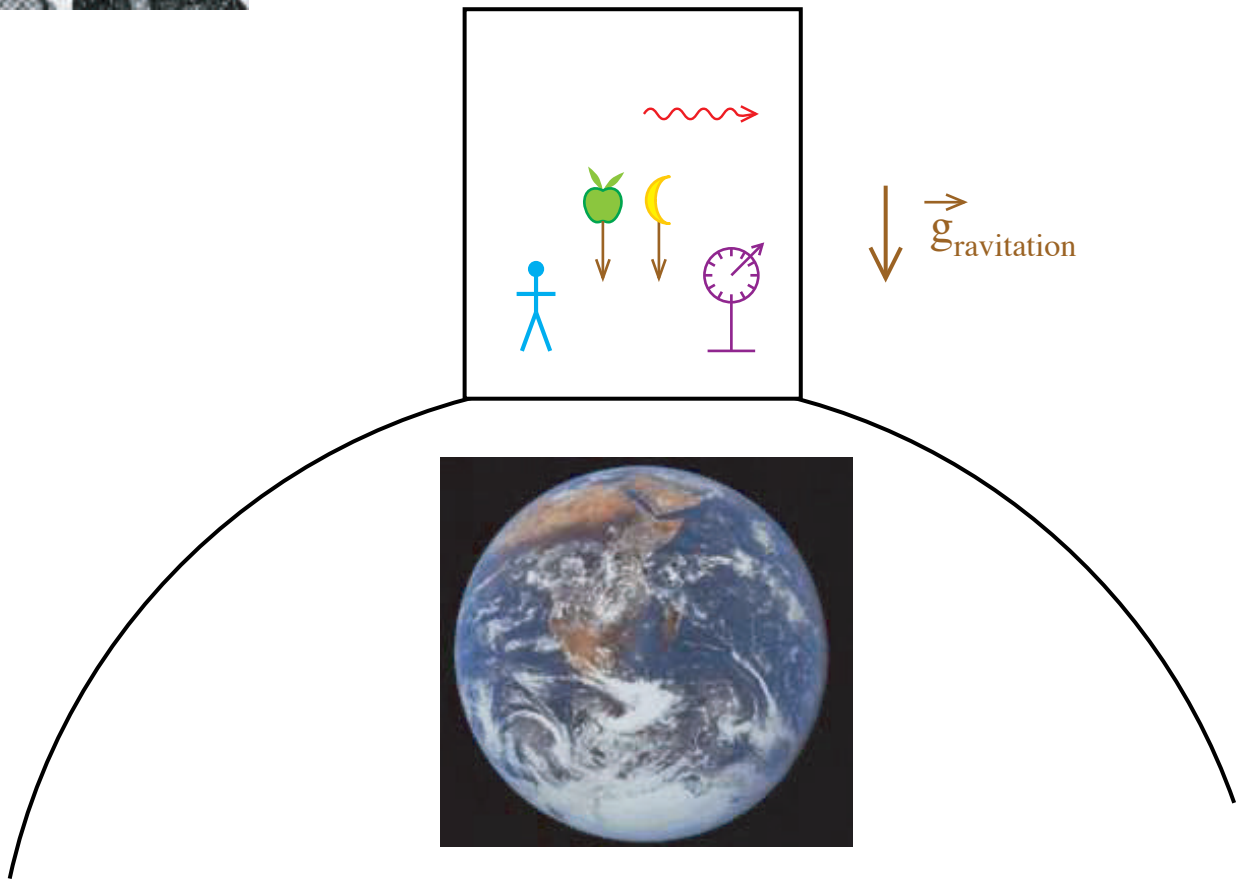
$S_{\text{matter}}\,[\ \text{matter}\ ,\ g_{\square\square}]$

Metric coupling chosen to satisfy the (weak) **equivalence principle**





Impossible to determine
from a ***local*** experiment
if there is **acceleration**
or **gravitation**
(Einstein 1907)



MATTER–GRAVITY COUPLING $S_{\text{matter}}\,[\,\text{matter}\,,\,g_{\square\square}]$

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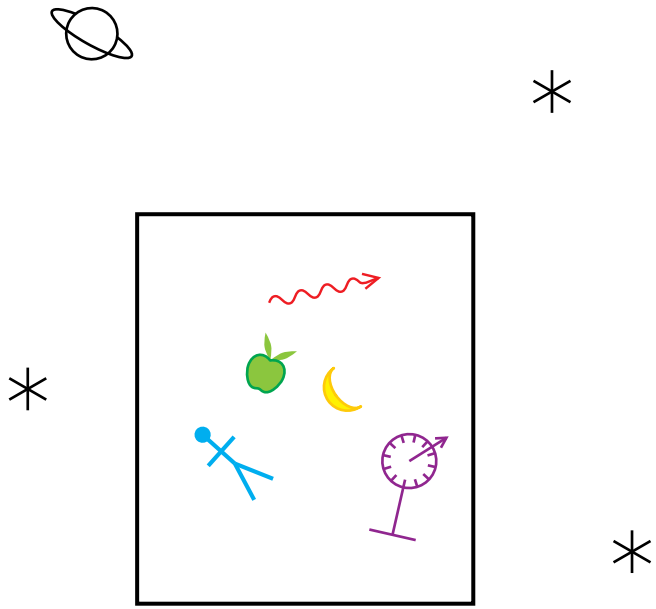
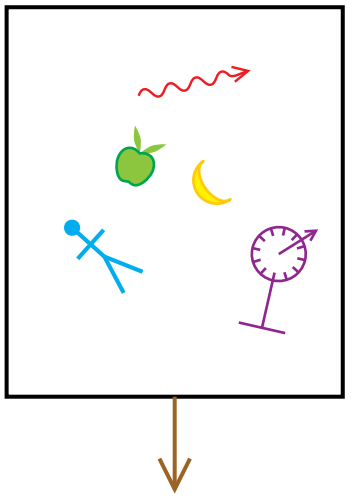
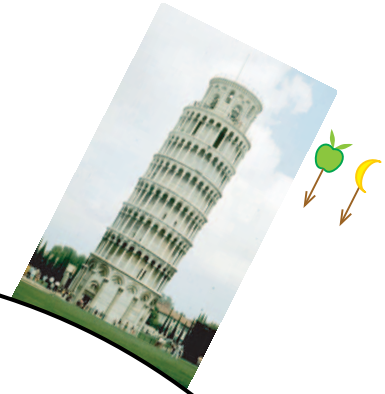


(special relativity)

freely falling elevator



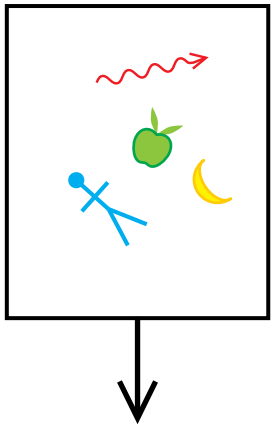


Earth

MATTER–GRAVITY COUPLING

Metric coupling: $S_{\rm matter}\,[\,{\rm matter}\,,\,g_{\mu\nu}\,]$

Freely falling elevator
(= Fermi coordinate system)



$$g_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$\Gamma^{\lambda}_{\mu\nu} = 0$$

$\Rightarrow$

(1) Constancy of the constants

Space & time independence of coupling constants and mass scales of the Standard Model

Oklo natural fission reactor
$|\dot\alpha/\alpha| < 7\times10^{-17}\,{\rm yr}^{-1} << 10^{-10}\,{\rm yr}^{-1}$ (cosmo)
[Shlyakhter 76, Damour & Dyson 96]

(2) Local Lorentz invariance

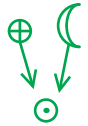
Local non-gravitational experiments are Lorentz invariant

Isotropy of space verified at the 10^{-27} level
[Prestage et al. 85, Lamoreaux et al. 86, Chupp et al. 89]

(3) Universality of free fall

Non self-gravitating bodies fall with the same acceleration in an external gravitational field

Laboratory: 4×10^{-13} level [Baessler et al. 99]

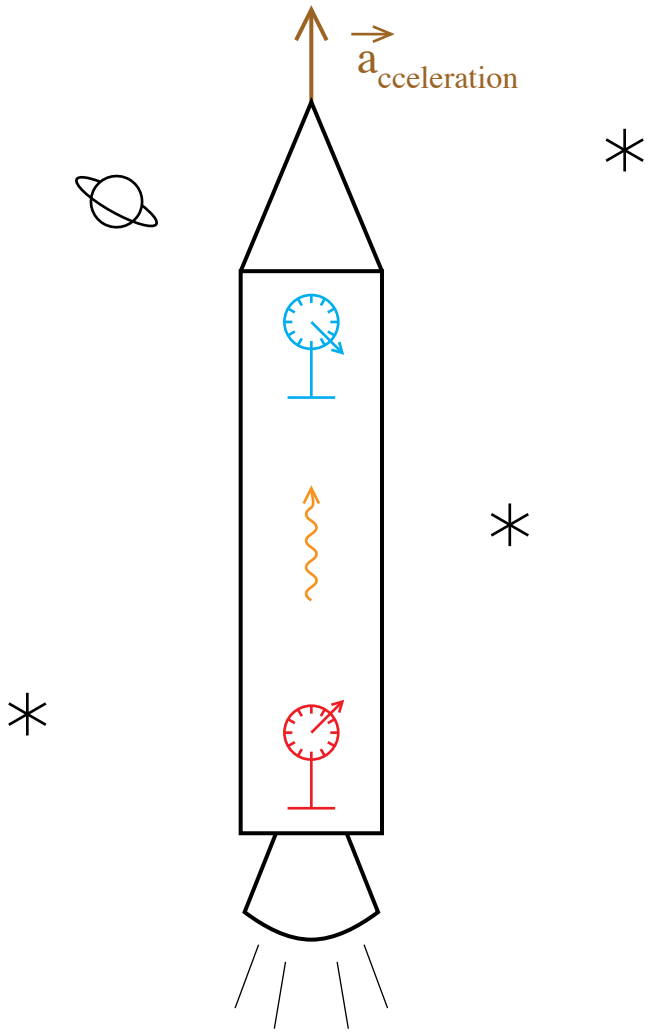


: 2×10^{-13} level [Williams et al. 04]

(4) Universality of gravitational redshift

In a static Newtonian potential
$$g_{00} = -1 + 2\,U(x)/c^2 + O(1/c^4)$$
the time measured by two clocks is
$$\tau_1/\tau_2 = 1 + [U(x_1)–U(x_2)]/c^2 + O(1/c^4)$$
Flying hydrogen maser clock: 2×10^{-4} level
[Vessot et al. 79–80, Pharao/Aces will give 5×10^{-6}]

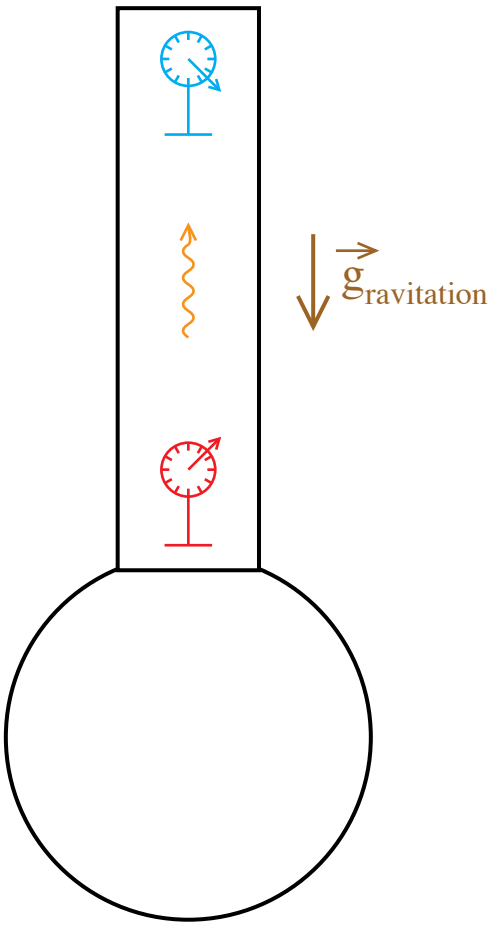
④ Universality of gravitational redshift (time dilation)



Doppler effect
(cf. fire-truck siren)

⇒ Whatever their composition,
lower clocks are slower

(⇒ impossible to
synchronize
even static clocks)



Theoretical motivations for non-metric coupling?

- SUPERstrings: ∃ dilaton φ, partner of graviton in 10 dimensions
- Dimensional reduction: ∃ moduli φ
- …

$$g_{mn} = \left(\begin{array}{c|c} g_{\mu\nu} & A_\mu \\ \hline A_\nu & \varphi \end{array}\right)$$

⇒ ∃ dilatonic coupling of a scalar field to gauge fields

$$S_{EM} = \int \frac{e^{\varphi}}{4\,k_0^2} F_{\mu\nu}^2 \sqrt{-g}\, d^4x$$

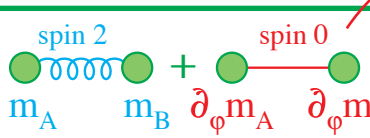
conformal invariant ⇒ e^{φ} cannot be eliminated by redefining $\tilde{g}_{\mu\nu} = f(\varphi)\, g_{\mu\nu}$

⇒ Effective coupling constant $k_{eff} = k_0\, e^{-\varphi/2}$ depends on $\vec{x}$

⇒ Masses $m(\varphi)$ depend also on $\vec{x}$

⇒ Violations of universality of free fall:

$$\delta \vec{a} = -\vec{\nabla} \ln m(\vec{x})$$

Newton force = 

depends on composition of m_A and m_B

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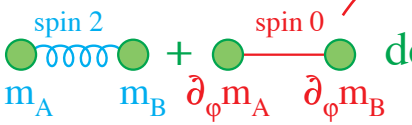
Newton force = 

depends on composition of m_A and m_B

	geometry	coupling constants
Newton	rigid	rigid
Einstein	soft	rigid
Strings	soft	soft

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$$\delta \vec{a} = -\vec{\nabla} \ln m(\vec{x})$$

Newton force = spin 2 (m_A, m_B) + spin 0 ($\partial_\varphi m_A$, $\partial_\varphi m_B$) depends on composition of m_A and m_B

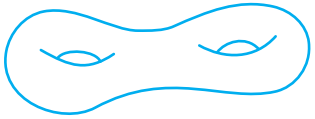
	geometry	coupling constants
Newton	rigid	rigid
Einstein	soft	rigid
Strings	soft	soft

Tree-level predictions of strings		Experiment
$\Delta a/a \sim 10^{-5}$	>>	5×10^{-13}
$\dot{\alpha}/\alpha \sim H_0 \sim 10^{-10}\ \mathrm{yr}^{-1}$	>>	$7\times10^{-17}\ \mathrm{yr}^{-1}$
$\gamma^{PPN}-1 \sim O(1)$, $\beta^{PPN}-1 \sim O(40)$	>>	10^{-5}

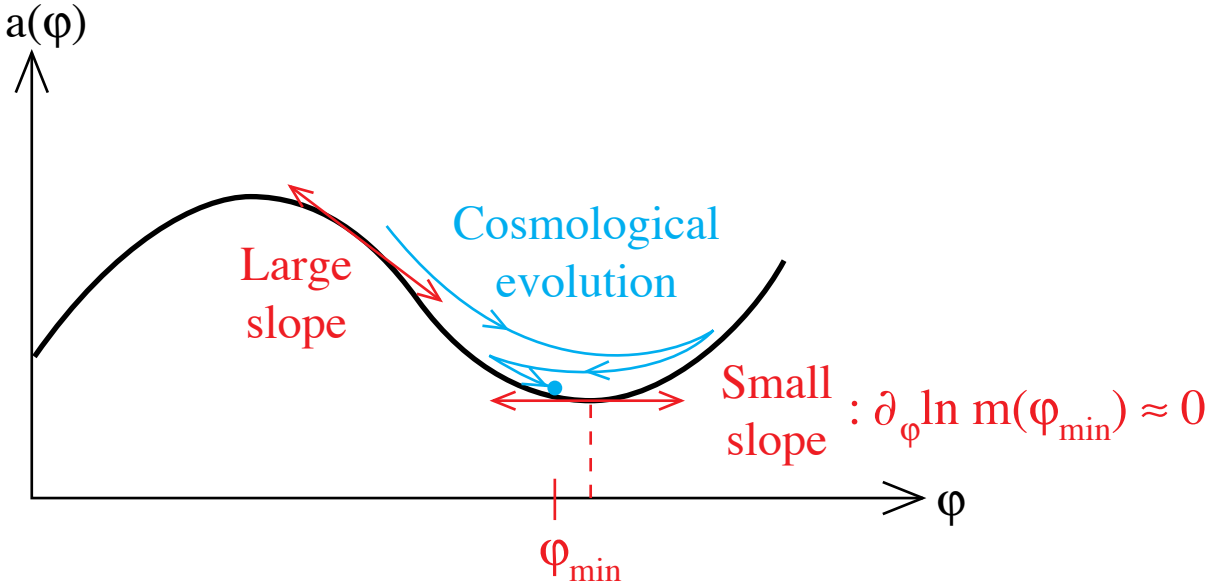
How can strings be saved?

- Add a mass to dilaton?
  BUT – no natural mechanism to generate masses for all scalar fields in the theory
  – difficult cosmological problems
  [e.g. Polonyi: too much energy stored in cosmological oscillations of $\varphi(t)$]

- String loops!



Transform e^{φ} into $e^{a(\varphi)}$

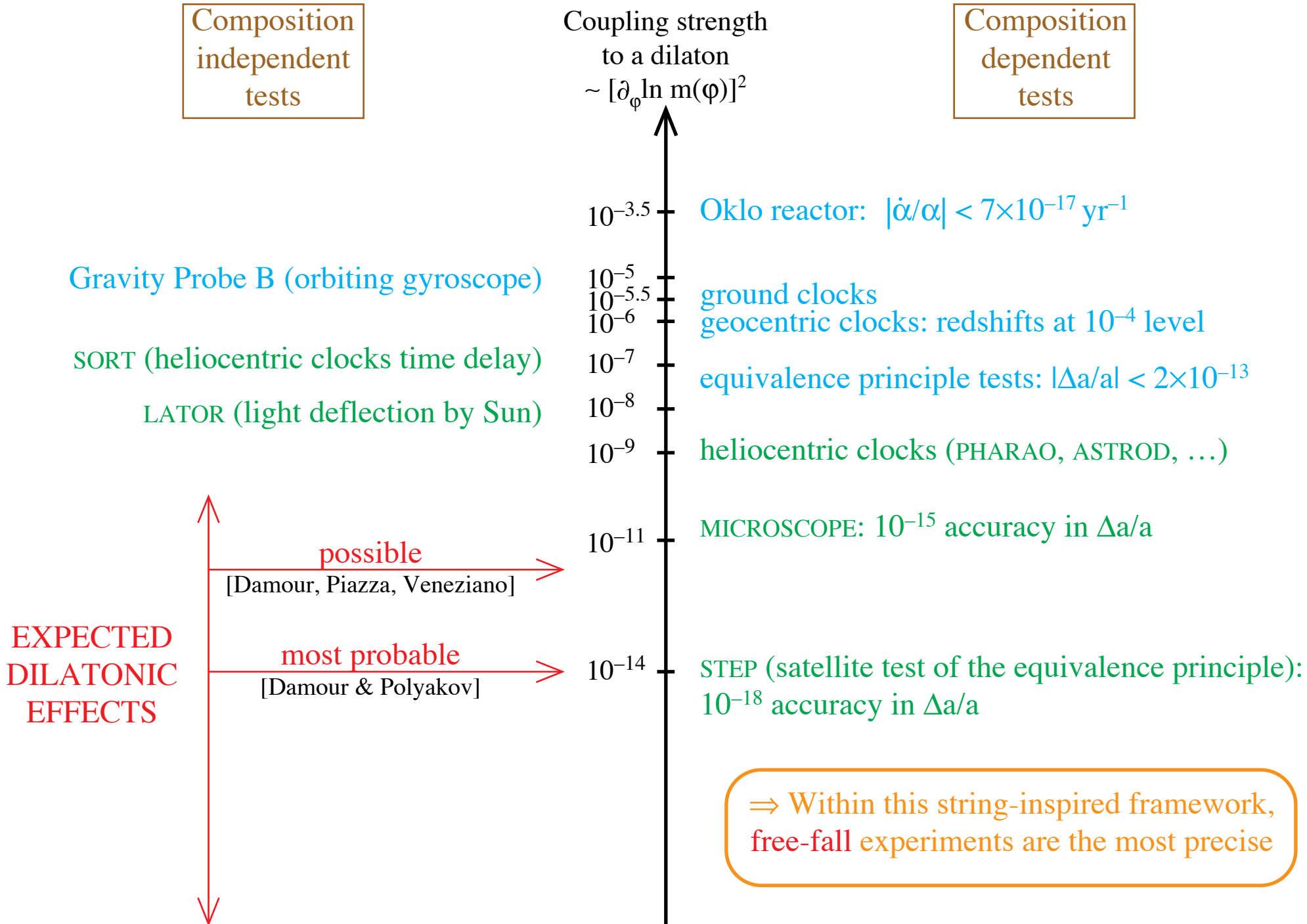


If ≈ same φ_{min} for all elementary particles [cf. S-duality, i.e., symmetry under $g_{string} = e^{\varphi} \to 1/g_{string} = e^{-\varphi}$], then expected deviations from general relativity are

$$[\partial_{\varphi} \ln m(\varphi_{now})]^2 \sim 10^{-10} \to 10^{-19}$$

[Damour & Polyakov 94, Damour & Vilenkin 95–96, Damour, Piazza, Veneziano 02]

Experimental data in the context of this string model



DYNAMICS OF GRAVITY

Now, assume metric coupling of matter to gravity: $S = S_{\rm gravity} + S_{\rm matter}\left[\,{\rm matter}\,,\, g_{\mu\nu}\right]$

?

Phenomenological approach: PPN formalism

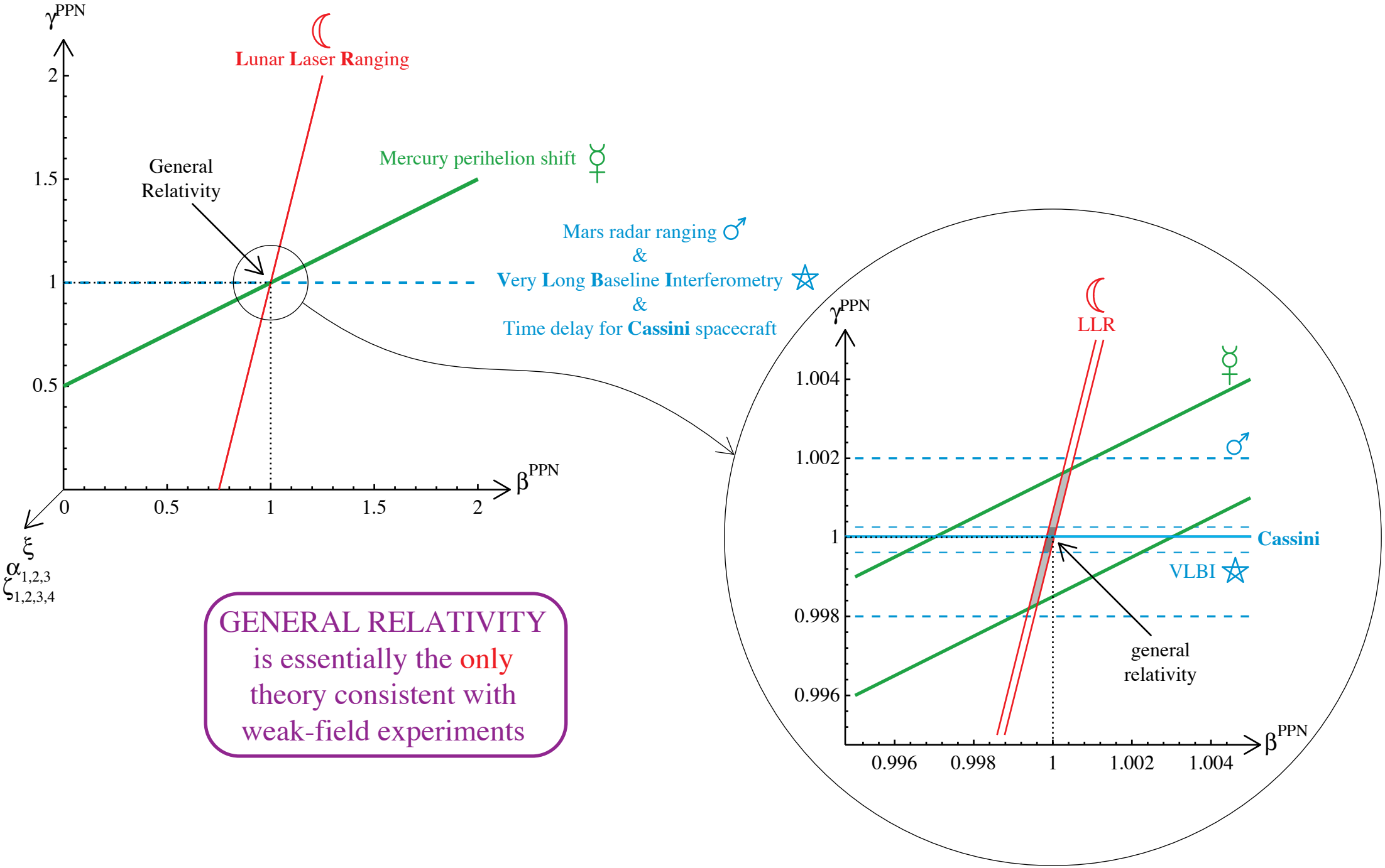
- Do not assume anything about $S_{\rm gravity}$
- Write the most general form that $g_{\mu\nu}$ can take in presence of matter, at the first post-Newtonian order [Newton $\times$ $1/c^2$]

Basic idea [Eddington 1923]:

$$\begin{cases} -\,g_{00} = 1 - 2\,\dfrac{Gm}{rc^2} + 2\,\beta^{\rm PPN}\left(\dfrac{Gm}{rc^2}\right)^2 + \ldots \\ g_{ij} = \delta_{ij}\left[\,1 + 2\,\gamma^{\rm PPN}\,\dfrac{Gm}{rc^2} + \ldots\,\right] \end{cases}$$

Generalization [Will & Nordtvedt 1972]: 10 parameters including $\beta^{\rm PPN}$ and $\gamma^{\rm PPN}$

Conclusion of experimental tests in the **P**arametrized **P**ost-**N**ewtonian formalism



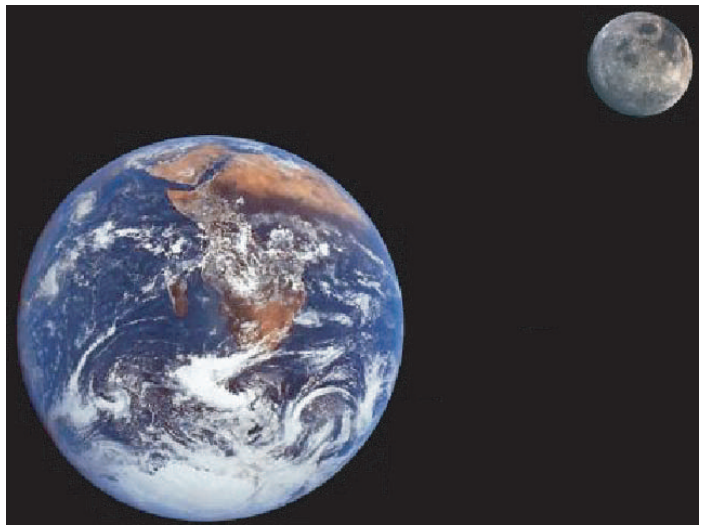
GENERAL RELATIVITY is essentially the only theory consistent with weak-field experiments

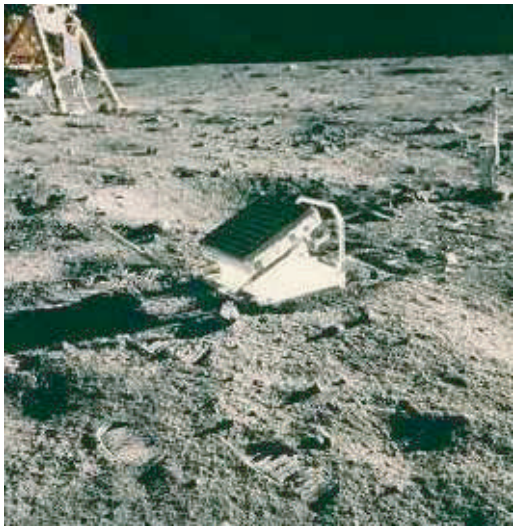
[Table from C.M. Will gr-qc/0504086]

Table 1. Current Limits on the PPN Parameters

	Parameter	Effect	Limit	Remarks
space curvature created by mass	$\gamma - 1$	(i) time delay	2.3×10^{-5}	Cassini tracking
		(ii) light deflection	3×10^{-4}	VLBI
nonlinearity in superposition law	$\beta - 1$	(i) perihelion shift	3×10^{-3}	$J_2 = 10^{-7}$ from helioseismology
		(ii) Nordtvedt effect	5×10^{-4} 10^{-4}	$\eta = 4\beta - \gamma - 3$ assumed
preferred-location effects	ξ	Earth tides	10^{-3}	gravimeter data
preferred-frame effects	α_1	orbital polarization	10^{-4}	Lunar laser ranging PSR J2317+1439
	α_2	solar spin precession	4×10^{-7}	alignment of Sun and ecliptic
	α_3	pulsar acceleration	2×10^{-20}	pulsar $\dot{P}$ statistics
combination of other parameters	η^1	Nordtvedt effect	10^{-3} 5 10^{-4}	Lunar laser ranging
violation of conservation of total momentum	ζ_1	–	2×10^{-2}	combined PPN bounds
	ζ_2	binary motion	4×10^{-5}	$\ddot{P}_p$ for PSR 1913+16
	ζ_3	Newton's 3rd law	10^{-8}	Lunar acceleration
	ζ_4	–	–	not independent

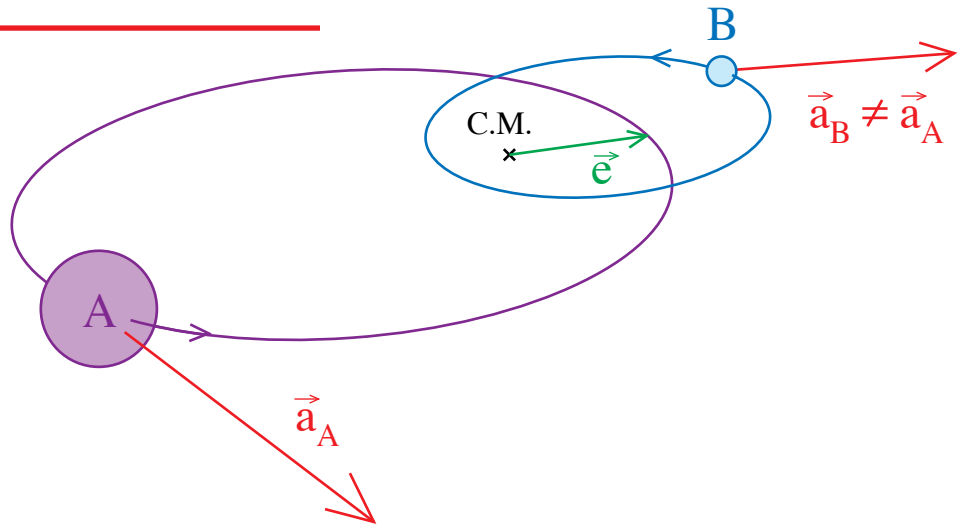
[1] Here $\eta = 4\beta - \gamma - 3 - 10\xi/3 - \alpha_1 - 2\alpha_2/3 - 2\zeta_1/3 - \zeta_2/3$

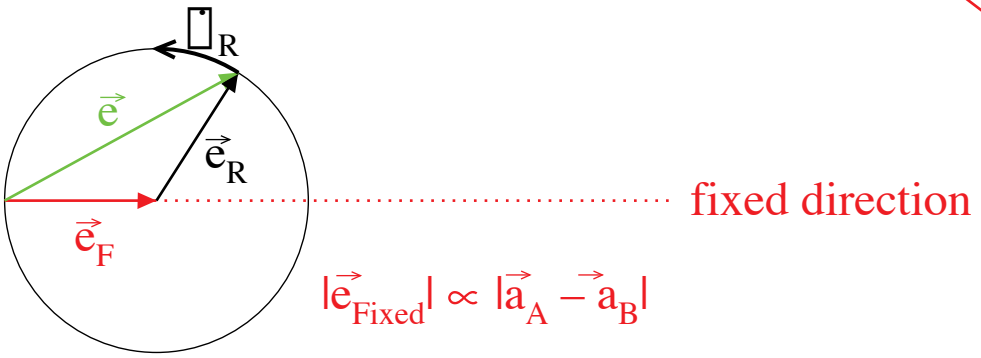




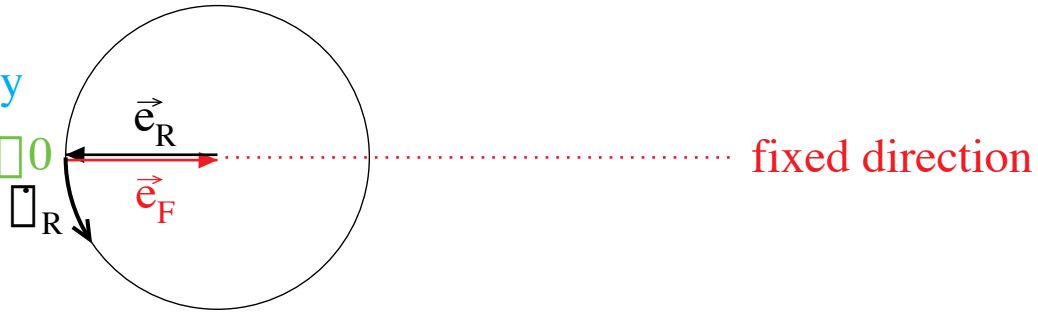
Tests of the "strong equivalence principle" and of preferred-frame effects

• The different accelerations (due to a third body or to their absolute velocity with respect to a preferred frame) induce a polarization of the periastron towards a precise direction





• several binary pulsars with $\vec{e} \neq 0$



→ statistical argument to constrain PPN parameters

[Damour, Schäfer, GEF, Bell, Camilo, Wex, …]

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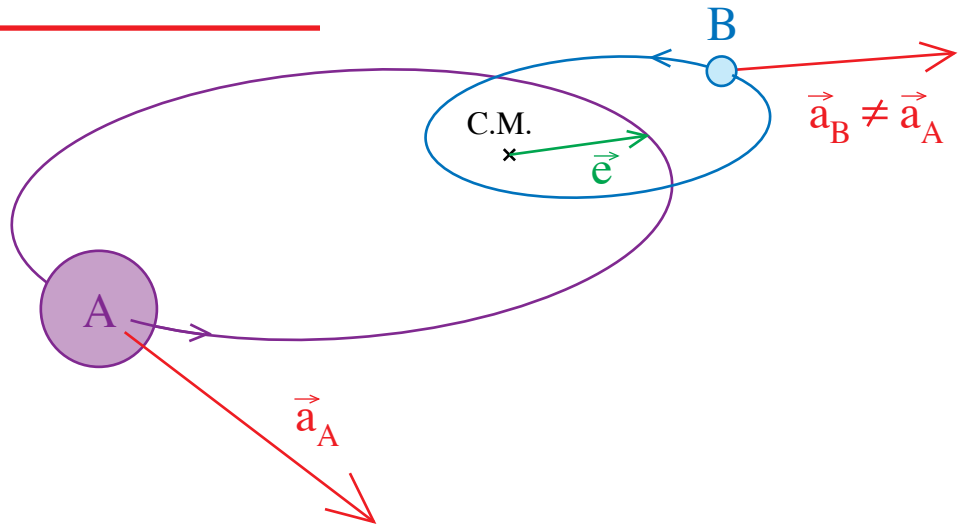
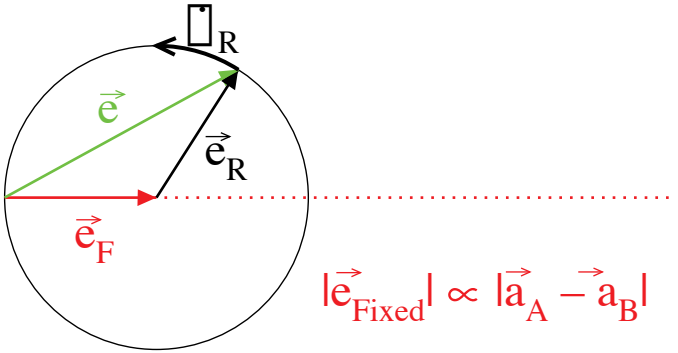
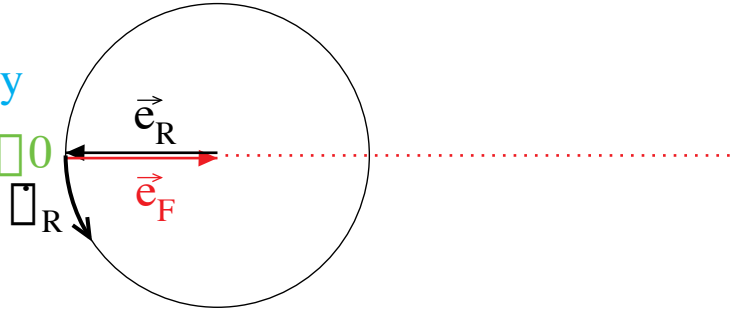




fixed direction

$|\vec{e}_{Fixed}| \propto |\vec{a}_A - \vec{a}_B|$

• several binary pulsars with $\vec{e} \neq 0$



statistical argument to constrain PPN parameters

[Damour, Schäfer, GEF, Bell, Camilo, Wex, …]

• Earth-Moon-Sun system [Nordtvedt]

$G_{\oplus\odot} = G\,(1 + \eta_{\oplus} + \eta_{\odot})$

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η_A = $\hat{\eta}_A$ + $(4\beta - \gamma - 3)\, E_A^{grav.}/m_A c^2$

η_A: equivalence principle violation

$\hat{\eta}_A$: due to dilaton coupling

$(4\beta - \gamma - 3)$: PPN contribution

$E_A^{grav.}/m_A c^2 \sim 10^{-10}$

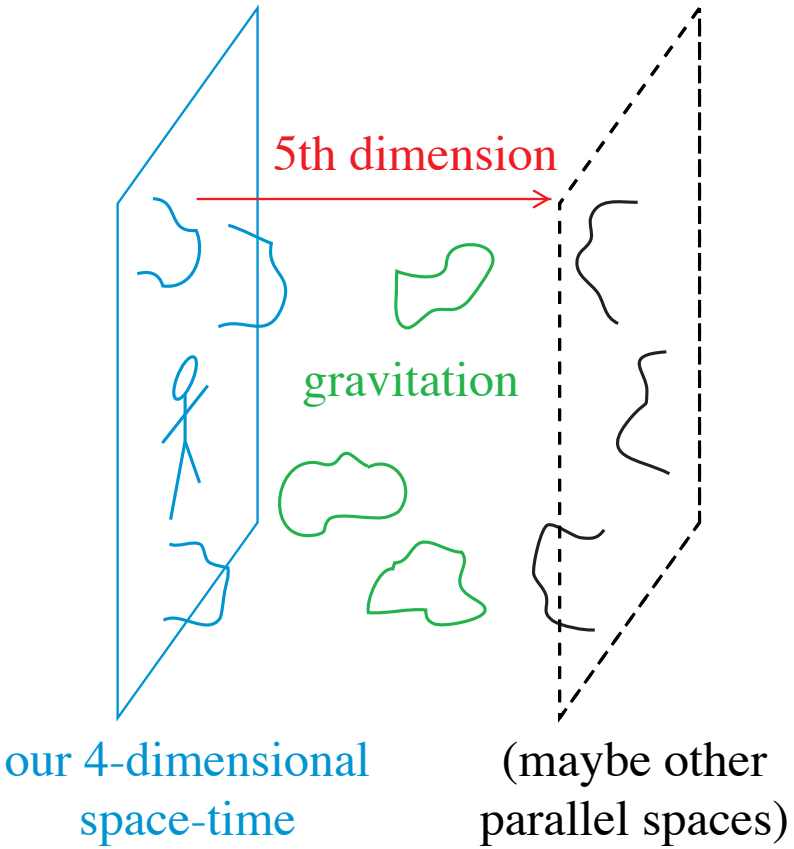
$|\Delta a/a| < 2 \times 10^{-13}$ experiment

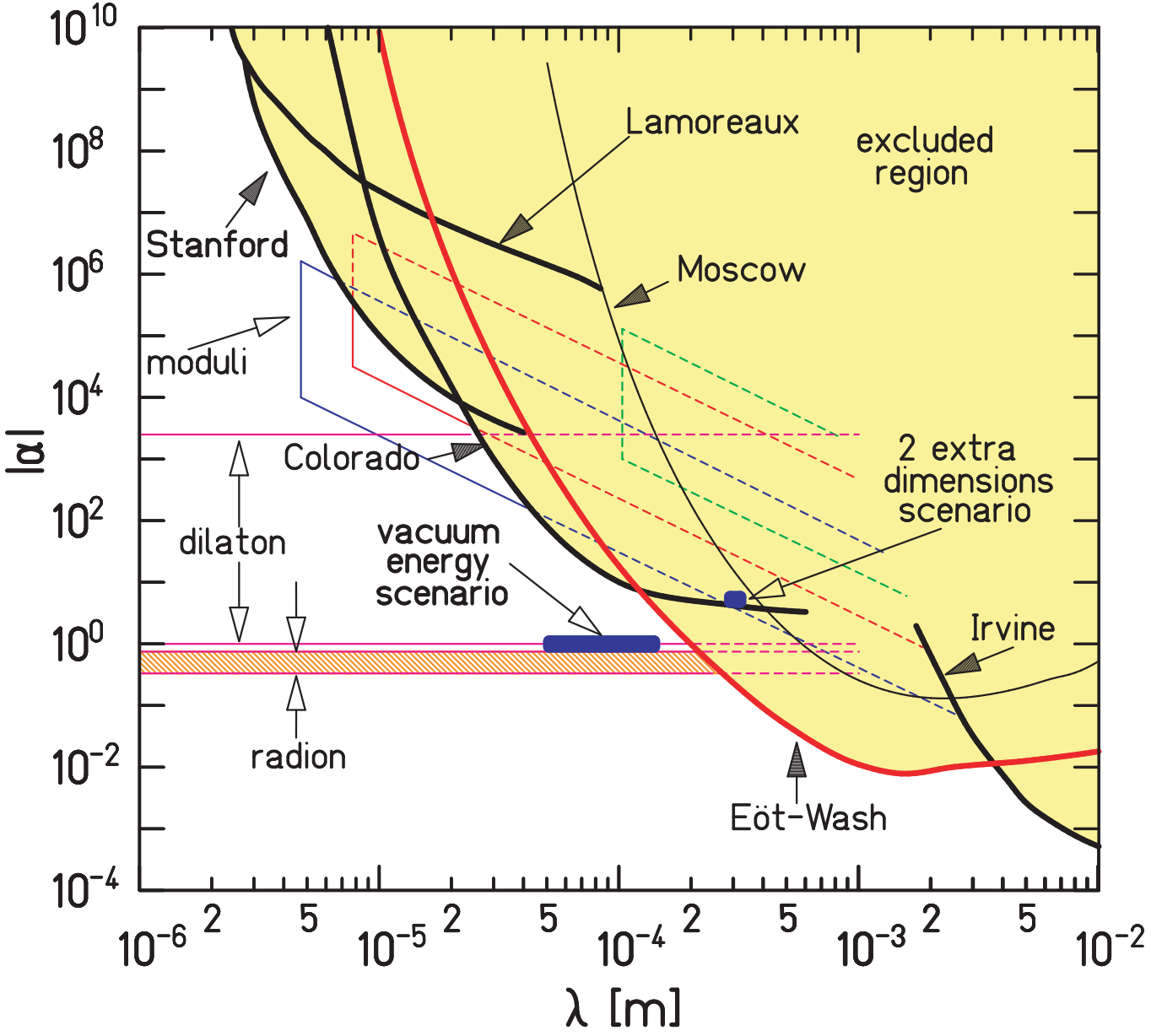
- dilaton coupling $< 10^{-8}$
- PPN constraint $|4\beta - \gamma - 3| < 10^{-3}$

DYNAMICS OF GRAVITY (continued)

Brane models imply (long and) short-distance modifications of Newton's law

$$V = \frac{GM}{r}\left(1 + \alpha\, e^{-r/\lambda}\right)$$





[C.D. Hoyle *et al.*, Phys. Rev. **D70** (2004) 042004, hep-ph/0405262]

Constraining the graviton mass?

⚠ ∃ no clean theory of massive graviton
("vDVZ" discontinuity, ghosts, or predictions not yet worked out)
⇒ phenomenological point of view…

• Solar system [C. Talmadge *et al.*, Phys. Rev. Lett. **61** (1988) 1159]

Yukawa-type Newtonian potential $V_N = \frac{Gm}{r} e^{-r/\lambda_g}$

$\Rightarrow m_g < 4\times10^{-22}\ eV/c^2 \Leftrightarrow \lambda_g = h/(m_g c) > 3\times10^{12}\ km$

• Binary pulsars

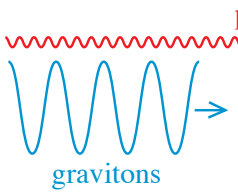
[L.S. Finn and P.J. Sutton, Phys. Rev. D **65** (2002) 044022; Class. Quantum Grav. **19** (2002) 1355]

$\Rightarrow m_g < 10^{-19}\ eV/c^2 \Leftrightarrow \lambda_g = h/(m_g c) > 10^{10}\ km$

• LISA correlated with optical observations

$E = \gamma mc^2 \Leftrightarrow v_g^2/c^2 = 1 - m_g^2 c^4/E^2$ (dispersion relation)

$\Rightarrow m_g < 6\times10^{-24}\ eV/c^2 \Leftrightarrow \lambda_g = h/(m_g c) > 2\times10^{14}\ km$



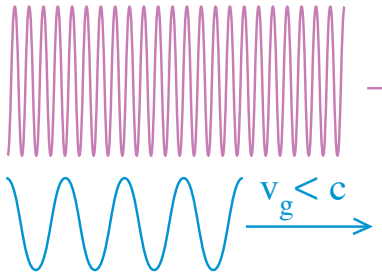
[S.L. Larson, W.A. Hiscock, Phys. Rev. D **61** (2000) 104008;
C. Cutler, W.A. Hiscock, S.L. Larson, Phys. Rev. D **67** (2003) 024015;
A. Cooray, N. Seto, Phys. Rev. D **69** (2004) 103502]

• GW interferometers alone

[C.M. Will, Phys. Rev. D **57** (1998) 2061;
E. Berti, A. Buonanno, C.M. Will, Phys. Rev. D **71** (2005) 084025]

LIGO/VIRGO $\Rightarrow m_g < 2\times10^{-22}\ eV/c^2 \Leftrightarrow \lambda_g = h/(m_g c) > 6\times10^{12}\ km$

LISA (BH-BH) $\Rightarrow m_g < 2\times10^{-26}\ eV/c^2 \Leftrightarrow \lambda_g = h/(m_g c) > 6\times10^{16}\ km$



DYNAMICS OF GRAVITY (continued)

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?

Field-theoretical approach

- Now, $g_{\mu\nu}$ is assumed to be a combination of **fields** which **propagate** in a consistent way:

$$g_{\mu\nu} = \underbrace{A^2(\varphi)}_{\text{spin 0}}\Big[\underbrace{g^*_{\mu\nu}}_{\text{spin 2}} + \underbrace{a_1\, B_\mu B_\nu + a_2\, g^*_{\mu\nu}\, B_\rho B^{*\rho}}_{\text{vector fields (spin 1)}} + \underbrace{a_3\, B_{\mu\rho}B^{*\,\rho}_{\nu}}_{\substack{\text{antisymmetric}\\ \text{tensor field}}} + \ldots\Big]$$

- However, vector (B_μ) or tensor ($B_{\mu\nu}$) partners to graviton ($g^*_{\mu\nu}$) lead in general to many flaws:
  - discontinuities in the field degrees of freedom
  - negative-energy modes
  - causality violations
  - ill-posedness of the Cauchy problem
  - no theoretical motivations for being coupled to matter in the "metric" way $S_{\rm matter}\left[\,{\rm matter}\,,\, g_{\mu\nu}\right]$
  - …

$\Rightarrow$ best-motivated and consistent alternatives to General Relativity:

Tensor–scalar theories

$$S = \frac{1}{16\,\pi\,G}\int \sqrt{-g^*}\left\{R^* - 2\left(\partial_\mu\varphi\right)^2\right\} + S_{\rm matter}\left[\,{\rm matter}\,,\, g_{\mu\nu} \equiv A^2(\varphi)\, g^*_{\mu\nu}\right]$$

R^*: spin 2; $\left(\partial_\mu\varphi\right)^2$: spin 0; $g_{\mu\nu}$: physical metric

[N.B.: more than one scalar and massive scalars are also possible]

Simplest example:
Nordström's purely scalar theory (1913)

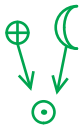
$$S = \frac{1}{8\pi G}\int d^4x\,\eta^{\mu\nu}\,\partial_\mu\varphi\partial_\nu\varphi + S_{\rm matter}\left[\text{matter},\, g_{\mu\nu} \equiv \varphi^2\eta_{\mu\nu}\right]$$

↑ spin 0

↑ conformally flat physical metric

– Correct Newtonian limit

– Satisfies weak and **strong equivalence principle**s (⇒ no Nordtvedt effect on Moon's orbit)

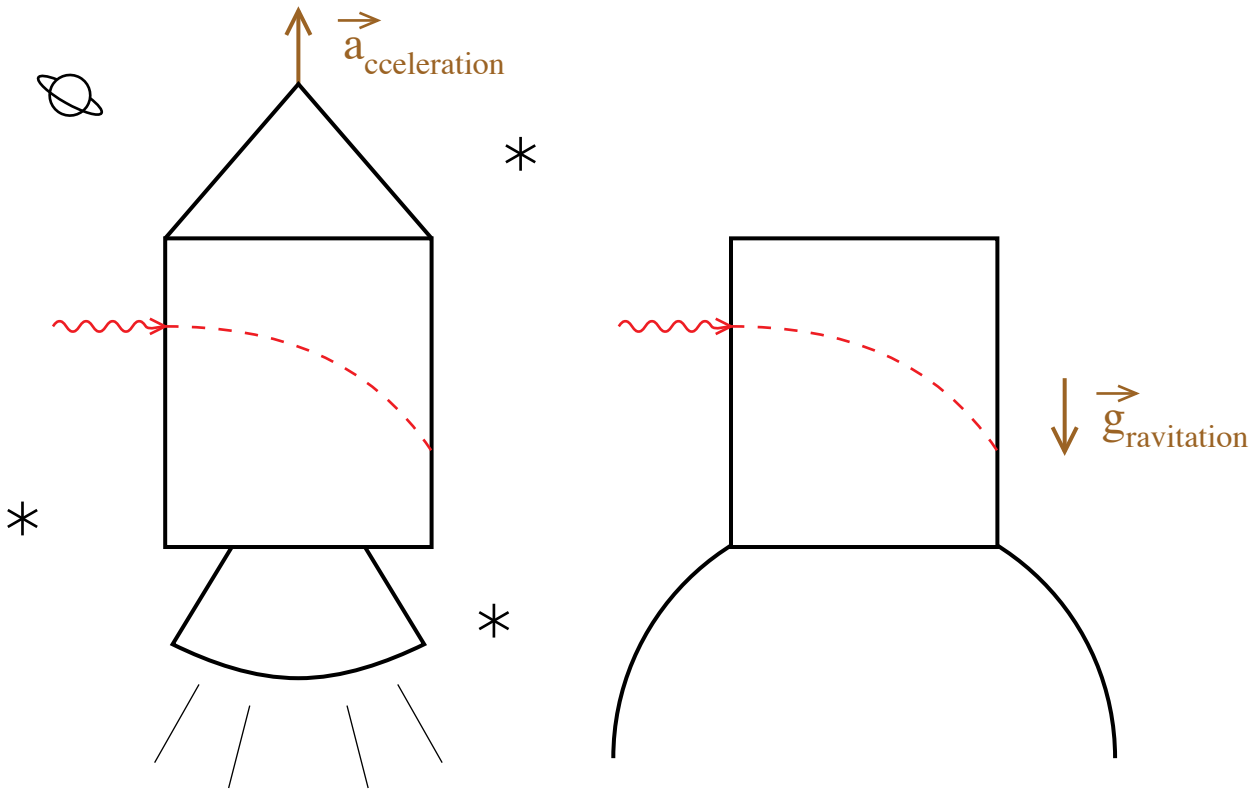


– But post-Newtonian predictions inconsistent with experiment:

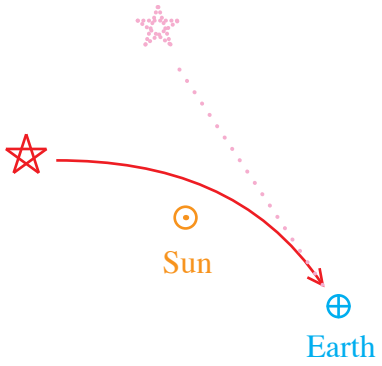
$$ds^2 = g_{\mu\nu}\,dx^\mu dx^\nu = 0 \Rightarrow \eta_{\mu\nu}\,dx^\mu dx^\nu = 0$$

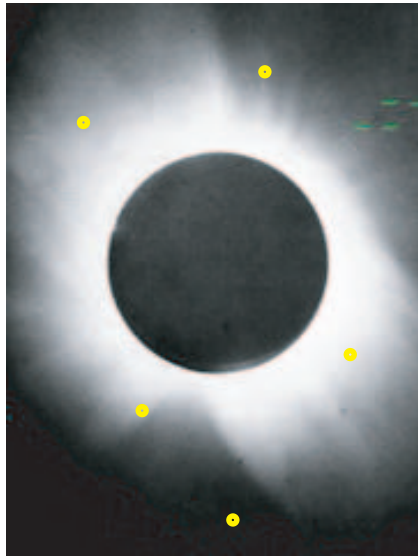
Light rays follow geodesics of flat spacetime ⇒ no light deflection!

■ Light deflection and the equivalence principle

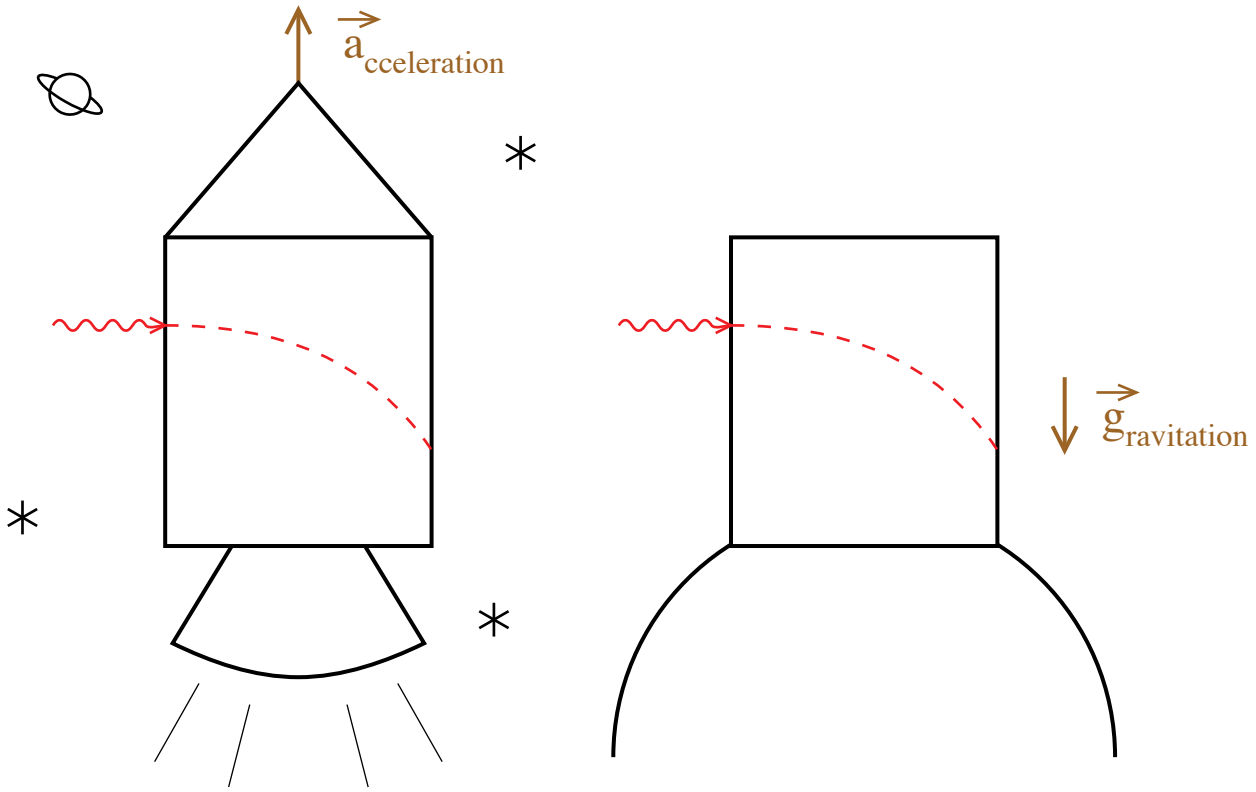


⇒ Modification of the stars' apparent position





■ Light deflection and the equivalence principle



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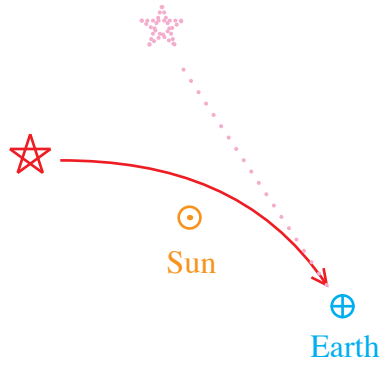

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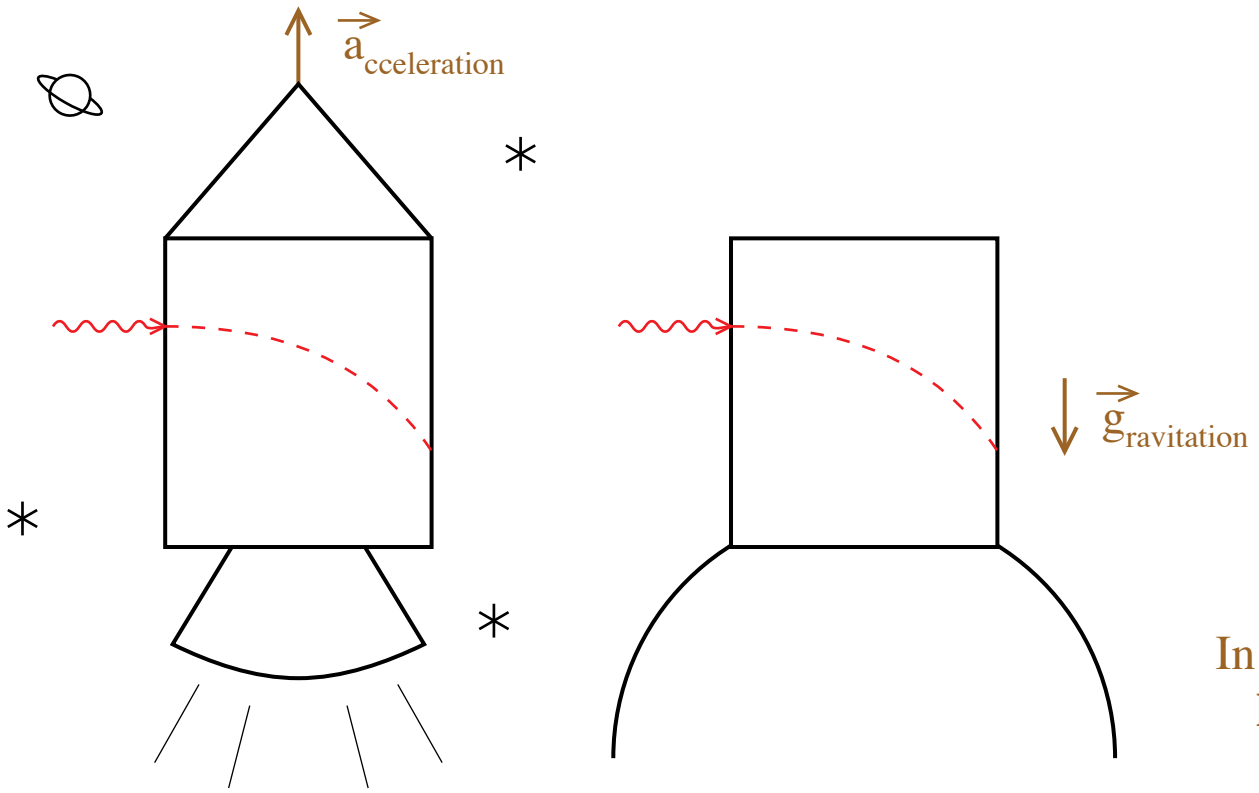

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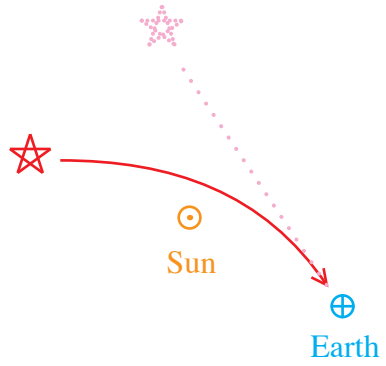
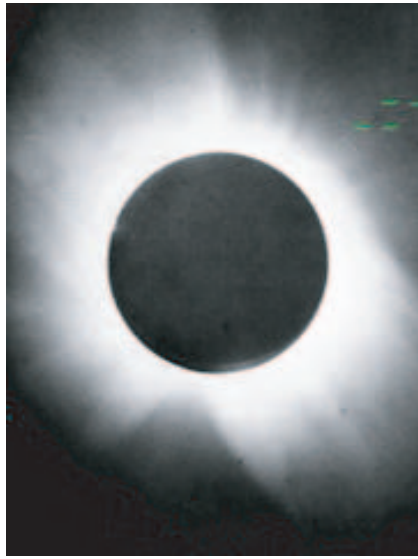


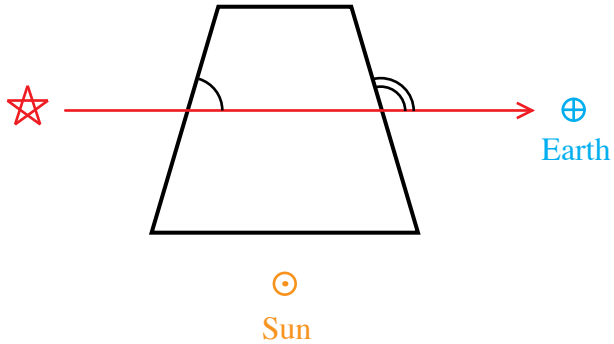


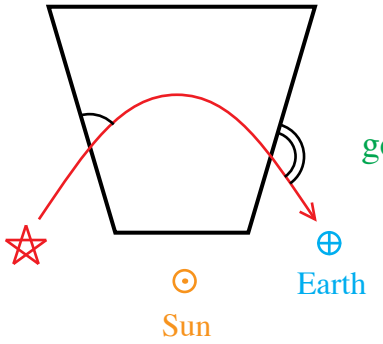
[Eddington 1919]

In 1914, Einstein predicts **half** the correct value

This is because ∃ also a deformation of *space*:

Nordström's theory 1913





Einstein's general relativity 1915

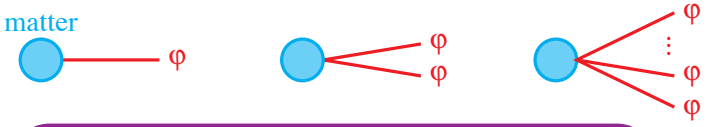
For general tensor–scalar theories

$$S = \frac{1}{16\pi G}\int \sqrt{-g^*}\left\{ R^* - 2\left(\partial_\mu \varphi\right)^2 \right\} + S_{\text{matter}}\left[\text{matter}\,,\; g_{\mu\nu} \equiv A^2(\varphi)\, g^*_{\mu\nu}\right]$$

spin 2 — spin 0 — physical metric

- Matter–scalar interaction:

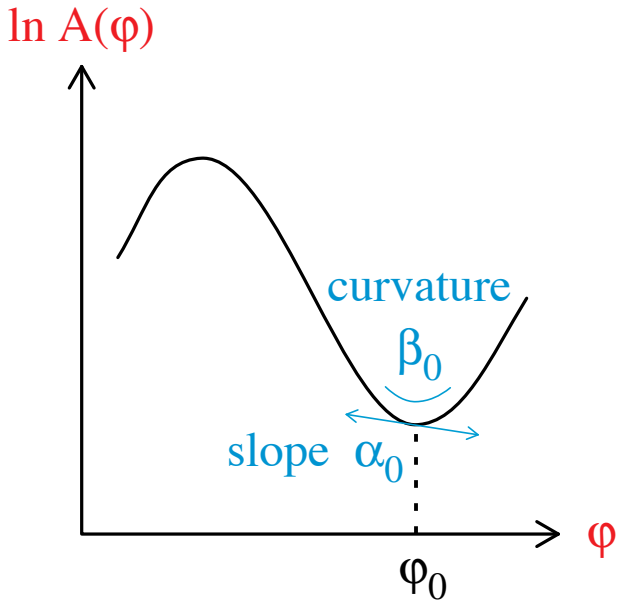
$$\ln A(\varphi) = \alpha_0\,(\varphi-\varphi_0) + \frac{1}{2}\beta_0\,(\varphi-\varphi_0)^2 + \ldots$$



Effective Newton's constant:

$$G_{\text{eff}} = G\,(1 + \alpha_0^{\,2})$$

graviton — scalar (α_0 — α_0)



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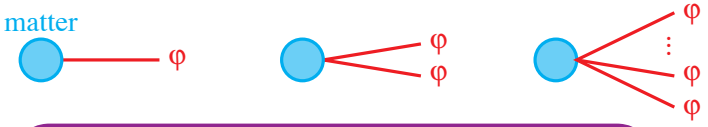

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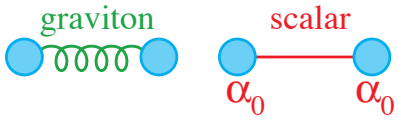
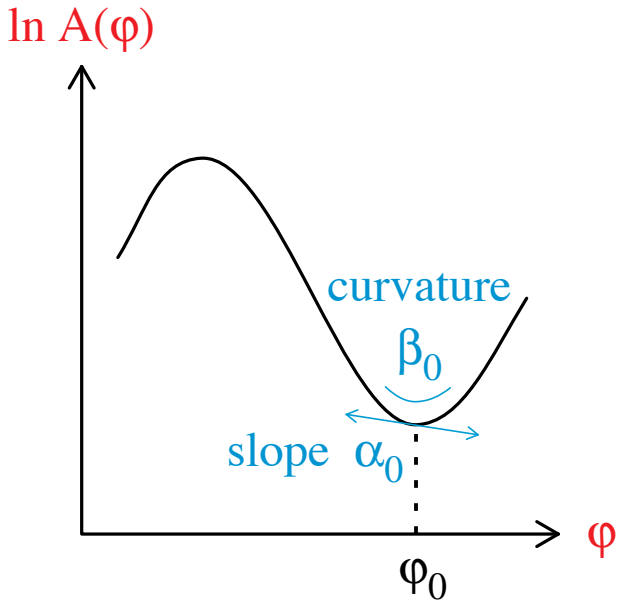
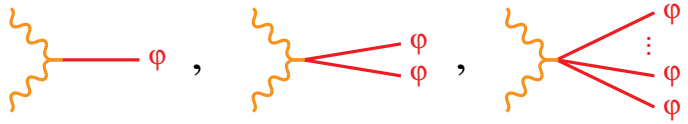




- Photon–scalar interaction:

conformal invariance $\quad S_{\text{EM}} = \int \sqrt{-g}\, g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} = \int \sqrt{-g_*}\, g_*^{\mu\rho} g_*^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma}$

$\Rightarrow$ no photon–scalar vertex ⚠

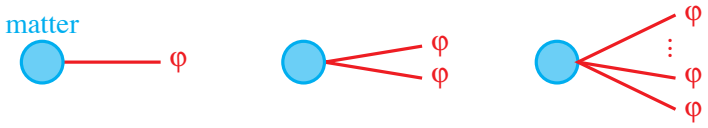


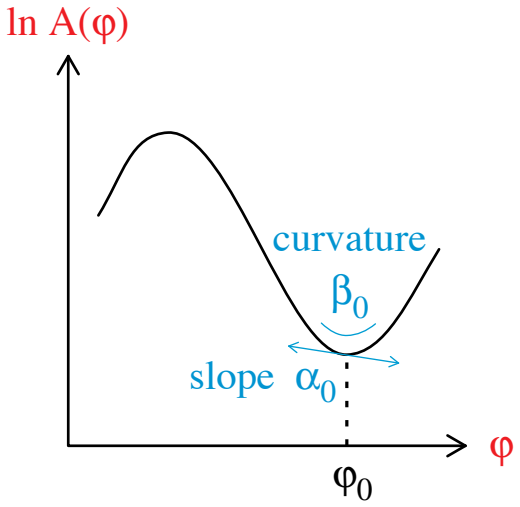
$= 0$

$\Rightarrow$ light deflection $\propto \dfrac{GM}{rc^2} = \dfrac{G_{\text{eff}} M}{(1+\alpha_0^2) rc^2}$ < G.R. result

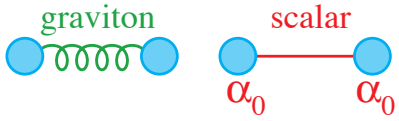
Other post-Newtonian predictions

$$\ln A(\varphi) = \alpha_0 (\varphi - \varphi_0) + \frac{1}{2}\beta_0 (\varphi - \varphi_0)^2 + \dots$$



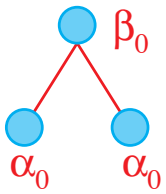


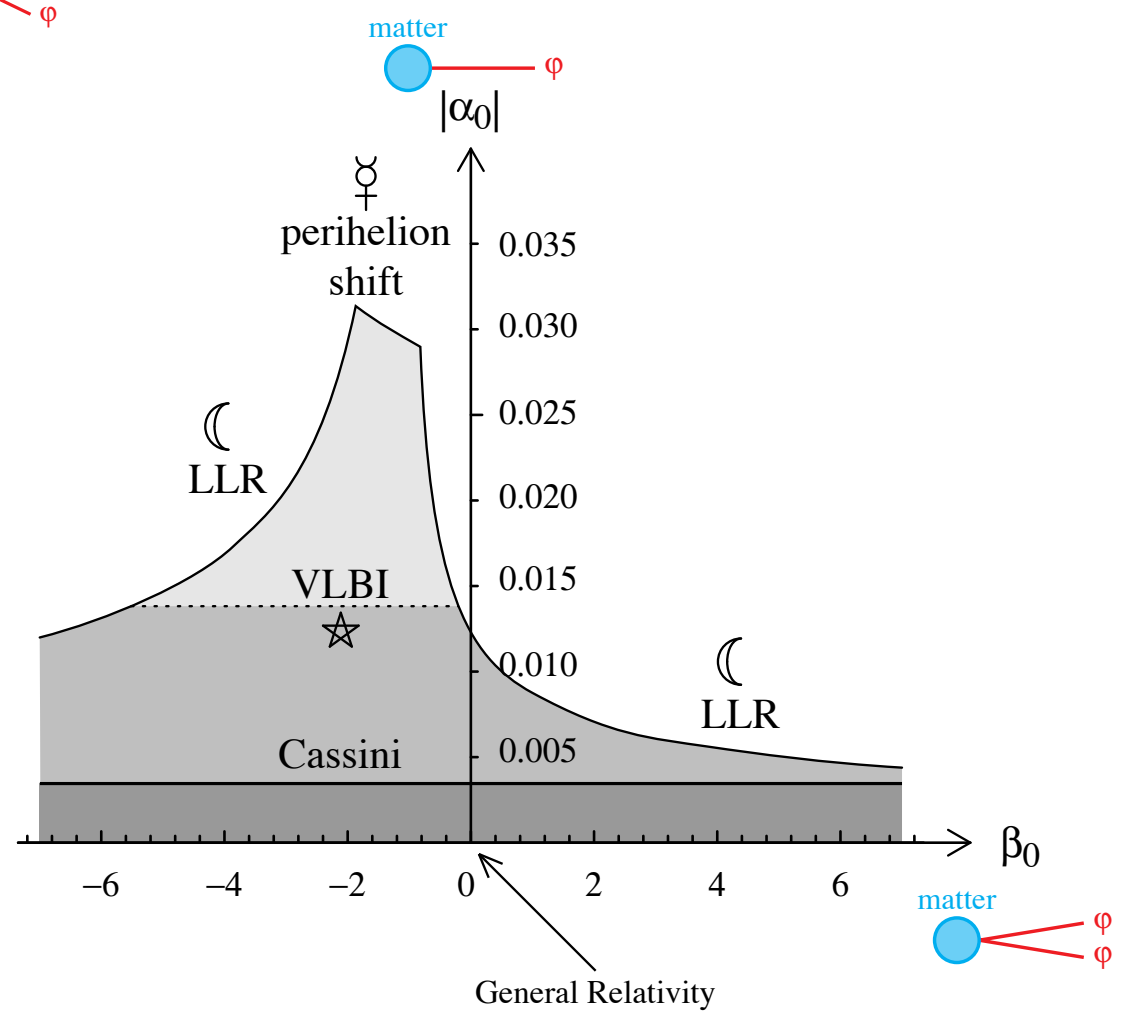
$$G_{eff} = G\,(1 + \alpha_0^{\,2})$$



$$\gamma^{PPN} - 1 \propto \alpha_0^{\,2}$$

$$\beta^{PPN} - 1 \propto \alpha_0^{\,2}\,\beta_0$$



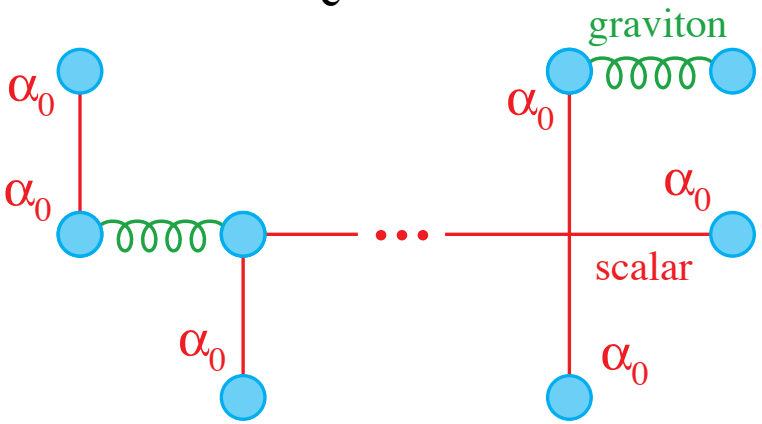


Vertical axis ($\beta_0 = 0$) : Jordan–Fierz–Brans–Dicke theory $\alpha_0^2 = \frac{1}{2\,\omega_{BD} + 3}$

Horizontal axis ($\alpha_0 = 0$) : perturbatively equivalent to G.R.

■ Higher-order deviations from G.R.

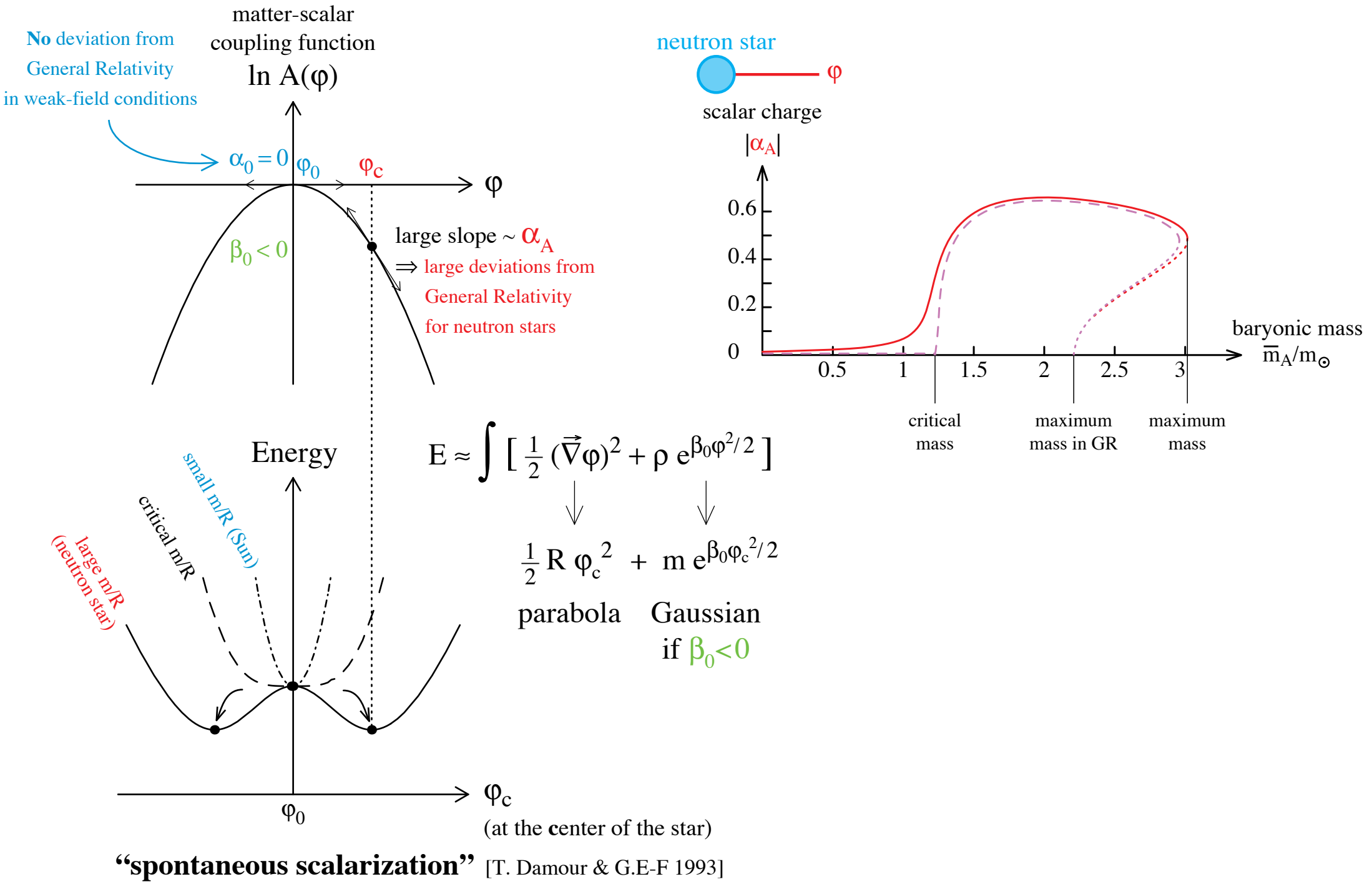
- At any order in $\frac{1}{c^n}$, the deviations involve at least two α_0 factors:



= small deviations!

- But **nonperturbative** strong-field effects may occur:

$$\text{deviations} = \underbrace{\alpha_0^2}_{<10^{-5}} \times \underbrace{\left[a_0 + a_1 \frac{Gm}{Rc^2} + a_2 \left(\frac{Gm}{Rc^2} \right)^2 + \ \ldots \right]}_{\text{LARGE for } \frac{Gm}{Rc^2} \approx 0.2 \ ?}$$

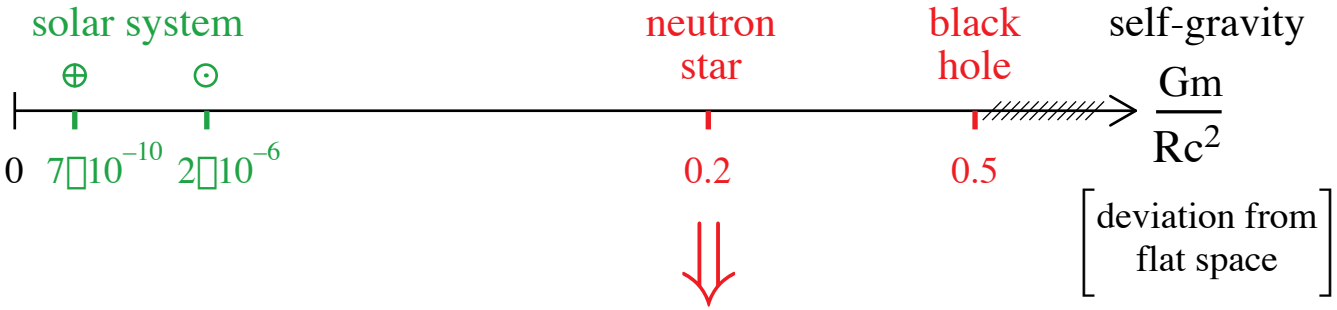


"spontaneous scalarization" [T. Damour & G.E-F 1993]

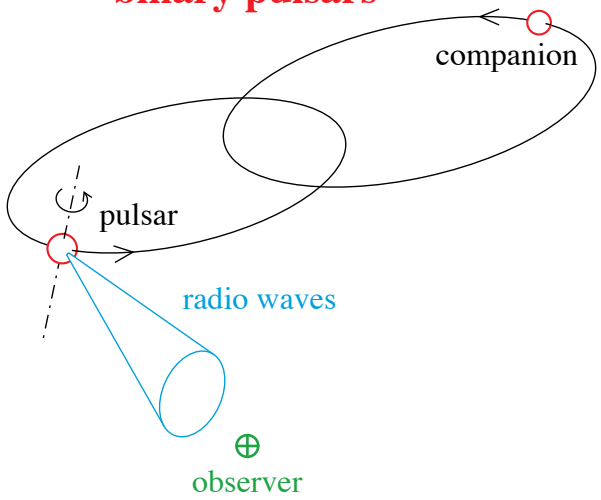
Weak-field experiments

$$\begin{cases} -g_{00} = 1 - 2\dfrac{Gm}{rc^2} + 2\beta^{PPN}\left(\dfrac{Gm}{rc^2}\right)^2 + \dots \\ g_{ij} = \delta_{ij}\left[\,1 + 2\gamma^{PPN}\dfrac{Gm}{rc^2} + \dots\right] \end{cases}$$

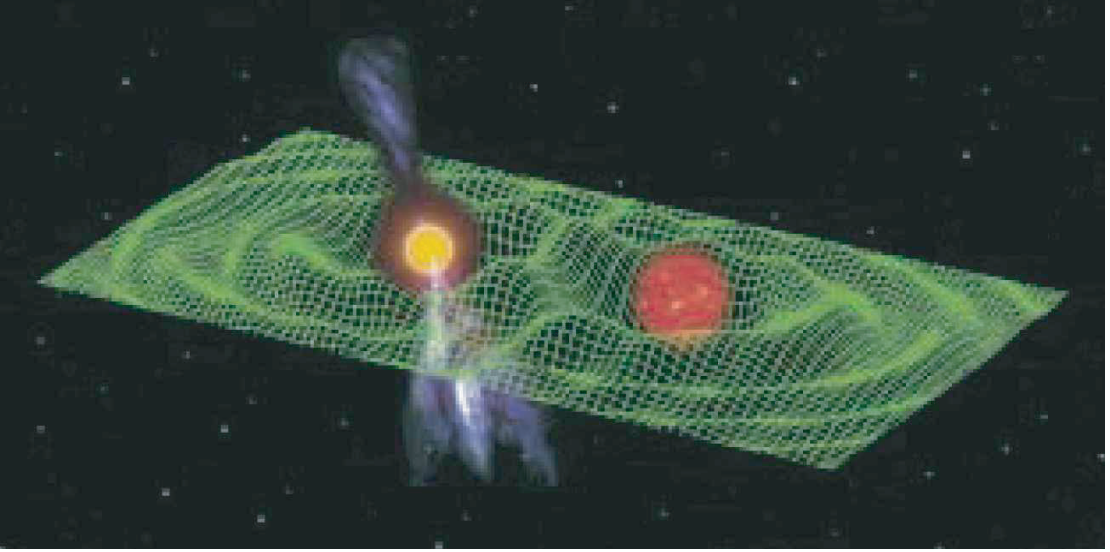
Strong-field tests ?



binary pulsars



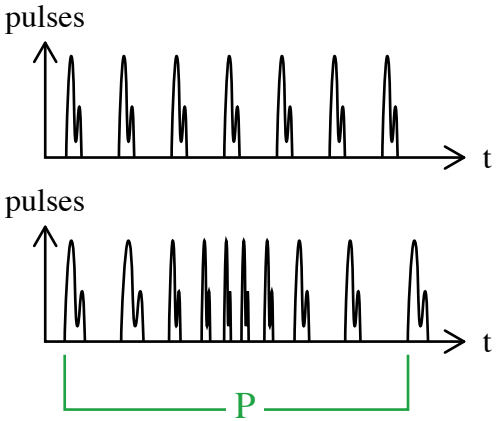
moving clock giving information about this strong-gravity region



Binary-pulsar tests

pulsar = (very stable) clock

binary pulsar = moving clock



- Time of flight across orbit $\propto \frac{\text{size of orbit}}{c}$ ("Roemer time delay")

  – orbital period P
  – eccentricity e
  – periastron angular position ω
  – ...

  } "Keplerian" parameters

- Redshift $\propto \frac{G\, m_B}{r_{AB}\, c^2}$ + second order Doppler effect $\propto \frac{\vec{v}_A^{\,2}}{2\, c^2}$ ("Einstein time delay")

  – parameter γ_{Timing}

- Time evolution of Keplerian parameters

  – periastron advance $\dot{\omega}$ (order $\frac{1}{c^2}$)
  – gravitational radiation damping $\dot{P}$ (order $\frac{1}{c^5}$)

  } "post-Keplerian" observables
  [PSR B1913+16 • Hulse & Taylor]

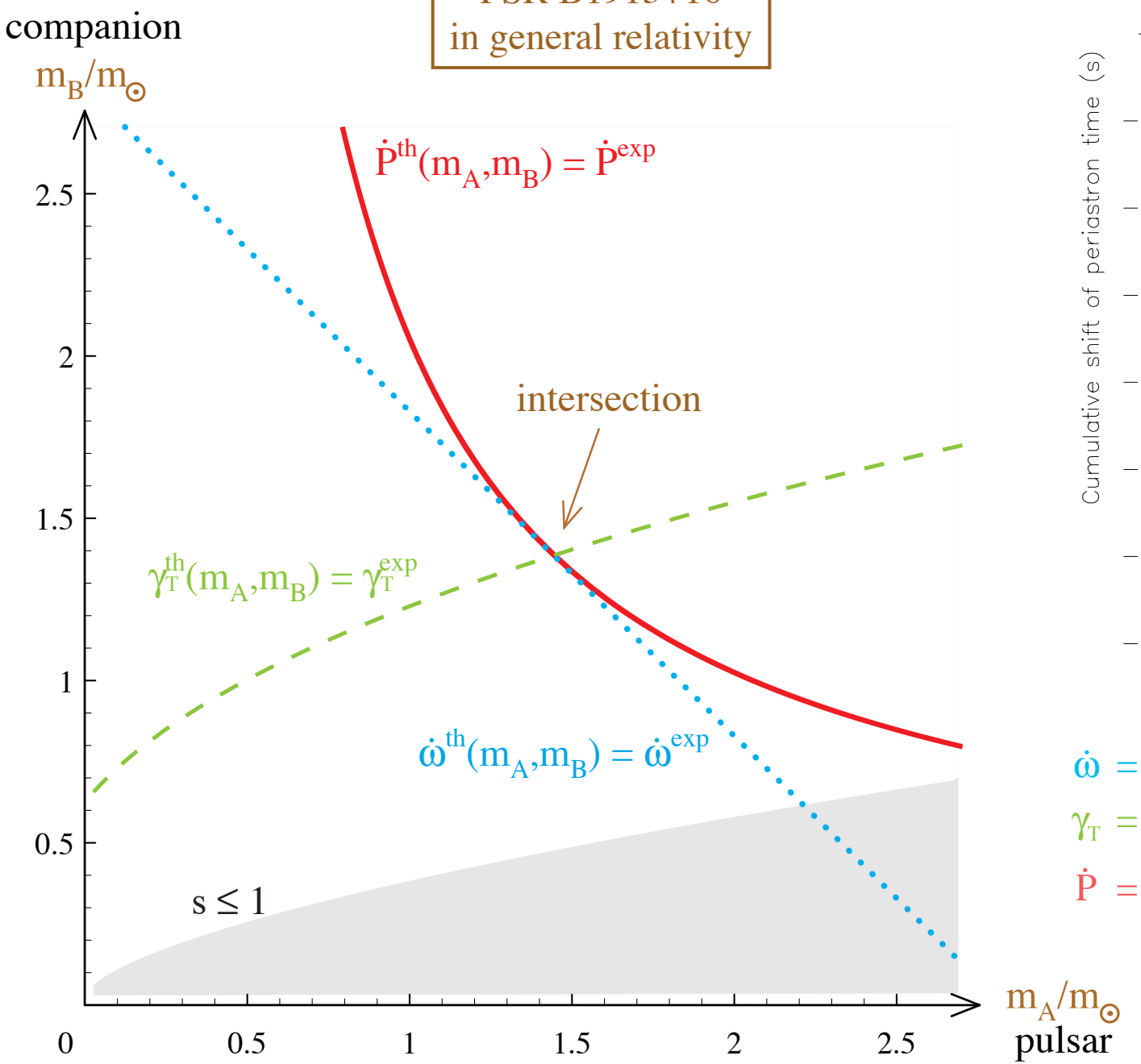
3	–	2	=	1
observables		unknown masses m_A, m_B		test

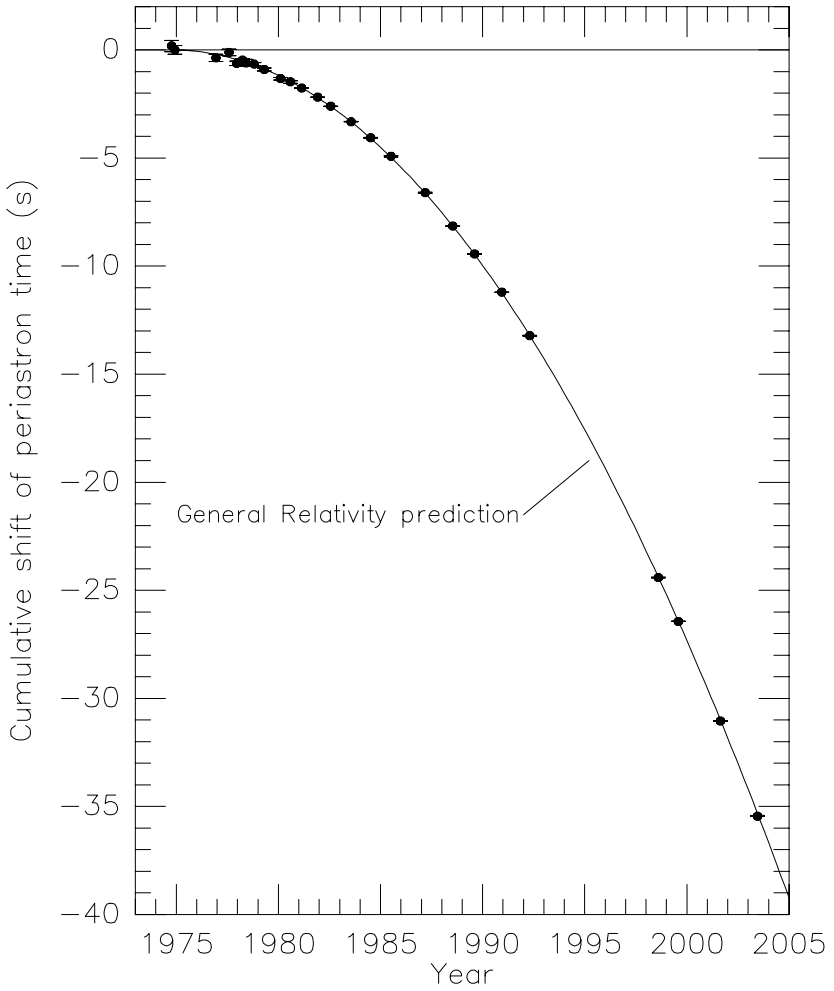
Plot the three curves [strips]

$\gamma_{Timing}^{theory}(m_A, m_B) = \gamma_{Timing}^{observed}$
$\dot{\omega}^{theory}(m_A, m_B) = \dot{\omega}^{observed}$
$\dot{P}^{theory}(m_A, m_B) = \dot{P}^{observed}$

} "γ - $\dot{\omega}$ - $\dot{P}$ test"

PSR B1913+16
in general relativity





$\dot{\omega} = 4.22661°/\text{yr}$

$\gamma_T = 4.294$ ms

$\dot{P} = -2.421 \times 10^{-12}$

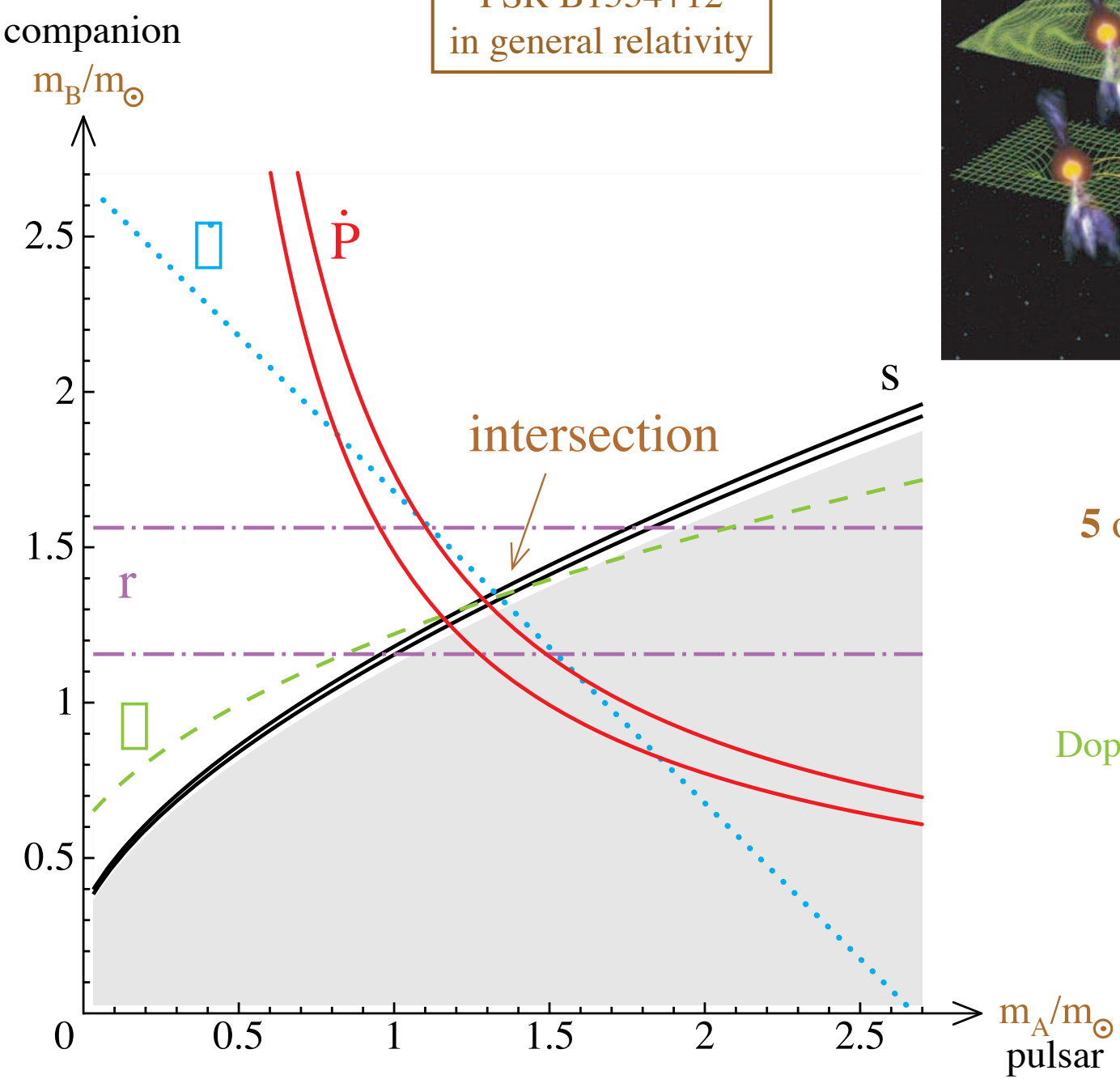
GR ⟹

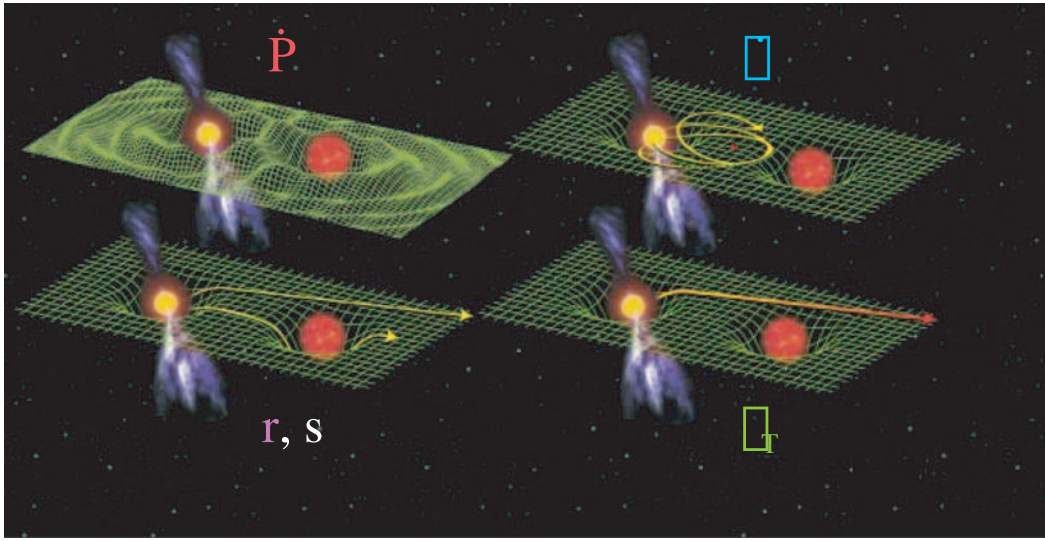
$m_A = 1.4408\ m_\odot$

$m_B = 1.3873\ m_\odot$

Discovered by R. Hulse and J. Taylor in 1974

PSR B1534+12
in general relativity





5 observables − **2** masses = **3** tests

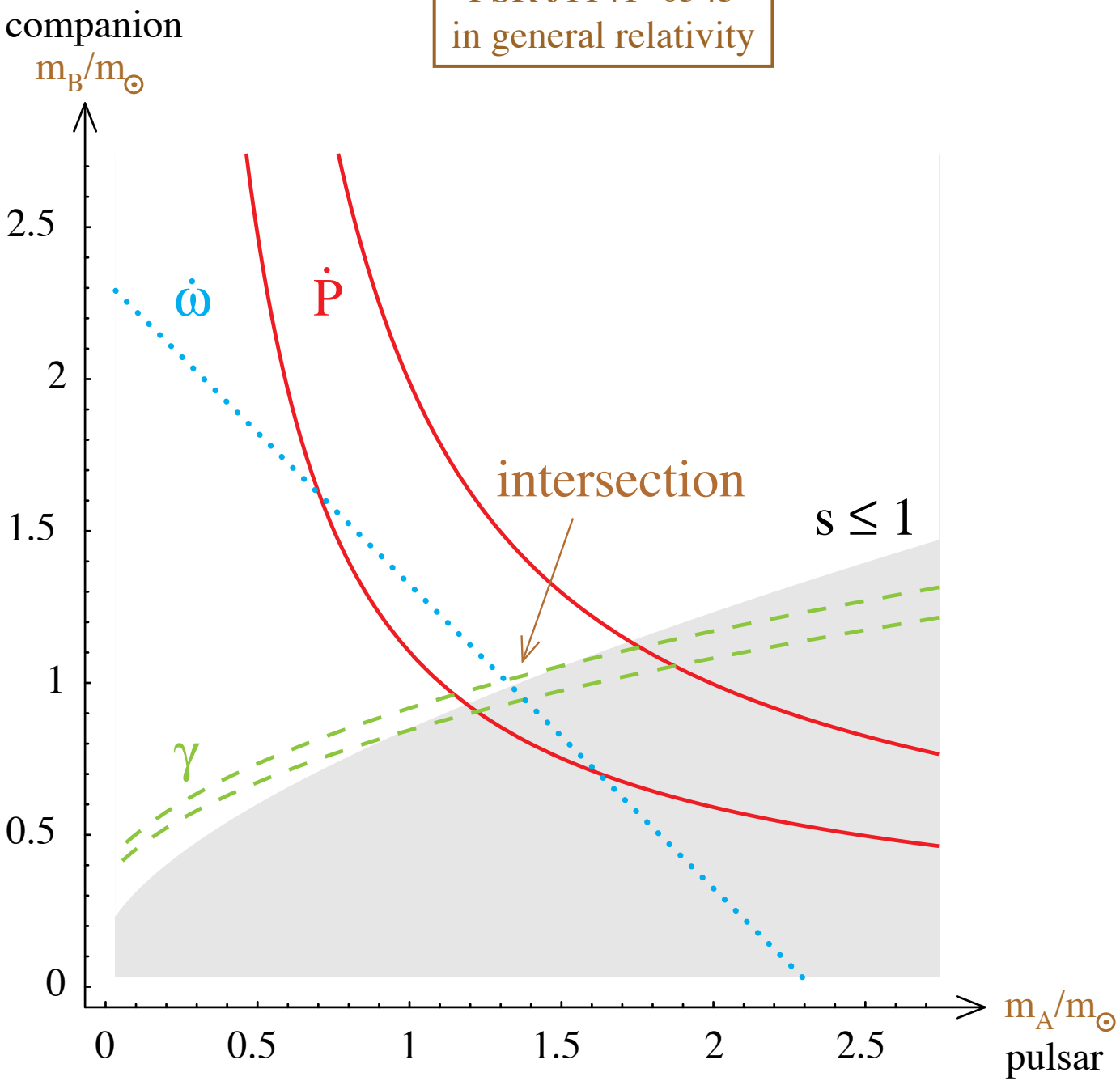
"Galactic" contribution to $\dot{P}$

[Damour–Taylor 1991]

$$\text{Doppler} \propto n.v \quad \Rightarrow \quad \frac{d\,\text{Doppler}}{dt} \propto n.a + \frac{v_{\odot}^2}{d_{\odot PSR}}$$

Discovered by A. Wolszczan in 1991

PSR J1141–6545
in general relativity



Discovery Kaspi *et al.* 1999, Timing Bailes *et al.* 2003

Asymmetrical system
neutron star – **white dwarf**

Neutron star born *after* white dwarf

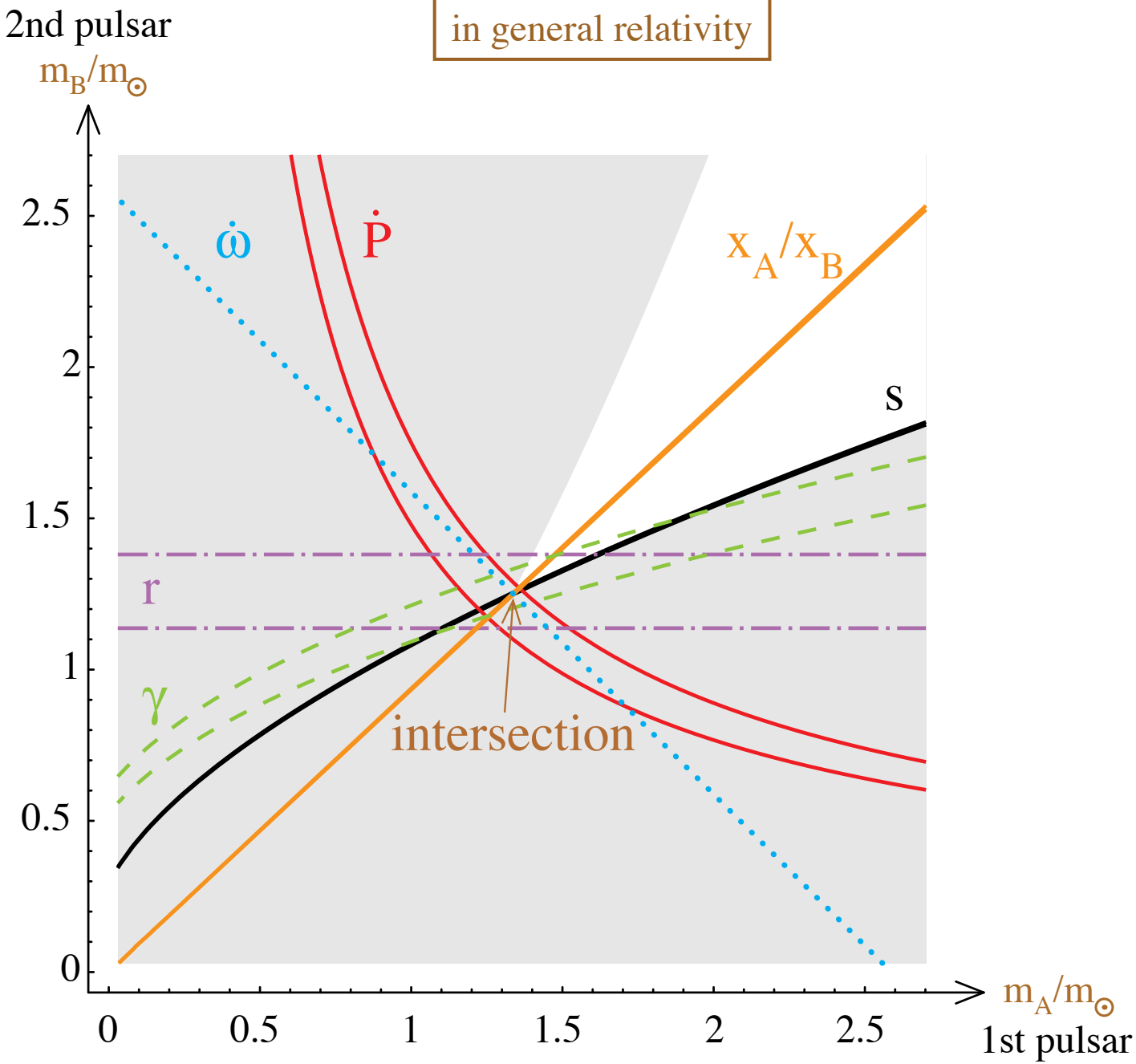
⇒ eccentricity e = 0.17 large
and nonrecycled pulsar

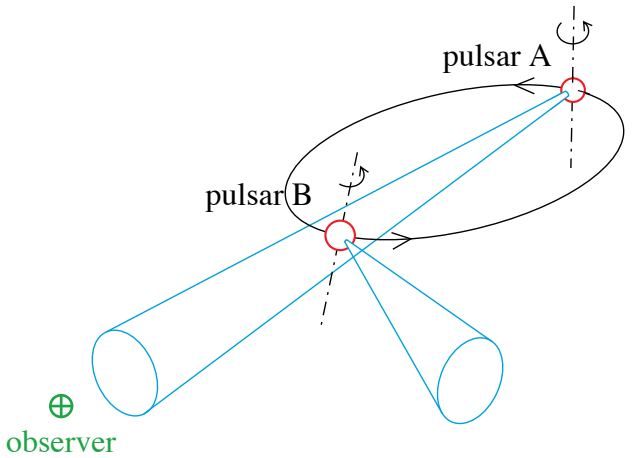
$\dot{P} = -4 \times 10^{-13}$

Mass function

$$\frac{(m_B \sin i)^3}{(m_A + m_B)^2} = \left(\frac{2\pi}{P}\right)^2 \frac{(x\,c)^3}{G}$$

PSR J0737–3039
in general relativity





$P = 2\text{ h }27\text{ min }14.5350\text{ s}$

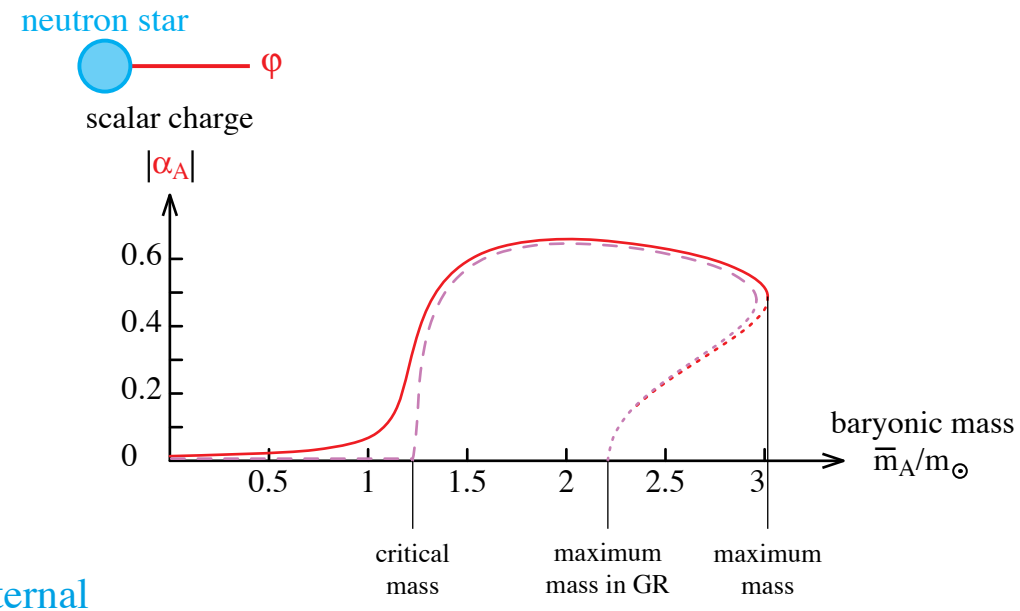
$\dot{\omega} = 16.90°/\text{yr}$

$$\frac{x_B}{x_A} = \frac{m_A}{m_B} = 1.07$$

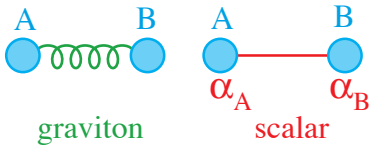
6 observables – **2** masses = **4** tests

Timing Burgay *et al.* 2003, **Double pulsar** Lyne *et al.* 2004

Strong-field effects

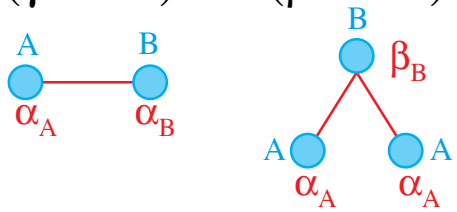


- $G_{AB}^{eff} = G\,(1 + \alpha_A\,\alpha_B)$



depends on internal structure of bodies A & B

- Similarly for $(\gamma^{PPN} - 1)$ and $(\beta^{PPN} - 1)$ $\Rightarrow$ all post-Newtonian effects



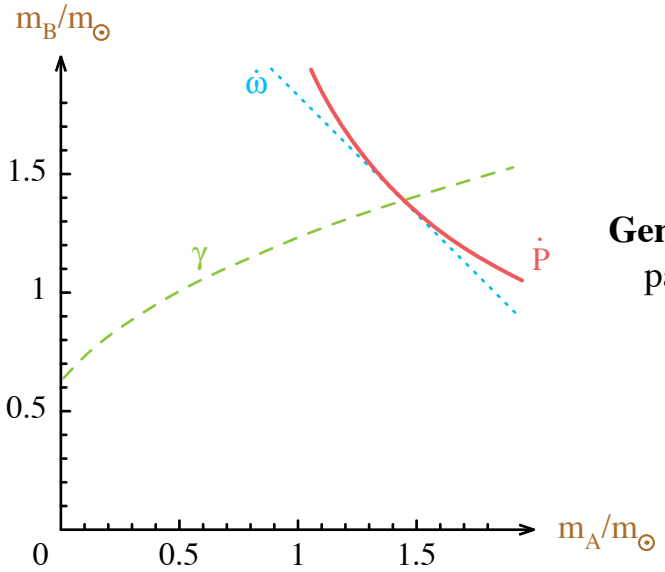
- Energy flux =

$$\frac{\text{Quadrupole}}{c^5} + O\left(\frac{1}{c^7}\right) \qquad \text{spin 2}$$

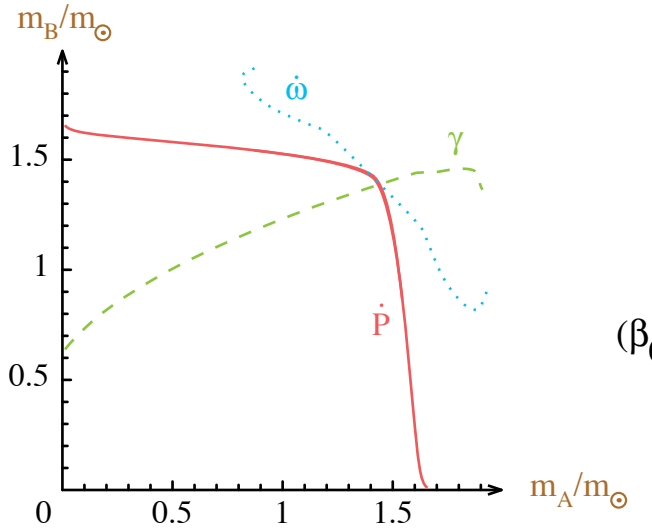
$$+\frac{\text{Monopole}}{c}\left(0+\frac{1}{c^2}\right)^2 + \frac{\text{Dipole}}{c^3} + \frac{\text{Quadrupole}}{c^5} + O\left(\frac{1}{c^7}\right) \qquad \text{spin 0}$$

$$\propto (\alpha_A - \alpha_B)^2$$

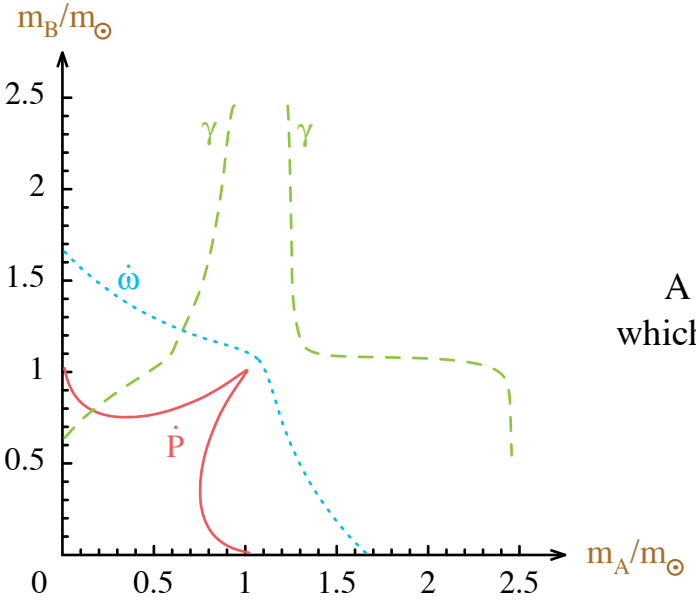
PSR B1913+16 in scalar-tensor theories



General relativity
passes the test

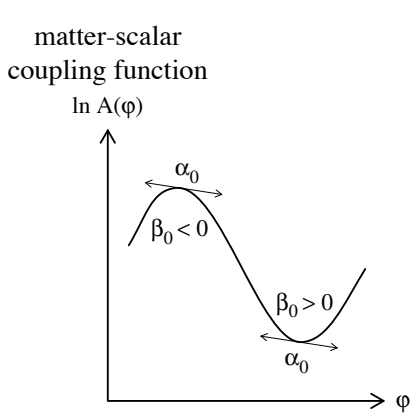


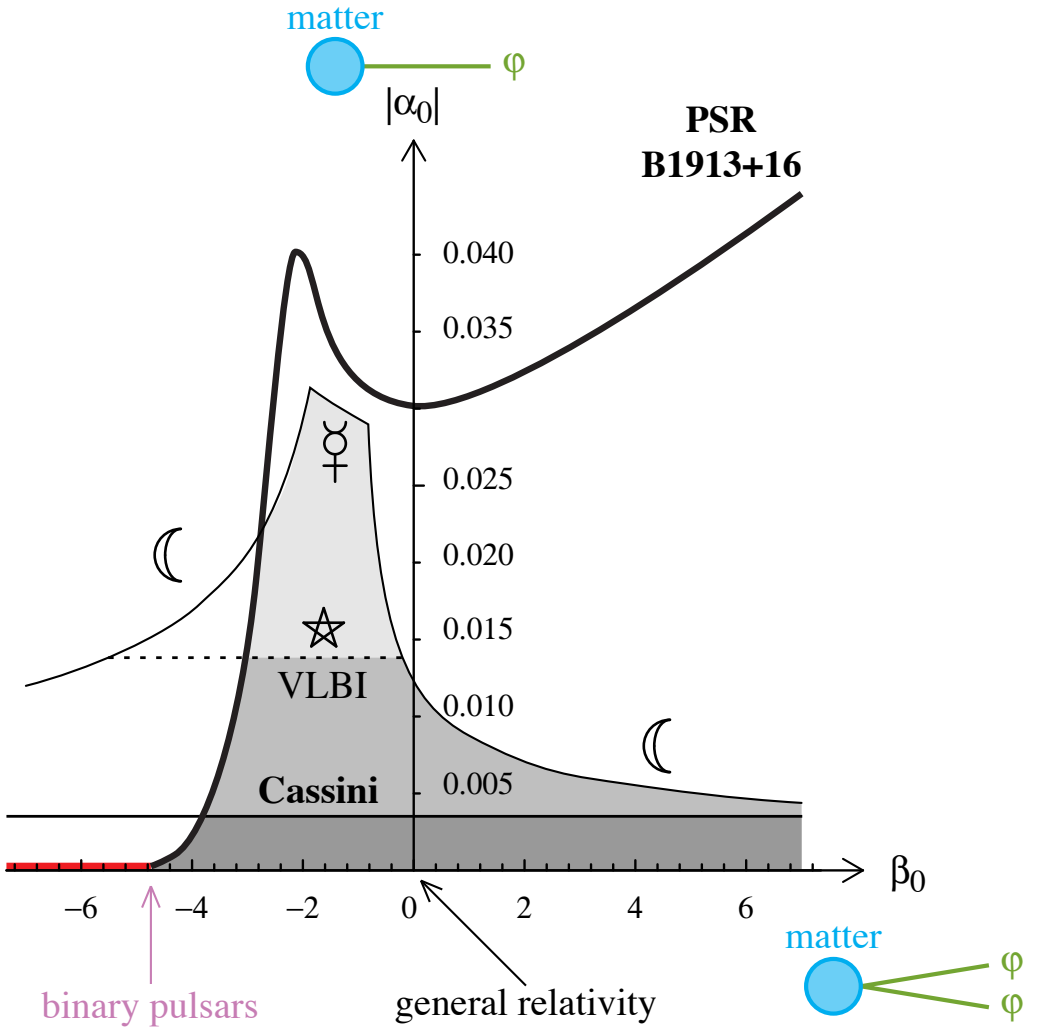
A tensor–scalar theory
which **passes the test**
($\beta_0 = -4.5$, α_0 small enough)



A tensor–scalar theory
which **does not pass the test**
($\beta_0 = -6$, any α_0)

Solar-system & PSR B1913+16 constraints on scalar-tensor theories of gravity





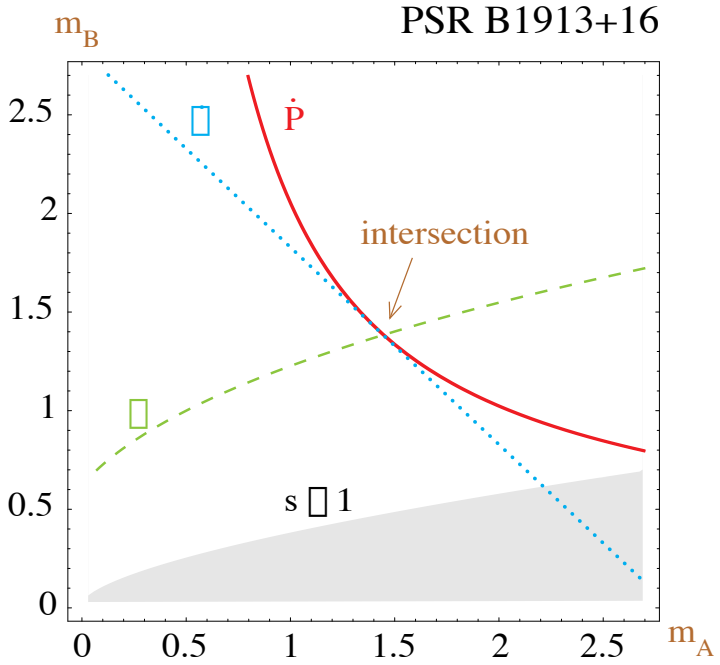
i.e. $\dfrac{\beta^{PPN}-1}{\gamma^{PPN}-1} < 1.1$

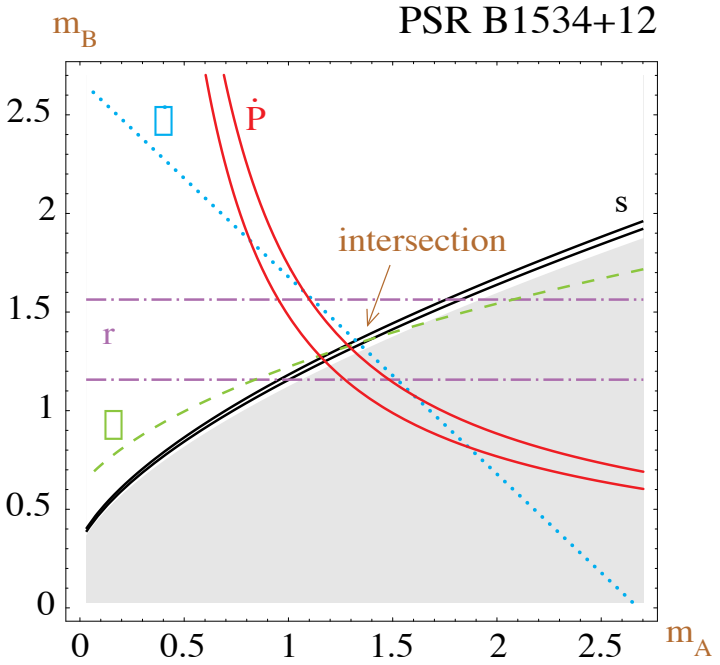
[T. Damour & G.E-F 1998]

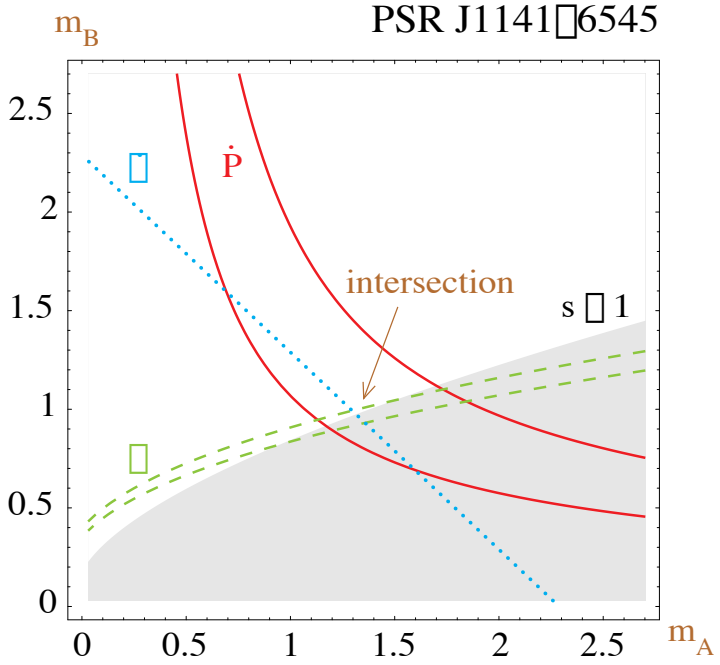
Vertical axis ($\beta_0 = 0$) : Jordan–Fierz–Brans–Dicke theory $\alpha_0^2 = \dfrac{1}{2\,\omega_{BD} + 3}$

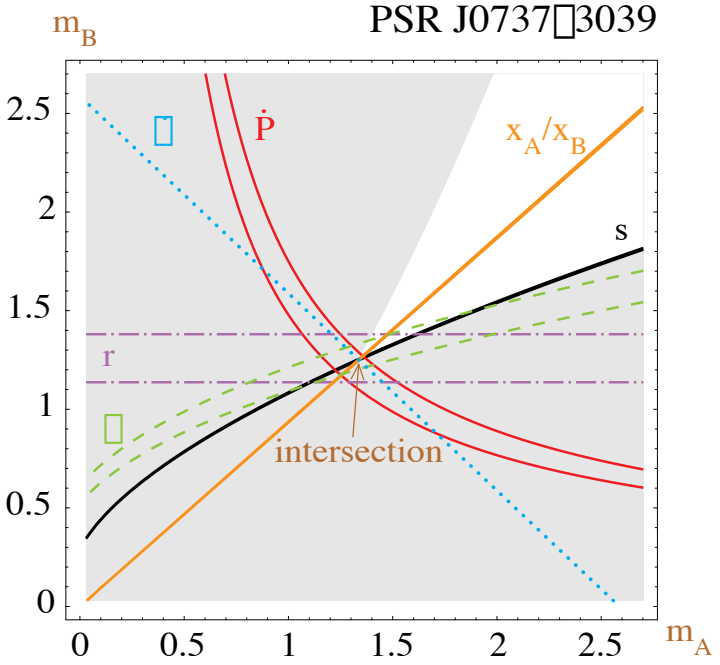
Horizontal axis ($\alpha_0 = 0$) : perturbatively equivalent to G.R.

The four accurately timed binary pulsars in general relativity

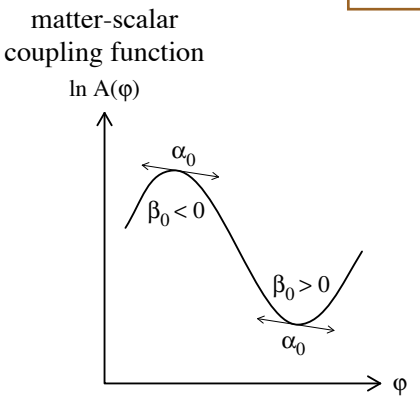


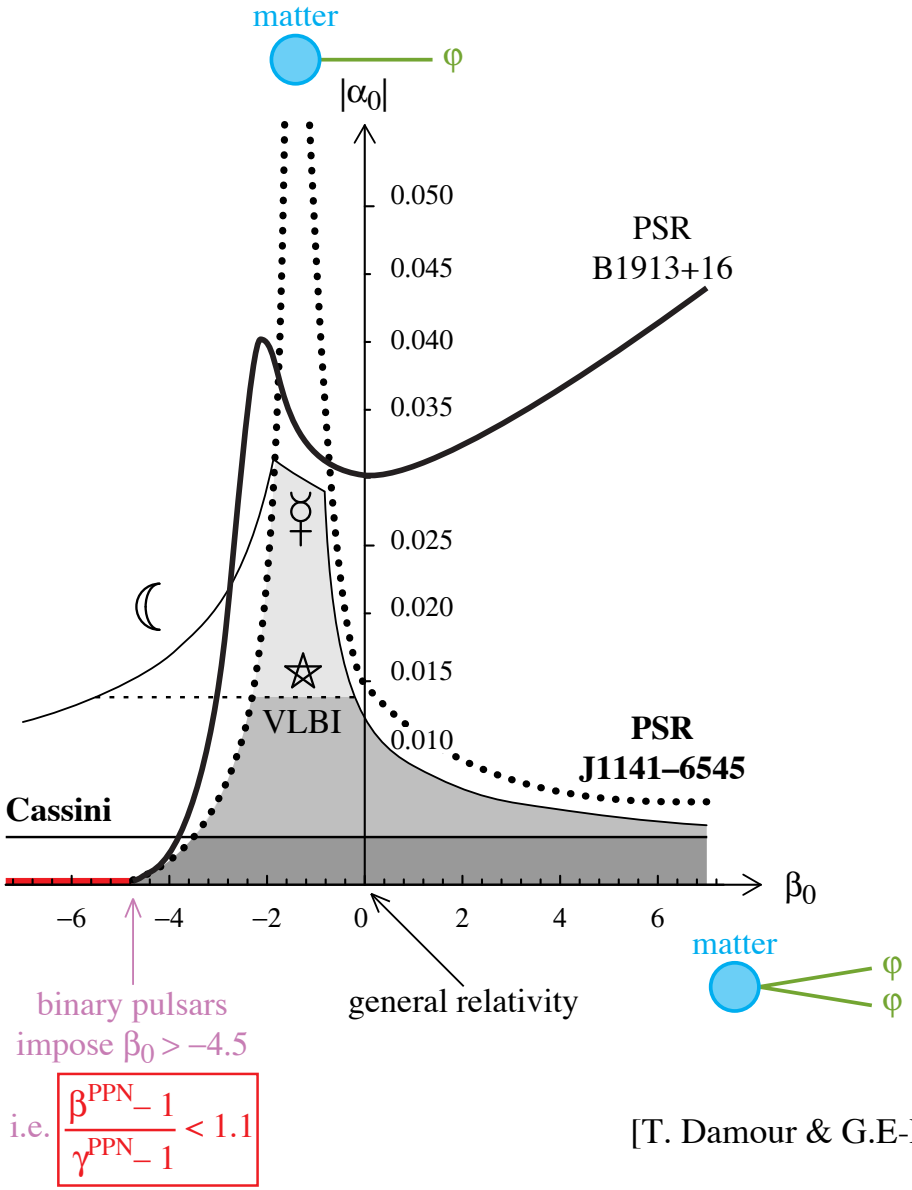






Solar-system & best binary-pulsar constraints on scalar-tensor theories of gravity



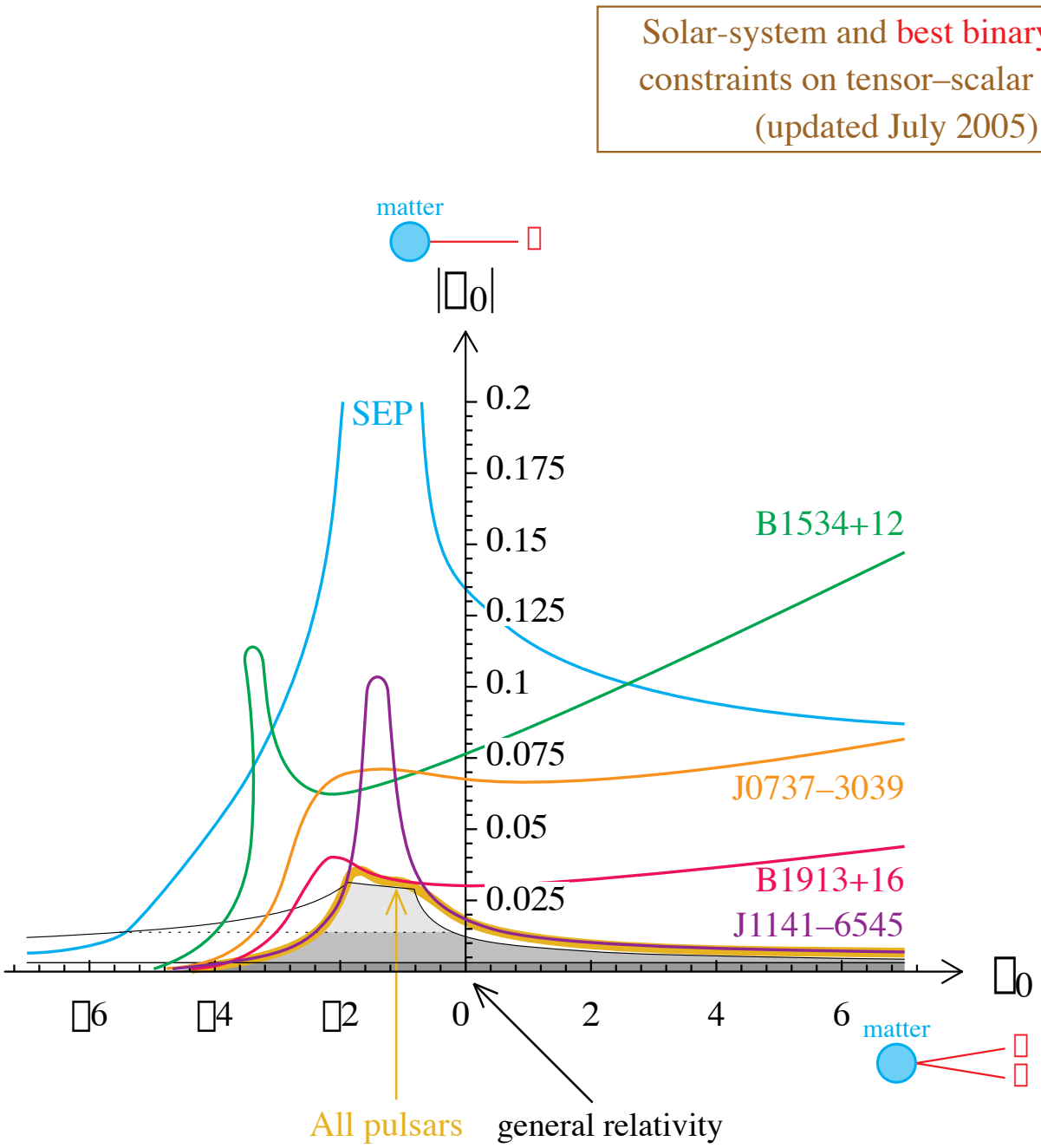


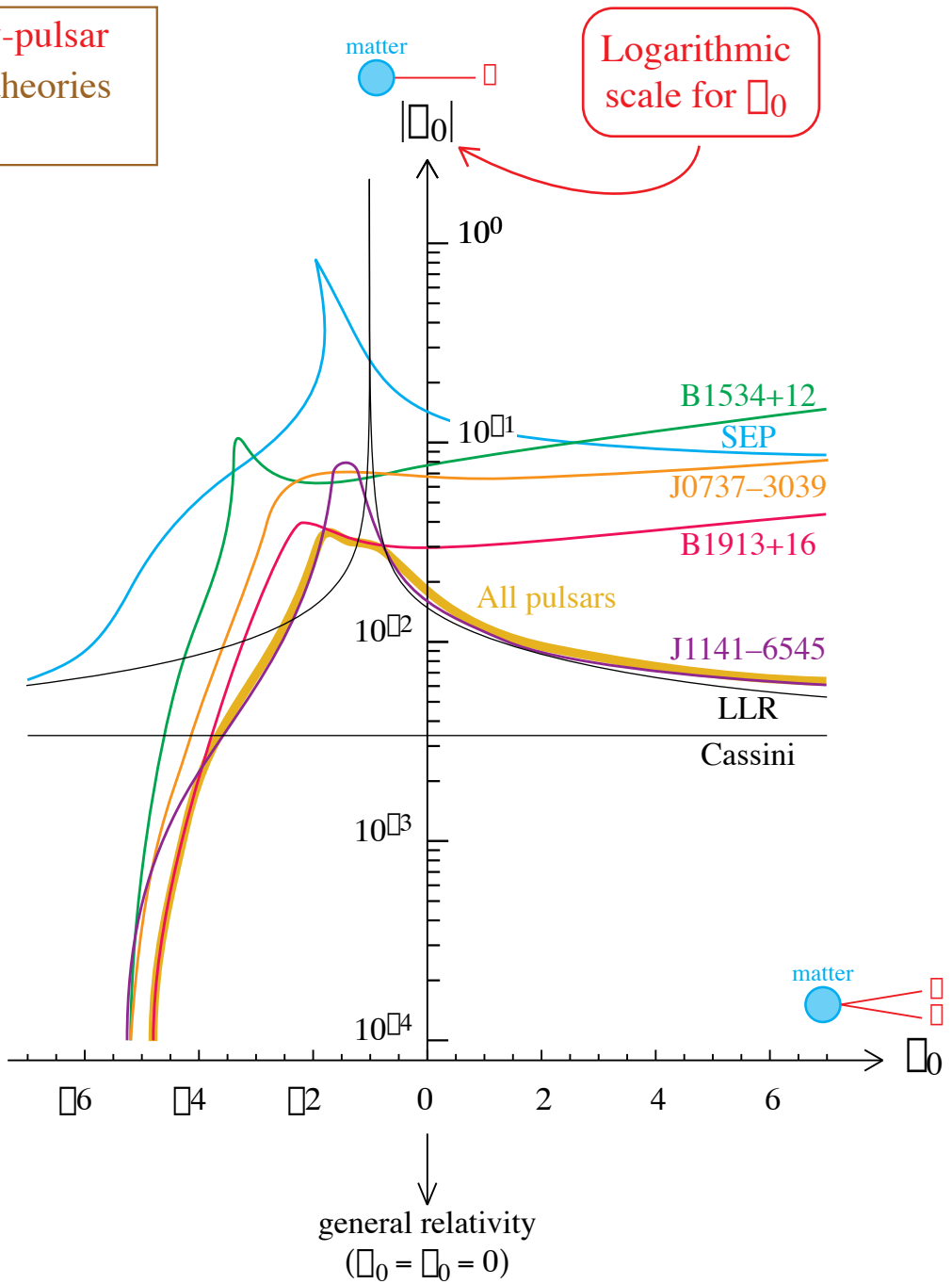
[T. Damour & G.E-F 2005]

Vertical axis ($\beta_0 = 0$) : Jordan–Fierz–Brans–Dicke theory $\alpha_0^2 = \frac{1}{2\,\omega_{BD} + 3}$

Horizontal axis ($\alpha_0 = 0$) : perturbatively equivalent to G.R.

Solar-system and best binary-pulsar constraints on tensor–scalar theories (updated July 2005)

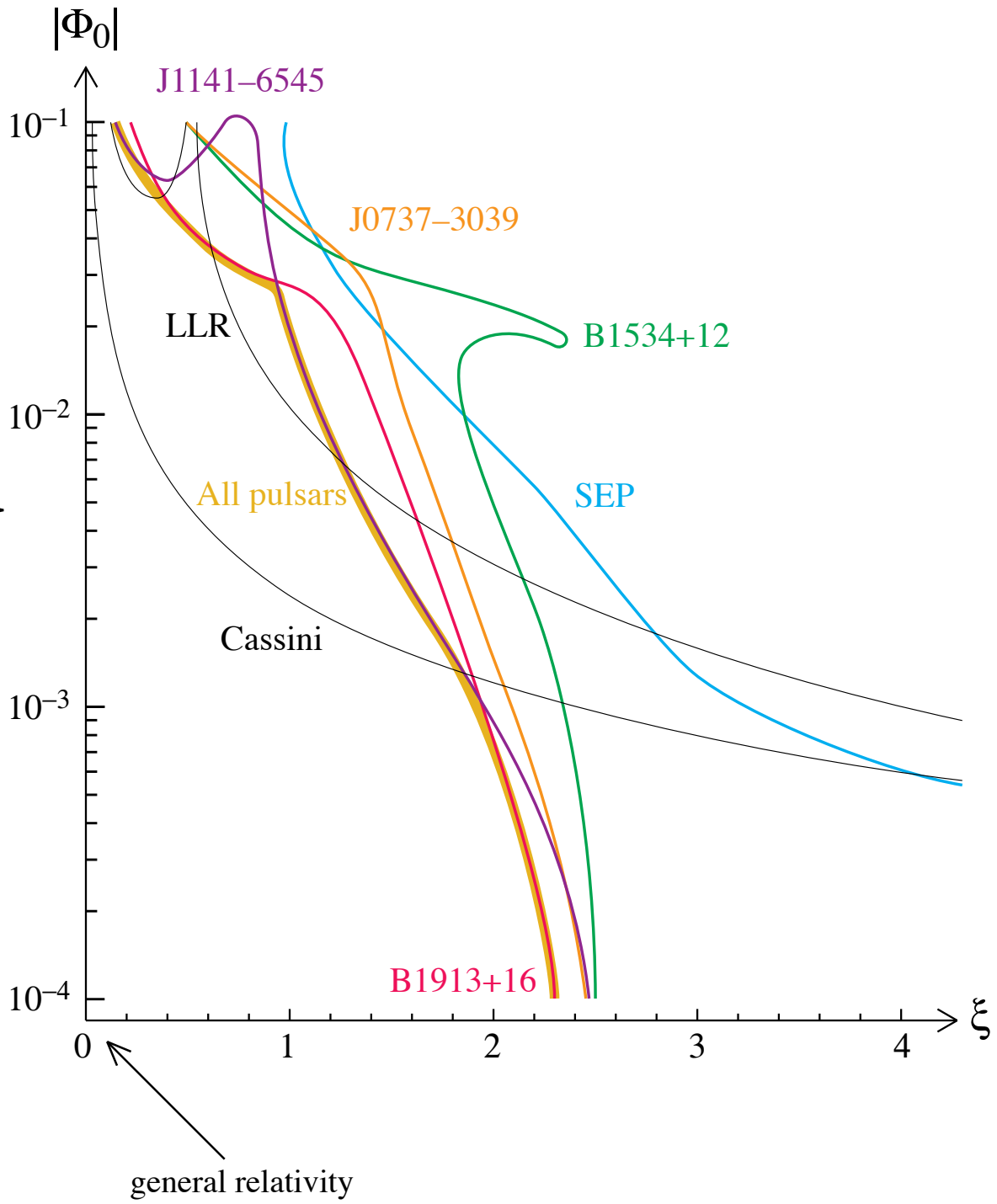




Experimental constraints on another natural class of scalar-tensor theories:

$$S = \frac{1}{16\,\pi\,G}\int \sqrt{-g}\,\left\{R\left(1+\xi\Phi^2\right) - \left(\partial_\mu\Phi\right)^2\right\}$$

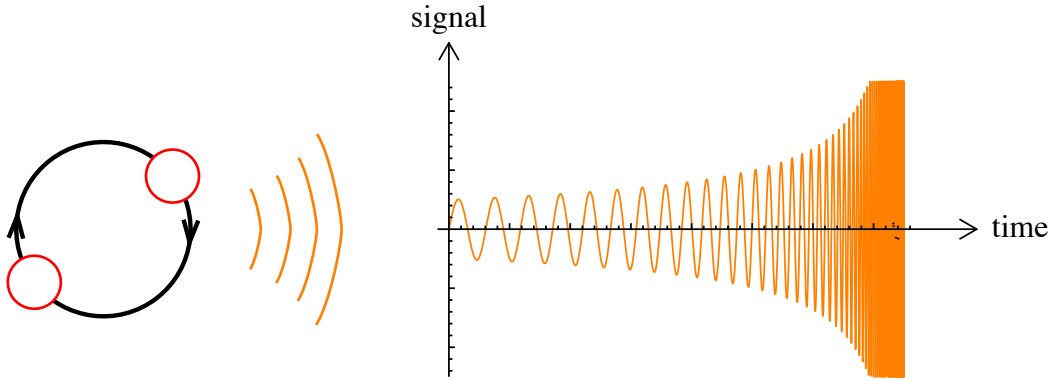
$$+\,S_{\text{matter}}\left[\text{matter}\,,\,g_{\mu\nu}\right]$$



N.B.: ∃ other classes of scalar-tensor theories
(e.g., some where the SEP tests are the most constraining, whereas PSR J1141–6545 does not tell us much!)

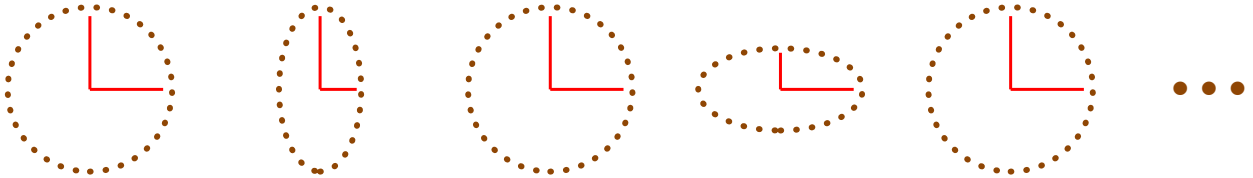
Gravitational wave antennas
LIGO/VIRGO/LISA

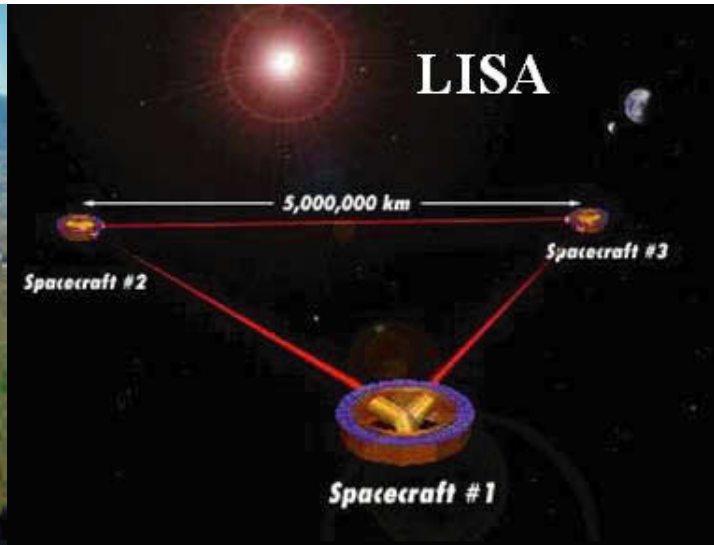
- Signal emitted by an inspiralling binary system:



(depends on hundreds of post-Newtonian coefficients)

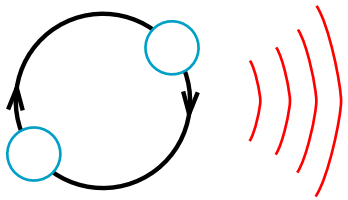
- Effect on a detector:





Gravitational wave antennas LIGO/VIRGO/LISA

One needs accurate (3.5 PN) templates to extract the signal from the noise

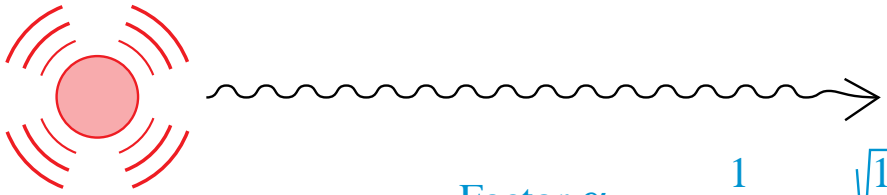


Gravitational waves
in scalar-tensor gravity

$$\text{Energy flux} = \frac{\text{Quadrupole}}{c^5} + O\left(\frac{1}{c^7}\right) \quad \text{spin 2}$$

$$+ \frac{\text{Monopole}}{c}\left(\dot{\sigma} + \frac{1}{c^2}\right)^2 + \frac{\text{Dipole}}{c^3} + \frac{\text{Quadrupole}}{c^5} + O\left(\frac{1}{c^7}\right) \quad \text{spin 0}$$

■ Collapsing star



$$\text{Factor } \alpha_0 = \frac{1}{\sqrt{2\omega_{BD}+3}} \approx \sqrt{\frac{1-\gamma^{PPN}}{2}} < 0.003$$

Energy flux
= (strong field)^2
= Monopole/c
≫ usual Quadrupole/c^5

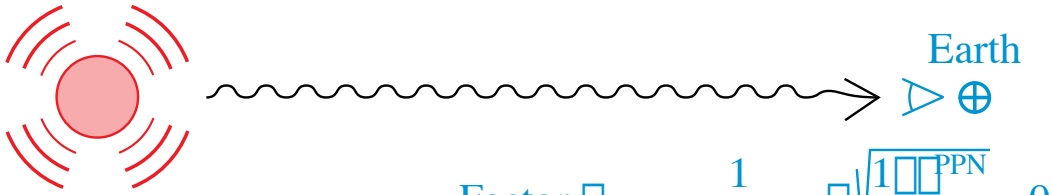
Detection
= (strong field) × (weak field)
= too small for LIGO/VIRGO
[J. Novak's thesis, PRD **57,** 4789; **58,** 064019 (1998)]
and not in LISA's frequency band

Gravitational waves
in scalar-tensor gravity

$$\text{Energy flux} = \frac{\text{Quadrupole}}{c^5} + O\left(\frac{1}{c^7}\right) \quad \text{spin 2}$$

$$+ \frac{\text{Monopole}}{c}\left(\Box + \frac{1}{c^2}\right)^2 + \frac{\text{Dipole}}{c^3} + \frac{\text{Quadrupole}}{c^5} + O\left(\frac{1}{c^7}\right) \quad \text{spin 0}$$

■ Collapsing star

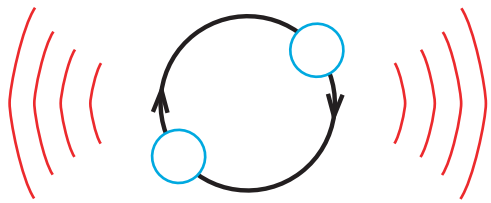


$$\text{Factor } \Box_0 = \frac{1}{\sqrt{2\Box_{BD}+3}}\Box\sqrt{\frac{1\Box\Box^{PPN}}{2}} < 0.003$$

Energy flux
= (strong field)^2
= Monopole/c
≫ usual Quadrupole/c^5

Detection
= (strong field) □ (weak field)
= too small for LIGO/VIRGO
[J. Novak's thesis, PRD **57,** 4789; **58,** 064019 (1998)]
and not in LISA's frequency band

■ Inspiralling binary



Even if no helicity-0 wave is detected, the time-evolution of the (helicity-2) chirp depends on the Energy flux = (strong field)^2

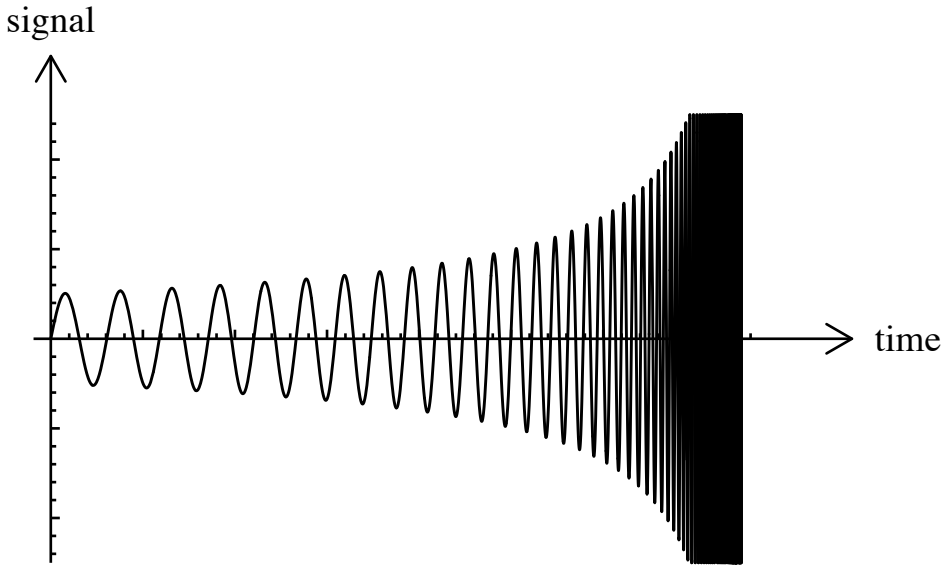
□ A priori possible to detect indirectly the presence of □:

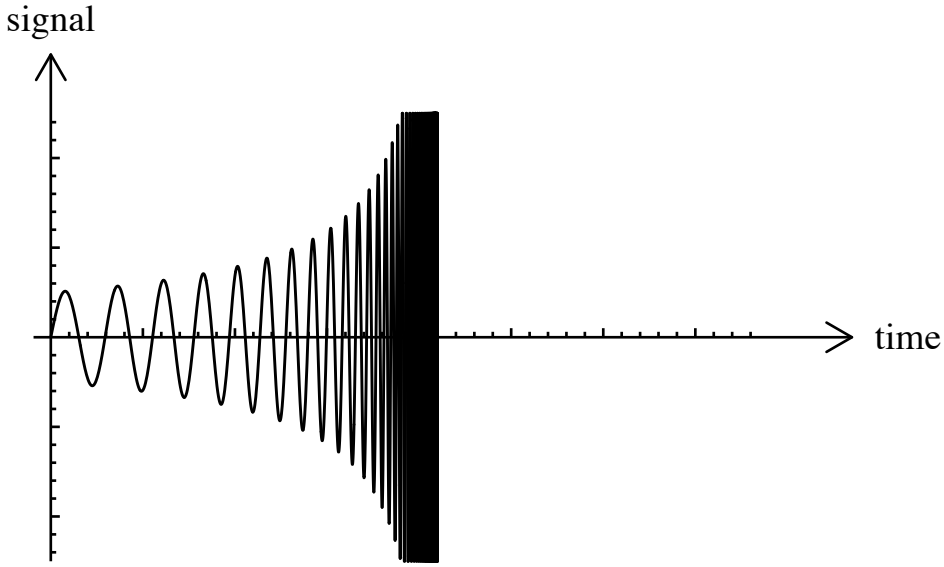
If binary inspiral detected with GR templates
□ bound on matter–scalar coupling strength

[matched-filter analysis: C.M. Will, Phys.Rev. D **50** (1994) 6058]

For a given binary system

Chirp evolution in general relativity

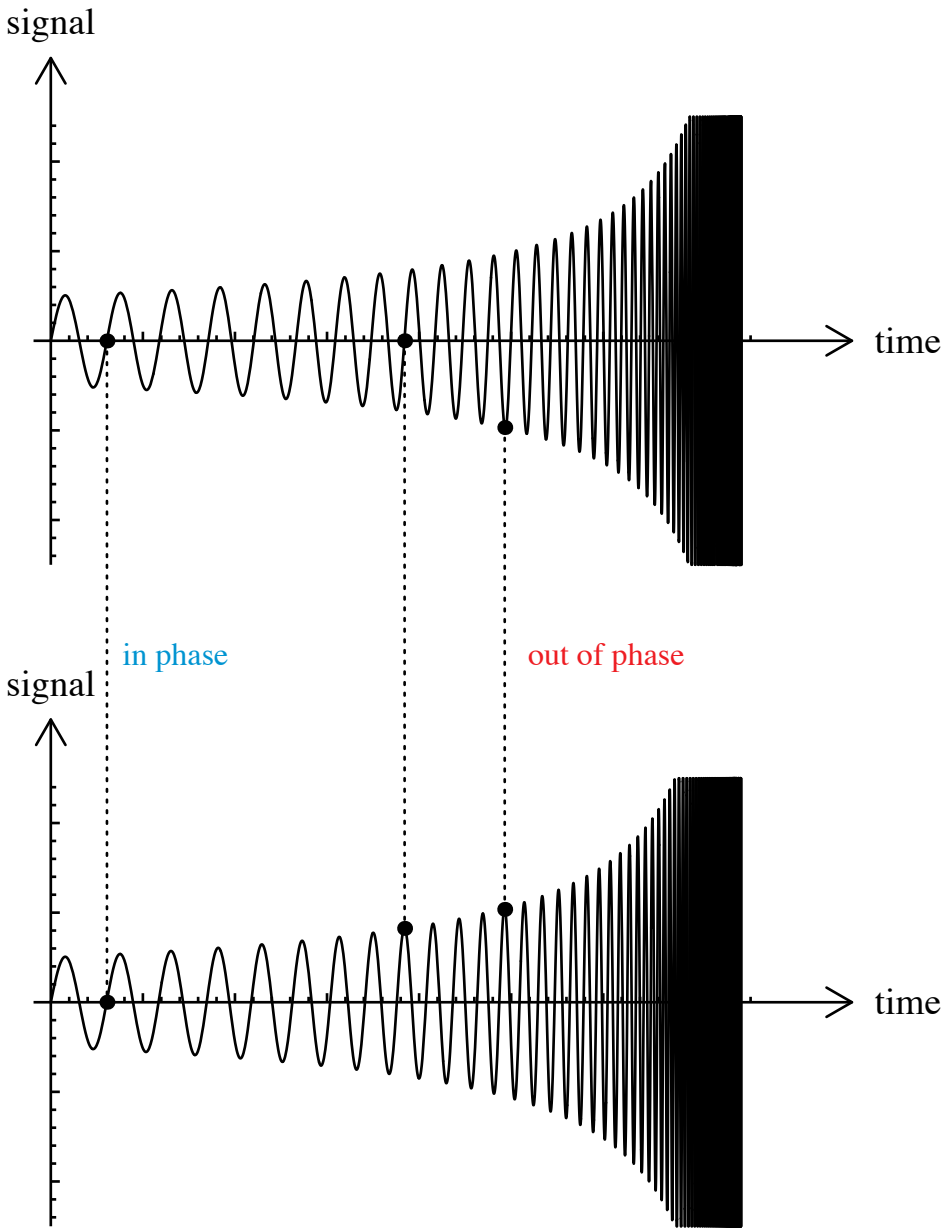




Chirp evolution in a tensor–scalar theory

For an unknown mass of the system

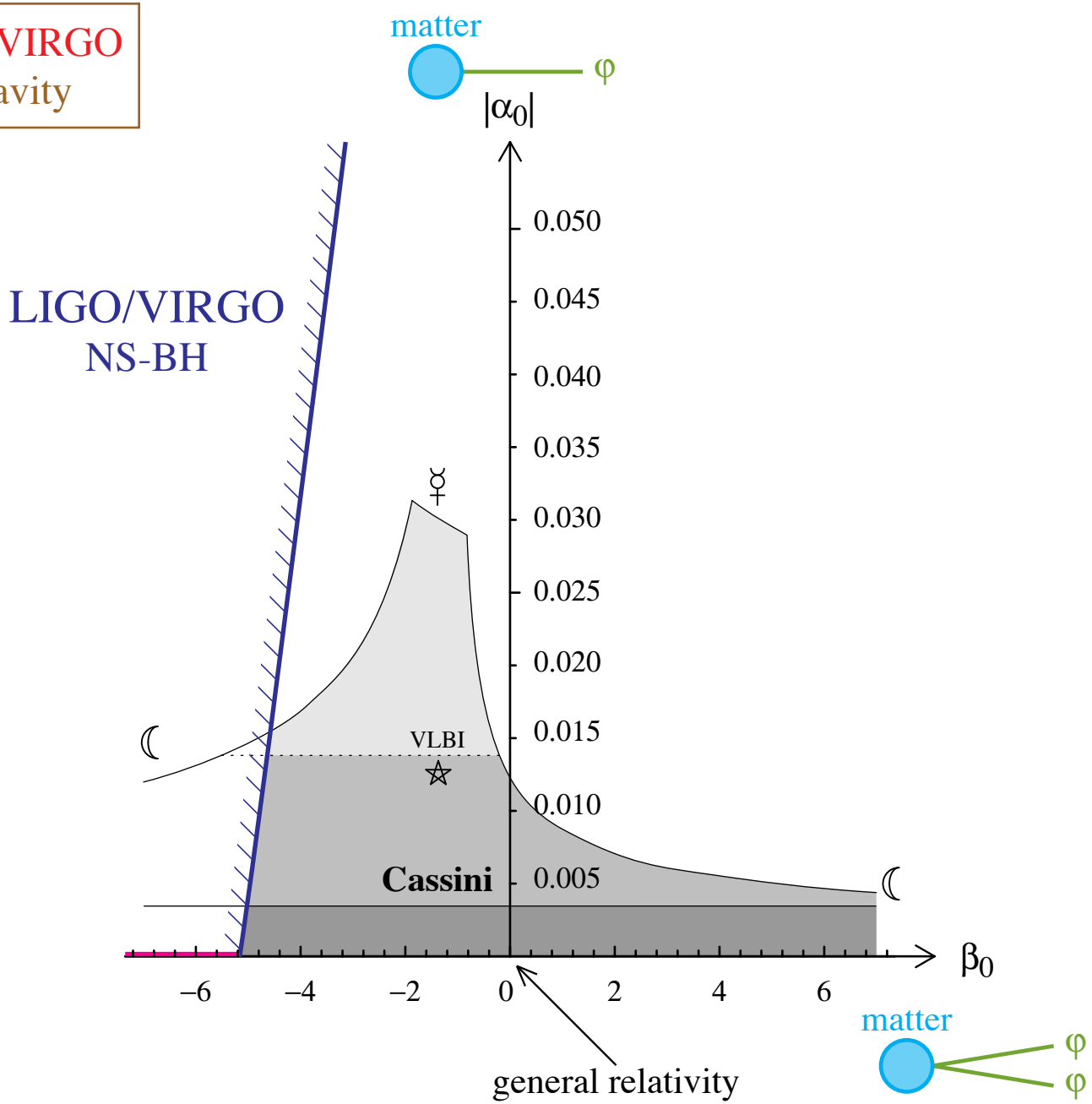
Chirp evolution in general relativity



Chirp evolution in a tensor–scalar theory

Solar-system and possible LIGO/VIRGO constraints on scalar-tensor gravity

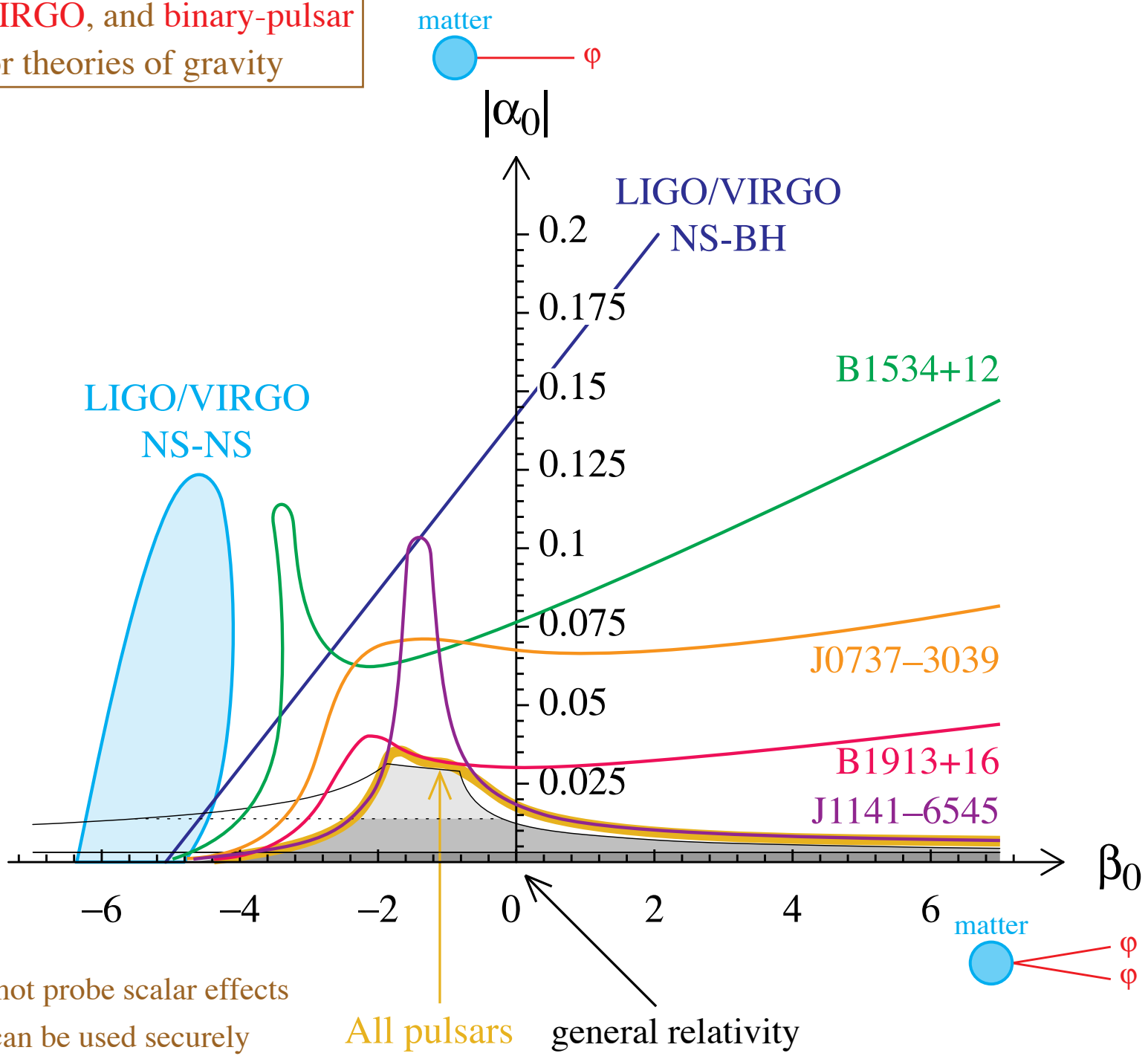
[Damour & GEF 1998]



Vertical axis ($\beta_0 = 0$) : Jordan–Fierz–Brans–Dicke theory $\alpha_0^2 = \frac{1}{2\,\omega_{BD} + 3}$

Horizontal axis ($\alpha_0 = 0$) : perturbatively equivalent to G.R.

Solar-system, possible LIGO/VIRGO, and binary-pulsar constraints on scalar–tensor theories of gravity



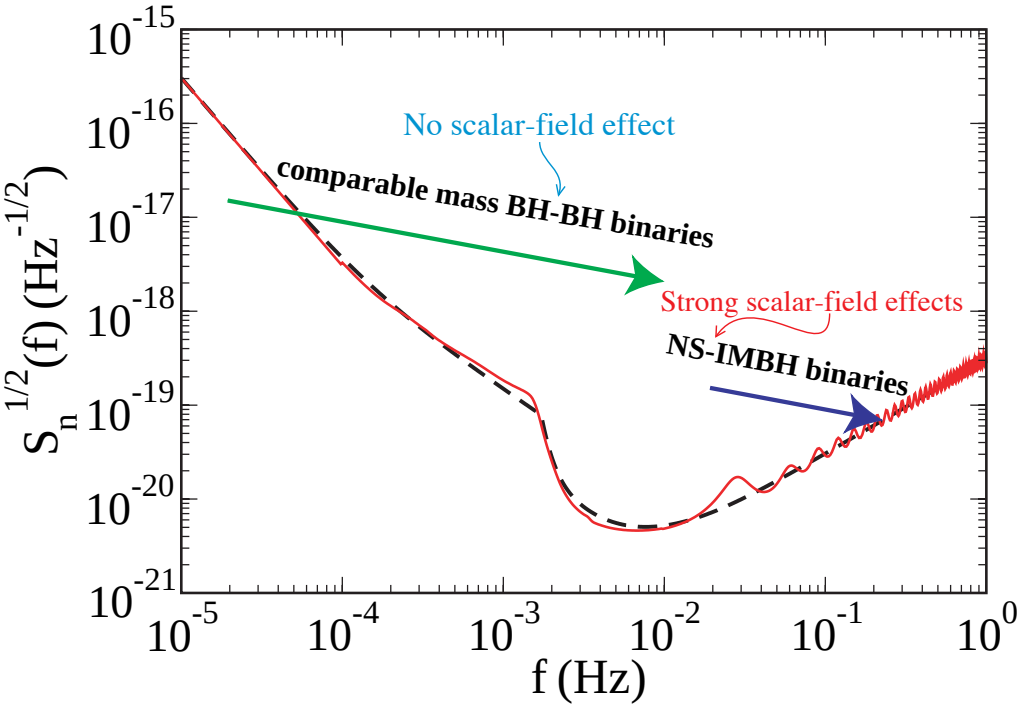
Bad news: LIGO/VIRGO will not probe scalar effects

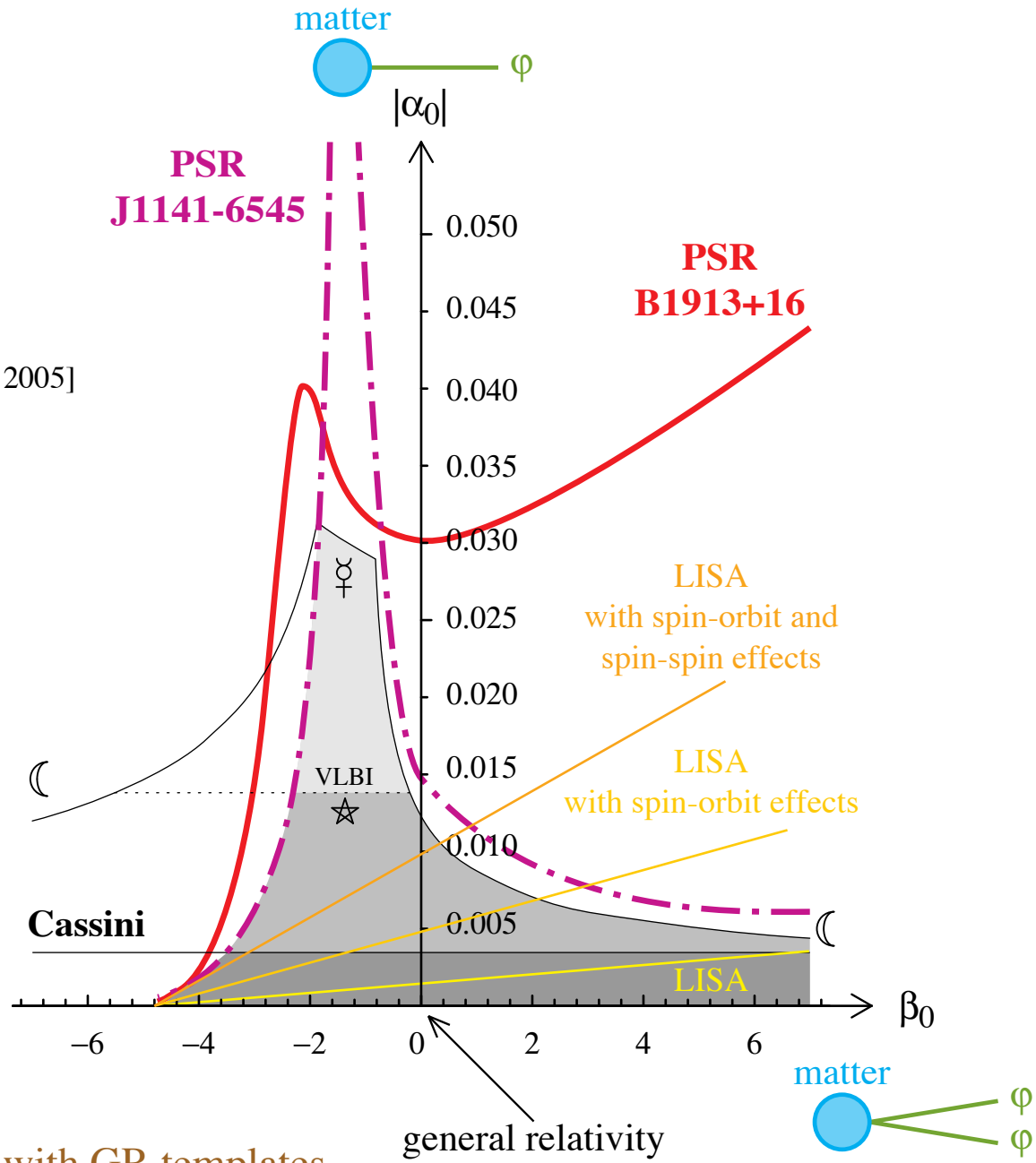
Good news! ⇒ GR templates can be used securely

Possible LISA constraints on scalar-tensor theories of gravity

LISA will probe $|\alpha_0| \sim 1.5 \times 10^{-3}$ if 1.4 $m_\odot$ NS – 1000 $m_\odot$ BH observed with S/N = 10

[Scharre & Will 2002; Will & Yunes 2004; Berti, Buonanno & Will 2005]

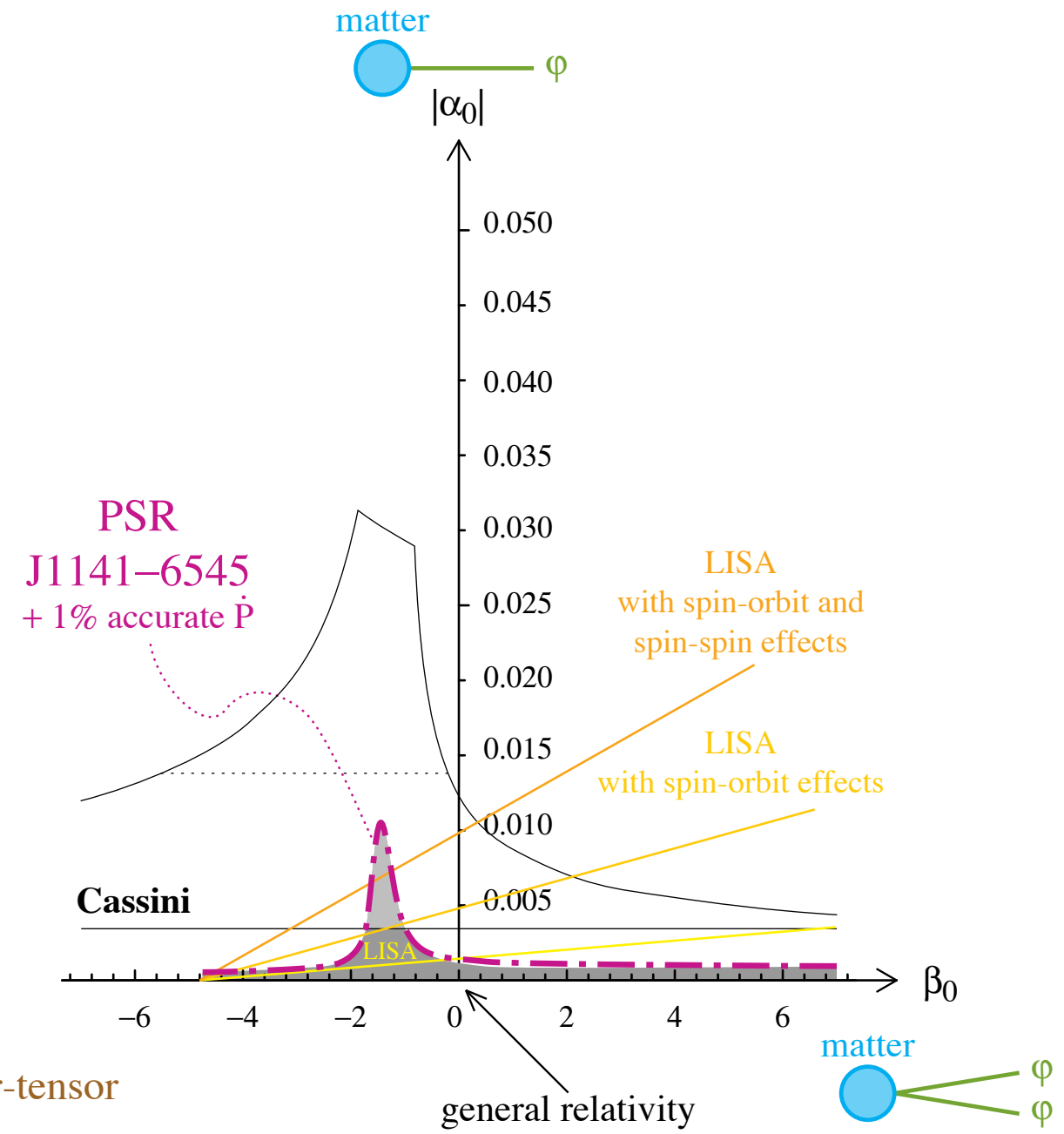




⇒ Tight constraints if detection of binary inspirals with GR templates

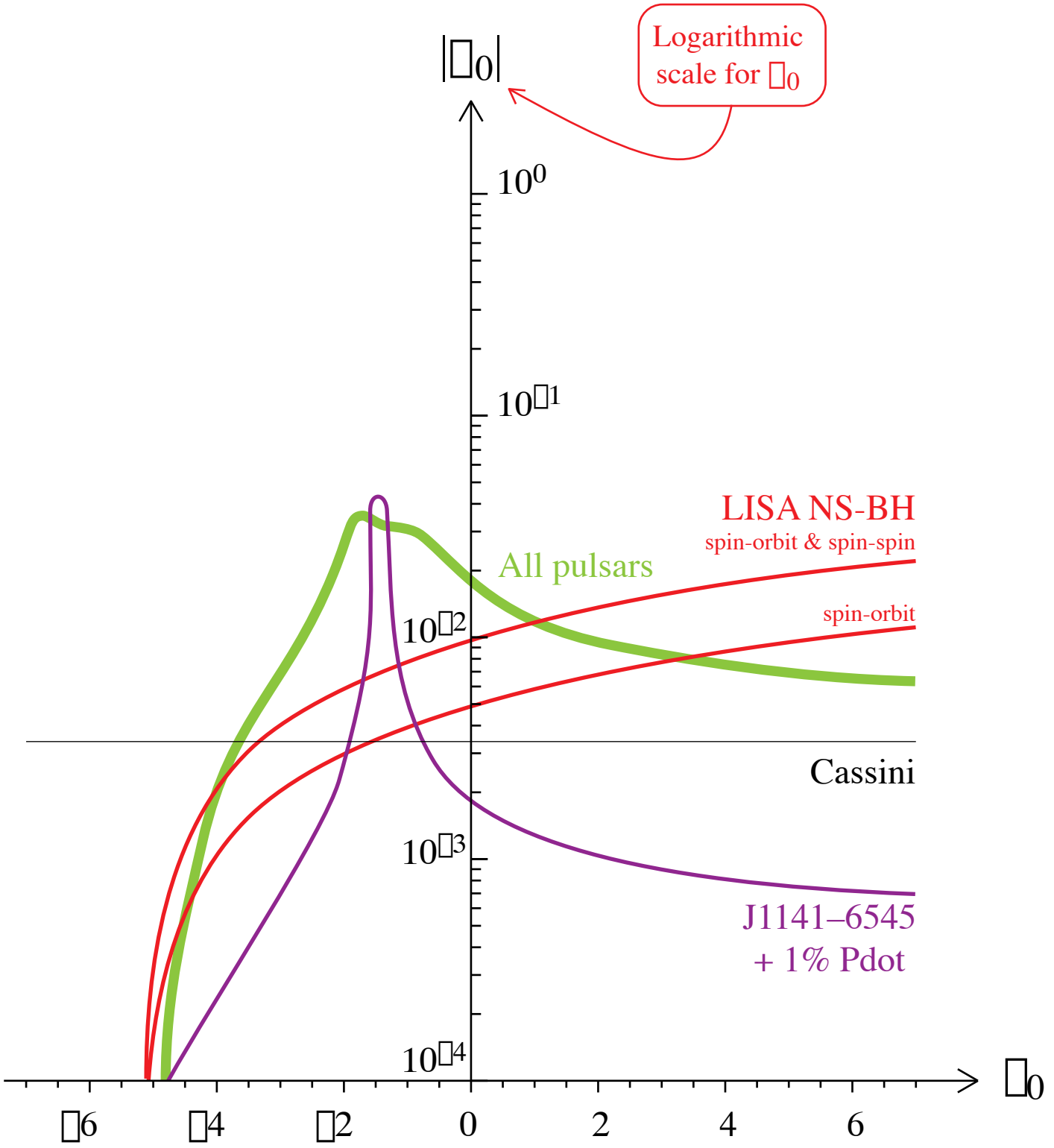
But if no detection, what would we conclude?

Future binary-pulsar constraints
on scalar-tensor theories of gravity



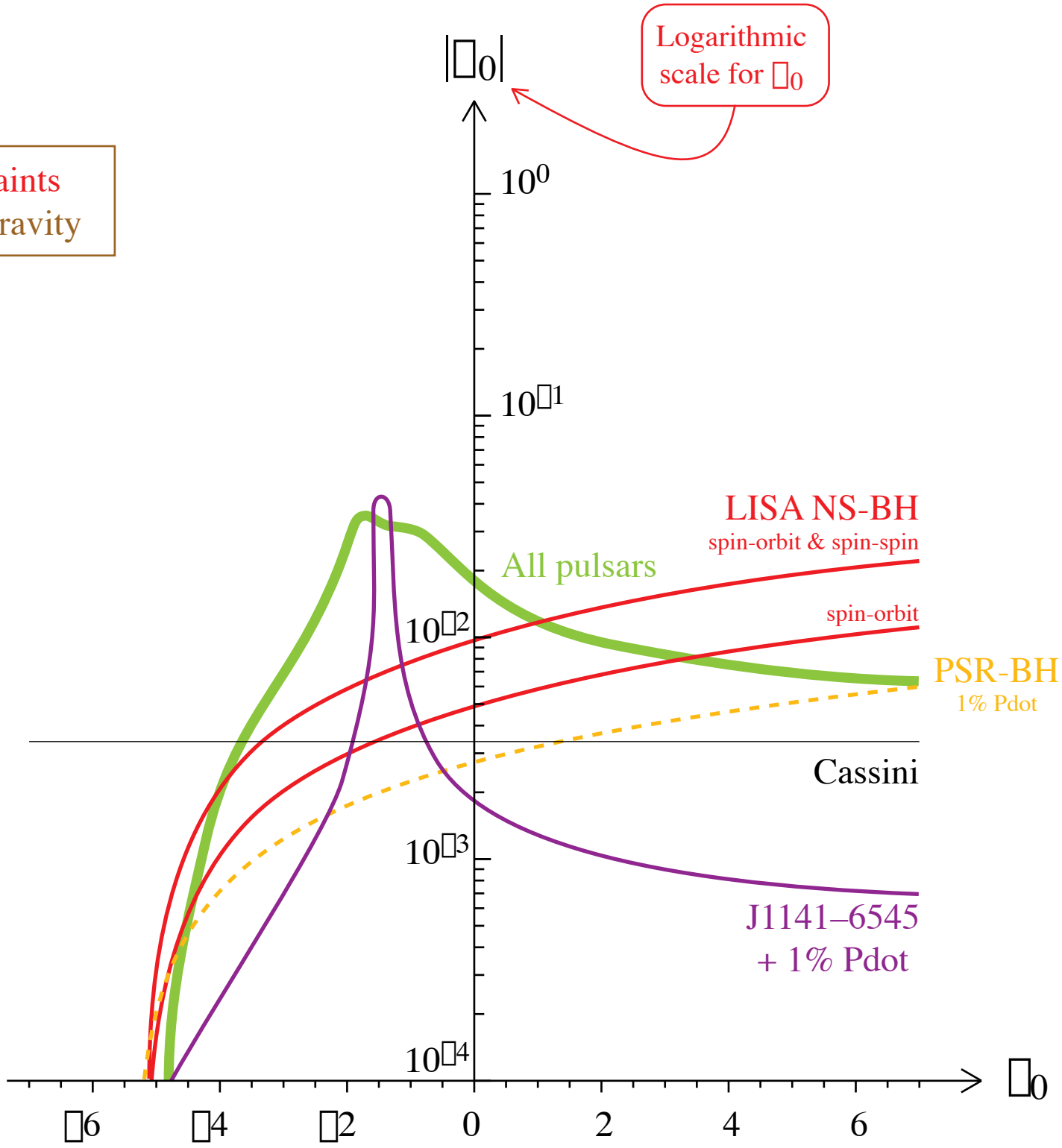
Binary pulsars will probably probe such scalar-tensor theories before LISA is launched

Good news: GR templates can be used securely

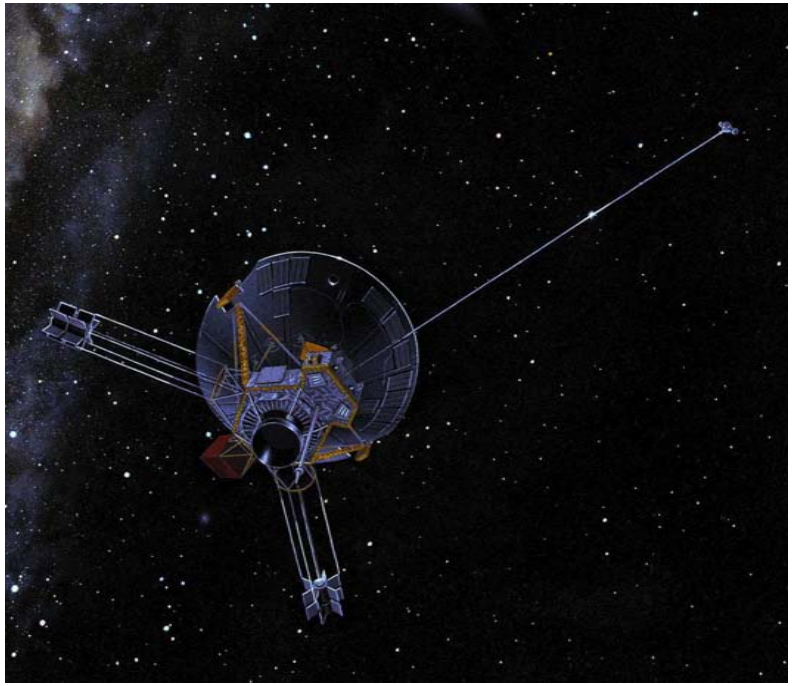

|□0|
Logarithmic scale for □0
10^0
10^□1
10^□2
10^□3
10^□4
LISA NS-BH
spin-orbit & spin-spin
spin-orbit
All pulsars
Cassini
J1141–6545
+ 1% Pdot
□0
□6
□4
□2
0
2
4
6

Future binary-pulsar constraints
on scalar-tensor theories of gravity

Pulsar-white dwarf
and
Pulsar-black hole
are the most constraining systems
for alternative theories
(large dipolar radiation)

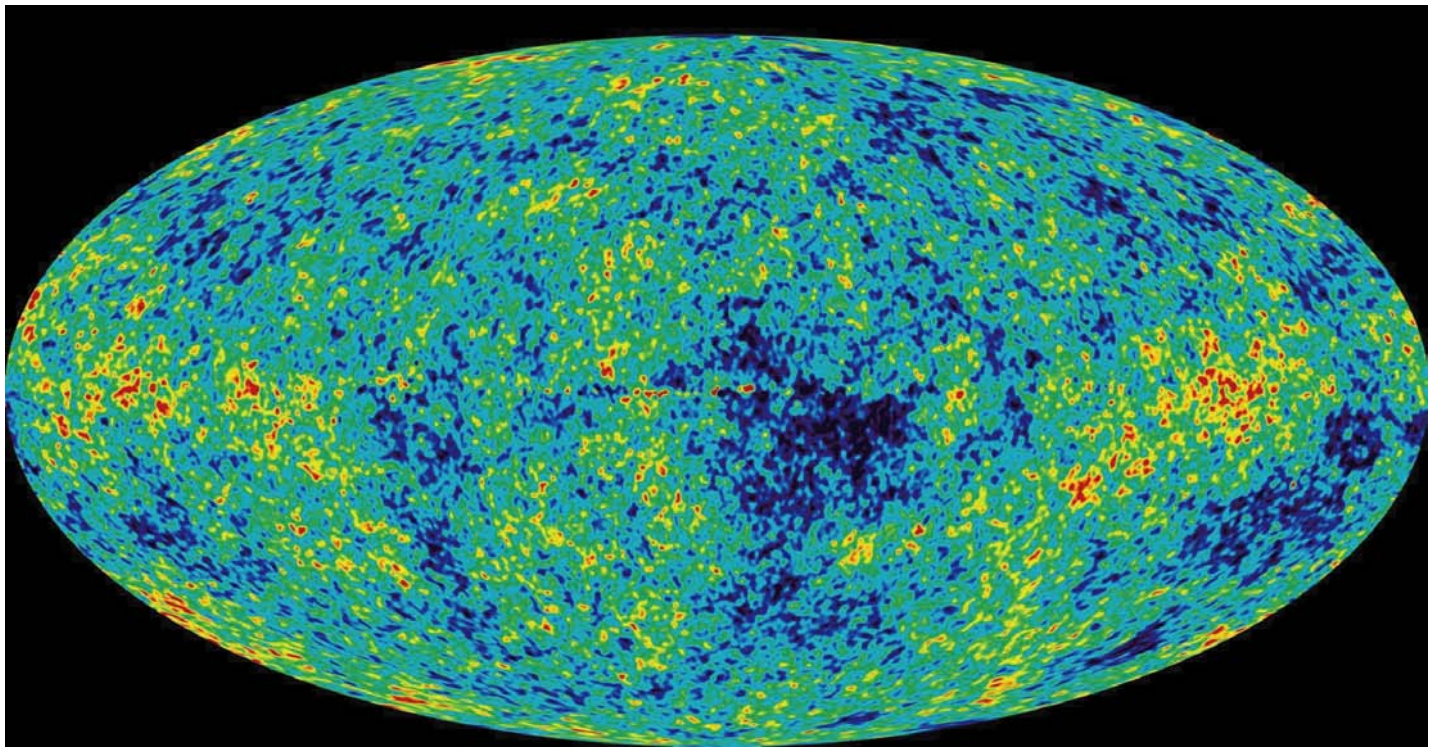


Puzzling issues



- Pioneer 10 & 11 anomaly in solar system (~70 AU):
  extra acceleration ~ 8.5 $\square 10^{-10}$ m s^{-2}

- Cosmological observations:
  72% of "dark energy"
  24% of "dark matter"
  4% of baryonic matter



"photograph" of Universe at 380 000 years (now 13.7 billion years)

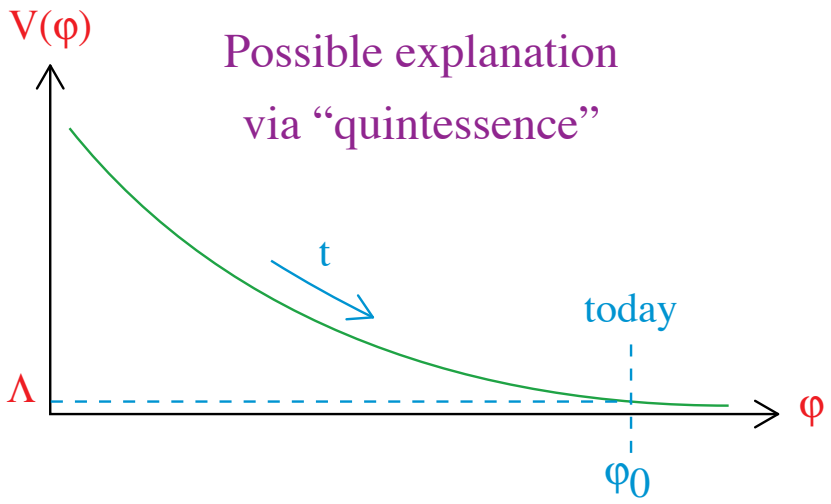
■ Dark energy

– Why $\Omega_\Lambda = 0.72 \sim \Omega_m = 0.28$ today? (type Ia supernovae combined with CMB)

∃ hints from some models

but no clean answer yet

– Why is $\Lambda \approx 3 \times 10^{-122} \dfrac{c^3}{\hbar G}$ so small? (type Ia supernovae notably)



• New qualitative difference between cosmological observations and solar-system/binary-pulsar ones

Cosmological observations give access
to the full shape of matter-scalar coupling $A(\varphi)$
and/or scalar-field potential $V(\varphi)$

• **Usual cosmology:**

– Assume particular forms of $V(\varphi)$ [and $A(\varphi)$] for theoretical reasons
– Predict all observable quantities
– Compare them to experimental data

• **Phenomenological approach:**

Reconstruct $A(\varphi)$ & $V(\varphi)$ from observational data.

Result:
If luminosity distance $D_L(z)$ and density fluctuations $\delta_m(z) = \frac{\delta\rho}{\rho}$ are both known as functions of the redshift z, then $A(\varphi)$ & $V(\varphi)$ can be reconstructed.

[Boisseau, GEF, Polarski & Starobinsky 2000]

N.B.: A priori obvious, since one "fits" **two** observed functions [$D_L(z)$ & $\delta_m(z)$] with **two** unknown ones [$A(\varphi)$ & $V(\varphi)$] !

• **Semi-phenomenological approach:**

[$\delta_m(z)$ not yet well measured]

– Theoretical hypotheses on $V(\varphi)$ or $A(\varphi)$
– Reconstruct the other one from $D_L(z)$

N.B.: A priori obvious too, since one fits **one** observed function [$D_L(z)$] with **one** unknown function [$A(\varphi)$ or $V(\varphi)$]. However, this naive reasoning works only locally (small interval).

Result:
$\exists$ tight constraints if $D_L(z)$ measured on a wide interval $z \in [0, \sim 2]$, even with large error bars!

[GEF & Polarski 2001]

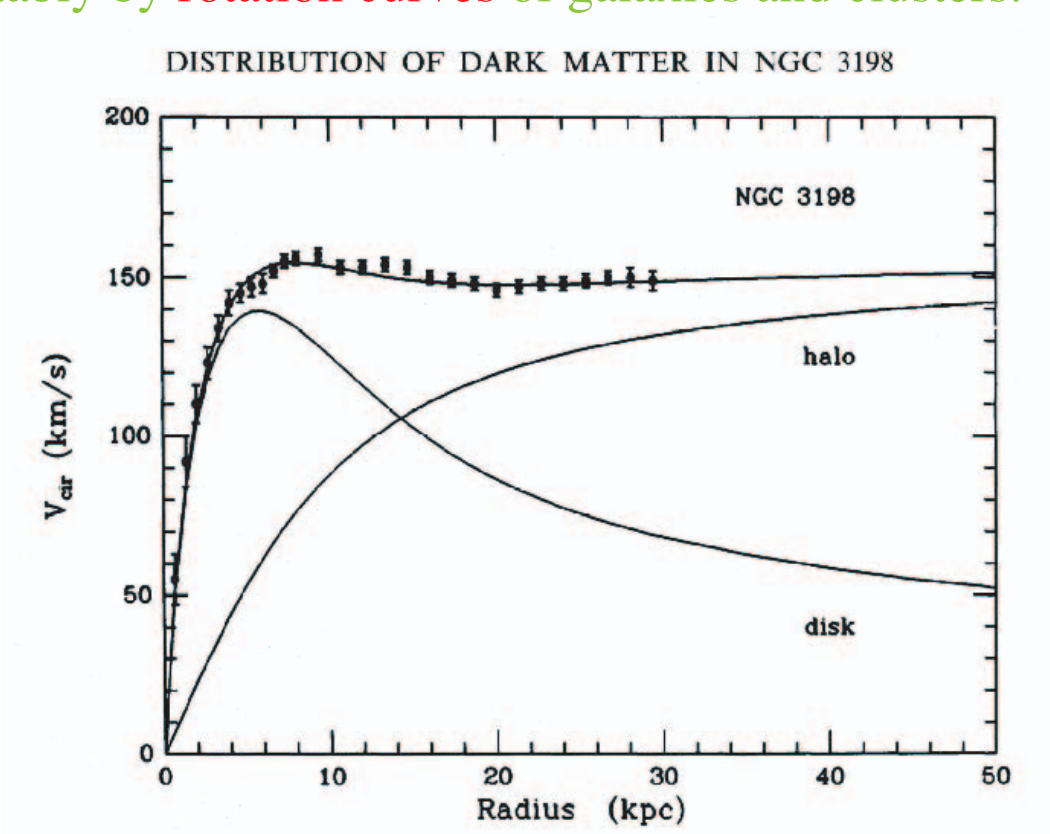
Constraints come mainly from positivity of energy :

$$E_{graviton} \geq 0 \Leftrightarrow A^2 > 0 \Leftrightarrow \Phi_{BD} > 0$$

$$E_\varphi \geq 0 \Leftrightarrow -(\partial_\mu \varphi)^2 \Leftrightarrow \omega_{BD} > -3/2$$

■ Dark matter = pressureless and noninteracting component of matter

- Imposed notably by rotation curves of galaxies and clusters:



- ? really some dark matter (many theoretical candidates notably from SUperSYmmetry), or modification of Newton's law at large distances?

- Milgrom's phenomenological "MOND" proposal:

$$a = \quad a_{\rm N} \quad = GM/r^2 \qquad \text{if} \quad a > a_0 \approx 1.2 \times 10^{-10} \ \text{m.s}^{-2}$$

$$a = \sqrt{a_0\, a_{\rm N}} = \sqrt{GM a_0}/r \qquad \text{if} \quad a < a_0$$

(OK for galaxies, not for clusters)

■ Relativistic formulations of MOND?

- various attempts but
  - some change the field-theory action depending on galaxy
  - some predict e.g. $a = kM^2/r$ instead of $\sqrt{M}/r$,
    and assume then $k = M^{3/2}$
  - many contain tachyons or ghosts (unstable)

■ Relativistic formulations of MOND?

Best present candidate: Bekenstein-Sanders model

– Many years of research seems very fine tuned but works (even for Pioneer anomaly if one wishes).

- 1 usual (spin-2) graviton
- 1 (spin-1) vector field
- 2 (spin-0) scalar fields

Idea: use one of the scalar fields as a "Lagrange parameter" to create a nonlinear F[(∂)^2] for the other one obtain the right $\sqrt{M}$ dependence

$$S_s = -\frac{1}{2}\int\left[\sigma^2 h^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} + \frac{1}{2}G\ell^{-2}\sigma^4 F(kG\sigma^2)\right] \times (-g)^{1/2} d^4x,$$

$$F = \frac{3}{8}\,\frac{\mu(4 + 2\mu - 4\mu^2 + \mu^3) + 2\ln[(1-\mu)^2]}{\mu^2}$$

■ Relativistic formulations of MOND?

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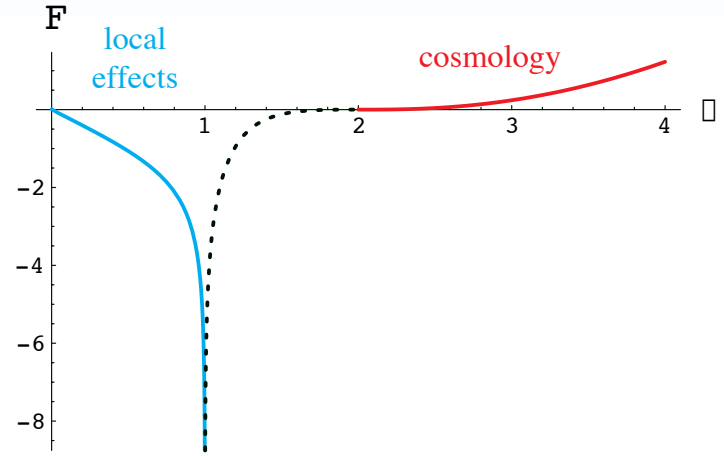
Idea: use one of the scalar fields as a "Lagrange parameter" to create a nonlinear $F[(\partial\phi)^2]$ for the other one obtain the right $\sqrt{M}$ dependence

– Stability not yet fully clear (tachyons & ghosts?).

$$S_s = -\frac{1}{2}\int\left[\sigma^2 h^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta} + \frac{1}{2}G\ell^{-2}\sigma^4 F(kG\sigma^2)\right] \times (-g)^{1/2} d^4x,$$

$$F = \frac{3}{8}\,\frac{\mu(4 + 2\mu - 4\mu^2 + \mu^3) + 2\ln[(1-\mu)^2]}{\mu^2}$$



■ Relativistic formulations of MOND?

Best present candidate: Bekenstein-Sanders model

– Many years of research seems very fine tuned but works (even for Pioneer anomaly if one wishes).

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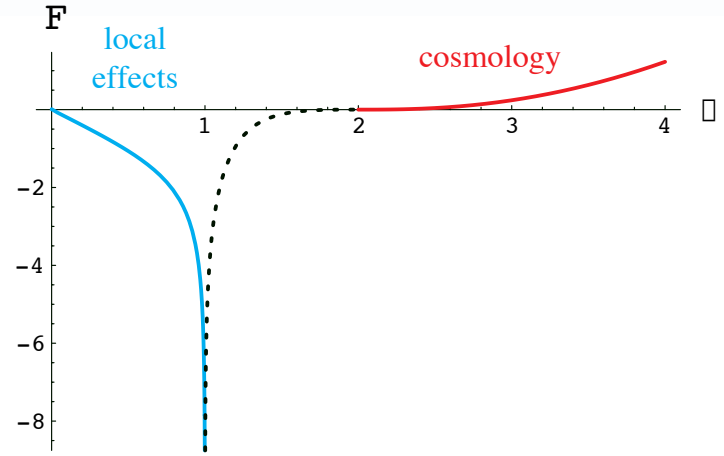

– Stability not yet fully clear (tachyons & ghosts?).

– Matter coupled to a physical metric involving the vector field preferred frame (ether).

Idea: couple differently the scalar field to "g_{00}" and "g_{ij}" in order to get the right light deflection
(cf. conformal coupling no extra deflection)

$$\tilde{g}_{\alpha\beta} = e^{-2\phi}(g_{\alpha\beta}+\mathcal{U}_\alpha\mathcal{U}_\beta) - e^{2\phi}\mathcal{U}_\alpha\mathcal{U}_\beta,$$

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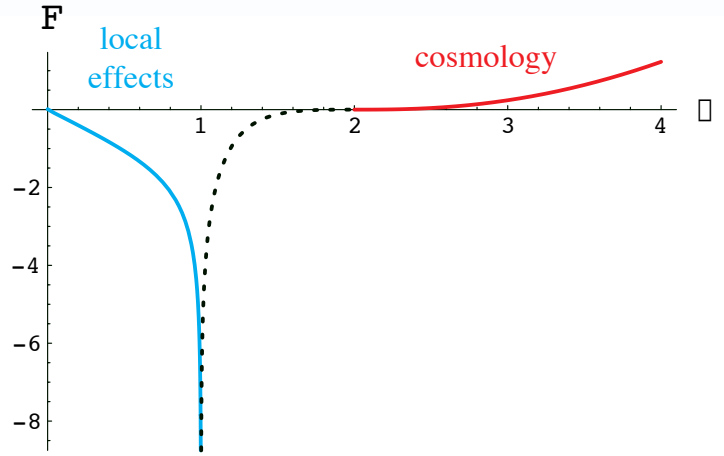

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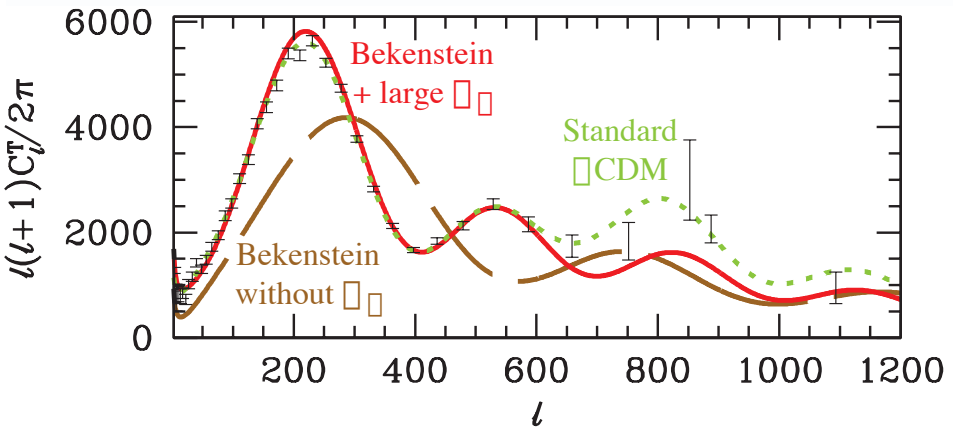
– CMB predictions [Skordis et al., astro-ph/0505519] need $\Omega_{\text{B}} = 0.17$ (not far from $\Omega_{\text{DM}} = 0.24$) dark matter!



Conclusions

- **$\exists$ theoretical motivations for considering alternatives to General Relativity.**

- **Contrasting G.R. with alternatives** $\Rightarrow$ understand which features have been tested

  $\Rightarrow$ compare *probing power* of different observations

Qualitative difference between

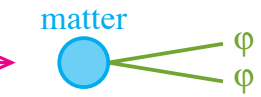
solar-system,
[weak field regime tested at the 10^{-5} level]

first derivative of $\ln A(\varphi)$

binary-pulsar,
[strong field regime]

nonperturbative effects

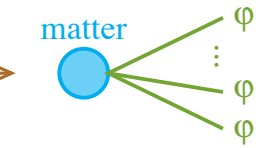
$\Rightarrow$ second derivative of $\ln A(\varphi)$



and cosmological observations.
[time evolution]

a priori full shape of $\ln A(\varphi)$

but much more noisy



- **General Relativity passes all *tests* with flying colors.**

- **$\exists$ still some puzzling experimental facts** $\Rightarrow$ understand them either theoretically

  or experimentally