

String Theory for Physicists



Lecture 3

J. Lykken
Fermilab

SSIØ5

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Last time we said we could make consistent string theories with gravity + 10 dim supersymmetry

These will have 10 dim. supergravity as their massless content

In 2, 6, or 10 dimensions, gauge-gravity-matter theories are very restricted by potential anomalies

↳ we can enumerate all possible anomaly-free 10 dim. supergravity theories!

There are Exactly 4 of them:

- 1) 10 dim. Type IIA supergravity
- 2) 10 dim. Type IIB supergravity
- 3) 10 dim. $N=1$ supergravity + $E_8 \times E_8$ Super-Yang-Mill,
- 4) 10 dim. $N=1$ supergravity + $SO(32)$ Super-Yang-Mill,

1) 10 dim. Type IIA supergravity

16 Majorana-Weyl supercharges $Q_L^\alpha, Q_R^\alpha \quad \alpha, \dot{\alpha} = 1,..8$

(compare 4 dim. N=4 SUSY also has 16 supercharges)

left-right pairing \Rightarrow theory is vectorlike

massless fields	# of components	massless field	# of components
g_{MN}	$\frac{8 \cdot 7}{2} + 7 = 35$	ψ_m^α	$7 \times 8 = 56$
A_M	8	ψ_n^α	56
ϕ	1	χ^α	8
A_{MNP}	$\frac{8 \cdot 7 \cdot 6}{3!} = 56$	$\chi^{\dot{\alpha}}$	8
B_{MN}	$\frac{8 \cdot 7}{2} = 28$		
<hr/>		<hr/>	
total bosons = 128		total fermions = 128	

This theory has brane sources!

A_{MNP} has field strength F_{MNPQ}

and dual field strength $\epsilon^{n_1 .. n_{10}} F_{n_7 n_8 n_9 n_{10}}$

Compare to ordinary gauge field in 4d

A_M has field strength $F_{\mu\nu}$, dual field strength $\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$

A_{MNP} has D2 branes as "electric sources"

D4 branes as "magnetic sources"

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A_M has field strength F_{MN}

dual field strength $\epsilon^{M_1 \dots M_{10}} F_{M_9 M_{10}}$

\Rightarrow electric sources are D ϕ branes (particles!!)

magnetic sources are D6 branes

B_{MN} has field strength F_{MNP}

dual field strength $\epsilon^{M_1 \dots M_{10}} F_{M_9 M_{10}}$

\Rightarrow electric sources are closed strings

magnetic sources are NS5 branes

So altogether d=10 Type IIA supergravity
should have the following kinds of objects as sources:

D ϕ branes closed strings

D2 " NS5 branes

D4 "

D6 "

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2) 10 dim. Type IIB supergravity

16 supercharges $Q_L^{(1)}, Q_L^{(2)}$ $\alpha=1,..8$ same chirality

massless fields # of components

$g_{\mu\nu}$	35	ψ_M^α	56
$A_{\mu\nu}$	28	$\tilde{\psi}_n^\alpha$	56
$B_{\mu\nu}$	28	λ^α	8
self-dual A_{MNPQ}	$\frac{8 \cdot 7 \cdot 6 \cdot 5}{4!} = 70 \div 2$ $= 35$	$\tilde{\lambda}^\alpha$	8
A, \tilde{A}	2		
total bosons	128	total fermions	128

→ self-dual means $F^{M_1 M_2 M_3 M_4 M_5} = \epsilon^{M_1 \dots M_{10}} F_{M_6 M_7 M_8 M_9 M_{10}}$
 \Rightarrow no Lorentz invariant Lagrangian!

- brane sources:
- $A_{\mu\nu}$: D1 electric D5 magnetic
 - A_{MNPQ} : D3 electric + magnetic
 - A : D-1 electric D7 magnetic
 - $B_{\mu\nu}$: closed string NS5 brane

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3) 10 dim $N=1$ supergravity + $SU(32)$ or $E_8 \times E_8$ super Yang-Mills
 $+ 4)$

$N=1$ supergravity

8 supercharges $Q_\alpha \alpha = 1..8$

$g_{mn} \ 35 \ \Psi_m^\alpha \ 5_6$

$B_{mn} \ 28 \ X^\alpha \ 8$

ϕ	1	$\frac{1}{64}$	$\frac{1}{64}$
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$SU(32)$ or $E_8 \times E_8$
 pure gauge super Yang-Mills

$A_m^a \ 8 \ X^{a\alpha} \ 8$

the gauge index "a"
 runs over the 496
 gauge bosons of either
 $SU(32)$ or $E_8 \times E_8$

From these 4 $d=10$ supergravities, we can
 construct 5 "obviously different" $d=10$ superstrings:

Type IIA \rightarrow Type IIA superstring

Type IIB \rightarrow Type IIB superstring

$N=1$ supergravity
 $+ E_8 \times E_8$ SYM \rightarrow $E_8 \times E_8$ heterotic superstring
 (gauge fields from closed strings)

$N=1$ supergravity
 $+ SU(32)$ SYM \rightarrow $SU(32)$ heterotic superstring
 (gauge fields from closed strings)

\rightarrow $SU(32)$ Type I superstring
 (gauge fields from open strings)

T-duality

Consider string theory with one dimension compactified on a circle of radius R .

Then any string excitation, e.g. the graviton, will have a whole tower of Kaluza-Klein copies, with masses

$$\frac{n}{R} \quad n=1, 2, \dots$$

Another way to make a tower of copies is to wrap the string m times around the circle. This costs energy

$$2\pi R m T = \frac{mR}{\alpha'}$$

These two sets of excitations are related by a symmetry of string theory called T-duality. Obviously T-duality involves a transformation

$$R \rightarrow \frac{\alpha'}{R}$$

so string spectra on a circle of radius R can be exactly mapped to the spectra on a circle of radius $\frac{\alpha'}{R}$

If T-duality is really an invariance of the full theory, including branes, then nontrivial things have to happen e.g.

Consider D1 brane in Type IIB string compactified on a circle

If the circle is in ^{the} direction that the D1 brane extends, then T-duality relates it to a D \emptyset brane (wrapped state \rightarrow unwrapped state)

If the circle is in any other direction, then T-duality relates it to a D2 brane

\Rightarrow T-duality maps Type IIB string into Type IIA

Type IIB

D-1

D1

D3

D5

D7

Type IIA

D \emptyset

D2

D4

D6

(must be D8 too)

brane tensions are

related by $T_p = \frac{T_{p-1}}{2\pi\sqrt{\alpha'}}$

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In D=11 there is a unique supergravity theory.
 It has 32 supercharges, which is both the minimum
 and the maximum amount of supersymmetry.

$$\text{Lagrangian: } -\int d^9x \sqrt{-\det g} \left\{ \frac{1}{2k^2} R + \frac{1}{2} \bar{\Psi}_M \Gamma^{NP} D_N \Psi_P + \frac{1}{48} F_{MNPQ} F_{MNPO} \right\} + \dots$$

$$g_{MN} \quad \frac{9 \cdot 8}{2} + 8 = 44$$

$$A_{MNP} \quad \frac{9 \cdot 8 \cdot 7}{3!} = 84 \quad \bar{\Psi}_M \quad 128$$

$$= \frac{1}{128}$$



Sources:

- M2 "electric" brane
- M5 "magnetic" brane

M-theory: the D=11 theory of dynamical
 M2 and M5 branes whose
 low energy limit is D=11 supergravity

this theory is hard to get a
 straight forward handle on



D=10 Type IIA supergravity is just a dimensional reduction of D=11 supergravity

↳ means compactify the 11th dimension on a circle, keep only the Kaluza-Klein zero modes

D=11

g_{MN}

\rightarrow

D=10 Type IIA

g_{MN}

$A_M = g_{M11}$

$\phi = g_{1111}$

A_{MNP}

\rightarrow

A_{MNP}

$B_{MN} = A_{MN11}$

ψ_n^α

\rightarrow

$\psi_M^\alpha, \psi_N^{\dot{\alpha}}$

$x^\alpha, \lambda^{\dot{\alpha}} = \psi_{11}^\alpha$

How is the Type IIA $d=10$ superstring theory related to $d=11$ M-theory compactified on a circle?

M theory

Type IIA

closed

M_2 brane wrapped on circle \rightarrow string

M_2 brane not wrapped \rightarrow D_2 brane

M_5 brane wrapped on circle \rightarrow D_4 brane

M_5 brane not wrapped \rightarrow $NS5$ brane

Let T_{M_2} be the tension of the M_2 brane

wrap it on a circle of radius R_{11} to get a closed string

$$\text{so } \frac{1}{2\pi\alpha'} = 2\pi R_{11} T_{M_2}$$

$$\text{and } T_{D_2} = T_{M_2}$$

$$\text{but T-duality tells us } T_{D_2} = \frac{T_{D_2}}{4\pi^2\alpha'}$$

$$\text{so } T_{D_2} = 4\pi^2\alpha' \frac{1}{2\pi\alpha'} \frac{1}{2\pi R_{11}}$$

$$= \frac{1}{R_{11}}$$

So the supermultiplet of D_2 brane particles looks like the 1st Kaluza-Klein excitation of the massless supergravity multiplet!!!

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This is a strong \leftrightarrow weak coupling duality

10d Type IIA \equiv M theory on a circle

There are more:

10d Type I \equiv (10d heterotic SO(32))

10d heterotic $E_8 \times E_8$ \equiv M theory on a line interval!

10d Type IIB \equiv itself

E Pluribus Unum

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Black holes and strings

recall strings can reproduce the correct behavior of high energy small angle scattering of hadrons

$$M_{\text{abacted}} \sim s^{\text{constant}}$$

and give an exponentially damping behavior in high energy hard scattering

$$M_{\text{abacted}} \sim e^{-x/s}$$

This "soft" high energy behavior is because at high energies you start to produce actual string excited states

The number of distinct string states of a given mass increases exponentially fast as a function of the mass (because the string is an extended object)

The number of ordinary particle excitations increases like a power of the energy (mostly just phase space)

Since Entropy $S = \log N$ N number of microstates available
we have:

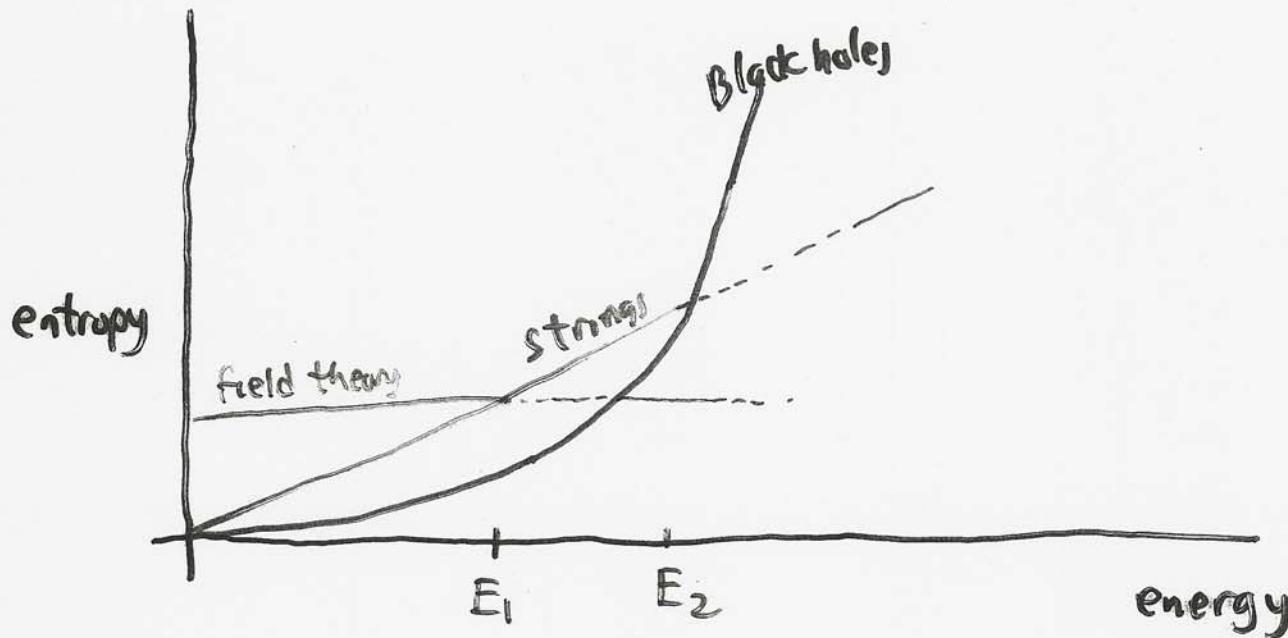
field theory: $S \sim \log M$ strings: $S \sim M$

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Beckenstein + Hawking showed that black holes have entropy proportional to their area

$$S = \frac{1}{4} A \sim R_{\text{horizon}}^2 \sim \frac{M^2}{M_{\text{Pl}}^4}$$

so



We expect, just on the grounds of entropy, that high energy collider physics has 3 regimes:

for $\sqrt{s} < E_1$ you see normal field theory behavior e.g. Standard Model

for $E_1 < \sqrt{s} < E_2$ collisions are dominated by production of stringy excitations

$\sqrt{s} > E_2$ collisions dominated by production of black holes

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So from a purely high energy physicist's viewpoint, strings are just the intermediate behavior between standard particle production and black hole production

Since we can compute a lot about black holes from very general arguments, we can check if highly excited strings really turn into black holes in a smooth way

Even better, we can try to make the black holes out of stringy things...

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Example: 10dim Type IIB string theory compactified on a 5 dim torus T^5

- Since Type IIB has D5 branes, wrap some around the torus.

the low energy limit is some 5d theory with a curved background, which actually has the metric of a 5dim. black hole

So wrapped D-branes are a stringy model for some black holes

wrap a Type IIB D1 brane also around one circle of the torus. If the circle is large (in string units) then there are lots of open strings that can be anywhere on the big circle, attached to the D1 brane.

this is like a one-dimensional gas of open strings, either left or right moving



occasionally two open strings coalesce to make a closed string, which then moves off the circle

This is a stringy model for Hawking radiation from a black hole

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Because we have lot's of SUSY and these are D-branes, this stringy Hawking radiation rate can be computed:

$$\Gamma(h) \sim \frac{1}{(e^{\frac{\omega}{2T_L}} - 1)(e^{\frac{\omega}{2T_R}} - 1)}$$

↑ ↑
 thermodynamic factors for a
 left and right moving gas of
 open strings

T_L, T_R are "temperatures" which are known in terms of parameters appearing in the black hole metric

$$T_L = \frac{1}{\pi} \frac{r_0 e^\sigma}{2r_1 r_5} \quad T_R = \frac{1}{\pi} \frac{r_0 e^{-\sigma}}{2r_1 r_5} \quad \sinh^2 \sigma = \frac{r_h^2}{r_0^2}$$

5dim. black hole metric:

$$ds^2 = -f(r)^{-2/3} \left(1 - \frac{r_0^2}{r^2}\right) dt^2 + f(r)^{1/3} \left[\left(1 - \frac{r_0^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2\right]$$

where $f(r) = \left(1 + \frac{r_1^2}{r^2}\right) \left(1 + \frac{r_5^2}{r^2}\right) \left(1 + \frac{r_h^2}{r^2}\right)$

Compare to 4d Schwarzschild Black Hole

$$ds^2 = -\left(1 - \frac{r_s}{r}\right) dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

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Black holes know about strings

Now start over with just this 5d black hole, and forget all about string theory.

Let's compute the rate of Hawking radiation, using standard black hole techniques

$$d\Gamma_{\text{emit}} = \frac{\nu \sigma_{\text{absorb}}}{e^{\frac{\omega}{T_H}} - 1} \frac{d^3 k}{(2\pi)^3}$$

↑
thermodynamic factor for black hole
with Hawking temperature $T_H = \frac{M_{\text{Pl}}^2}{4\pi M}$

σ_{absorb} (the "greybody factor")
is the cross section for the same particle (with freq ω)
to be absorbed by the black hole

$\sigma_{\text{absorb}} = A$ if the Hawking radiation
is perfectly thermal, which it isn't.

So far this looks nothing like the stringy result

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5d black hole metric:

$$ds^2 = -f(r)^{-2/3} z dt^2 + f(r)^{1/3} [\bar{z}^{-1} dr^2 + r^2 d\Omega_3^2]$$

$$\text{where } z \equiv 1 - \frac{r_0^2}{r^2}; \quad f = \left(1 + \frac{r_1^2}{r^2}\right) \left(1 + \frac{r_5^2}{r^2}\right) \left(1 + \frac{r_6^2}{r^2}\right)$$

$$\text{so } \sqrt{g} = r^3 f^{1/3}$$

$$g^{rr} = \bar{f}^{1/3} z$$

$$g^{tt} = \frac{f^{2/3}}{z}$$

so a scalar living in this background has an action

$$\int d^5x \sqrt{g} g^{mn} \partial_m \phi \partial_n \phi$$

giving an EOM

$$\left\{ \frac{d}{dr} \left(\sqrt{g} g^{rr} \frac{d}{dr} \right) + \sqrt{g} g^{tt} w^2 \right\} \phi = 0$$

$$\text{or } \left\{ \frac{d}{dr} \left(z r^3 \frac{d}{dr} \right) + w^2 \frac{f r^3}{z} \right\} \phi = 0$$

Solve this in terms of hypergeometric functions
to get Σ_{absorb} :

$$\Sigma_{\text{absorb}} \sim \frac{e^{\frac{w}{2T_H} - 1}}{(e^{\frac{w}{2T_L} - 1})(e^{\frac{w}{2T_R} - 1})}$$

!!!

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So the black hole result is

$$\boxed{\text{Hawking radiation}} \sim \frac{e^{\frac{w}{T_H}} - 1}{(e^{\frac{w}{2T_L}} - 1)(e^{\frac{w}{2T_R}} - 1)} \frac{1}{e^{\frac{w}{T_H}} - 1} \sim \frac{1}{(e^{\frac{w}{2T_L}} - 1)(e^{\frac{w}{2T_R}} - 1)}$$

, including the overall factors, matches exactly to the stringy result

How does the black hole know it should mimic a one-dimensional gas of left and right moving strings on a hidden circle?

If these kind of results can be extended to the kind of black holes that actually exist out there in the universe, it would be almost a proof that string theory is correct.