Lunar Laser Ranging

A Local Laboratory for Gravity

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Why Test Gravity?

- Dark Matter *could* be misunderstood gravity (nobody say MOND)
- Dark Energy *could* be misunderstood gravity
- Gravity and Quantum Mechanics do not play well together
- **Fundamental constants** (like $G$) may change with time
- Scalar fields may lead to Equivalence Principle violation
- **Extra Dimensions** may impact the gravity sector
- A theory is only as believable as the backing experiment
The Full PPN Metric

- Generalized metric where little is sacred
  - GR is a special case
  - Allows violations of conservations, Lorentz invariance, etc.

\[
g_{00} = -1 + 2U - 2\beta U^2 - 2\xi \phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\phi_1 \\
+ 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\phi_2 + 2(1 + \zeta_3)\phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\phi_4 \\
- (\zeta_1 - 2\xi)A - (\alpha_1 - \alpha_2 - \alpha_3)w^2U - \alpha_2 w^i w^j U_{ij} + (2\alpha_3 - \alpha_1)w^i V_i \\
+ O(\epsilon^3)
\]

\[
g_{0i} = -\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i \\
- \frac{1}{2}(\alpha_1 - 2\alpha_2)w^i U - \alpha_2 w^j U_{ij} + O(\epsilon^{5/2})
\]

\[
g_{ij} = (1 + 2\gamma U + O(\epsilon^2))\delta_{ij}
\]
Simplified (Conservative) PPN Equations of Motion

\[ \ddot{r}_{i, \text{point mass}} = \sum_{j \neq i} \frac{\mu_j (r_j - r_i)}{r_{ij}^3} \left\{ 1 - \frac{2(\beta + \gamma)}{c^2} \sum_{k \neq i} \frac{\mu_k}{r_{ik}} \right. \\
- \frac{2\beta - 1}{c^2} \sum_{k \neq j} \frac{\mu_k}{r_{jk}} + \gamma \left( \frac{v_i}{c} \right)^2 + (1 + \gamma) \left( \frac{v_j}{c} \right)^2 \\
- \frac{2(1 + \gamma)}{c^2} \dot{r}_i \cdot \dot{r}_j - \frac{3}{2c^2} \left[ \frac{(r_i - r_j) \cdot \dot{r}_j}{r_{ij}} \right]^2 \\
+ \frac{1}{2c^2} (r_j - r_i) \cdot \ddot{r}_j \right\} + \frac{1}{c^2} \sum_{j \neq i} \frac{\mu_j}{r_{ij}^3} \\
\times \{ (r_i - r_j) \cdot [(2 + 2\gamma)\dot{r}_i - (1 + 2\gamma)\dot{r}_j] \} (\dot{r}_i - \dot{r}_j) \\
+ \frac{3 + 4\gamma}{2c^2} \sum_{j \neq i} \frac{\mu_j \ddot{r}_j}{r_{ij}} \]
- The Post-Newtonian Parameterization (PPN) describes deviations from GR
- The main parameters are $\gamma$ and $\beta$
  - $\gamma$ tells us how much curvature is produced per unit mass
  - $\beta$ tells us how nonlinear gravity is (self-interaction)
  - $\gamma$ and $\beta$ are identically 1.00 in GR
- Current limits have:
  - $(\gamma-1) < 2.3 \times 10^{-5}$ (Cassini)
  - $(\beta-1) < 1.1 \times 10^{-4}$ (LLR)
Relativistic Observables in the Lunar Range

- Lunar Laser Ranging provides a comprehensive probe of gravity, boasting the best tests of:
  - Equivalence Principle (two flavors)
    - WEP: $\Delta a/a < 1.4 \times 10^{-13}$
    - SEP: $\eta < 4.5 \times 10^{-4}$
  - time-rate-of-change of $G$: $< 9 \times 10^{-13}$ fractional change per year
  - geodetic precession: confirmed to 0.6%
  - $1/r^2$ force law: $< 10^{-10}$ times strength of gravity at $10^8$ m scales
  - gravitomagnetism (frame-dragging): 0.1%
- Future sensitivity to extra-dimensional modifications to gravity as proposed by Dvali et al.
  - result would be precession of lunar orbit
Two Flavors of the Equivalence Principle

- **Weak EP**
  - Composition difference: e.g., iron in earth vs. silicates in moon
  - Probes all interactions but gravity itself
    - Currently tested by LLR and torsion balance to $\Delta a/a < 1.4 \times 10^{-13}$
  - the LLR $\Delta a/a$ value is tied with torsion balance results (e.g., Eöt-Wash group), though laboratory measurements constrain WEP more meaningfully

- **Strong EP**
  - Applies to gravitational “energy” itself
    - Earth self-energy has equivalent mass ($E = mc^2$)
      - Amounts to $4.6 \times 10^{-10}$ of earth’s total mass-energy
      - Does this mass have $M_G/M_I = 1.00000$?
  - Another way to look at it: gravity pulls on gravity
    - This gets at *nonlinear* aspect of gravity (PPN $\beta$)
The Strong Equivalence Principle

- Earth’s energy of assembly amounts to $4.6 \times 10^{-10}$ of its total mass-energy

$$M_{\text{S.E.}} = \frac{G}{c^2} \int \int \rho(\mathbf{r}_1)\rho(\mathbf{r}_2) \frac{d^3\mathbf{r}_1 d^3\mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|} \approx \frac{GM^2}{Rc^2}$$

The ratio of gravitational to inertial mass for this self energy is

$$\frac{M_G}{M_I} = 1 - (4\beta - 3 - \gamma) \frac{M_{\text{S.E.}}}{M} \equiv 1 - \eta \frac{M_{\text{S.E.}}}{M}$$

The resulting range signal is then

$$\Delta r = 13.1\eta \cos D \text{ meters}$$

Currently $\eta$ is limited by ~cm LLR to be $\leq 4.5 \times 10^{-4}$

LLR is the best way to test the strong EP
If, for example, Earth has greater inertial mass than gravitational mass (while the moon does not):

- Earth is sluggish to move
- Alternatively, pulled weakly by gravity
- Takes orbit of larger radius (than does Moon)
- Appears that Moon’s orbit is shifted toward sun: \( \cos D \) signal
WHAT COULD BE FOUND IN THE ORBITS

If the equivalence principle is true, the sun’s gravity pulls equally on the Earth and the moon. Therefore Earth’s orbit and the moon’s average orbit follow the same path.

The moon orbits the Earth, but it also orbits the sun, giving its actual path this wavy shape.

If the equivalence principle isn’t true, gravity treats the objects differently, and one orbit would be skewed.

This would disprove the equivalence principle, and scientists would have to go back to the drawing board.

moon close

moon far

Graphic excerpt from San Diego Union Tribune
Aside on Gravitomagnetism

- Stems from “motional” term in equation of motion:

\[ a_i = -\frac{\mu_j (2 + 2\gamma)}{c^2 r_{ij}^3} v_i \times (v_j \times r_{ij}) \]

- If earth has velocity \( V \), and moon is \( V + u \), two terms of consequence emerge:
  - One proportional to \( V^2 \) with 6.5 meter \( \cos 2D \) signal
  - One proportional to \( Vu \) with 6.1 meter \( \cos D \) signal
- LLR determines \( \cos D \) to 4 mm precision and \( \cos 2D \) to < 8 mm
  - Constitutes a \( \approx 0.1\% \) measurement of effect
- The same exact \( v \times v \times g \) term can be used to derive the precession of a gyroscope in the presence of a spinning mass
  - Recovers the full effect (0.042 mas/yr) sought by Gravity Probe-B
Lunar Retroreflector Arrays

Corner cubes

Apollo 14 retroreflector array

Apollo 11 retroreflector array

Apollo 15 retroreflector array
Previously 200 meters
APOLLO: the next big thing in LLR

- APOLLO offers order-of-magnitude improvements to LLR by:
  - Using a 3.5 meter telescope
  - Gathering multiple photons/shot
  - Operating at 20 pulses/sec
  - Using advanced detector technology
  - Achieving millimeter range precision
  - Tightly integrating experiment and analysis
  - Having the best acronym
Gigantic Laser Pointer
Blasting the Moon
## APOLLO Random Error Budget

<table>
<thead>
<tr>
<th>Error Source</th>
<th>Time Uncert. (ps) (round trip)</th>
<th>Range error (mm) (one way)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retro Array Orient.</td>
<td>100–300</td>
<td>15–45</td>
</tr>
<tr>
<td>APD Illumination</td>
<td>60</td>
<td>9</td>
</tr>
<tr>
<td>APD Intrinsic</td>
<td>&lt;50</td>
<td>&lt; 7</td>
</tr>
<tr>
<td>Laser Pulse Width</td>
<td>45</td>
<td>6.5</td>
</tr>
<tr>
<td>Timing Electronics</td>
<td>20</td>
<td>3</td>
</tr>
<tr>
<td>GPS-slaved Clock</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td><strong>Total Random Uncert</strong></td>
<td><strong>136–314</strong></td>
<td><strong>20–47</strong></td>
</tr>
</tbody>
</table>

Ignoring retro array, APOLLO system has 93 ps (14 mm) error *per photon*
Example Range Return

03 Dec., 2006
Apollo 15
2000 photons
1670 in last half
last half average: 0.33 photons per pulse
c.f. OCA: 0.01
MLRS: 0.002
More Examples

9 Jan., 2007: Apollo 15
1700 photons

03 Dec., 2006: Apollo 11
1100 photons

Apollo 11 is 3$\times$ smaller than Apollo 15
We learn:

We can optimize the signal given time.

We can get 1 mm range precision in single 10-minute “runs”.

The scatter about a linear function is small: *almost* within estimated error.

We’re producing mm-quality data.
In poor conditions, we tend to dwell on Apollo 15 (3× bigger)

This gives us a useful internal check on scatter: is scatter consistent with estimated errors?

Generally, it is, with occasional outliers thought to result from crude initial form of data reduc.
APOLLO data points processed together with 16,000 ranges over 35 years shows consistency with model orbit

Some outliers likely APOLLO’s fault (early form of data reduc.)

Some trends likely model deficiencies

Weighted RMS is about 6–8 mm

$\chi \approx 3$ for this fit
APOLLO Superlatives

- More lunar return photons in **10 minutes** than the McDonald station gets in **three years**
  - best single run: >2500 photons in 10,000 shots (8 minutes)
- Peak rates of **>0.6 photons per shot** (12 per second)
- Sustained rates typically **0.1–0.3 photons per shot**
  - compare to typical 1/500 for McDonald, 1/100 for France
- Range with ease at **full moon**
  - APOLLO’s *very first* returns were at full moon
  - other stations can’t fight the high background
- As many as **9 photons** detected in a single pulse!
  - In best runs, *half* of detected photons in multi-photon clumps
- Expect substantial gains in EP, gravitomagnetism within the year
  - longer for other physics
Future Directions for LLR

- **Current Problems:**
  - Only one one-millimeter capable station in the world
  - Array tilts dominate error budget (designed in the day of few-ns lasers)
  - Array response appears to suffer a factor of > 10 attenuation

- **Future Option 1:**
  - deploy new arrays that allow one to resolve individual corner cubes
  - thus take full advantage of ground capability (spurring improvements)
  - fresh arrays allow greater participation

- **Future Option 2:**
  - deploy transponders on lunar surface
  - $1/r^4$ passive loss becomes $1/r^2$: everyone can play
  - essential practice for more ambitious solar system ranging experiments
Future Directions Beyond LLR

- LLR tests gravity on our **doorstep**
- There’s also a **back yard**: the solar system
- Interplanetary laser ranging offers another order-of-magnitude over APOLLO’s capabilities:
  - Measure $\gamma$ via Shapiro delay
  - Measure strong equivalence principle as Sun falls toward Jupiter
- Techniques:
  - Multi-task laser altimeters as asynchronous transponders
    - incredible demonstration to MESSENGER: 24 million km 2-way link
  - Piggyback on optical communications/navigation
- Other methods for probing local spacetime:
  - Weak equivalence principle tests (e.g., STEP)
  - Solar-induced curvature via interferometric angular measurements
  - Clocks in space to test Lorentz invariance/SME
Interplanetary Laser Link Milestone

- Farthest laser link established May 2005: 24 million km
- MESSENGER spacecraft en-route to Mercury
- Used laser altimeter as asynchronous transponder
- 20 cm range precision achieved
  - 10^(-11) precision
- path-breaking result important for interplanetary ranging, laser communications, navigation

Smith et al., 2006: *Science*, 311, 53
LATOR: the Laser Test of Relativity

3 lengths & 1 angle measured

Geometric redundancy allows for accurate measurement of relativistic gravitational light deflection to 1 part in $10^8$. 

$D_{R-Earth} \sim 2 \text{ AU} \approx 0.3 \cdot 10^9 \text{ km}$

$D_{R-T} \sim 0.03 \text{ AU} \approx 5 \cdot 10^6 \text{ km}$

$D_{T-Earth} \sim 2 \text{ AU} \approx 0.3 \cdot 10^9 \text{ km}$

$\theta \sim 1^\circ$

Expected accuracies:

$\Delta D_{R,T-Earth} \approx 15 \text{ cm}$

$\Delta D_{R-T} \approx 1 \text{ cm}$

$\Delta \theta \approx 0.1 \text{ picorad}$