Large Scale Structure (Galaxy Correlations)

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Overview

- Redshift Surveys
- Theoretical expectations
- Statistical measurements of LSS
- Observations of galaxy clustering
- Baryon Acoustic Oscillations (BAO)
- ISW effect

I will borrow heavily from two recent reviews:
2. Nichol et al. 2007 (see my webpage)
Random galaxy

\( Z = 0.182 \)

\[
\begin{align*}
  z &= \frac{\lambda_{\text{obs}}}{\lambda_{\text{emitted}}} - 1 \\
  \frac{cz}{H_0} &\approx d \\
  (\text{locally})
\end{align*}
\]
Redshift Surveys II

Now on an industrial scale

SDSS (DR6 ~800k galaxies)
Large Scale Structure (LSS)

Observational Definition

Structures larger than clusters, typically > 10Mpc
(larger than a galaxy could have moved in a Hubble time)
Only gain insight from the combination of LSS & CMB

$0.6 < h < 0.8$
We define a dimensionless overdensity as
\[ \delta(x) = \frac{\rho(x) - \bar{\rho}}{\bar{\rho}} \]
where \( \delta << 1 \) on large scales. Then, the autocorrelation function is
\[ \xi(x_1, x_2) = \langle \delta(x_1) \delta(x_2) \rangle \]
where \( <> \) denote the average over an ensemble of points or densities. Assuming homogeneity and isotropy, then
\[ \xi(x_1, x_2) = \xi(|x_1 - x_2|) = \xi(r) \]
In practical terms, the correlation function is given by
\[ P_{12} = n^2 (1 + \xi(r))dV^2 \]
The equivalent in Fourier space is the power spectrum

\[ P(k) = \frac{1}{(2\pi)^3} \langle \delta(k_1)\delta(k_2) \rangle = \delta_{\text{dirac}}(k_1 - k_2)P(k) \]

we have suppressed the volume term on the P(k) and therefore it has units of Mpc\(^3\)

The correlation function and power spectrum are related

\[ P(k) = \int \xi(r)e^{ik.r} d^3r \]
We do not consider inflation here. We just assume an adiabatic scale-invariant power spectrum of fluctuation created by quantum processes deep in the Inflationary era,

\[ P(k) = k^n \quad (n \approx 1) \]
After inflation, the evolution of density fluctuations in the matter depend on scale and the composition of that matter (CDM, baryons, neutrinos, etc.)

An important scale is the Jeans Length which is the scale of fluctuation where pressure support equals gravitational collapse,

$$\lambda_J = \frac{c_s}{\sqrt{G\rho}}$$

where $c_s$ is the sound speed of the matter, and $\rho$ is the density of matter.
The ratio of the time-integrated growth on a particular scale compared to scales much larger than the Jeans Length is known as the Transfer Function:

\[ T(k) \equiv \frac{\delta(k, z = 0)}{\delta(k, z = \infty)} \frac{\delta(0, z = \infty)}{\delta(0, z = 0)} , \quad (1) \]

where \( \delta(k,z) \) are density perturbations at \( k \) and \( z \). By definition, \( T(k) \) goes to 1 on large scales. Therefore,

\[ P(k) \approx \langle |\delta(k)|^2 \rangle \approx T(k)^2 \]

_Eisenstein & Hu, 1998, 496, 605
_Eisenstein & Hu, 1999, 511, 5_
For $\lambda > \lambda_J$, the fluctuations grow at the same rate independent of scale and therefore, the primordial power spectrum is maintained.

For $\lambda < \lambda_J$, the fluctuations cannot grow in the radiation era because of pressure support (or velocities for collisionless particles). In matter dominated era, pressure disappears and thus all scales can grow.

Therefore, in a Universe with CDM and radiation the Jeans length of the system grows to the particle horizon at matter-radiation equality and then falls to zero. Critical scale of $\lambda_{eq}$ or $k_{eq}$
Matter-radiation equality II

The redshift of equality is

\[ z_{eq} \approx 25000\Omega_m h \]

The particle horizon therefore is

\[ k_{eq} \approx 0.075\Omega_m h^2 \]

which is imprinted on the power spectrum as fluctuations smaller than this scale are suppressed in the radiation era compared to scales on larger scales.

The existence of dark energy and curvature today is insignificant. \( \Omega_m \) is the density of CDM and baryons.
P(k) models

Full boltzmann codes now exist or fitting formulae

Primordial P(k) on large scales

$T(k) \sim 1$

$k_{eq}$

[k [hMpc$^{-1}$]

P(k) [h$^{-3}$Mpc$^3$]

[Units of Volume]
Although they make-up a small fraction of the density (which is dominated by CDM), they dominate the pressure term. Baryons have two effects on the $T(k)$:

1. Suppression of the growth rates on scales smaller than the Jeans length

2. Baryon Acoustic Oscillations (BAO), which are sound waves in the primordial plasma
Initial fluctuation in DM. Sound wave driven out by intense pressure at 0.57c.

Courtesy of Martin White
After $10^5$ years, we reach recombination and photons stream away leaving the baryons behind
Baryon Acoustic Oscillations

Photons free stream, while baryons remain still as pressure is gone

Courtesy of Martin White
Photons almost fully uniform, baryons are attracted back by the central DM fluctuation.

Courtesy of Martin White
Today. Baryons and DM in equilibrium. The final configuration is the original peak at the center and an echo (the sound horizon) $\sim 150$Mpc in radius

Courtesy of Martin White
Baryon Acoustic Oscillations

Many superimposed waves. See them statistically

Sound horizon can be predicted once (physical) matter and baryon density known

\[ \approx \Omega_m h^2 \]
Lack of strong BAO means low baryon fraction.
The existence of massive neutrinos can also introduce a suppression of $T(k)$ on small scales relative to their Jeans length. Degenerate with the suppression caused by baryons.
We have discussed linear perturbations but on small scales we must consider non-linear effects. These can be modeled or numerically solved using N-body simulations etc.
Measuring $\xi(r)$ or $P(k)$

Simple estimator:
$$\xi(r) = \frac{DD(r)}{RR(r)} - 1$$

Advanced estimator:
$$\xi(r) = \frac{(DD-RR)^2}{RR-1}$$

*The latter does a better job with edge effects, which cause a bias to the mean density of points*

*Usually 10x as many random points over SAME area / volume*

Same techniques for $P(k)$ - take Fourier transform of density field relative to a random catalog over same volume. Several techniques for this - see Tegmark et al. and Pope et al. Also “weighted” and mark correlations
Measuring $\xi(r)$ II

Essential the random catalog looks like the real data!
$\xi(r)$ for LRGs

Comoving Separation ($h^{-1}$ Mpc)
Errors on $\xi(r)$

Hardest part of estimating these statistics

On small scales, the errors are Poisson

On large scales, errors correlated and typically larger than Poisson

• Use mocks catalogs
  • **PROS**: True measure of cosmic variance
  • **CONS**: Hard to include all observational effects and model clustering

• Use jack-knifes (JK)
  • **PROS**: Uses the data directly
  • **CONS**: Noisy and unstable matrices
Jack-knife Errors

Real Data

- Split data into \( N \) equal subregions
- Remove each subregion in turn and compute \( \xi(r) \)
- Measure variance between regions as function of scale

\[
\sigma^2 = \frac{(N-1)}{N} \sum_{i=1}^{N} (\xi_i - \bar{\xi})^2
\]

Note the \((N-1)\) factor because there are \( N-1 \) estimates of mean.
Latest P(k) from SDSS

\[ \Omega_m = 0.22 \pm 0.04 \]

\[ \Omega_m = 0.32 \pm 0.01 \]

\[ k_{eq} \approx 0.075 \Omega_m h^2 \]

No turn-over yet seen in data!

2\(\sigma\) difference

Depart from “linear” predictions at much lower k than expected

Strongest evidence yet for \(\Omega_m \ll 1\) and therefore, DE
There are two “hindrances” to the full interpretation of the $P(k)$ or $\xi(r)$

- *Redshift distortions*
- *Galaxy biasing*
Therefore we usually quote $\xi(s)$ as the “redshift-space” correlation function, and $\xi(r)$ as the “real-space” correlation function.

We can compute the 2D correlation function $\xi(\pi, r_p)$, then

$$w(r_p) = 2 \int_0^{\pi_{\text{max}}} \xi(r_p, \pi) d\pi$$

Redshift distortions
We only measure redshifts not distances

“Fingers of God”

Expected

Infall around clusters
Biasing
We see galaxies not dark matter

Maximal ignorance

$$\delta_{gal} = b \delta_{dm}$$

$$P(k)_{gal} = b^2 P(k)_{dm}$$

However, we know it depends on color & L

Is there also a scale dependence?

$$b/b_* = 0.85 + 0.15 \frac{L}{L_*} + 0.04(M_* - M_{0.1r})$$

$$b = 0.85 + 0.15 \frac{L}{L_*} + 0.04(M_* - M_{0.1r})$$
All galaxies reside in a DM halo

Elegant model to decompose intra-halo physics (biasing) from inter-halo physics (DM clustering)

\[ P(k) = P_{1\text{-halo}} + P_{2\text{-halo}} \]
\[ \langle N_{\text{cent}}(M) \rangle = \exp \left( -\frac{M_{\text{min}}}{M} \right) , \]
\[ \langle N_{\text{sat}}(M) \rangle = \exp \left( -\frac{M_{\text{min}}}{M} \right) \left( \frac{M}{M_1} \right)^\alpha , \]
\[ \langle N(M) \rangle = \langle N_{\text{cent}}(M) \rangle + \langle N_{\text{sat}}(M) \rangle . \]
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How will we solve these problems?

Higher-order statistics can help greatly