Direct Detection of WIMP Dark Matter: Part 1 - a primer

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SLAC Summer Institute
Aug 2, 2007
Standard Cosmology

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Colley, Turner & Tyson

from Perlmutter, Phys. Today

Tytler & Burles

Knop et al. (2003)
Spergel et al. (2003)
Allen et al. (2002)

(units of critical density)

vacuum energy density (cosmological constant)

mass density

Clusters
Supernovae
CMB
expands forever
recollapses eventually
open
closed
flat

No Big Bang

D  H  Ly-α

5555  5560
Wavelength (Å)

Normalized Flux
Non-Baryonic Dark Matter

- Matter density
  - $\Omega_{\text{Matter}} = 0.30 \pm 0.04$

- Big Bang Nucleosynthesis
  - $\Omega_{\text{Baryons}} = 0.05 \pm 0.005$

- Nature of dark matter
  - Non-baryonic
  - Large scale structure predicts DM is ‘cold’

- WIMPs – Weakly Interacting Massive Particle
  - ~10--1000 GeV Thermal relics
  - $T_{\text{FO}} \sim m/20$
  - $\sigma_A \sim$ electroweak scale

Production = Annihilation ($T \geq m_\chi$)

Production suppressed ($T < m_\chi$)

Freeze out: $H > \Gamma_A \sim n_\chi \langle \sigma_A v \rangle$

$\sim \exp(-m/T)$

Next week: Axions

Comoving Number Density

$N_{\text{EQ}}$

$m_\chi / T$ (time $\rightarrow$)
WIMPs in the Galactic Halo

The Milky Way

WIMP-Nucleus Scattering

Scatter from a Nucleus in a Terrestrial Particle Detector

\[ \langle E \rangle \sim 30 \text{ keV} \]
\[ \Gamma < 0.1/\text{kg/day} \]
SUSY Dark Matter: elastic scattering cross section

- The ‘standard’ progress plot
  - Direct-search experimental bounds
- Theory
  - Sample SUSY parameter space
  - Apply accelerator and model-specific particle physics constraints
  - Apply cosmological bound on relic density

⇒ Extract allowed region for WIMP-nucleon cross-section versus WIMP mass

Broad theoretical landscape: much of it testable with next and next-next generation DM searches and/or next and next-next generation accelerators
WIMPs in the Galactic Halo

- Exploit movements of Earth/Sun through WIMP halo
  - Direction of recoil -- most events should be opposite Earth/Sun direction (Spergel 1988)
  - Annual modulation -- harder spectrum when Earth travels with sun (Drukier, Freese, & Spergel 1986)

Erecoil

Log(rate)

WIMP-Nucleus Scattering
Scatter from a Nucleus in a Terrestrial Particle Detector

~2% seasonal effect

E_{recoil}

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Defining the Signal

- **Kinematics**
  - halo potential
  - WIMP mass
  - target mass & velocity

- **Rate**
  - halo density
  - cross section
    - SD/SI
    - coherence & form factors

- **Primary signal**

- **Secondary features**
  - annual modulation of rate
  - diurnal modulation of direction

- **Backgrounds**

- **Experimental methods & results**
  \{ mostly tomorrow \}
References and further reading

• References and notation - generally following the treatment of two key review articles:
  - J.D. Lewin and P.F. Smith, Astroparticle Physics 6 (1996)
  - G. Jungman, M. Kamionkowski and K. Griest, Physics Reports 267 (1996)

• A very nice pedagogic reference
    • 10th CDMS dissertation - first astrophysics results!
    • winner of first APS Tanaka dissertation prize!

• See also
Differential energy spectrum (simplified)

\[
\frac{dR}{dE_R} = \frac{R_0}{E_0 r} e^{-E_R/E_0 r}
\]

\[R = \text{event rate per unit mass}\]
\[E_R = \text{recoil energy}\]
\[R_0 = \text{total event rate}\]
\[E_0 = \text{most probable incident energy (Maxwellian)}\]
\[r = \frac{4M_W M_N}{(M_W + M_N)^2}\]
\[M_W = \text{mass of WIMP}\]
\[M_N = \text{mass of target nucleus}\]

\[\log \frac{dR}{dE_R} = \int_{\infty}^{0} \frac{dR}{dE_R} dE_R = R_0\]
\[\langle E_R \rangle = \int_{0}^{\infty} E_R \frac{dR}{dE_R} dE_R = E_0 r\]
WIMP search - c.1988

- Germanium ionization detector (UCSB/UCB/LBL)

from Jungman et al. (D.O. Caldwell et al.)
Typical numbers

For:

\[ M_W = M_N = 100 \text{ GeV}/c^2 \]
\[ \Rightarrow r = 1 \]

\[ \beta \sim 0.75 \times 10^{-3} = 220 \text{ km/s} \]

\[ \langle E_R \rangle = E_0 = \frac{1}{2} M_W \beta_0^2 c^2 \]
\[ = \frac{1}{2} 100 \frac{\text{GeV}}{c^2} (0.75 \times 10^{-3})^2 c^2 \]
\[ = 30 \text{ keV} \]
Refinements

\[ \frac{dR}{dE_R} \bigg|_{OBS} = R_0 \, S(E_R) \, F^2(E_R) \, I \]

\[ S(E_R) = \text{spectral function — masses and kinematics} \]
\[ F^2(E_R) = \text{form factor correction, with } E_R = \frac{q^2}{2M_W} \]
\[ I = \text{interaction type} \]
DM particle with velocity $v$ and incident KE $E_i = \frac{1}{2} M_W v^2$ scattered at angle $\theta$ in CM frame gives recoil energy in lab frame

$$E_R = E_i r \frac{(1 - \cos \theta)}{2}$$

where

$$r = \frac{4 m_r^2}{M_W M_N} = \frac{4 M_W M_N}{(M_W + M_N)^2}$$

and

$$m_r = \frac{M_W M_N}{M_W + M_N}$$

is the reduced mass
Isotropic scattering: uniform in $\cos \theta$
Incident WIMP with energy $E_i$ gives recoil energies uniformly in

$$E_R = 0 \rightarrow E_i r$$

Recall “familiar” case for equal masses ($r = 1$), target at rest, head-on collision

$$E_R = E_i$$

Overall spectrum? — sample incident spectrum
In each interval $E_i \rightarrow E_i + dE_i$
contribution to spectrum in $E_R \rightarrow E_R + dE_R$
at rate $dR(E_i)$ of

$$d \left( \frac{dR}{dE_R} (E_R) \right) = \frac{dR(E_i)}{E_i r}$$
Kinematics

Need to integrate over range of incident energies

\[ \frac{dR}{dE_R}(E_R) = \int_{E_{\text{min}}}^{E_{\text{max}}} \frac{dR(E_i)}{E_i r} \]

For \( E_{\text{max}} \) use \( \infty \) or \( v_{\text{esc}} \) (later...)

For \( E_{\text{min}} \), to get recoil of energy \( E_R \) need incident energy

\[ E_i \geq \frac{E_R}{r} \equiv E_{\text{min}} \]

and also need differential rate...
Differential rate

In a kilogram of detector of nuclear mass number $A$

$$dR = \frac{N_0}{A} \sigma v \, dn$$

where the differential density $dn$ is taken as a function $v$

$$dn = \frac{n_0}{k} f(\vec{v}, \vec{v}_E) \, d^3\vec{v}$$

with $n_0 = \rho_{DM}/M_W$ and normalization

$$k = \int f \, d^3\vec{v}$$
Coordinate system

Collision kinematics

\[ \vec{v} = \vec{v}_{W,E} = \text{WIMP velocity in the target/Earth frame} \]
\[ \vec{v}_{E,G} = \text{Earth velocity in the Galaxy frame} \]
\[ \vec{v}_{W,G} = \text{WIMP velocity in the Galaxy frame} \]

Galaxy dynamics

\[ \vec{v}_{W,G} = \vec{v}_{W,E} + \vec{v}_{E,G} \]
\[ = \vec{v} + \vec{v}_E \]

Maxwellian velocity distribution

\[ f(\vec{v}, \vec{v}_E) = e^{-\frac{(\vec{v}+\vec{v}_E)^2}{v_0^2}} \]
Differential rate

For simplified case of $v_E = 0$ and $v_{esc} = \infty$

$$dR = R_0 \frac{1}{2\pi v_0^4} v f(v, 0) \, d^3v$$

with $R_0 = 2\pi^{-\frac{1}{2}} \frac{N_0}{A} \frac{\rho_{DM}}{M_W} \sigma_0 v_0$

For Maxwellian $f(v, 0) = e^{-v^2/v_0^2}$, isotropic $d^3v \rightarrow 4\pi v^2 \, dv$, $E_i = \frac{1}{2} M_W v^2$ and $E_0 = \frac{1}{2} M_W v_0^2$:

$$\frac{dR}{dE_R}(E_R) = \int_{E_R}^{\infty} \frac{dR(E_i)}{E_i \, r} = \frac{R_0}{r(\frac{1}{2} M_W v_0^2)^2} \int_{v_{min}}^{\infty} e^{-v^2/v_0^2} \, v \, dv$$

$$= \frac{R_0}{E_0 \, r} e^{-E_R/E_0 \, r}$$

$v_{min} = \sqrt{2E_R/r M_W}$
Corrections: escape velocity

For finite $v_{esc}$

$$\frac{dR}{dE_R} = \frac{k_0}{k_1(v_{esc}, 0)} \frac{R_0}{E_0 r} \left( e^{-E_R/E_0 r} - e^{-v_{esc}^2/v_0^2} \right)$$

but $k_0/k_1 = 0.9965$ for $v_{esc} = 600$ km/s,

and for $M_W = M_N = 100$ GeV/c$^2$,

maximum $E_R = 200$ keV

$\Rightarrow$ cutoff energy $\gg \langle E_R \rangle = 30$ keV
Corrections: earth velocity

Clearly $\vec{v}_E \neq 0$ — but $\sim v_0 = 230 \text{ km/s}$. Full calculation yields:

$$\frac{dR(v_{esc}, v_E)}{dE_R} = \frac{k_0}{k_1} \frac{R_0}{E_0 r} \left( \frac{\sqrt{\pi} \frac{v_0}{4} v_E}{v_0} \right) \left[ \text{erf} \left( \frac{v_{min} + v_E}{v_0} \right) - \text{erf} \left( \frac{v_{min} - v_E}{v_0} \right) \right] - e^{-v_{esc}^2/v_0^2}$$

where $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$, $v_{min} = v_0 \sqrt{E_R/E_0 r}$, $k_0 = (\pi v_0^2)^{3/2}$ and

$$k_1 = k_0 \left[ \text{erf} \left( \frac{v_{esc}}{v_0} \right) - \frac{2}{\sqrt{\pi}} \frac{v_{esc}}{v_0} - e^{-v_{esc}^2/v_0^2} \right]$$

Fortunately, average value well approximated by numerical fit

$$\frac{dR(v_{esc} = \infty, v_E)}{dE_R} = c_1 \frac{R_0}{E_0 r} e^{-c_2 E_R/E_0 r}$$

$\sim 30\%$ increase in integrated rate $= c_1/c_2 = 0.751/0.561$ and harder spectrum
Signal modulations: annual effect

\[ v_E(t) \text{[km/s]} = 232 + 15 \cos \left( 2\pi \frac{t - 152.5}{365.25} \right) \]

\( t \) in days after January 1

~2% seasonal effect - need ~1000 events
Signal modulations: recoil direction

- Differential angular spectrum:

\[ \frac{d^2 R}{dE_R d(\cos \psi)} = \frac{1}{2} \frac{R_0}{E_0 r} e^{-\left(\frac{v_E \cos \psi - v_{min}}{v_0}\right)^2} \]

- Asymmetry → more recoils in forward direction by 5x: ~10 events
- Orientation of lab frame rotates relative to forward direction
  - eg, definition of forward/backward in lab frame changes as earth rotates
  - \( \perp \) versus \( \parallel \) reduces asymmetry to 20% effect: ~300 events
Refinements

\[ \frac{dR}{dE_R} \bigg|_{OBS} = R_0 \, S(E_R) \, F^2(E_R) \, I \]

✓ \( S(E_R) \) = spectral function — masses and kinematics
  time dependence

\( F^2(E_R) \) = form factor correction, with \( E_R = q^2/2M_W \)

\( I \) = interaction type

in zero-velocity limit (v<<c), scalar and axial vector
interactions dominate \( \rightarrow \) spin independent and
spin dependent couplings

these dominate
Nuclear form factor and Spin Ind. interactions

- Scattering amplitude: Born approximation
  \[ \vec{q} = \hbar (\vec{k}' - \vec{k}) \]
- Spin-independent scattering is coherent
  \[ \lambda = \hbar/q \sim \text{few fm} \]

\[
M(\vec{q}) = f_n A \int d^3x \rho(\vec{x}) e^{i \vec{q} \cdot \vec{x}} \Rightarrow \sigma \propto |M|^2 \propto A^2
\]

\[
F(qr_n) = \frac{3[\sin(qr_n) - qr_n \cos(qr_n)]}{(qr_n)^3} e^{-(qs)^2/2}
\]

- \( j_1 \) exact for ‘sharp’ density cutoff
  - \( r_n \) nuclear radius
  - \( s \) skin thickness parameter

swiped from Fraunfelder and Henley
Nuclear form factor and Spin Ind. interactions

- Loss of coherence as larger momentum transfer probes smaller scales
**SI cross section**

- Now have dependence on $q^2$ and nucleus $\rightarrow$ separate out fundamental WIMP-nucleon cross section

- Differential cross section can be written

\[
\frac{d\sigma_{WN}(q)}{dq^2} = \frac{\sigma_{0WN} F^2(q)}{4m_r^2 v^2}
\]

where $\sigma_{0WN}$ is total cross section for $F = 1$.

From Fermi’s Golden Rule

\[
\frac{d\sigma_{WN}(q)}{dq^2} = \frac{1}{\pi v^2} |M|^2 = \frac{1}{\pi v^2} f_n^2 A^2 F^2(q)
\]

- Can identify “unity-form-factor” cross sections:

\[
\sigma_{0WN} = \frac{4m_r^2}{\pi} f_n^2 A^2 = \frac{4}{\pi} m_n^2 f_n^2 \frac{m_r^2}{m_n^2} A^2
\]

\[
\sigma_{WN} = \text{all the particle physics, here}
\]
SI cross section and differential rate

- Putting this all together

\[
\frac{d\sigma_{WN}(q)}{dq^2} = \frac{1}{4m_n^2 v^2} \sigma_{WN} A^2 F^2(q)
\]

- Recall

\[
\frac{dR}{dE_R} = \int \frac{dR(E)}{E r} \quad \text{(where } dR(E) \text{ contained } \sigma)\]

- The \(E_r\) factor was from isotropic scattering - corresponds to the \(v^2\) in the differential cross section. Including now the FF:

\[
\frac{dR}{dE_R} = \frac{R_0}{E_0 r} e^{-E_R/E_0 r} F^2(q)
\]

\[
R_0 = 2 \sqrt{\pi} \frac{N_0}{A} n_0 \sigma_0 v_0 \quad \sigma_{WN} \frac{A^2}{m_n^2} \left( \frac{M_W M_N}{M_W + M_N} \right)^2
\]

- Particle physics
- Halo
- Detector
- Case Western Reserve University

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SI cross section and differential rate

\[ m_{\text{WIMP}} = 100 \text{ GeV}, \alpha_{W-N} = 1.0 \times 10^{-42} \text{ cm}^2 \]

Recoil Energy, \( E_r \) [keVr]

\begin{align*}
\text{(dash) Rate} &> \frac{E_r}{\text{[Kg/day]}} \\
\text{dN/dE}_r &< \text{[KeVr/kg/day]}\end{align*}

courtesy of R. Gaitskell
Nuclear form factor and Spin Dep. interactions

- Scattering amplitude dominated by unpaired nucleon
  - paired nucleons $\uparrow\downarrow$ tend to cancel -- couple to net spin $J$
    \[
    \frac{d\sigma}{dq^2} = \frac{8}{\pi v^2} \Lambda^2 G_F^2 J(J+1)F^2(q)
    \]

- Simplified model based on thin-shell valence nucleon
  \[
  F(qr_n) = j_0(qr_n) = \frac{\sin(qr_n)}{qr_n}
  \]

- Better: detailed nucleus specific calcs.
  - average over odd-group nucleons
  - use measured nuclear magnetic moment

\[131\text{Xe}\]
signal characteristics

- $A^2$ dependence
  - coherence loss
  - relative rates
- $M_W$ relative to $M_N$
  - large $M_W$ - lose mass sensitivity
  - if $\sim 100$ GeV
- Present limits on rate
- Following a detection (!), many cross checks possible
  - $A^2$ (or $J$, if SD coupling)
  - WIMP mass if not too heavy
    - different targets
    - accelerator measurements
  - galactic origin
    - annual
    - diurnal/directional - WIMP astronomy
Backgrounds: cosmic rays and natural radioactivity

WIMP scatters (< 1 evts /10 kg/ day) swamped by backgrounds ( > 10^6-7 evts/kg-d)

- Radioactive Nuclides in rock, surroundings: 238U, 232Th chains, 40K
- Cosmic Rays
  - Neutron capture
  - Muon capture
  - Photo fission
- Airborne Radioactivity: 222Rn
- Radioactive Nuclides in detector, shield
- Radioactive Nuclides in atmosphere
- Shield contaminants
  - Spontaneous fission
  - (α, n)

Electrons

Gammas

Neutrons

courtesy of S. Kamat
The Signal and Backgrounds

**Signal (WIMPs)**
- Nucleus Recoils
  - $v/c \approx 7 \times 10^{-4}$
  - $E_r \approx 10$'s KeV

**Background (gammas/betas)**
- Electron Recoils
  - $v/c \approx 0.3$

*Neutrons also interact with nuclei, but mean free path a few cm*
Nuclear-Recoil Discrimination

- Nuclear recoils vs. electron recoils
  - Division of energy
  - Timing
  - Stopping power

1% energy
- fastest
- no surface effects

Phonons/heat
- 100% energy
- slowest
- cryogenics

Ionization
- 10% energy

WIMP
- HPGe expts
- CDMS II, EDELWEISS I
- CDMS II, EDELWEISS II + timing
- Phantom II, EDELWEISS I
- Picasso, Simple, Coupp (superheated)

Light
- 1% energy
- fastest
- no surface effects

CRESST II
- HPGe expts
- ZEPLIN II/III/Max, XENON, LUX, WARP, ArDM
- DAMA/LIBRA
- KIMS
- ZEPLIN-I, DEAP, CLEAN, XMASS

ZEPLIN II/III/Max, XENON, LUX, WARP, ArDM
- PICASSO, SIMPLE, COUPP (superheated)
- HPGe expts
- ZEPLIN II/III/Max, XENON, LUX, WARP, ArDM
- DAMA/LIBRA
- KIMS
- ZEPLIN-I, DEAP, CLEAN, XMASS

ZEPLIN II/III/Max, XENON, LUX, WARP, ArDM
- PICASSO, SIMPLE, COUPP (superheated)
- HPGe expts
Next: WIMP search experiments