Cosmology for Particle Physicists: Anisotropies in the Cosmic Microwave Background

- Acoustic Oscillations
- Damping
- Inhomogeneities to Anisotropies
- Ramifications
Goal: Explain the Physics and Ramifications of this Plot

Bennett et al. 2006
Photon Distribution

- Distribution $\Theta$ depends on position $x$ (or wavenumber $k$), direction $n$ and time $t$ (or $\eta$).

- Moments

$$\Theta_{lm}(\vec{x}, \eta) = \int d^2n Y^*_l m(\hat{n}) \Theta(\vec{x}, \hat{n}, \eta)$$

$$\langle \Theta_{lm} \rangle = 0 \quad ; \quad \langle \Theta_{lm} \Theta_{l'm'} \rangle = \delta_{mm'} \delta_{ll'} C_l$$

- Monopole: $\Theta_0$

- Dipole: $\Theta_1$

- Quadrupole: $\Theta_2$

- You might think we care only about $\Theta$ at our position because we can’t measure it anywhere else, but …
We see photons today from last scattering surface at $z=1100$

$$\Theta(\vec{x} = 0, \hat{n}, \eta_0) \simeq [\Theta_0 + \Psi](D_* \hat{n}, \eta_*)$$
Pre-recombination: Acoustic Oscillations

- Pressure of radiation acts against clumping.
- If a region gets overdense, pressure acts to reduce the density: restoring force.

Oscillations:
- Slumping
- Pressure acts to reduce the density.
Before recombination, electrons and photons are tightly coupled; equations reduce to

Temperature perturbation

\[ \frac{\partial^2 \Theta_0}{\partial \eta^2} - c_s^2 \nabla^2 \Theta_0 = F \]

Very similar to …

Displacement of a string

\[ \frac{\partial^2 y}{\partial t^2} - c_s^2 \frac{\partial^2 y}{\partial x^2} = F \]
What spectrum is produced by a stringed instrument?

Middle C on a ukelele
Compare the piano spectrum to CMB spectrum

Bennett et al. 2006
CMB is different because ...

- Fourier Transform of spatial, not temporal, signal
- Time scale much longer (400,000 yrs vs. 1/260 sec)
- No finite length: all k allowed; ends not tied down?!
Why peaks and troughs?

- Vibrating String: Characteristic frequencies because ends are tied down

- Temperature in the Universe: Small scale modes begin oscillating earlier than large scale modes
Acoustic Oscillations

Solutions of the form:
\[ \Theta_0 \propto \cos(k r_s) \]

With the sound horizon defined as:
\[ r_s(\eta) = \int_0^\eta d\eta' c_s(\eta') \]

Expect a series of peaks at
\[ k_p = \frac{n \pi}{r_s} \]
Puzzle: Second Order Differential Eqn has Two Solutions

The perturbation corresponding to each wavevector can be either a cosine mode (zero initial velocity) or a sine mode (zero initial amplitude)

We implicitly assumed that each $\theta(k)$ started in the cosine mode.
It is worse than this: each wavenumber has an infinite number of wavevectors associated with it

Do all of these Fourier modes have cosine initial conditions?
If they do all start out with the same phase ...

First peak will be well-defined
As will first trough ...

And all subsequent peaks and troughs
If all modes are **not** synchronized though

First “Peak”

First “Trough”

We will NOT get series of peaks and troughs!
Inflation gives a beautiful explanation of synchronization

When modes leave the horizon, they cease to evolve; when they re-enter, only the **cosine** mode remains.
The spectrum at last scattering is:

\[ \Delta k = \pi / r_s(\eta_*) \]
Damping on small scales

\[ \lambda_D \sim \lambda_{\text{MFP}} N^{1/2} \]

But

\[ N \sim \frac{c}{\lambda_{\text{MFP}}} \times H^{-1} \]

So

\[ \lambda_D \sim \left( \frac{\lambda_{\text{MFP}}}{cH} \right)^{1/2} \]
On scales smaller than $\lambda_D$ (or $k > k_D$) perturbations are damped.
How do inhomogeneities at last scattering show up as anisotropies today?

Perturbation w/ wavelength $k^{-1}$ shows up as anisotropy on angular scale $\theta \sim k^{-1}/D_\ast \sim l^{-1}$
Quantitatively

$$\Theta(\hat{n}, \eta_0) \sim [\Theta_0 + \Psi] (D_\ast \hat{n}, \eta_*)$$

$$= \int \frac{d^3 k}{(2\pi)^3} e^{ik \cdot D_\ast \hat{n}} \left[ \tilde{\Theta}_0 + \tilde{\Psi} \right] (\vec{k}, \eta_*)$$

Use identity

$$e^{ik \cdot \vec{x}} = 4\pi \sum_{lm} i^l j_l (k \vec{x}) Y_{lm}(\hat{n}) Y_{lm}^*(\hat{k})$$

leading to

$$\Theta(\hat{n}, \eta_0) = 4\pi \sum_{lm} Y_{lm}(\hat{n}) \left\{ (-i)^l \int \frac{d^3 k}{(2\pi)^3} j_i (k D_\ast) \left[ \tilde{\Theta}_0 + \tilde{\Psi} \right] (\vec{k}, \eta_*) Y_{lm}^*(\hat{k}) \right\}$$
So the moments are

\[ \Theta_{lm} = 4\pi (-i)^l \int \frac{d^3 k}{(2\pi)^3} j_l(kD_*) \left[ \bar{\Theta}_0 + \bar{\Psi} \right] (\vec{k}, \eta_*) Y_{lm}(\vec{k}) \]

Each \( \Theta_{lm} \) is drawn from a Gaussian distribution with mean zero and variance

\[ 16\pi^2 \int \frac{d^3 k}{(2\pi)^3} Y^*_l(\hat{k}) \int \frac{d^3 k'}{(2\pi)^3} Y_{l'} m'(\hat{k'}) j_l(kD_*) j_{l'}(k'D_*) \left[ \bar{\Theta}_0 + \bar{\Psi} \right] (\vec{k}) \left[ \bar{\Theta}_0^* + \bar{\Psi}^* \right] (\vec{k}') \]

The expectation value sets \( k = k' \); then, by orthogonality of \( Y_{lm} \), the angular integral sets \( l = l' \) and \( m = m' \) leaving a variance

\[ C_l = 4\pi \int_0^\infty \frac{dk}{k} j_l^2(kD_*) \frac{k^3 P_{\Theta_0 + \Psi}(k, \eta_*)}{2\pi^2} \]

\( j_l^2 \) sharply peaked at \( kD^* \sim l \) with amplitude \( l^{-2} \)
Post Last Scattering: Integrated Sachs-Wolfe Effect

If potential wells decay, photons gain more energy falling in than they lose going out.
$C_l$ simply related to $[\Theta_0 + \Psi]^2(k = l/D^*)$
When we see this, we conclude that modes were synchronized during inflation!
We have good reason to believe we are working with the correct model

So let the fun begin: fit for cosmological parameters!
Parameter I: Curvature

- Same wavelength subtends smaller angle in an open universe
- Peaks appear on smaller scales in open universe
Second Chance?

As early as 1998, observations favored a flat universe.
By 2001, the case was closed

DASI, Boomerang, Maxima (2001)
The Universe is Flat!

Spergel et al. 2006
Reionization lowers the signal on small scales
A tilted primordial spectrum ($n<1$) increasingly reduces signal on small scales
Tensors reduce the scalar normalization, and thus the small scale signal
Parameters III

- Baryons accentuate odd/even peak disparity
- Less matter implies changing potentials, greater driving force, higher peak amplitudes
- Cosmological constant changes the distance to LSS
E.g.: Baryon density

\[ \ddot{\Theta} + \omega^2 \dot{\Theta} = F \]

Here, $F$ is forcing term due to gravity.

\[ \omega = k c_s = \frac{k}{\sqrt{3[1 + 3\rho_b/4\rho_\gamma]}} \]

As baryon density goes up, frequency goes down. Greater odd/even peak disparity.
Penultimate Slide

If you want to get your hands dirty check out ...

http://cosmologist.info/cosmomc/

Meet me at the bar tonight if you have a good idea about ...

Atomic physics of recombination
Fable

If you receive a letter in Rio from Sao Paulo on Monday, you know that it was sent on Sunday. Similarly if you receive a letter on Tuesday, you know it was sent from Sao Paulo on Sunday. Indeed, since mail carriers cannot traverse the full distance between the two cities on bicycle during the week, even if you receive a letter in Rio on Friday, you know that it must have been sent from Sao Paulo on Sunday.
Coherent picture of formation of structure in the universe

Quantum Mechanical Fluctuations during Inflation

\[ t = 10^{-34} \text{ sec} \]

Perturbation Growth: Pressure vs. Gravity

\[ t \sim 100,000 \text{ years} \]

Photons freestream: Inhomogeneities turn into anisotropies

\[ \Omega_{reion}, \Omega_{de}, W \]

Matter perturbations grow into non-linear structures observed today

\[ \Omega_m, \Omega_r, \Omega_b, f_v \]
What happens to photon perturbations when they re-enter the horizon?
To see how perturbations evolve, need to solve an infinite hierarchy of coupled differential equations.

Perturbations in metric induce photon, dark matter perturbations.