Black Holes: Edge of Infinity

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Outline

- Newton: Why he had to be wrong
- Einstein: Why he was right
  - Principles of relativity
  - Concept of space-time
- Black Holes:
  - What are they?
  - How do we find them?
  - Where do they come from?
  - What is their impact on evolution of Universe?
  - How do we use them to test Einstein’s GR?
- Black Holes not so black:
  - Actually brightest systems in universe
  - Ergosphere: drags space-time allowing energy out
  - Disks and Jets
  - Simulations and Movies
Problems with Newton’s Gravity

- Why inertial mass gravitates?
  - $F=ma$ vs. $F=GMm/r^2$

- Instantaneous force incompatible with special relativity

- Post-Newtonian required:
  - Perihelion shift of Mercury
  - Small effects add up secularly

- 1783: Mitchell: Escape velocity:

\[ \frac{mv^2}{2} = GMm/r \quad \rightarrow \quad v = \sqrt{2GM/r} \]
\[ v = c \quad \rightarrow \quad r = \frac{2GM}{c^2} \]
Black Hole Theorists

- 1916: Einstein: Equivalence, speed of light constant
- 1916: Schwarzschild: Exact static spherical solution
- 1931: Chandrasekhar: >1.4M☉ white dwarf -> neutron star
- 1935: Eddington: >>1.4M☉, then black hole must form
- 1963: Kerr: rotating black hole solution
- 1970’s: Wheeler: Better “PR” for black holes
- 1969: Penrose process: Spin energy extractable
- 1970: Bardeen: Last stable orbit and efficient emission
- 1973: Shakura & Sunyaev: Quasars powered by BHs
- 1974: Hawking: BH is a black body
- 1977+: Blandford: Jets powered by BH/Disks
- 80’s: Simulations of Disks
- 90’s: Simulations of BHs, novae, supernovae
- 00’s: Simulations of Formation of BHs, disks, jets
Einstein’s Gravity

- Freely-falling objects move straight in curved space-time
- Principles of Equivalence:
  - **Weak**: Test body motion independent of composition
  - **Einstein**: Non-gravitational experimental result same in any inertial frame
  - **Strong**: Applies to all experiments for entire Universe
  - Dimensionless numbers constant

- No known violations of relativity:
  - Perihelion **shift** of Mercury (+43arcsec/century)
  - Sun **bends** light 2X more than Newton (1.75arcsec)
  - Gravitational **redshift** (from equivalence principle)
  - Shapiro light **delay**: Mars by Sun (+250µs / 45mins)
  - Relativistic geodetic **precession** (7arcsec/year)
  - Gravitational wave decay of binary pulsars
  - Constancy of Newton’s G (lunar ranging): 1 in $10^{12}$/year
  - Gravitomagnetism (Venus-Sun: -0.0003 arcsec/century)
Metric: Space-time is a smoothly warping manifold

\[ d (x, y) = \sqrt{(x_1 - y_1)^2 + \ldots + (x_n - y_n)^2} \]

\[ ds^2 = g_{\alpha\beta} \, dx^\alpha \, dx^\beta \]

\[ = (dx^1)^2 + (dx^2)^2 + (dx^3)^2 \]

\[
(g)_{\alpha\beta} \equiv \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Minkowski metric/Light Cone:

\[
\eta = \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

Embedding Diagram: Mapping of full geometry

Parallel Transport:
- Describes motion of a vector on manifold.
- Many such transports can be used to build up metric
Black Holes

Wheeler’s No-Hair Theorem:
Mass: M, Spin: J, Charge: Q

Inner-most stable circular orbit (ISCO): Inside no circular orbits (3r_H for a=0, 1r_H for a=1)
Photon Sphere: Inside, objects cannot orbit at all, at (~3/2r_H for a=0)
Static Limit: Inside, objects cannot be static (varies from 1r_H to 2r_H for a=M)
Horizon or Schwarzschild radius: Inside r_H, objects must fall
Singularity: Near, physics breaks down (need quantum gravity), reached in finite time

Schwarzschild Metric (BH w/ only Mass):
\[ c^2 dt^2 = \left( 1 - \frac{r_s}{r} \right) c^2 dt^2 - \frac{dr^2}{1 - \frac{r_s}{r}} - r^2 \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right) \]
\[ g_{\mu\nu} = \begin{pmatrix} -\alpha^2 & 0 & 0 & 0 \\ 0 & \alpha^{-2} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2 \theta \end{pmatrix} \]
\[ \alpha = \left(1 - \frac{2M}{r}\right)^{1/2} \]
Horizon: \[ r_H = M \pm \sqrt{M^2 - a^2} \]
Static Limit: \[ r_S = M \pm \sqrt{M^2 - a^2 \cos^2 \theta} \]

Kerr Metric (BH w/ Spin):
\[ g_{\mu\nu} = \begin{pmatrix} -\alpha^2 + \omega^2 \omega^2 & 0 & 0 & -\omega \omega^2 \\ 0 & \rho^2 / \Delta & 0 & 0 \\ 0 & 0 & \rho^2 & 0 \\ -\omega \omega^2 & 0 & 0 & \omega^2 \end{pmatrix} \]
Horizon: \[ r_H = M \pm \sqrt{M^2 - a^2} \]
Static Limit: \[ r_S = M \pm \sqrt{M^2 - a^2 \cos^2 \theta} \]
Detecting Black Holes

- **Power and Efficiency**
  - Rule out fusion

- **Compactness:**
  - Rule out other extreme objects such as boson stars
  - Gravitational microlensing

- **Kepler’s Laws:**
  - Binary orbit with star
  - Many stellar orbits
  - Stellar velocity dispersion
  - Gas motion around BH
  - Maser emission

- **Empirical Relations**
  - Mass-Luminosity
  - Mass-Velocity

- **Gravitational Waves**
Gravitational Waves

- Generated by quadrupolar motion
- No direct detection yet!
- LIGO/advanced LIGO/LISA
Black Hole Collision Simulations

- **Purpose:**
  - Waveforms for LIGO/LISA
  - Kicks

- **Simulate BH/NSs**
  - Singularity/horizon evolution difficult

- **Simulate colliding BH-BH**
  - Slow progress in 90’s
  - Breakthroughs in 2006 full merger, ring down, waveforms

- **Kicks:**
  - 175km/s for $a=0$
  - 2500km/s for anti-aligned spins

Alcubierre (1999)

Pretorius (2005)

Black Hole Origins

- Death of massive star
- Collision of compact stars
- Single massive gaseous uniform collapse
- Many events (accretion or mergers)
- Primordially in early Universe
Supernovae and Gamma-Ray Bursts

- Supernova: Death of Massive Star to NS/BH
- 2/century in our galaxy
- BH Jet creates GRB lasting seconds seen about 2/day

NS-NS collision and GRB

NS-BH collision and GRB
Cosmology/Matter Evolution

- Friedmann-Robertson-Walker solution
  - Homogenous and Isotropic
  - Dark Energy is Cosmological Constant
  - $\Lambda$-CDM (cold dark matter)

\[
-c^2\,dt^2 = -c^2\,dt^2 + a(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta \,d\phi^2 \right)
\]

- First Stars and Quasars with black holes
  - Feedback from stars and black holes
  - Correlation between stellar velocity dispersion and black hole mass
  - Implies historical link or co-evolution
Quasars and Jets

• First Jet:
  • 1917: Heber Curtis: M87
• Quasars: Quasi-Stellar radio source
  • 1950s: Stars with odd lines
  • 1960s: 3C273: Optical Counterpart
    High redshift: $z=0.158\ 749\text{Mpc}$
• Problems:
  • Deep gravity or large distance?
  • Implied $>$ Fusion Efficiency
• BH with Accretion Disk Solution:
  • 1970s: Gravity to Radiation in Disks
  • 1980s: Unified models of diverse appearance of quasars
• 1990s: $>12,000$ observed quasars
• 2007: Highest redshift: $z=6.43\ 10\text{Gpc}$
Active Galactic Nuclei

- Million-Billion solar mass BHs
- Nearly every galaxy has BH
- BH surrounded by disk
- Stars can form in outer disk
- 10% are “active” with jets
- Jets observed mostly in radio
Sagittarius A*: In our Galaxy

- Center of galaxy
  - 2.6 million solar mass BH
  - Radio source
  - Accretion flow
- Near BH trace hot spots
- Back-lit BH
  - Silhouette: ~5M
  - 2.5X larger than horizon
- Used to measure:
  - mass, spin
  - Compare to Kerr space-time

Broderick
X-Ray Binaries

- Roughly 250 X-ray Binaries
- 3-25 Solar Mass BH
- Disk comes from companion star
- Disk emits in X-rays
- 25 BH X-ray Binaries
- 10 Produce Relativistic Jets
- 1 (GRS1915+105) may contain nearly maximally spinning BH
Black Hole Power (Energy/Time)

- 100 Watts: 1 Light Bulb
- $10^4$ Watts: Each US Citizen
- $10^9$ Watts (GigaWatt): Largest Power Station
- $10^{13}$ Watts: Human Total or 1 Hurricane
- $10^{15}$ Watts (PetaWatt): Most Powerful Laser
- $10^{24}$ Watts: Russian Tsar Bomba (50 Megatons)
- $10^{27}$ Watts: Sun ($10^{17}$ hits Earth)
- $10^{32}$ Watts ($10^8$ Tsar): X-Ray Binaries
- $10^{37}$ Watts: 1 Galaxy
- $10^{37}$ Watts: Active Galactic Nucleus
- $10^{45}$ Watts: Supernova
- $10^{45}$ Watts: Gamma-Ray Burst
- $10^{48}$ Watts: All stars in obs. universe
  - $(10^{11}$ galaxies/$10^{23}$ stars)
(Spinning) Black Holes Efficiency

Efficiency:
- Defined: \[\text{Usable Energy out} / \text{Energy in}\]
- Total free energy in rapidly spinning BH: 30% of its mass

Observed Efficiency of BHs:
- Quasars = old active galaxies with active galactic nucleus
- Quasars w/ BHs: ~20% efficient

Efficiency of Mechanisms to Extract Free Energy (E=mc²):
- **Antimatter**-matter: 100% of mass
- Nuclear **Fusion**: 0.07%
- **Gravitational Accretion Friction**: 5% (no spin) to 42% (max spin)
- Penrose **Particle Explosion** Process: up to 20%
- Blandford-Znajek **ElectroMagnetic** Process:
  - Typically 10% but can be >100%!
Penrose Process

Assumptions:
• Rotating BH
• Particle explodes into 2 parts inside ergosphere
• One particle goes backwards through rotating BH
• Second particle explodes off to infinity before reaching horizon

Find:
• One particle absorbs BH spin energy
• BH spins down
• 20% of particle mass-energy can be released
• No obvious astrophysical application
Assumptions:
• Rotating BH
• Electricity & Magnetism
• Hot Plasma in disk

Find:
• BH Emits Energy!
• 30% of total BH mass-energy releasable
• 10% of mass-energy can be released in Jet
• Jet power scales with BH mass
Energy Extraction

• Dimensional Analysis:

\[
\text{Power} \sim \left[B^2\right] \left[\frac{4\pi}{3} L^3\right] [T^{-1}]
\]

\[
\begin{align*}
L & \equiv GM/c^2 \\
T & \equiv L/c \\
j & \equiv J/M^2 \\
r_+ & \equiv L(1 + \sqrt{1 - j^2}) \\
\Omega_H & \equiv jc/2r_+
\end{align*}
\]

• Correct Answer:

\[
\text{Power} \sim \left[1\%\right] \left[j^2\right] \left[B^2\right] \left[\frac{4\pi}{3} L^3\right] [T^{-1}]
\]
General Relativistic Plasma Computer Simulations

Simulations:
- Rotating BH
- Hot Electromagnetic Plasma in Disk

Purpose:
- What is the structure of disks around black holes?
- How do jets form?
- Simulation show jets naturally collimated and accelerate to high Lorentz factors

\[ \gamma = \frac{1}{\sqrt{1 - (v/c)^2}} \]
Summary about Black Holes

- **Black Holes are more efficient than Fusion**
  - Spinning BH: up to 30% stored as spin energy
  - Gravity Accretion: 5 (0 spin) to 42% (max spin)
  - Magnetic field + BH: Typical: 10% , Maybe >100% during long periods

- **Creating and Colliding black holes:**
  - GRBs: NS-NS, NS-BH collisions and star death
  - Produce kicks up to 2500km/s
  - Gravitational waveforms for LIGO/LISA

- **Black Holes brightest objects in sky:**
  - Accretion converts Gravity to Radiation
  - BH threaded by magnetic field makes Jets/Outflows