Solar Neutrinos II: New Physics

- Helioseismology
- Matter-enhanced neutrino oscillations
- SNO and Super-Kamiokande

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Cl/Ga/Kamioka measured the three major solar ν fluxes and found they were difficult to reconcile with the SSM.

**Graph:**
- **Tc/Tc^SSM** vs. **\(\phi^S / \phi^S_{SSM}\)**
  - **pp**
  - **Be**
  - **B**
  - **Spp**
  - **OPA.**
  - **Z/X**
  - **Age**

- **90% C.L.**
- **95% C.L.**
- **99% C.L.**

**Legend:**
- Monte Carlo SSMs
- TC SSM
- Low Z
- Low Opacity
- WIMPs
- Large S11
- Dar-Shaviv Model
- Combined Fit

__Hata et al. (and Heeger and Robertson)___

Tuesday, August 3, 2010
• Under the assumption that the experiments were measuring EC line sources and continuous $\beta$-decay sources with allowed shapes

$$\phi(pp) \sim 9.9\phi_{SSM}(pp)$$

$$\phi(7\text{Be}) \sim 0$$

$$\phi(8\text{B}) \sim 0.43\phi_{SSM}(8\text{B})$$

• Steady-state models where the fluxes are largely controlled by the average temperature of the core cannot produce this pattern

$$\frac{\phi(8\text{B})}{\phi(pp)} \sim T^{23} \ll \frac{\phi_{SSM}(8\text{B})}{\phi_{SSM}(pp)} \Rightarrow T < T_{SSM} \text{ cooler Sun}$$

$$\frac{\phi(7\text{Be})}{\phi(8\text{B})} \sim T^{-12} \ll \frac{\phi_{SSM}(7\text{Be})}{\phi_{SSM}(8\text{B})} \Rightarrow T > T_{SSM} \text{ hotter Sun}$$

so the pattern is contradictory
In parallel, a second precise probe of the solar interior was being developed: helioseismology, the measurement and analysis of Doppler shifts of photospheric absorption lines. Amplitudes $\sim 30$ m and velocities $\sim 0.1$ m/s.

Turbulence within Sun’s convective zone acts as a random driver of sound waves propagating through the gas.

Specific frequencies are enhanced as standing waves -- normal modes whose frequencies depend on solar physics.

$n=14 \ l=20 \ m=16$ p-mode (acoustic)
How does this probe the SSM? For a spherical star

\[ p(r), \rho(r), T(r), s(r), \phi_{\text{gravity}}(r), \epsilon_{\text{nuclear energy}}(r) \]

Introduce adiabatic indices describing power-law behavior of \( p, T \) with \( \rho \)

\[ \Gamma_1 \equiv \left( \frac{\partial \log p}{\partial \log \rho} \right)_s \quad \Gamma_3 - 1 \equiv \left( \frac{\partial \log T}{\partial \log \rho} \right)_s \]

Define a total derivative

\[ \frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla} \]

Write down the equations for

**motion:** \[ \rho \frac{D\vec{v}}{Dt} = -\vec{\nabla} p - \rho \vec{\nabla} \phi \]

**continuity:** \[ \frac{D\rho}{Dt} + \rho \vec{\nabla} \cdot \vec{v} = 0 \]

**gravitational potential:** \[ \vec{\nabla}^2 \phi = 4\pi G \rho \]

**energy conservation:** \[ \frac{1}{p} \frac{Dp}{Dt} - \Gamma_1 \frac{1}{\rho} \frac{D\rho}{Dt} = \frac{\Gamma_3 - 1}{p} (\rho \epsilon - \vec{\nabla} \cdot \vec{F}) \]

(internal energy corrected for any work done due to volume change)

where \( \vec{F} \) is the energy flux. Static interior solution \( \Rightarrow \) SSM
• Now look for a variation around the SSM solution

$$\rho(\vec{r}, t) = \rho_0(r) + \rho'(\vec{r}, t)$$

where displacements are small, \( \delta(\vec{r}) \), \( v = \frac{\partial}{\partial t} \delta(\vec{r}) \)

• Plug into the stellar evolution equations (see Balantekin, WH nucl-th9903038)

\( \diamond \) try normal mode solution \( \rho'(\vec{r}, t) \sim \rho'(r) Y_{lm}(\theta, \phi) e^{-i\omega t} \)

\( \diamond \) introduce the adiabatic sound speed c, \( p = \frac{1}{\Gamma_1} \rho c^2 \)

\( \diamond \) and the field \( \Psi(r) = c^2 \sqrt{\rho} \vec{\nabla} \cdot \delta\vec{r} \)

and one finds that the equations reduce to a Schroedinger-like form
\[
\frac{d^2 \Psi(r)}{dr^2} + \frac{1}{c^2} \left[ \omega^2 - \omega_{co}^2 - \frac{l(l+1)c^2}{r^2} \left( 1 - \frac{N^2}{\omega^2} \right) \right] \Psi(r) \sim 0
\]

\(\omega_{\text{eff}} > 0\) propagating \quad \(\omega_{\text{eff}} < 0\) damped

an eigenvalue problem, governed by two frequencies

the bouyancy frequency

\[N(r) = \sqrt{\frac{Gm(r)}{r}} \left( \frac{1}{\Gamma_1} \frac{d \log p}{dr} - \frac{d \log \rho}{dr} \right)\]

vanishes in the convective zone, \(\sim\) constant in radiative zone

acoustic cutoff frequency

\[\omega_{co} = \frac{c}{2H} \sqrt{1 - 2 \frac{dH}{dr}}\]

where \(H^{-1} = -\frac{1}{\rho} \frac{d\rho}{dr}\)

propagating modes are those where one does not “see” a change in the density over a wavelength

p-modes: surface modes, \(N = 0, \quad \omega > \omega_{co}, \quad \omega > \frac{l(l+1)c^2}{r^2}\)

different modes propagate to different depths depending on \(l\)
the turning-point defined by

\[ \frac{\omega^2 - \omega_{co}^2}{l(l+1)} = \frac{c(r)^2}{r^2} \]

so large-l acoustic modes (p-modes) less penetrating

similar arguments for the gravity modes, those that propagate in the radiative zone, controlled by the buoyancy frequency (\( \omega < N \) guarantees propagation at sufficiently small \( r \)): difficult to see, because the surface is in the forbidden region
sound speed $c(r)$ derived from mode inversion, compared to SSM

Bahcall: agreement at 0.2% over 80% of Sun a more severe test than the $\nu$
The neutrino flux discrepancy -- the fact it was not compatible with any adjustment of $T$ in steady-state solar models -- combined with the SSM success in helioseismology made a “new-physics” solution more credible.

Another development that changed viewpoints was a theoretical step, the recognition that solar matter could enhance $\nu$ oscillations.
Vacuum flavor oscillations: mass and weak eigenstates

\begin{align*}
|\nu_e\rangle & \leftrightarrow |\nu_L\rangle \quad m_L \\
|\nu_\mu\rangle & \leftrightarrow |\nu_H\rangle \quad m_H
\end{align*}

Noncoincident bases ⇒ oscillations downstream:

\begin{align*}
|\nu_e\rangle &= \cos \theta |\nu_L\rangle + \sin \theta |\nu_H\rangle \\
|\nu_\mu\rangle &= -\sin \theta |\nu_L\rangle + \cos \theta |\nu_H\rangle
\end{align*}

\begin{align*}
|\nu_k^e(x = 0, t = 0)\rangle &= E^2 = k^2 + m_i^2 \\
|\nu_k^k(x \sim ct, t)\rangle &= e^{ikx} \left[ e^{-iE_L t} \cos \theta |\nu_L\rangle + e^{-iE_H t} \sin \theta |\nu_H\rangle \right] \\
|< \nu_\mu |\nu_k^k(t)\rangle|^2 &= \sin^2 2\theta \sin^2 \left( \frac{\delta m^2}{4E} t \right), \quad \delta m^2 = m_H^2 - m_L^2
\end{align*}

$\nu_\mu$ appearance downstream ⇔ vacuum oscillations

(some cheating here: wave packets)
Can slightly generalize this

\[ |\nu(0)\rangle \rightarrow a_e(0)|\nu_e\rangle + a_\mu(0)|\nu_\mu\rangle \]

yielding

\[
i \frac{d}{dx} \begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\delta m^2 \cos 2\theta & \delta m^2 \sin 2\theta \\ \delta m^2 \sin 2\theta & \delta m^2 \cos 2\theta \end{pmatrix} \begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix}
\]

vacuum \( m^2 \) matrix
solar matter generates a flavor asymmetry

- modifies forward scattering amplitude
- explicitly dependent on solar electron density
- makes the electron neutrino heavier at high density

\[ m_{\nu_e}^2 = 4E\sqrt{2}G_F \rho_e(x) \]
inserting this into mass matrix generates the 2-flavor MSW equation

\[
i \frac{d}{dx} \left( \begin{array}{c} a_e(x) \\ a_\mu(x) \end{array} \right) = \frac{1}{4E} \left( \begin{array}{cc} -\delta m^2 \cos 2\theta + 4E\sqrt{2}G_F \rho_e(x) & \delta m^2 \sin 2\theta \\ \delta m^2 \sin 2\theta & \delta m^2 \cos 2\theta \end{array} \right) \left( \begin{array}{c} a_e(x) \\ a_\mu(x) \end{array} \right)
\]

or equivalently

\[
i \frac{d}{dx} \left( \begin{array}{c} a_e(x) \\ a_\mu(x) \end{array} \right) = \frac{1}{4E} \left( \begin{array}{cc} -\delta m^2 \cos 2\theta + 2E\sqrt{2}G_F \rho_e(x) & \delta m^2 \sin 2\theta \\ \delta m^2 \sin 2\theta & -2E\sqrt{2}G_F \rho_e(x) + \delta m^2 \cos 2\theta \end{array} \right) \left( \begin{array}{c} a_e(x) \\ a_\mu(x) \end{array} \right)
\]

the \( m_\nu^2 \) matrix’s diagonal elements vanish at a critical density

\[
\rho_c : \quad \delta m^2 \cos 2\theta \equiv 2E\sqrt{2}G_F \rho_c
\]
Alternately this in terms of local mass eigenstates

\[ |\nu(x)\rangle = a_H(x)|\nu_H(x)\rangle + a_L(x)|\nu_L(x)\rangle \]

\[ i \frac{d}{dx} \begin{pmatrix} a_H(x) \\ a_L(x) \end{pmatrix} = \frac{1}{4E} \begin{bmatrix} m_H^2(x) & i\alpha(x) \\ -i\alpha(x) & m_L^2(x) \end{bmatrix} \begin{pmatrix} a_H(x) \\ a_L(x) \end{pmatrix} \]

observe:

- mass splittings small at \( \rho_c \): avoided level crossing

- \( \nu_H(x) \sim \nu_e \) at high density

- if vacuum \( \theta \) small, \( \nu_H(0) \sim \nu_{\mu} \) in vacuum

thus there is a local mixing angle \( \theta(x) \) that rotates from \( \sim \pi/2 \rightarrow \theta_v \) as \( \rho_e(x) \) goes from \( \infty \rightarrow 0 \)
\[ \frac{m_i^2}{2E} \]

\[ \theta(x) \sim \pi/2 \]

\[ |\nu_H\rangle \sim |\nu_e\rangle \]

\[ |\nu_L\rangle \sim |\nu_\mu\rangle \]

\[ |\nu_L\rangle \sim |\nu_e\rangle \]

\[ \rho \rightarrow \infty \]

\[ \rho(x_c) \]

\[ \rho \rightarrow 0 \]
• it must be that $\alpha(x) \sim \frac{d\rho}{dx}$

• if derivative gentle (change in density small over one local oscillation length) we can ignore: matrix then diagonal, easy to integrate

$$P_{\nu_e}^{adiabatic} = \frac{1}{2} + \frac{1}{2} \cos 2\theta_v \cos 2\theta_i \rightarrow 0 \text{ if } \theta_v \sim 0, \theta_i \sim \pi/2$$

• most adiabatic behavior is near the crossing point: small splitting
  ⇒ large local oscillation length ⇒ can “see” density gradient

• derivative at $\rho_c$ governs nonadiabatic behavior (Landau Zener)

$$P_{\nu_e}^{LZ} = \frac{1}{2} + \frac{1}{2} \cos 2\theta_v \cos 2\theta_i (1 - 2P_{hop})$$

so $\rightarrow 1$ if $\theta_v \sim 0, \theta_i \sim \pi/2, P_{hop} \sim 1$
\( \frac{\rho(r)}{\rho(0)} \)

\( r_c \)

\( \sin^2 2\theta = 0.005 \)

\( \delta m^2 / E = 10^{-6} \text{ eV}^2 / \text{MeV} \)

we can do this problem analytically
\[ P_{\text{hop}}^{\text{linear}} = e^{-\pi \gamma_c / 2} \]

\[ \gamma_c = \frac{\sin^2 2\theta \, \delta m^2}{\cos 2\theta \, 2E} \left| \frac{1}{\rho_c} \frac{d\rho}{dx} \right| \]

\( \Upsilon_c \gg 1 \Leftrightarrow \text{adiabatic, so strong flavor conversion} \)

\( \Upsilon_c << 1 \Leftrightarrow \text{nonadiabatic, little flavor conversion} \)

so two conditions for strong flavor conversion:

sufficient density to create a level crossing

adiabatic crossing of that critical density

**MSW mechanism** is about passing through a level crossing
Mathematica HW problem

a) vacuum oscillations $\theta=15^\circ$
R from -20 to +20

$$R \text{ in units of } \frac{4E \cos 2\theta}{\delta m^2 \sin^2 2\theta}$$

b) matter oscillations

add $\rho_e(R) \propto 1 - \frac{2}{\pi} \arctan aR$

normalize so that crossing occurs at $R = 0$

note $\rho_e(R) \to 0$ as $R \to \infty$

So $\nu_e$ is produced as a heavy eigenstate, then propagates toward the vacuum, where it is the light eigenstate
solving the solar neutrino problem

\[ \sin^2 2\theta_v \]

\[ \frac{\delta m^2}{E} \text{(eV}^2/\text{MeV}) \]

\[ \gamma \ll 1 \]

nonadiabatic

no level crossing

Flavor conversion here

\[ \text{pp} \]

\[ \text{Be} \]

\[ \text{B} \]
$\sin^2 2\theta_v$

$10^{-4}$ $10^{-2}$ $1$ $\delta m^2/E (eV^2/MeV)$

$10^{-8}$ $10^{-6}$

no level crossing

nonadiabatic

Low solution

$^7\text{Be}$

$^8\text{B}$

PP
\[ \sin^2 2\theta_v \]

\[ \frac{\delta m^2}{E} \text{ (eV}^2/\text{MeV)} \]

- no level crossing
- nonadiabatic

Small angle solution

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Tuesday, August 3, 2010
Large angle solution

this is the solution matching SNO and SuperK results + Ga/Cl/KII

\[ \tan^2 \theta_v \approx 0.40 \]
distinctive energy-dependent suppression

\[ P(E_{\nu}) \]

\[ \sin^2 2\theta = 0.006 \]

\[ \sin^2 2\theta = 0.6 \]

Borexino

Super-Kamiokande and SNO
Neutrino oscillations had been one of the early suggestions for solving the solar neutrino puzzle (Pontecorvo) -- but the apparent need for nearly complete mixing of three neutrino species to produce the needed factor-of-three reduction in the Cl counting rate seemed a stretch. The known quark mixing angles are small.

The MSW mechanism provided a means for suppressing the flux even if the mixing angle were small; and the energy-dependent reductions that the data seemed to demand.

By the mid-1980s planning was underway for two next-generation experiments to resolve the solar neutrino puzzle, Super-Kamiokande and SNO.
SK increase in fiducial volume (from 0.68 to 22 ktons) provided the potential to see spectrum distortions or day-night matter effects -- reconstructed from spectrum of scattered electrons in $\nu_e(\nu_x) + e \rightarrow \nu'_e(\nu'_x) + e'$
Preliminary

SK-III 289 days
Full Final sample
6.5 - 20 MeV, 22.5 kton
Signal: $3378.9^{+82.7}_{-81.1}$ stat. only

- Data
- Background
- Best fit ($^8$B MC + background)

\[
\phi(^8\text{B}) = \left[ 2.38 \pm 0.05(\text{stat})^{+0.16}_{-0.15}(\text{sys}) \right] \cdot 10^6 \text{ cm}^{-2} \text{s}^{-1}
\]

SKII

\[
\delta(\text{day/night}) = -6.3 \pm 4.2(\text{stat}) \pm 3.7(\text{sys})\%
\]

Ratio of SKII observed to SSM energy spectrum. Purple: 1σ level of energy-correlated systematic errors.
Low-energy turnup predicted is potentially a 10% effect, detectable with proper attention to energy-correlated systematic errors, and with a reduced threshold.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Energy response</th>
<th>Energy threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>SK-I</td>
<td>6p.e./MeV</td>
<td>5.0MeV</td>
</tr>
<tr>
<td>SK-II</td>
<td>3p.e./MeV</td>
<td>7.0MeV</td>
</tr>
<tr>
<td>SK-III</td>
<td>6p.e./MeV</td>
<td>5.0MeV → 4.5MeV</td>
</tr>
<tr>
<td>SK-IV</td>
<td>6p.e./MeV</td>
<td>4.0MeV</td>
</tr>
</tbody>
</table>
Sudbury Neutrino Observatory

• Suggested by Herb Chen in mid-1980s: replacement of the ordinary water in a Cerenkov detector with heavy water

• Provides three complementary detection channels

<table>
<thead>
<tr>
<th>reaction</th>
<th>detected</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) $\nu_x + e \rightarrow \nu'_x + e'$</td>
<td>scattered electron</td>
</tr>
<tr>
<td>2) $\nu_e + D \rightarrow p + p + e'$</td>
<td>produced electron</td>
</tr>
<tr>
<td>3) $\nu_x + D \rightarrow p + n + \nu'_x$</td>
<td>produced neutron</td>
</tr>
</tbody>
</table>

1) isolated from the forward-peaking of the scattering: energy shared among outgoing leptons -- sensitive to $\nu_e s$, reduced sensitive to $\nu_{\mu,\tau} s$

2) detected by the scattered electron, hard spectrum with $E_e \sim E_\nu - 1.44$ MeV, as GT strength concentrated near threshold; angular distribution $1 - 1/3 \cos \theta$; only sensitive to $\nu_e s$
3) detected by capture of the produce neutron: total cross section measured; sensitive equal to $\nu_s$ of any flavor
• Detection of electrons weakly correlated with direction, and especial of neutrons, placed exception requirements on background reduction
  ◊ cavity at exceptional depth of 2 kilometers to reduce muons
  ◊ construction under cleanroom conditions: tiny quantities of dust in 12-story cavity would have produced neutrons above the expected solar rate, 8/day

• Experiment proceeded in three phases, depending on the neutral current detection scheme
  ◊ capture on deuterium d(n,γ) producing a 6.25 MeV γ Phase I
  ◊ capture on 2 tons of dissolved salt: $^{35}\text{Cl}(n,\gamma)$ 8.6 MeV energy release Phase II
  ◊ capture in $^3\text{He}$ proportional counters Phase III

• Recent low-energy re-analysis of Phases I and II, reaching to electron kinetic energies of 3.5 MeV
Figure 2: Flux of $^8$B solar neutrinos is divided into $\nu_\mu/\nu_\tau$ and $\nu_e$ flavors by the SNO analysis. The diagonal bands show the total $^8$B flux as predicted by the SSM (dashed lines) and that measured with the NC reaction in SNO (solid band). The widths of these bands represent the $\pm 1\sigma$ errors. The bands intersect in a single region for $\phi(\nu_e)$ and $\phi(\nu_\mu/\nu_\tau)$, indicating that the combined flux results are consistent with neutrino flavor transformation assuming no distortion in the $^8$B neutrino energy spectrum.
• SNO thus definitively resolved the solar neutrino problem

• The detector is dismantled, making space for SNO+, but the analysis continues. The best-fit combined-analysis two-flavor parameters are

\[ \delta m_{12}^2 = 7.59^{+0.20}_{-0.21} \times 10^{-5} \text{ eV}^2 \]

\[ \theta_{12} = 34.06^{+1.16}_{-0.84} \text{ degrees} \]

• The SSM was found to be consistent with the measurements

\[ \phi(^{8}\text{B}) = \left( 5.046^{+0.169}_{-0.152} (\text{stat}) +^{0.107}_{-0.123} (\text{syst}) \right) \times 10^6 \text{ cm}^{-2}\text{s}^{-1} \]

\[ \text{BPS08(OP; GS)} \ 5.95 \times 10^6 \text{ cm}^{-2}\text{s}^{-1} \]

\[ \text{BPS08(OP; AGS)} \ 4.72 \times 10^6 \text{ cm}^{-2}\text{s}^{-1} \]

``I was called right after the[SNO] announcement was made by someone from the New York Times and asked how I felt. Without thinking I said `I feel like dancing I'm so happy.' ... It was like a person who had been sentenced for some heinous crime, and then a DNA test is made and it's found that he isn't guilty. That's exactly the way I felt.'''  

JNB