Constraining Dark Matter Through a Cosmic Index of Refraction

David C. Latimer

Department of Physics and Astronomy
University of Kentucky
Lexington, KY 40506

latimer@pa.uky.edu

In collaboration with Susan Gardner from the University of Kentucky.

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Indirect DM searches look for the products of DM annihilation: $\nu$, $e^+$, $\bar{p}$, $\gamma$. E.g., IceCube, Fermi, PAMELA, ATIC.

In many models, DM annihilation into photons is permitted (if only at the loop level). Such models will also result in light-DM scattering engendering the cosmos with a refractive index.

The real part of the refractive index will result in a time lag between simultaneously emitted pulses of light. The imaginary part of the index results in attenuation of a signal.

An advantage to this new method of DM detection is that effects are not sensitive to local fluctuations of DM density; instead, they rely upon the average density, measured by WMAP, over a very long baseline.
In the DM rest frame, the relationship between the refractive index and the forward Compton amplitude is locally

\[ n(\omega) = 1 + \frac{\rho}{4m_{dm}^2\omega^2} M_{\text{fwd}} \quad \text{in the limit} \quad |n - 1| \ll 1. \]

Here, \( \omega \) is the measured photon frequency, and \( \rho \approx 1.2 \times 10^{-6} \text{ GeV/cm}^3 \) the present day DM density. [ E. Komatsu et al. [WMAP Collab.], ApJ Suppl. 180(2009).]  

Note: For low mass (\( m_{dm} < 5 \text{ GeV} \)) DM candidates, the number density of DM particles can exceed that of normal matter.

Assuming Lorentz invariance, we may decompose the amplitude as follows

\[ M_{\text{fwd}} = f_1(\omega)\epsilon' \cdot \epsilon + i f_2(\omega) S \cdot \epsilon' \times \epsilon, \]

where \( S \) is the spin operator associated with the dark-matter particle and \( \epsilon (\epsilon') \) is the photon polarization in its initial (final) state. Only \( f_1 \) will affect the phase speed of the light in the medium.
Forward Compton amplitude

- From causality, analyticity, and unitarity, dispersion relations show that $f_1(\omega)$ is a real and even function of $\omega$ for photon energies below the inelastic threshold. Additionally, the coefficients of the $\mathcal{O}(\omega^{2k})$ terms are positive, $k \geq 1$.

  [Gell-Mann, Goldberger, and Thirring, PR 95 (1954); Goldberger, PR 97 (1955).]

- As the photon energy approaches zero, the leading order behavior of the forward amplitude is known exactly for particles of arbitrary spin regardless of their structure.

  [Low, PR 96 (1954); Gell-Mann and Goldberger, PR 96 (1954); Lapidus and Kuang-Chao, Sov. Phys. JETP 12 (1961); Brodsky and Primack, Ann. Phys. 52 (1969).]

- The fully coherent amplitude has the form

  $$\mathcal{M}_{\text{fwd}} = A_0 + A_2\omega^2 + A_4\omega^4 + \mathcal{O}(\omega^6).$$

  The low energy theorem sets $A_0 = -2e^2 e^2$, and the remaining coefficients $A_{2k}$ are positive.
The refractive index governing the phase speed is

\[ n(\omega) = 1 + \frac{\rho}{4m^2_{dm}} \left[ \frac{A_0}{\omega^2} + A_2 + A_4\omega^2 + O(\omega^4) \right]. \]

The group speed governs the propagation time \( v_g = d\omega/dk = (n + \omega(dn/d\omega))^{-1} \). The propagation time for a pulse to travel a distance \( L \) is

\[ t(\omega) = L \left( 1 + \frac{\rho}{4m^2_{dm}} \left( \frac{-A_0}{\omega^2} + A_2 + 3A_4\omega^2 + O(\omega^4) \right) \right). \]
At cosmological scales, the dispersive time lag for simultaneously emitted photons of energy $\omega_1$ and $\omega_2$ at redshift $z$ is

$$\Delta t(\omega_1, \omega_2, z) \approx \frac{\rho}{4m_{dm}^2} \left[-A_0 \left(\frac{1}{\omega_1^2} - \frac{1}{\omega_2^2}\right) K_0(z) + 3A_4(\omega_1^2 - \omega_2^2)K_4(z)\right].$$

where $K_0(z) = \int_0^z (1 + z')H(z')^{-1}dz'$ and $K_4(z) = \int_0^z (1 + z')^5H(z')^{-1}dz'$. We solve for the Hubble constant at redshift $z$ using the Friedman equation with

$$H(z) = H_0 \sqrt{(1 + z)^3 \Omega_M + \Omega_\Lambda}$$

and the present day value for the Hubble constant $H_0 = 70.5 \pm 1.3$ km/s/Mpc.


The DM density accrues a scale factor of $(1 + z)^3$, and the observed frequency requires a blueshift factor of $1 + z$ when looking into the past.
Searches for Lorentz violation


- Deviations from the normal dispersion relation for a photon can take the form
  \[ E^2 = p^2 \left( 1 + \xi_1 \frac{p}{M} + \xi_2 \frac{p^2}{M^2} + \cdots \right) \]
  [Jacob and Piran, JCAP 0801 (2008).]

- The factors of momenta \( p^k \) accrue factors \((1 + z)^k\) at cosmological scales. (Note: There is no density factor in this time lag formula.) [Jacob and Piran, JCAP 0801 (2008).]

- LV signals could be confused with (dark) matter interactions though the energy dependence of the LV terms can be different and the \( z \) dependence of the two time lags will be different.
What observations would allow one to determine the $A_k$ from the time lag?

$$\Delta t(\omega_1, \omega_2, z) \approx \frac{\rho}{4m^2_{dm}} \left[ -A_0 \left( \frac{1}{\omega_1^2} - \frac{1}{\omega_2^2} \right) K_0(z) + 3A_4(\omega_1^2 - \omega_2^2) K_4(z) \right].$$

- $A_4$ is fixed by the higher order terms in the DM polarizability. This energy dependence of this term goes as $\omega^2$ and is best assessed through observations of optical and gamma ray photons.

- $A_0$ is fixed by the DM electric charge. The energy dependence of this term goes as $\omega^{-2}$ so that low frequency observations provide the best limits. The radio afterglow of GRBs fix the size of this coefficient.
We plot the observed time lag of the radio afterglow of GRBs, and fit with the following equation allowing a frequency dependent time lag (and require $\tilde{A}_0 \geq 0$)

$$\frac{\tau}{1+z} = \tilde{A}_0 \frac{K(z)}{\nu^2} + \delta((1+z)\nu).$$

The observational limit on millicharged DM at 95% CL is $|\varepsilon|/m_{dm} < 1 \times 10^{-5}$ eV$^{-1}$.

The points correspond to frequencies (in GRB rest frame) of 4-12 GHz (▼, green), 12-30 GHz (■, maroon), 30-75 GHz (♦, blue).

Our limit on charged DM is comparable with limits on the existence of millicharges from terrestrial experiments: $|\varepsilon| < 3 - 4 \times 10^{-7}$ for $m \lesssim 0.05$ eV. [Ahlers et al., PRD 77 (2008).]
Some other limits on millicharges

- Stellar evolution provides stringent indirect limits on the existence of millicharges. [Davidson, Hannestad, and Raffelt, JHEP 05 (2000).]

- The most stringent limits on $U(1)$ charged dark matter come from elastic scattering constraints in conjunction with the formation of small scale structure in the universe. [Ackerman et al. arXiv:0810.5126; Feng et al. arXiv:0905.3039.]

- These constraints are indirect and can be evaded. Our limits should improve with further radio observations of GRBs (e.g., GASE, LOFAR).
We have developed a new method to detect DM through its modification of the propagation of light. In principle, time lags accrued by light passing through a DM medium should be distinguishable from Lorentz violation effects.

Improved constraints on the frequency dependence of the speed of light from distant GRBs lead to stronger limits on DM models.

From the time lags of radio afterglows associated with GRBs, we find a limit on the charge-to-mass ratio of DM at 95% CL of $|\varepsilon|/m_{dm} < 1 \times 10^{-5}$ eV$^{-1}$. This can further improve with observations.