Propagation of High Energy Neutrinos in a weakly magnetized environment

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The coherent forward scattering of low energy neutrinos in a medium give the matter potential

\[ V_{\nu_e} = \sqrt{2} G_F (N_e - N_\bar{e}) \]  for \[ q^2 \ll M_W^2 \]

This is derived by considering the contact interaction and assuming the momentum transfer smaller than the vector boson mass. For \[ q^2 \sim M_W^2 \] this is no more valid. In this case the energy limit is

\[ \nu_e + e^+ \rightarrow W^+ \hspace{1cm} \bar{\nu}_e + e^- \rightarrow W^- \]

\[ E_\nu \approx M_W^2 / 2 m_e \approx 10^7 \text{ GeV} \]

Take into account threshold effect

\[ W_{\mu\nu}(q) = \frac{-g_{\mu\nu}}{(q^2 - M_W^2 + i\Gamma_W M_W)} \]

\[ W_{\mu\nu}(q) = \frac{-g_{\mu\nu}}{(q^2 - M_W^2 + iq^2 \Gamma_W M_W)} \]
Using finite temperature field theory as a tool to calculate the neutrino self energy in a background medium in the presence of a magnetic field, we calculate the neutrino effective potential.

Dispersion relation of the neutrino is

\[ k_0^2 - K^2 = m^2_\nu + V_{\text{eff}} \]
Neutrino self-energy in the magnetized e+e- medium is given by

\[ \Sigma(k) = \frac{g^2}{2} \int \frac{d^4 p}{(2\pi)^4} R \gamma_\mu S_1(p) \gamma_\nu L W^{\mu\nu}(q) \]

\[ i S(p) = \int_0^\infty ds e^{\phi(p,s)} G(p,s) \]

Schwinger Propagator for fermion

Decomposition of Fermion Self-Energy

\[ \Sigma = R \tilde{\Sigma} L = a \gamma_\mu k^\mu + b \gamma_\mu u^\mu + c \gamma_\mu b^\mu \]

\[ u^\mu = (1,0), \quad b^\mu = (0, \hat{b}) \quad \tilde{B} = B \hat{b} \]
The dispersion relation of the neutrino in the background is given by

\[ k_0 - k \simeq b - c \cos \theta \]

\[ V_{\nu_e} = V = \left( 1 - \frac{1}{2} \frac{eB}{m_e T} \cos \theta \right) \sqrt{2} G_F \left( N_e f(s_w) - N_{\bar{e}} f(-s_w) \right) \]

Reduction due to B field, B\( \sim 0.01 m_e^2/e \), T=0.05 me, 10% reduction

\[ f(\pm s_w) = \frac{(1 \pm s_w)}{(1 \pm s_w)^2 + s_w^2 \gamma_w^2} \]

\[ s_w = 2 E_{\nu} m_e / M_W^2 \]

\[ \gamma_w = \Gamma_w / M_W \simeq 0.0266 \]
The discontinuity in the function $f(-sw)$ at $sW=1$ corresponds to W-boson production.

\[ \nu_e + e^- \rightarrow \nu_e + e^- \]
\[ \bar{\nu}_e + e^+ \rightarrow \bar{\nu}_e + e^+ \]

\[ \nu_e + e^+ \rightarrow W^+ \]
\[ \bar{\nu}_e + e^- \rightarrow W^- \]
Application to Magnetars

**SGR 1806-20**

27\(^{th}\) of October 2004 in our galaxy with ~ 15 Kpc distance

\[ E_\gamma \approx 3 \times 10^{46} \text{ erg} , \ 0.1 \text{ sec} \]

\[ B \sim 1.6 \times 10^{15} \text{ G} \]

Presence of baryons in the fireball are responsible for the production of high energy neutrinos through

\[ p \gamma \to \Delta \to n \pi^+ \to n \nu_\mu e^+ \nu_e \bar{\nu}_\mu \]

also pp interaction

Zhang.. 03
Neutrino oscillation in magnetar atmosphere

\[ \nu_e \leftrightarrow \nu_\mu, \tau \]

\[ P(t) = \frac{\Delta^2 \sin^2 \theta}{\omega^2} \sin^2 \left( \frac{\omega t}{2} \right) \]

\[ \omega = \sqrt{V - \Delta \cos 2\theta + \Delta^2 \sin^2 2\theta} \]

\[ \Delta = \frac{\Delta m^2}{2 E_\nu} \]

Resonance condition

\[ V = \Delta \cos 2\theta \]
At resonance, the length is

\[
\ell_{\text{res}} = \frac{2.5 \times 10^{10} \ E_{\nu,14} \ cm}{\Delta \hat{m}_\nu^2 \sin 2 \theta}, \quad E_{\nu,14} = 10^{14} \ eV
\]

On the surface of the magnetar, the photon number density is

\[
n_\gamma \approx 9.73 \times 10^{29} L_{47.5} \ r_{0,6}^{-2} \ cm^{-3} \quad T \sim 313 \ keV
\]
We have to estimate the number density of e+e-pair in the fireball.

To create an e+e- pair, the minimum energy of the photon should be

\[ h \nu \approx 2 m_e \]
By equating the $e^+e^-$ annihilation rate to the local expansion rate, the remaining pair is given by

$$N_\pm \approx 6.1 \times 10^{44} \ E_{46.5}^{3/4} \ r_{0.6}^{-1/2} \ t_{-1}^{1/4}$$

Pair temperature is

$$T \approx 18 \text{ KeV at a distance } r_\pm \approx 2.1 \times 10^9 \text{ cm}$$
Number density of photon at this point is

\[ n_\gamma \approx 1.8 \times 10^{26} \text{ cm}^{-3} \]

Number density of pair is

\[ N_e \approx 2.2 \times 10^{30} \left( \frac{T}{m_e} \right)^{3/2} e^{-m_e/T} \text{ cm}^{-3} \approx 6.8 \times 10^{15} \text{ cm}^{-3} \]

The radius is

\[ r_\pm \approx 2.1 \times 10^9 \text{ cm} \]
\[ \Delta \hat{m}^2 \cos 2 \theta \approx 1.7 \times 10^{-7} \]

Analysis is done by considering the large and small mixing.

**Large Mixing range** \[ 0.64 \leq \sin^2 2 \theta \leq 0.96 \]

We get
\[ 2.8 \times 10^{-7} \text{ eV}^2 \leq \Delta m^2 \leq 8.5 \times 10^{-7} \text{ eV}^2 \]
\[ 3.16 \times 10^{16} \text{ cm} \leq l_{\text{res}} \leq 1.1 \times 10^{17} \text{ cm} \]

**Small Mixing range** \[ 2 \times 10^{-3} \leq \sin^2 2 \theta \leq 7 \times 10^{-3} \]

We get
\[ \Delta m^2 \sim 10^{-7} \text{ eV}^2 \]
\[ l_{\text{res}} \sim 10^{18} \text{ cm} \]

The \( l_{\text{res}} \) obtained are much larger than \( r_{\text{(-+,-)}} \) and the Temp also will be very small at \( l_{\text{res}} \). So High Energy Neutrinos will never satisfy the resonance condition and no suppression of their flux due to matter and magnetic field effect.
But if the Neutrinos are of GeV energy or less, Temp ~ 50 keV or more
For large mixing we get:

\[ 10^{-3} \text{eV}^2 \leq \Delta m^2 \leq 3.1 \times 10^{-3} \text{eV}^2 \]

\[ (6.7 \times 10^7 \text{cm} \leq l_{res} \leq 3 \times 10^8 \text{cm}) < r_\pm \]

Resonant oscillation is possible if GeV neutrinos are produced deep inside the Magnetar atmosphere where Temperature is of Order 50 keV or so.
$E_\nu = 10^{14} \, eV$

$X = \log[\Delta m^2_\nu]$, \quad $Y = \log[\sin^2 2\theta]$
Summary:

We have studied the neutrino propagation in a medium, where we found:

- Resonant Oscillation of TeV-PeV neutrinos in the Magnetar Atmosphere is very much suppressed.
- GeV neutrinos resonant oscillation is possible.