Theory and Observation of Atomic Line Polarization

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- Electron impact excitation of line emission from atoms and ions may be polarized if the electron distribution function is anisotropic.
- How does this arise?
- How can it be observed?
- Where might this be interesting?

Impact Polarization – A Simple Case 1s-2p Excitation

<u>Impact Polarization</u> (direct excitation)

From Coulomb Born/Bethe approximation

$$f(\hat{k}_q, \hat{k}_q') = \frac{-8\pi}{\sqrt{k_q k_q'}} \sum_{\substack{lm \\ l'm'}} Y^*_{lm}(\hat{k}_q) Y_{l'm'}(\hat{k}_q') \, i^{l-l'} e^{i(\sigma_l + \sigma_{l'})} \sum_\lambda \langle l'm' l_a' m_a' | P_\lambda | l m l_a m_a \rangle D_\lambda(n'l_a' k_q'; n l k_q)$$

Form $\sigma = \frac{k_q'}{4\pi k_q} \iint |f(\hat{k}_q, \hat{k}_q')|^2 d\hat{k}_q' d\hat{k}_q$: integrals over spherical harmonics select only diagonal terms in $|\sum_{\substack{lm \\ l'm'}} \cdots|^2$

For impact excitation by particle beam, $Y^*_{lm}(\hat{k}_q) = Y^*_{lm}(0) = \sqrt{\frac{2l+1}{4\pi}}$

$$\Rightarrow \sigma = \frac{k_q'}{k_q} \int |f(\hat{k}_q, \hat{k}_q')|^2 d\hat{k}_q'$$

$$\Rightarrow \sigma_{l'} = \frac{16\pi}{k_q^2} \Big| \sum_l \sqrt{2l+1} \, i^l e^{i\sigma_l} \sum_\lambda \langle l'm' l_a' m_a' | P_\lambda | l 0 l_a m_a \rangle D_\lambda(l', l) \Big|^2$$

$$= \frac{16\pi}{k_q^2} \sum_{\substack{l l'' L L'' \\ M M''}} i^{(l-l'')} e^{i(\sigma_l - \sigma_{l''})} \sqrt{(2l+1)(2l''+1)} \langle l' l_a' L M | P_\lambda | l l_a L M \rangle \langle l' l_a' L'' M'' | P_\lambda | l'' l_a L'' M'' \rangle D_\lambda(l', l) D_\lambda(l', l'')$$
$$\times \langle l l_a L M | l_a m_a l 0 \rangle \langle l'm' l_a' m_a' | l' l_a' L M \rangle \langle l'' l_a L'' M'' | l_a m_a l'' 0 \rangle \langle l'm' l_a' m_a' | l' l_a' L'' M'' \rangle$$

$$= \frac{16\pi}{k_q^2} \sum_{\substack{l l'' L L'' \\ M M''}} i^{(l-l'')} e^{i(\sigma_l - \sigma_{l''})} \sqrt{(2l+1)(2l''+1)} \, (2L+1)(2L''+1) (-1)^{-L-L''} \begin{pmatrix} l & l_a & L \\ 0 & m_a & -M \end{pmatrix} \begin{pmatrix} l' & l_a' & L \\ m' & m_a' & -M \end{pmatrix} \begin{pmatrix} l'' & l_a & L'' \\ 0 & m_a & -M'' \end{pmatrix} \begin{pmatrix} l' & l_a' & L'' \\ m' & m_a' & -M'' \end{pmatrix}$$
$$\times \begin{Bmatrix} L & l_a' & l' \\ \lambda & l & l_a \end{Bmatrix} \begin{Bmatrix} L'' & l_a' & l' \\ \lambda & l'' & l_a \end{Bmatrix} \begin{pmatrix} l & \lambda & l' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l'' & \lambda & l' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_a & \lambda & l_a' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_a & \lambda & l_a' \\ 0 & 0 & 0 \end{pmatrix}$$
$$\times (2l'+1)(2l_a'+1)(2l_a+1) \sqrt{(2l+1)(2l''+1)} \, D_\lambda(l', l) D_\lambda(l', l'')$$

Simplifications: $l_a = 0$, $m_a = 0 \Rightarrow L = l$, $L'' = l''$, $M = M'' = 0$, $m' = -m_a'$

$$\Rightarrow \sigma_{l'} = \frac{16\pi}{k_q^2} \sum_{l l''} i^{(l-l'')} e^{i(\sigma_l - \sigma_{l''})} (2l+1)^2 (2l''+1)^2 (2l'+1)(2l_a'+1) \begin{pmatrix} l & 0 & l \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l' & l_a' & l \\ -m_a' & m_a' & 0 \end{pmatrix} \begin{pmatrix} l'' & 0 & l'' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l' & l_a' & l'' \\ -m_a' & m_a' & 0 \end{pmatrix}$$
$$\times \begin{Bmatrix} l & l_a' & l' \\ \lambda & l & 0 \end{Bmatrix} \begin{Bmatrix} l'' & l_a' & l' \\ \lambda & l'' & 0 \end{Bmatrix} \begin{pmatrix} l & \lambda & l' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l'' & \lambda & l' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & \lambda & l_a' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & \lambda & l_a' \\ 0 & 0 & 0 \end{pmatrix} D_\lambda(l', l) D_\lambda(l', l'')$$

now $l_a' = \lambda$, hence $\begin{pmatrix} 0 & \lambda & l_a' \\ 0 & 0 & 0 \end{pmatrix} = \frac{(-1)^{l_a'}}{\sqrt{2l_a'+1}}$, $\begin{Bmatrix} l & l_a' & l' \\ \lambda & l & 0 \end{Bmatrix} = (-1)^{(l+l_a'+l')} \frac{1}{\sqrt{(2l+1)(2l_a'+1)}}$

$$\Rightarrow \sigma_{l'} = \frac{16\pi}{k_q^2} \sum_{l l''} i^{(l-l'')} e^{i(\sigma_l - \sigma_{l''})} \frac{(2l+1)(2l''+1)(2l'+1)}{(2l_a'+1)} \begin{pmatrix} l & l' & l_a' \\ 0 & -m_a' & m_a' \end{pmatrix} \begin{pmatrix} l'' & l' & l_a' \\ 0 & -m_a' & m_a' \end{pmatrix}$$
$$\times \begin{pmatrix} l & l_a' & l' \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l'' & l_a' & l' \\ 0 & 0 & 0 \end{pmatrix} D_\lambda(l', l) D_\lambda(l', l'')$$

Further simplification, put $l_a' = 1$ $\quad \begin{pmatrix} l' & l & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad (-1)^{l-1} \sqrt{\frac{\max(l', l)}{(2l'+1)(2l+1)}}$

$$\Rightarrow \sigma_{l'} = \frac{16\pi}{3k_q^2} \sum_{l l''} i^{(l-l'')} e^{i(\sigma_l - \sigma_{l''})} \sqrt{\max(l', l) \max(l', l'') (2l+1)(2l''+1)} \begin{pmatrix} l & l' & 1 \\ 0 & -m_a' & m_a' \end{pmatrix} \begin{pmatrix} l'' & l' & 1 \\ 0 & -m_a' & m_a' \end{pmatrix} D_\lambda(l', l) D_\lambda(l', l'')$$

for $m_a' = 0$ $\quad \sigma_{l'} = \frac{16\pi}{3k_q^2} \sum_{l l''} i^{(l-l'')} e^{i(\sigma_l - \sigma_{l''})} \sqrt{\max(l', l) \max(l', l'') (2l+1)(2l''+1)} \sqrt{\frac{\max(l, l')}{(2l+1)(2l'+1)}} \sqrt{\frac{\max(l'', l')}{(2l''+1)(2l'+1)}} D_\lambda(l', l) D_\lambda(l', l'')$

$$= \frac{16\pi}{3k_q^2} \frac{1}{2l'+1} \Big\{ (l'+1)^2 D_1^2(l', l'+1) + l'^2 D_1^2(l', l'-1) - D_1(l', l'+1) D_1(l', l'-1) \big[\underbrace{e^{i(\sigma_{l'-1} - \sigma_{l'+1})} + e^{i(\sigma_{l'+1} - \sigma_{l'-1})}}_{2\cos(\sigma_{l'-1} - \sigma_{l'+1})} \big] l'(l'+1) \Big\}$$

as $l' \to \infty$ $\quad \sigma_{l'}(m_a' = 0) = \frac{16\pi}{3k_q^2} D^2 \frac{1}{2l'+1}$

for $m_a' = 1$ $\quad \sigma_{l'} = \frac{16\pi}{3k_q^2} \Big\{ \frac{l'(l'+1)}{2(2l'+3)(2l'+2)(2l'+1)} D_1^2(l', l'+1) \cdot (l'+1)\cancel{(2l'+3)} + \frac{l'(l'+1)}{(2l'+1)(2l')(2l'-1)} D_1^2(l', l'-1) \cdot l' \cancel{(2l'-1)}$

$$+ \frac{l'(l'+1)}{[(2l'+3)(2l'+2)(2l')(2l'-1)]^{1/2}} \frac{1}{2l'+1} D_1(l', l'+1) D_1(l', l'-1) \big[\underbrace{e^{i(\sigma_{l'-1} - \sigma_{l'+1})} + e^{i(\sigma_{l'+1} - \sigma_{l'-1})}}_{2\cos(\sigma_{l'-1} - \sigma_{l'+1})} \big]$$
$$\times \sqrt{l'(l'+1)(2l'-1)(2l'+3)} \Big\}$$

(remember phases of 3j symbols)

$$\simeq \frac{16\pi}{3k_q^2} \times l'(l'+1) \Big\{ \frac{D_1^2(l', l'+1)}{2(2l'+1)} + \frac{D_1^2(l', l'-1)}{2(2l'+1)} + \frac{2}{2(2l'+1)} D_1(l', l'+1) D_1(l', l'-1) \cos(\sigma_{l'-1} - \sigma_{l'+1}) \Big\}$$

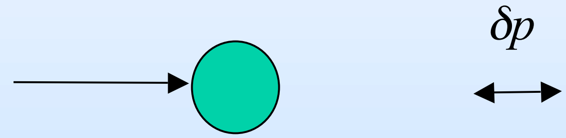
$\Rightarrow \sigma_{l'}(m_a' = 1) \simeq \frac{16\pi}{3k_q^2} D^2 \frac{2l'(l'+1)}{(2l'+1)}$ as $l' \to \infty$

Check $\quad \sigma_{l'}(m_a' = 0) + 2\sigma_{l'}(m_a' = 1) = \frac{16\pi}{3k_q^2} \frac{D^2}{2l'+1} [1 + 4l'(l'+1)] = \frac{16\pi}{3k_q^2} D^2 (2l'+1)$

as $l' \to 0$, $\quad \sigma(m_a' = 0) = \frac{16\pi}{5k_q^2} D_1^2(0, 1)$, $\quad \sigma(m_a' = 1) = 0$

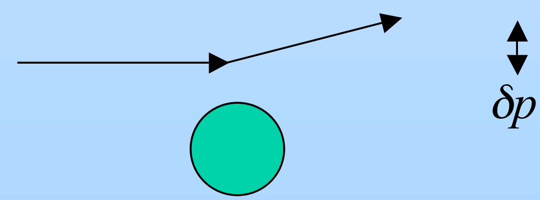
Close to threshold

low l, $\delta m_l = 0$, pol>0



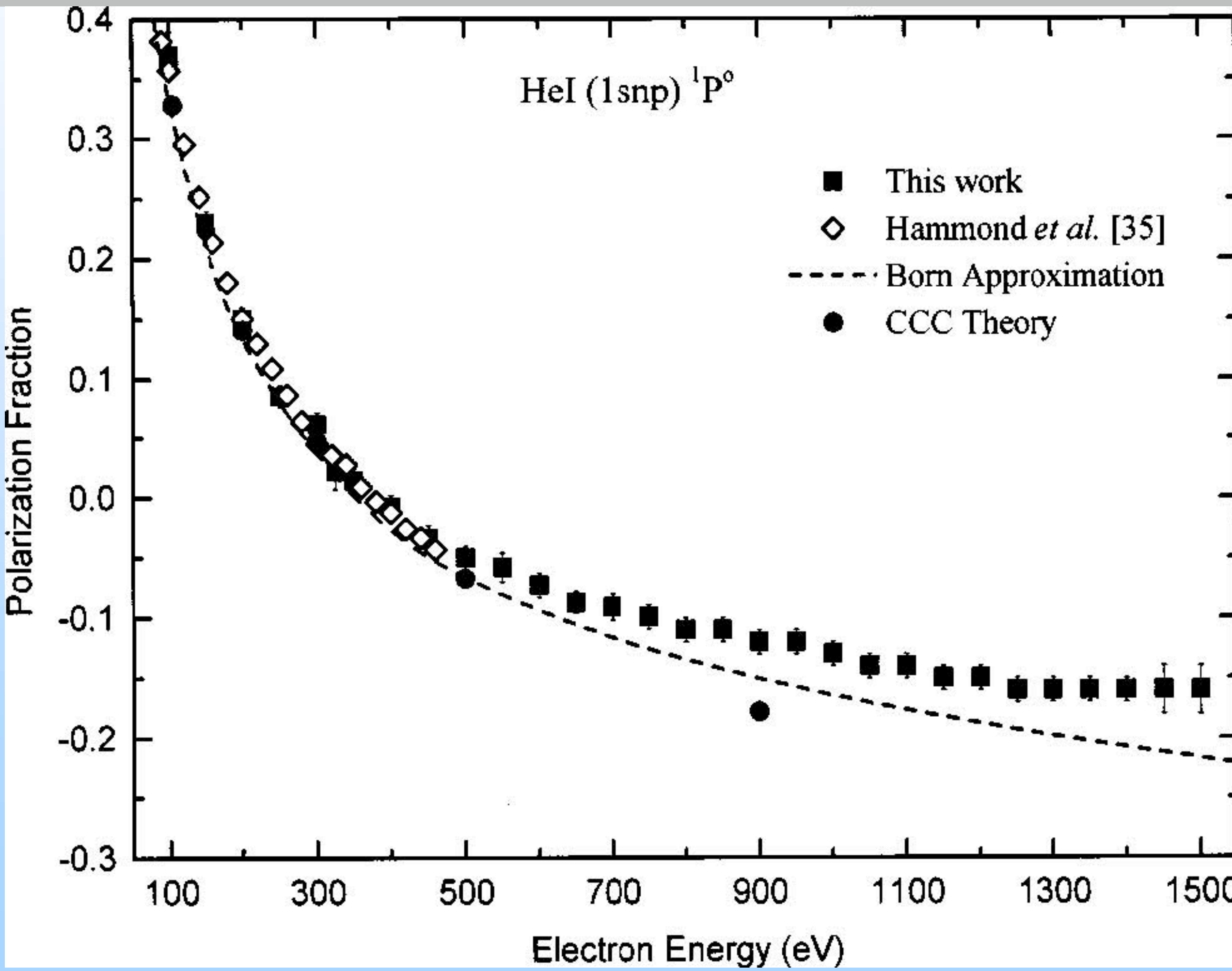
Well above threshold

high l, $\delta m_l = +/-1$, pol<0



Variation of Polarization with Energy

(for He I from Merabet et al. 1999, Phys. Rev. A 60, 1187)



pol = $(I_{||}-I_{+})/(I_{||}+I_{+})$

δm_l=0 gives $I_{||}$
δm_l=+/-1 gives I_{+}

threshold pol = +1 in this case

fine structure and varying ang. mom. alter behavior

Depolarization by Resonances;

capture electron into doubly excited state, then autoionize
(Fano & Macek, 1973, Rev. Mod. Phys. 45, 553, Ba II)
(Takacs et al. 1996, PRA, 54, 1342, Ne-like Ba XLVII)

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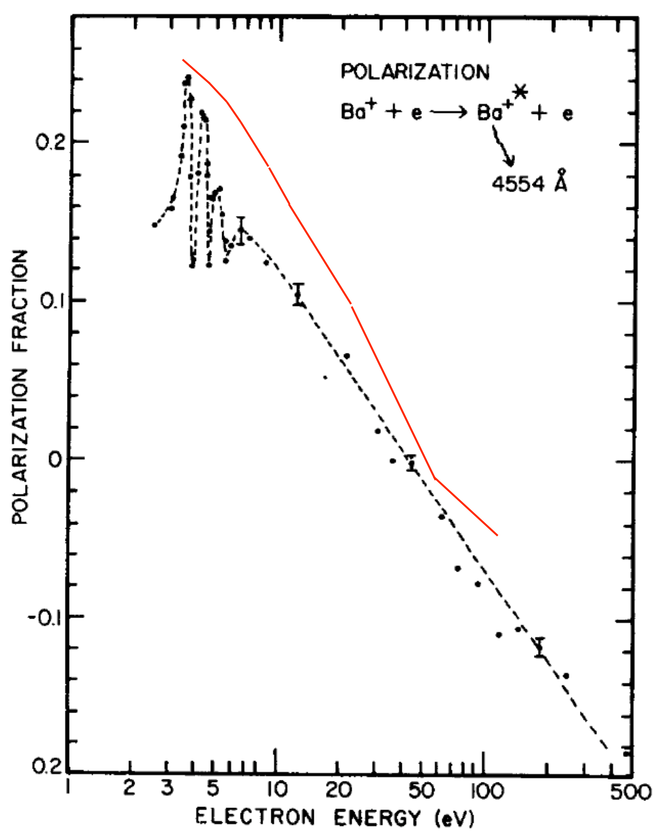
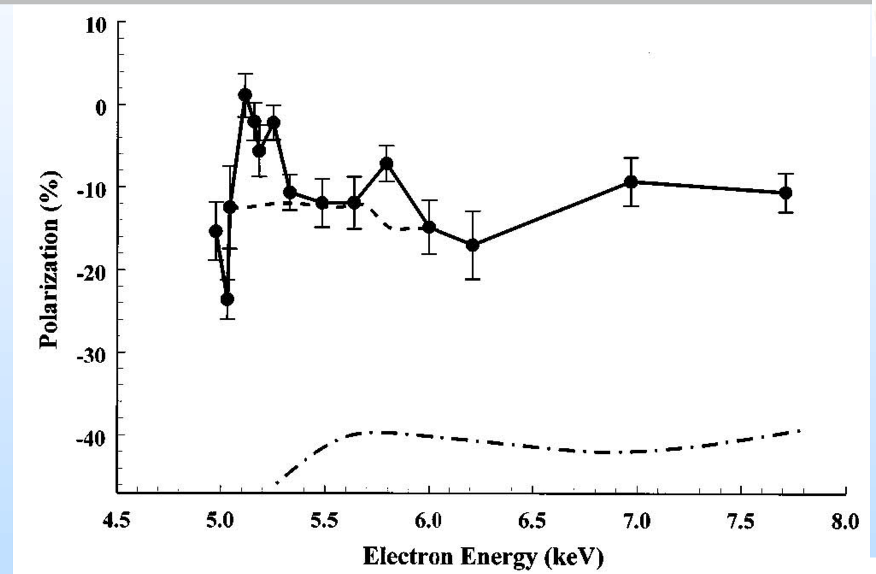
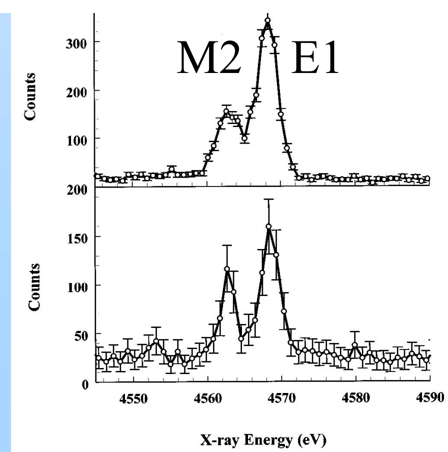


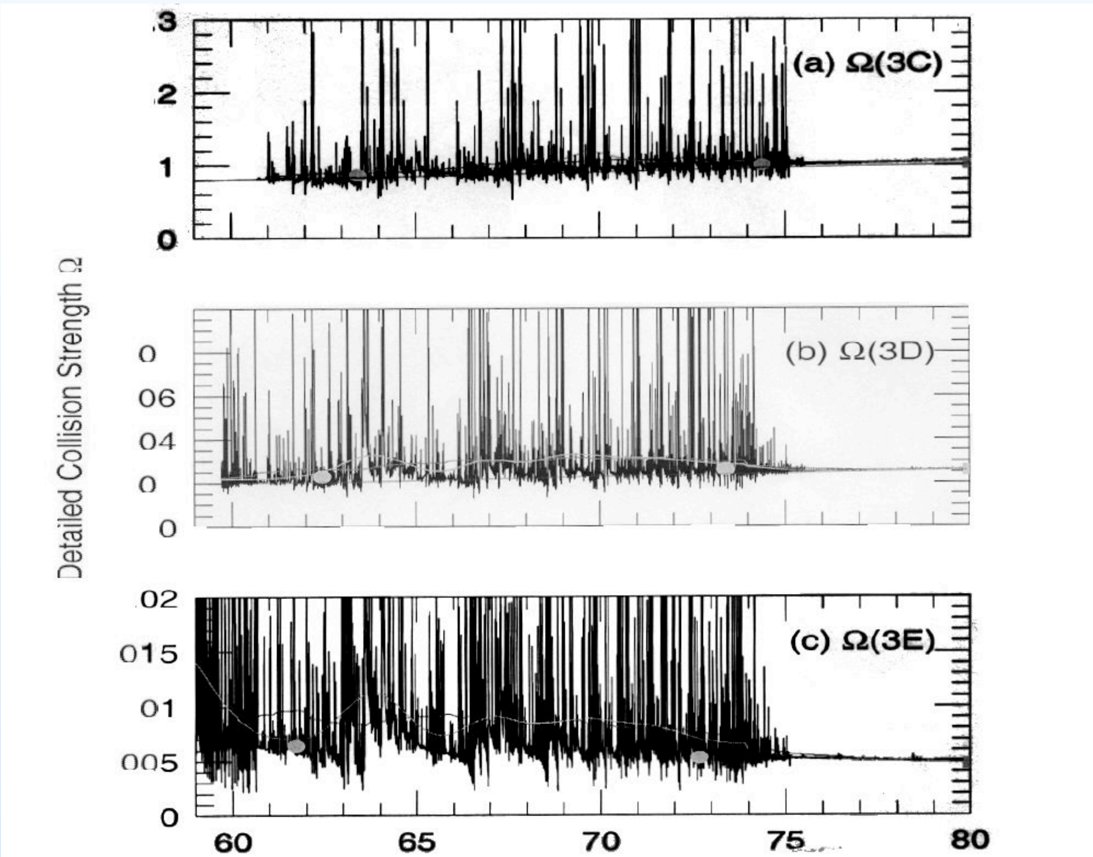
FIG. 4. Polarization of the 4554-Å Ba^+ resonance line ($^2P_{3/2}{}^\circ \rightarrow {}^2S_{1/2}$) as a function of the energy of exciting electrons (CTD73). Note the clearly resolved resonances and the smooth variation and sign reversal of the polarization on the logarithmic scale.



M2 line of Ne-like Ba, excited mainly by radiative cascade



What Will Happen Here?



Electron Energy (Rydbergs)

Resonance structures in Fe XVII by Chen and Pradhan 2002, Phys. Rev. Lett. 89, 013202

Best option for observations likely to be strong lines with few resonances

Efficiency of 6000 gr/mm Off-Plane Gratings (Goray 2003): Application to Con-X?

Gold coated, triangular grooves. Optimized for 15 Å incident wavelength:

Left: 7.65° facet angle, 7.65° polar angle, 88° azimuthal angle.

Right: 10.3° facet angle, 10.3° polar angle, 88.5° azimuthal angle.

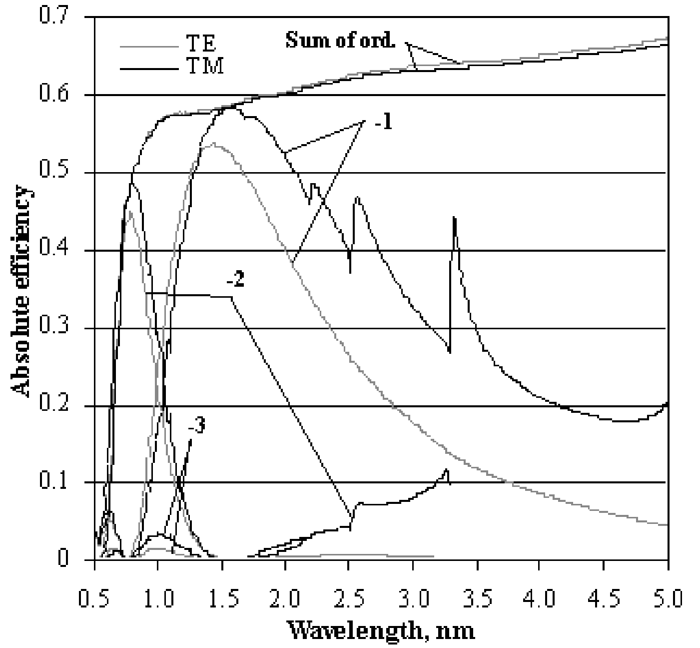


Fig. 14. Absolute efficiency in the –1, –2, and –3 orders of a 6000-gr/mm triangular grating with 7.65° working facet angle and 5-Å rms roughness calculated as a function of wavelength for the 7.65° polar and 88° azimuth incidence angles and the off-plane mounting.

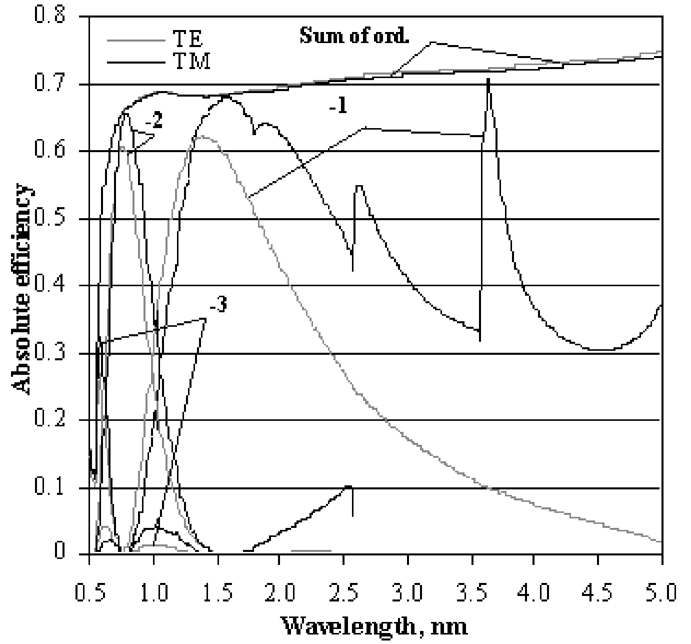
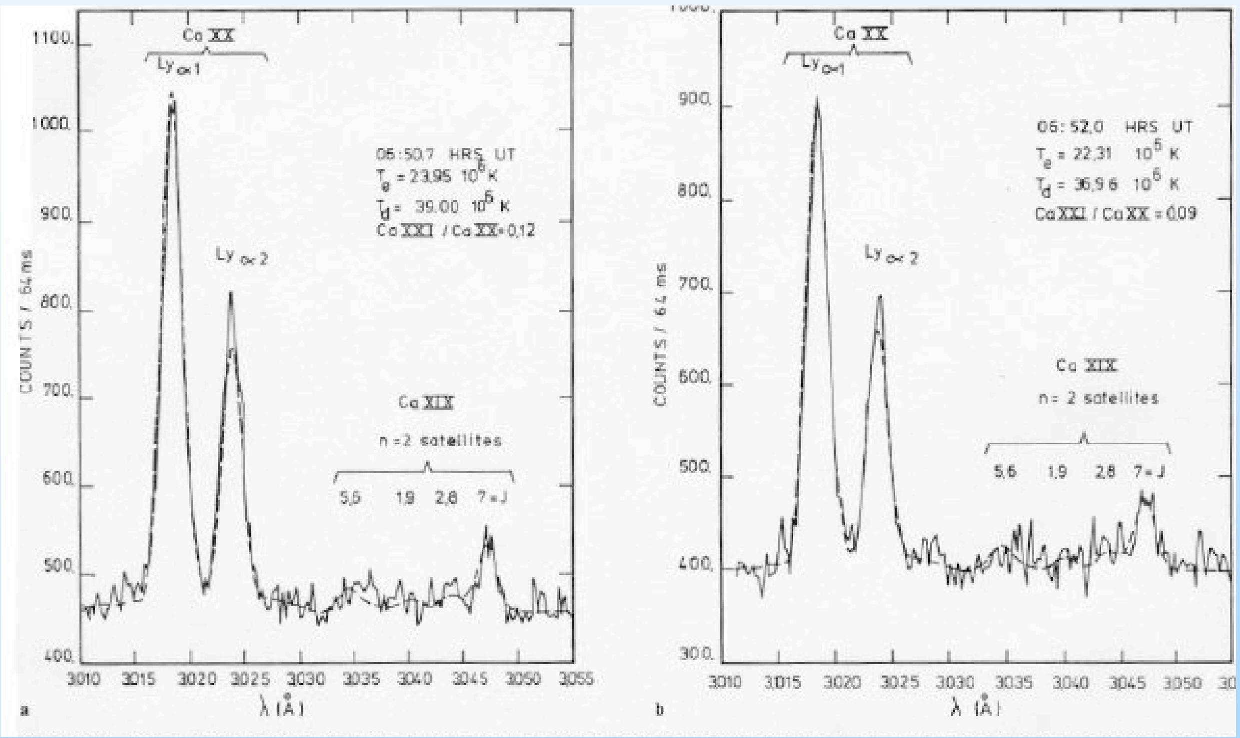


Fig. 15. Absolute efficiency in the –1, –2, and –3 orders of a 6000-gr/mm triangular grating with 10.3° working facet angle and 5-Å rms roughness calculated as a function of wavelength for the 10.3° polar and 88.5° azimuth incidence angles and the off-plane mounting.

Polarization in Solar/Stellar Flares?



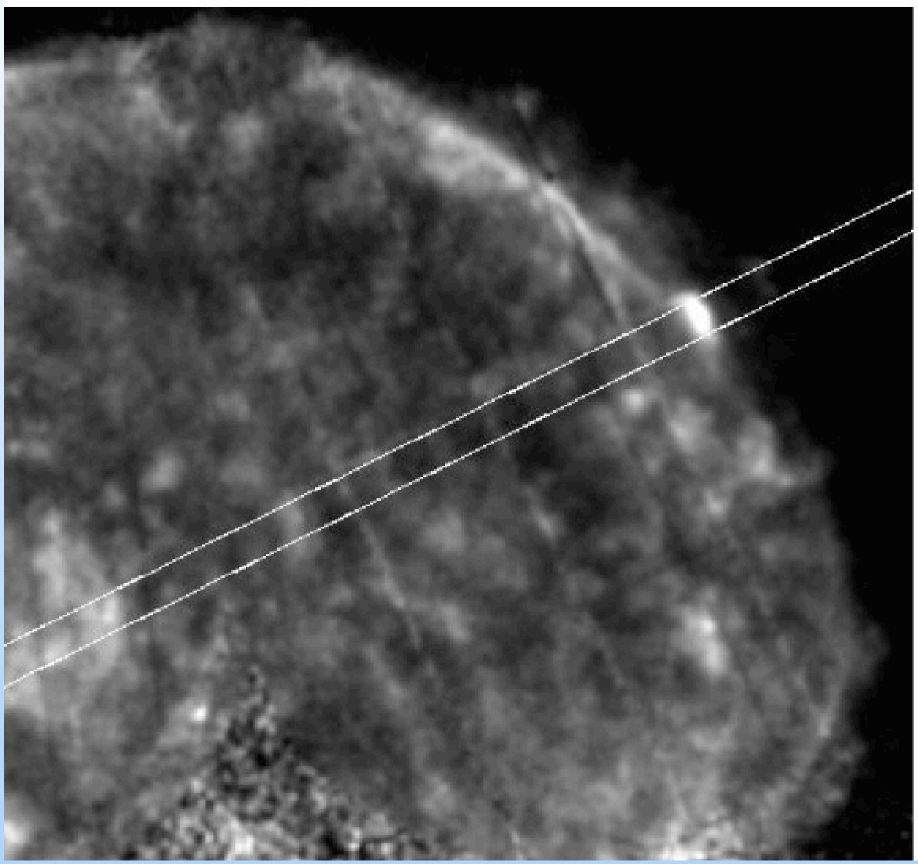
NRL SOLFLEX on P78-1 (Blanchet et al. 1985, A&A, 152, 417)

Anomalous ratio of Lyα_1:Lyα_2 in Ca XX

Polarization or opacity?

No direct polarization detections so far in solar flare observations.

Polarization at Collisionless Shock Fronts (SN 1006)



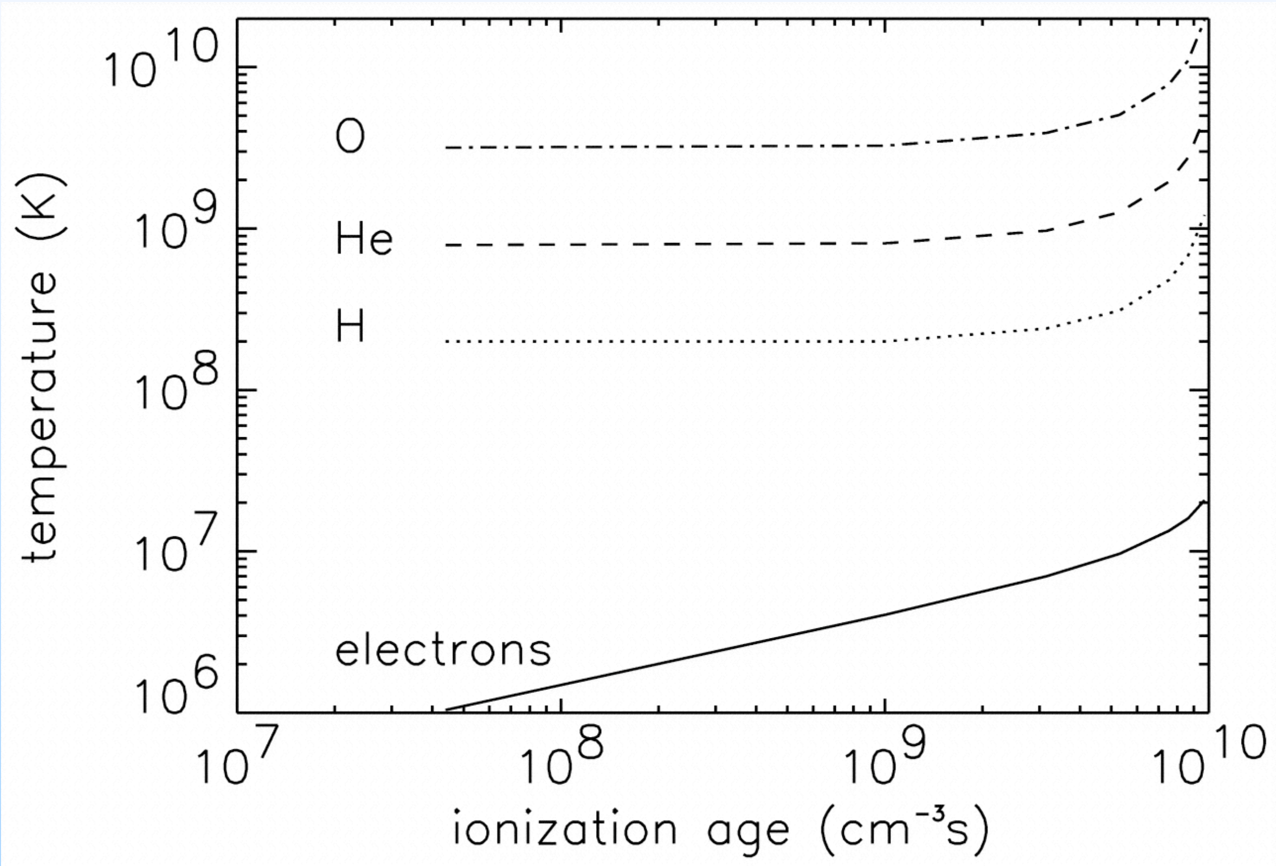
SN 1006 NW limb imaged in 0.50-0.61 keV by XMM-Newton (O I-VII K shell lines, Vink et al. 2003, ApJ, 587, L31).

Evidence for collisionless electron heating to about 1 keV by several authors (Laming et al. 1996, Ghavamian et al. 2002, Vink et al. 2003).

Energized electrons are non-Maxwellian and almost certainly "beamed" close to the shock front.

Expect to pick this up in polarization of lines formed close to shock (O I, II, III, etc K shell emission).

Simplest Ideas about Electron Heating



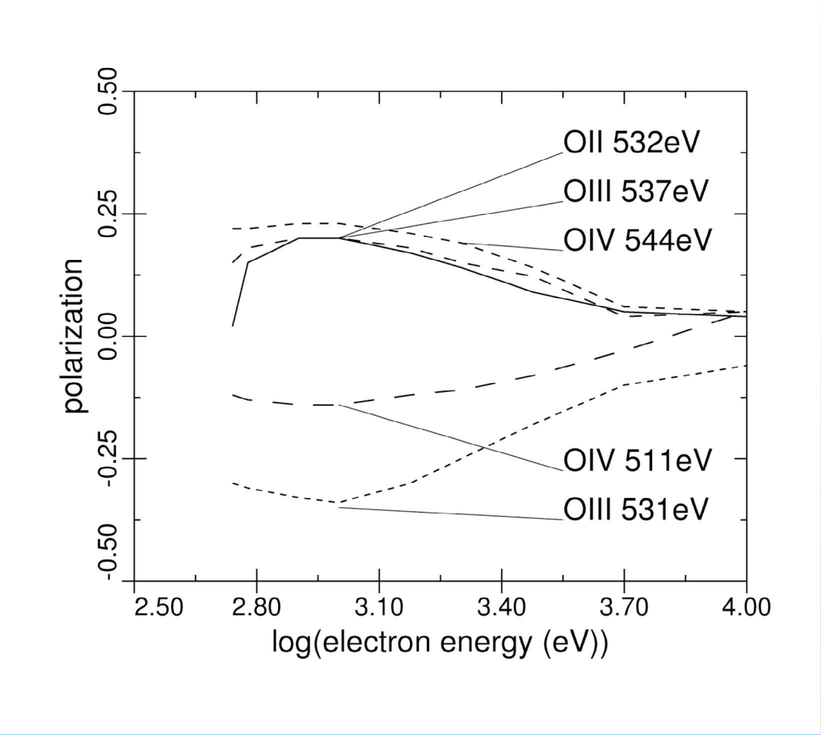
Shock jump conditions give T~mass, followed by Coulomb equilibration. O emission at $n_e t$~ *few x* 10^9 $cm^{-3}s$ gives T_e=600 eV.

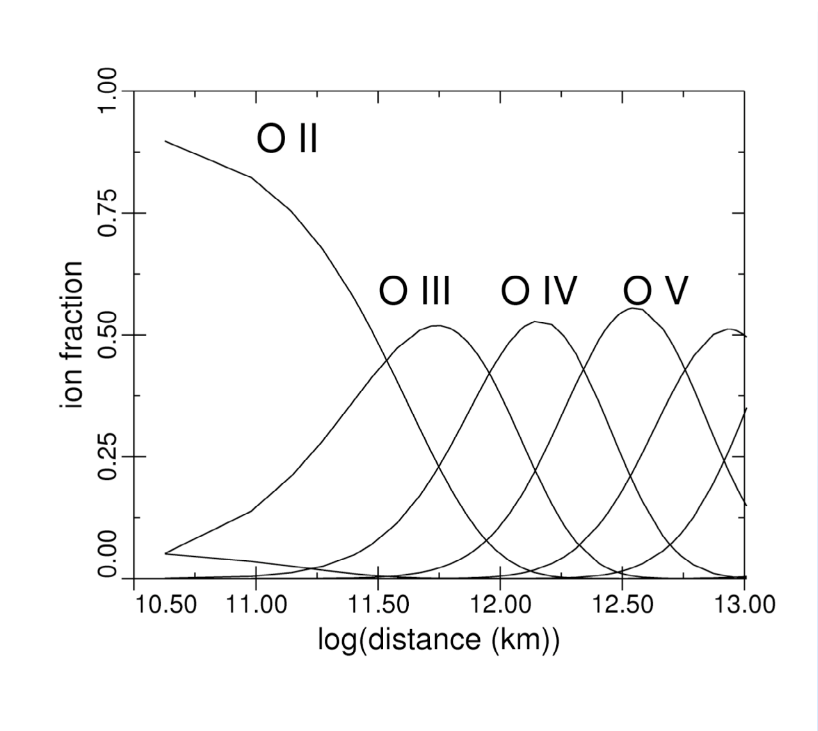
How do we get to T > 1keV?

Collisionless Electron Heating

- ~20% preshock ions reflected back upstream and generate plasma turbulence by (modified) two stream instability…
- Cargill & Papadopoulos 1988, electron plasma and ion acoustic waves which accelerate electrons along shock velocity vector, possibly followed by betatron acceleration in shock compressed magnetic field.
- McClements, Bingham & others, lower hybrid waves accelerate electrons along magnetic field vector, i.e. orthogonally to CP88 model for a perpendicular shock.
- Also discussed in connection with electron-ion equilibration in ADAFS, solar wind, other solar system bodies…

O inner shell lines in SN 1006 with FAC (Gu 2003)

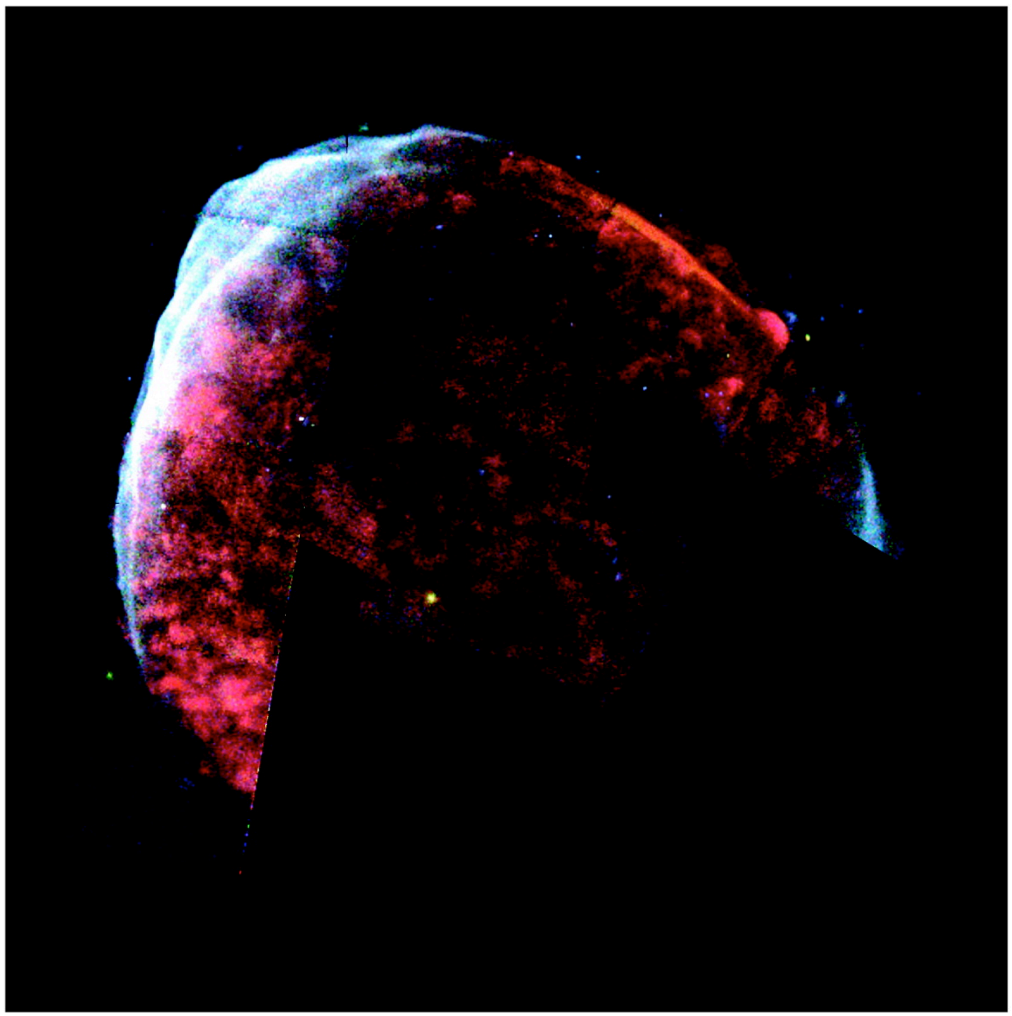




ignores inner shell ionization, resonances, electron pitch angle distribution,…

O ionization balance against distance behind shock front, (from BLASPHEMER)

NW and NE Limbs of SN 1006



Long et al. (2003)
ApJ, 586, 1162

Chandra/ACIS image of SN 1006 showing X-ray synchrotron emission on NE limb

Could we detect the electron injection spectrum?

Summary: Spectral Line Polarization

- Any plasma heated by magnetic/plasma instabilities must show impact polarization at some level as electrons are accelerated.
- Electron thermalization time is usually very short (order of seconds in solar/stellar coronae) so one has to work hard to catch it.
- Most promising sources are those with the most nonthermal line emission, e.g. supernova remnants – real prospect of understanding electron heating/acceleration to x-ray thermal energies and beyond.