Calculating Higher Order Corrections: An Update

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Outline

- NLO (EW+QCD) corrections to Higgs production
  \[ e^+ e^- \rightarrow ZH, \; \nu\nuH, \; t\bar{t}H \]

- NNLO QCD corrections to event shapes
  \[ e^+ e^- \rightarrow 3\text{-jets}, \; \text{thrust}, \; \text{EEC}, ... \]
NLO corrections to Higgs production

- Important for extracting Higgs couplings
- Electroweak corrections at 500 GeV can be ~ 10%
- Processes Considered:
  - Higgstrahlung: \( g_{HZZ} \)
  - WW fusion: \( g_{HWW} \)
  - Top-associated production: \( g_{Htt} \)
NLO ZH and $\nu\nu H$ production

One-loop corrections are all electroweak.

$\nu\nu H$ process is particularly challenging due to:
1) large number of virtual diagrams (1350 in all; “only” 249 lacking $\lambda_e$)
2) complicated 5-point integrals (98 in all; “only” 15 lacking $\lambda_e$)

First $\nu\nu H$ calculation used GRACE-LOOP to generate, evaluate loop diagrams; nonlinear gauge for diagram checking.

Belanger et al., hep-ph/0212261

Second independent calculation: Denner et al., hep-ph/0301189
NLO ZH and $\nu\nu H$ production results

Denner et al., hep-ph/0301189

IBA scheme (?): set $\alpha$ to $\alpha(M_Z)$; similarly for $g_2(M_W)$

“Blips” due to 2-particle intermediate state rescatterings (WW, ZZ, tt).

Largest corrections to IBA are near end of phase space, where Born cross section varies rapidly (and is tiny anyway).
Conclusion: Theoretical corrections to $\sigma(e^+e^- \rightarrow ZH)$ are under good control; they only get large (vs. IBA) where experimental accuracy suffers.

Is the same true of WW fusion process? Probably, at least at 500 GeV…
Now there are QCD and electroweak corrections.

QCD corrections were computed first. “Standard” NLO QCD formalism, but also IR divergent 5-point integrals.

They are modest away from threshold.

Electroweak corrections turn out to be comparable

Dawson, Reina, hep-ph/9808443
Dittmaier et al., hep-ph/9808433
You et al., hep-ph/0306036
Belanger et al., hep-ph/0307029
(latter two agree well)
Conclusion: Theoretical corrections to $\sigma(e^+e^- \rightarrow ttH)$ are also now under good control: 10-20% NLO corrections are $\sim$ experimental uncertainty in $g^2_{ttH}$
Prospects for NNLO corrections to $e^+e^-$ event shapes

- Important for extracting a precise $\alpha_s$
- at large momentum transfer $Q$
- Many fewer hadronic events than at the $Z$ resonance
- Also more backgrounds
- However, nonperturbative corrections $\sim 1/Q$
Motivation

- $e^+e^-$ annihilation to hadrons is experimentally clean
  - At full energy LC, not quite as clean as at $M_Z$:
  - Use $e_R^-$ to suppress $WW$ background
  - Use anti-$b$-tagging to suppress $tt$ background
    (Schumm, hep-ex/9612013; Schumm, Truitt, hep-ex/0102010; Burrows, hep-ex/0112036)

- Excellent opportunity for high $Q^2$, clean $\alpha_s(Q)$ measurement
  - $\alpha_s(M_Z)$ is biggest uncertainty now in testing GUT coupling constant unification.
  - $\alpha_s(M_Z)$ an important input into other LC physics;
    e.g. extraction of $m_t$ from $tt$ threshold scan.

- With Giga-Z (or LEP/SLC archival data), compare full energy results to $M_Z$ results to fit out nonperturbative $(1/Q)$ corrections
Need for NNLO

- Measurements of $\alpha_s$ in 3-jet rate, other $e^+e^-$ event shapes, dominated by theoretical uncertainty due mainly to truncation of perturbative series at NLO:

$$O_i = A_i \frac{\alpha_s(\mu)}{\pi} + [B_i + 4\beta_0 \ln(\mu/Q)A_i] \left( \frac{\alpha_s(\mu)}{\pi} \right)^2 + [C_i + \ldots] \left( \frac{\alpha_s(\mu)}{\pi} \right)^3$$

- Fit to NLO theory leads to:

  - large scatter in $\alpha_s$ values from different event shapes,
  - large renormalization scale dependence

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Burrows, hep-ex/9709010
What about NLO EW corrections?

- Sudakov effects lead to growth with $Q$:
- Result for Thrust distribution at 1 TeV:

$$\delta\sigma \propto \frac{\alpha_w}{\pi} \ln^2\left(\frac{Q}{M_W}\right)$$

However, it depends on how result is normalized; normalizing to $\sigma$ instead of $\sigma_0$ will give much smaller EW corrections, except at very large $1-T$.

- True momentum scale is $\sim (1-T)^2 Q$. 

Maina et al., hep-ph/0210015
NNLO QCD Diagram Types

\[ A_i \supset e^+ e^- + 1 \text{ more diagram} \]

\[ B_i \supset e^+ e^- + e^+ e^- + \cdots \]

\[ C_i \supset e^+ e^- + e^+ e^- + e^+ e^- + \cdots \]

+ hundreds more diagrams
Status of NNLO Diagrams (Amplitudes)

Tree $e^+e^- \rightarrow q\bar{q}ggg$, $q\bar{q}q'\bar{q}'g$
completed in 1980s


1-loop $e^+e^- \rightarrow q\bar{q}gg, q\bar{q}q'\bar{q}'$
completed in 1990s

Glover, Miller, hep-ph/9609474
Bern, LD, Kosower, Weinzierl, hep-ph/9610370
Campbell, Glover, Miller, hep-ph/9706297
Bern, LD, Kosower, hep-ph/9708239

2-loop $e^+e^- \rightarrow q\bar{q}g$
completed in 2000s

Garland, Gehrmann, Glover, Koutkoutsakis, Remiddi,
Remaining Obstacle: Phase-space integration

- 3 types of terms, each contributes $1/\varepsilon^4 + \ldots$ singularities using dimensional regularization, $D=4-2\varepsilon$, for all integrals.
  - 5 partons, tree x tree
  - 4 partons, 1-loop x tree
  - 3 partons, 2-loop x tree and 1-loop x 1-loop

- 3-parton phase-space integrations trivial (all partons visible)

- 5-partons (doubly unresolved) the hardest.

Several types of singular regions, behavior of $\sigma$ understood:
  - double soft Berends, Giele (1989)
  - 2 collinear + 1 soft Campbell, Glover, hep-ph/9710255
  - 3 collinear Catani, Grazzini, hep-ph/9810389, 9908523
  - 2 collinear, twice Del Duca, Frizzo, Maltoni, hep-ph/9909464
  - plus overlaps
Recent Advances in Phase-space integration

- Analytic behavior of 5-parton cross section in individual singular regions understood for a while; more recently formulas interpolating between the different regions have been found:

- If such formulae were simple enough to integrate analytically, they could be subtracted from the exact integrand, and the finite difference integrated numerically over phase-space in 4-dimensions.

- However, this has not yet been done.
More recently, the technique of sector decomposition, originally developed for loop integrals, has been applied to doubly unresolved phase-space integrals:

- Gehrmann-De Ridder et al., hep-ph/0311276
- Anastasiou et al., hep-ph/0311311

Anastasiou et al. have been able to compute integrals for the NNLO $e^+e^- \rightarrow 2$-jet rate in this way.
Sector decomposition

- One-dimensional singular integrals are easily expanded in $\varepsilon$ using “plus” distributions:

$$I_0 = \int_0^1 dx \ x^{-1+\varepsilon} = \frac{1}{\varepsilon} \delta(x) + \sum_{n=0}^{\infty} \frac{\varepsilon^n}{n!} \left[ \ln^n x \right]_+$$

where

$$\int_0^1 dx \ f(x) \left[ \frac{\ln^n(x)}{x} \right]_+ = \int_0^1 dx \ [f(x) - f(0)] \frac{\ln^n(x)}{x}$$

- To expand a multi-dimensional singular integral such as

$$I = \int_0^1 dx \ dy \ x^{-1-\varepsilon} \ y^{-1-\varepsilon} \ (x+y)^{-\varepsilon}$$

first split into sectors: $x > y$ and $x < y$

$$I_1 = \int_0^1 dx \ \int_0^x dy \ x^{-1-\varepsilon} \ y^{-1-\varepsilon} \ (x+y)^{-\varepsilon}$$

$$I_2 = \int_0^1 dy \ \int_0^y dx \ x^{-1-\varepsilon} \ y^{-1-\varepsilon} \ (x+y)^{-\varepsilon}$$
Sector decomposition (cont.)

- Let \( y = y'x \) in \( I_1 \) \( x = x'y \) in \( I_2 \)

\[
I_1 = \int_0^1 dx\ dy\ x^{-1-3\varepsilon}(y')^{-1-\varepsilon}(1+y')^{-\varepsilon}
\]

\[
I_2 = \int_0^1 dx'\ dy\ y^{-1-3\varepsilon}(x')^{-1-\varepsilon}(1+x')^{-\varepsilon}
\]

- Now that the singularities are separated, expansion in \( \varepsilon \) can be carried out as in 1-dimensional case.

- Anastasiou et al. found a parametrization of 4-parton phase space (5 variables) which is amenable to automated sector decomposition:

\[
\hat{I}_4 = \mathcal{N}_4 \int_0^1 d\lambda_1 d\lambda_2 d\lambda_3 d\lambda_4 d\hat{\lambda}_5 \delta(\lambda_1 - \lambda'_1)\delta(\lambda_2 - \lambda'_2)\delta(\lambda_3 - \lambda'_3)\delta(\lambda_4 - \lambda'_4)\delta(\hat{\lambda}_5 - \hat{\lambda}'_5)
\]

\[
\times [\lambda_1 (1 - \lambda_1) (1 - \lambda_2)]^{1-2\varepsilon} [\lambda_2 \lambda_3 (1 - \lambda_3) \lambda_4 (1 - \lambda_4)]^{-\varepsilon} [\hat{\lambda}_5 (1 - \hat{\lambda}_5)]^{-1/2-\varepsilon}
\]

\[
\times s_{13}(\hat{\lambda}_5) [s_{13}^+ s_{13}^-]^{-1/2-\varepsilon} \left\{ \hat{\lambda}_5 \left( s_{13}^+ - s_{13}^- \right) + s_{13}^- \right\}^{2\varepsilon}.
\]
As an example, they numerically integrated the 4-parton contribution to the $N_f$-dependent part of the NNLO 2-jet rate (JADE algorithm, $y=0.1$) order-by-order in $\epsilon$:

$$\frac{\sigma_R}{\sigma_0} = N_f \left( \frac{\alpha_s}{\pi} \right)^2 \left( \delta_{j,2} \left[ -\frac{(5.5553 \pm 0.0005) \cdot 10^{-2}}{\epsilon^3} - \frac{0.20369 \pm 0.00005}{\epsilon^2} + \frac{0.4180 \pm 0.0005}{\epsilon} + 4.855 \pm 0.003 \right] - \delta_{j,3} \left( \frac{0.41005 \pm 0.00016}{\epsilon} + 2.9832 \pm 0.0018 \right) + (1.4561 \pm 0.0018) \cdot 10^{-3} \delta_{j,4} \right).$$

(51)

Adding the (2,3)-parton contributions, the $1/\epsilon$ poles cancel, and the right answer is obtained ($\sigma_2 + \sigma_3 + \sigma_4 = \sigma_{\text{tot}}$ is known):

$$\frac{\sigma^{(2)}}{\sigma_0} = N_f \left( \frac{\alpha_s}{\pi} \right)^2 \left[ \delta_{j,2} \left( \frac{(2.6 \pm 4.6) \cdot 10^{-6}}{\epsilon^3} + \frac{(1.4 \pm 5.5) \cdot 10^{-5}}{\epsilon^2} - \frac{(3.1 \pm 5.2) \cdot 10^{-4}}{\epsilon} + 1.846 \pm 0.003 \right) + \delta_{j,3} \left( \frac{(-0.5 \pm 2.6) \cdot 10^{-4}}{\epsilon} - 1.963 \pm 0.017 \right) + (1.456 \pm 0.002) \cdot 10^{-3} \delta_{j,4} \right].$$
Most important lessons of this technique:
- It is not necessary (for a human) to understand the behavior of the integrand in the various singular regions.
- It is not necessary to integrate singular terms analytically to verify cancellations.

Method looks generalizable to the NNLO 3-jet problem
- Either by a direct attack on the 5-particle phase space (5-variables \(\rightarrow\) 8-variables)
- Or by using it to integrate subtraction terms over 4-particle subspaces.
Conclusions

- Much recent progress in computing NLO QCD and particularly electroweak corrections for important multi-body final-state electroweak processes at a linear collider.

- Here, surveyed a few relevant for precision extraction of Higgs couplings. With the one-loop EW corrections, theoretical precision generally appears adequate.

- For precision measurement of $\alpha_s$, NNLO QCD corrections to hadronic event shapes are required, but not yet available. However, I predict they will be within a year, due to recent progress and the upcoming KITP workshop in collider physics.