Overview and Innerview of Black Holes

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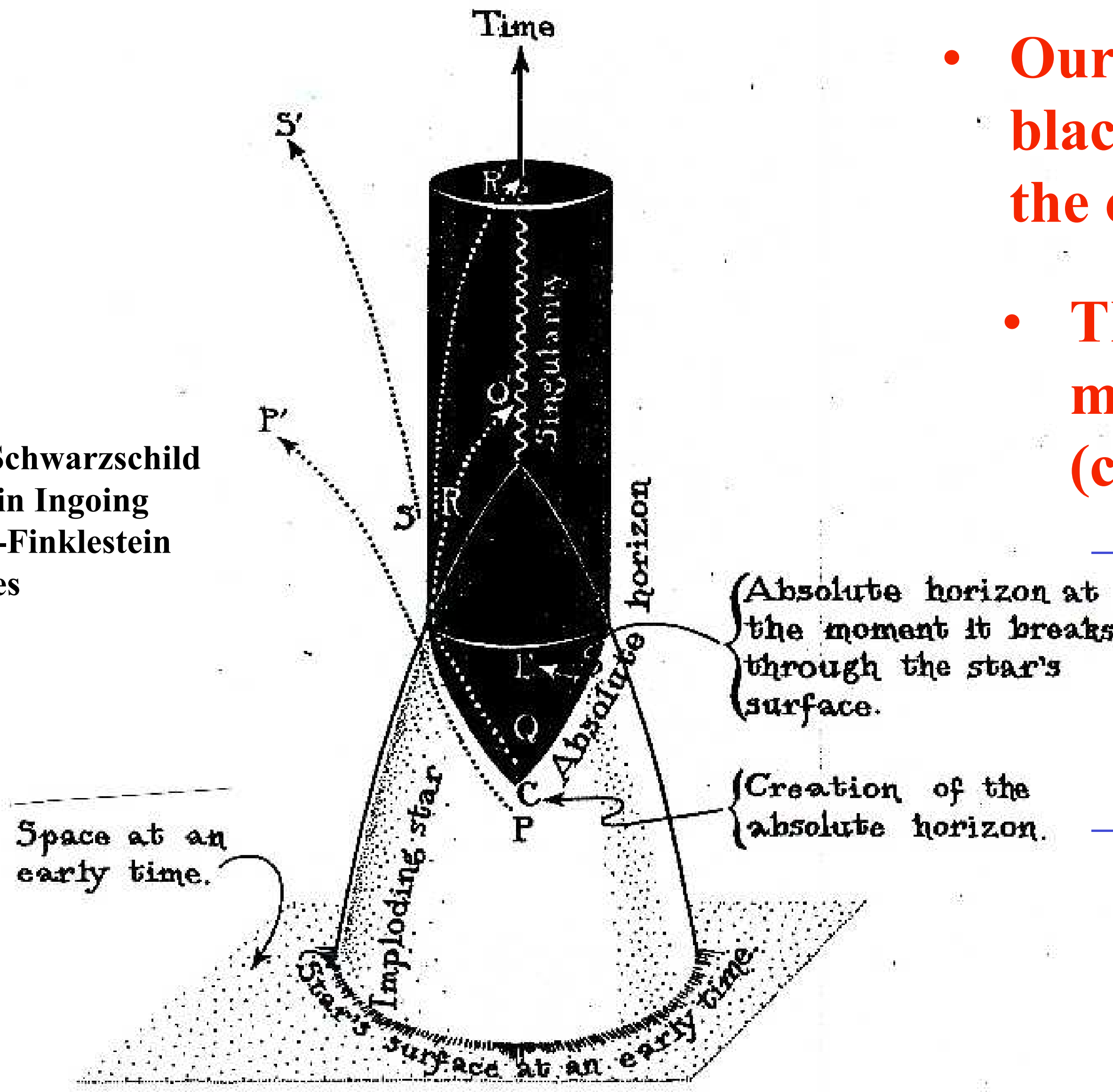
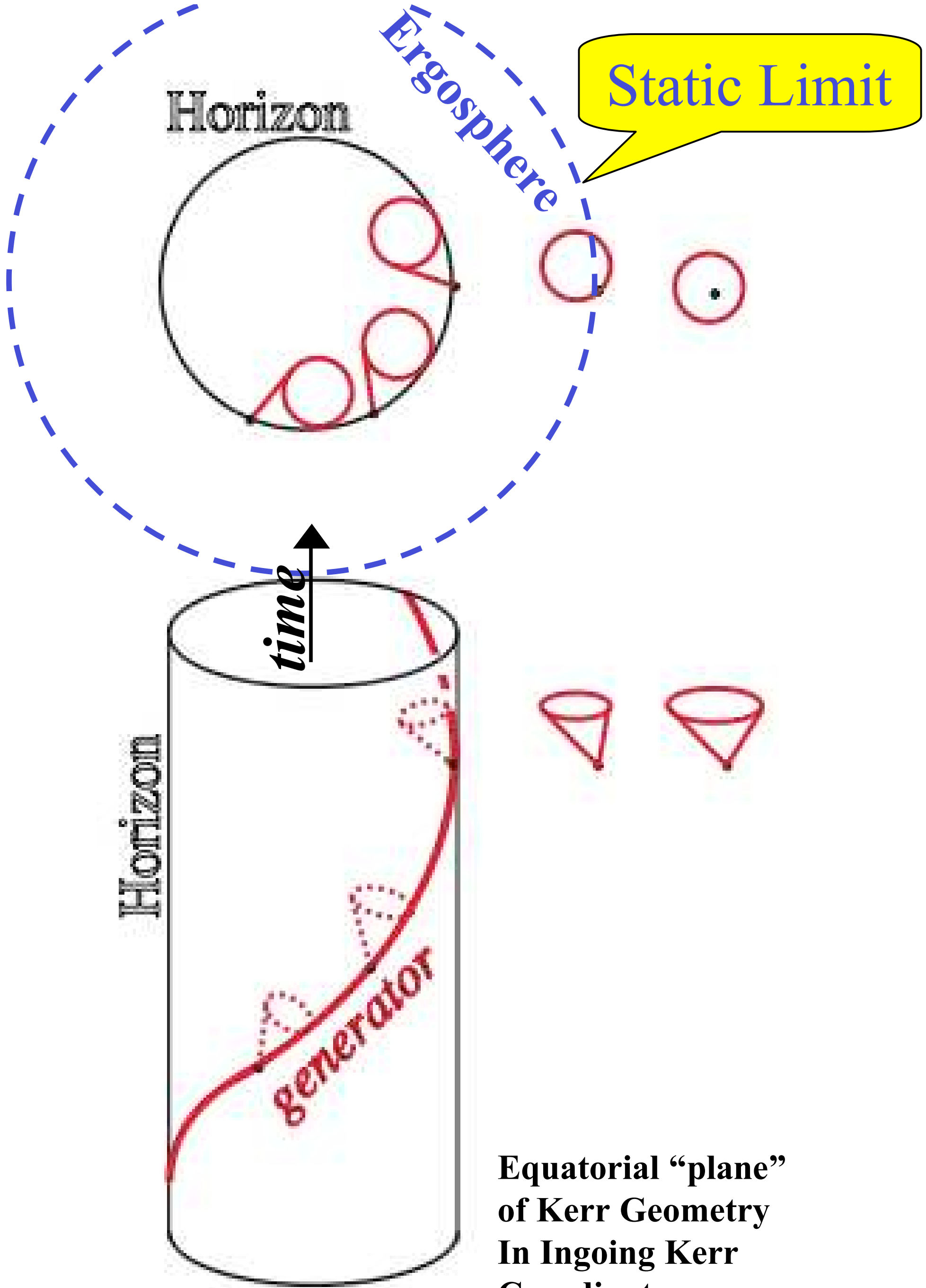
Beyond Einstein: From the Big Bang to Black Holes
SLAC, 14 May 2004

Black Hole Created by Implosion of a Star

Exterior: Schwarzschild Geometry in Ingoing Eddington-Finklestein Coordinates



- Our Focus: quiescent black hole long after the collapse.
  - The black hole is made from warped (curved) spacetime
    - The energy that produces the warpage is in the warpage itself
    - [nonlinearity of Einstein's equations]



Spinning Black Hole

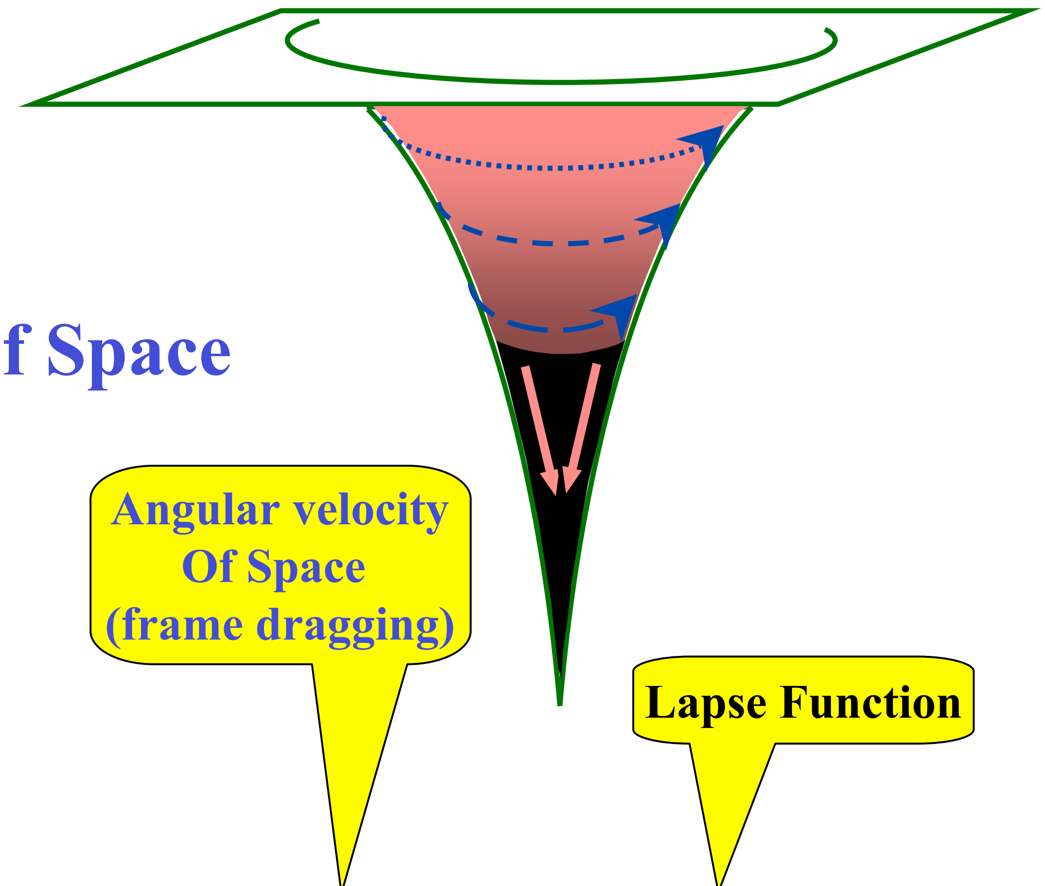
Kerr Solution of Einstein's Equations

Split Spacetime into Space + Time

Equatorial "plane" of Kerr Geometry In Ingoing Kerr Coordinates

Three Aspects of Spacetime Warpage for Quiescent Black Hole

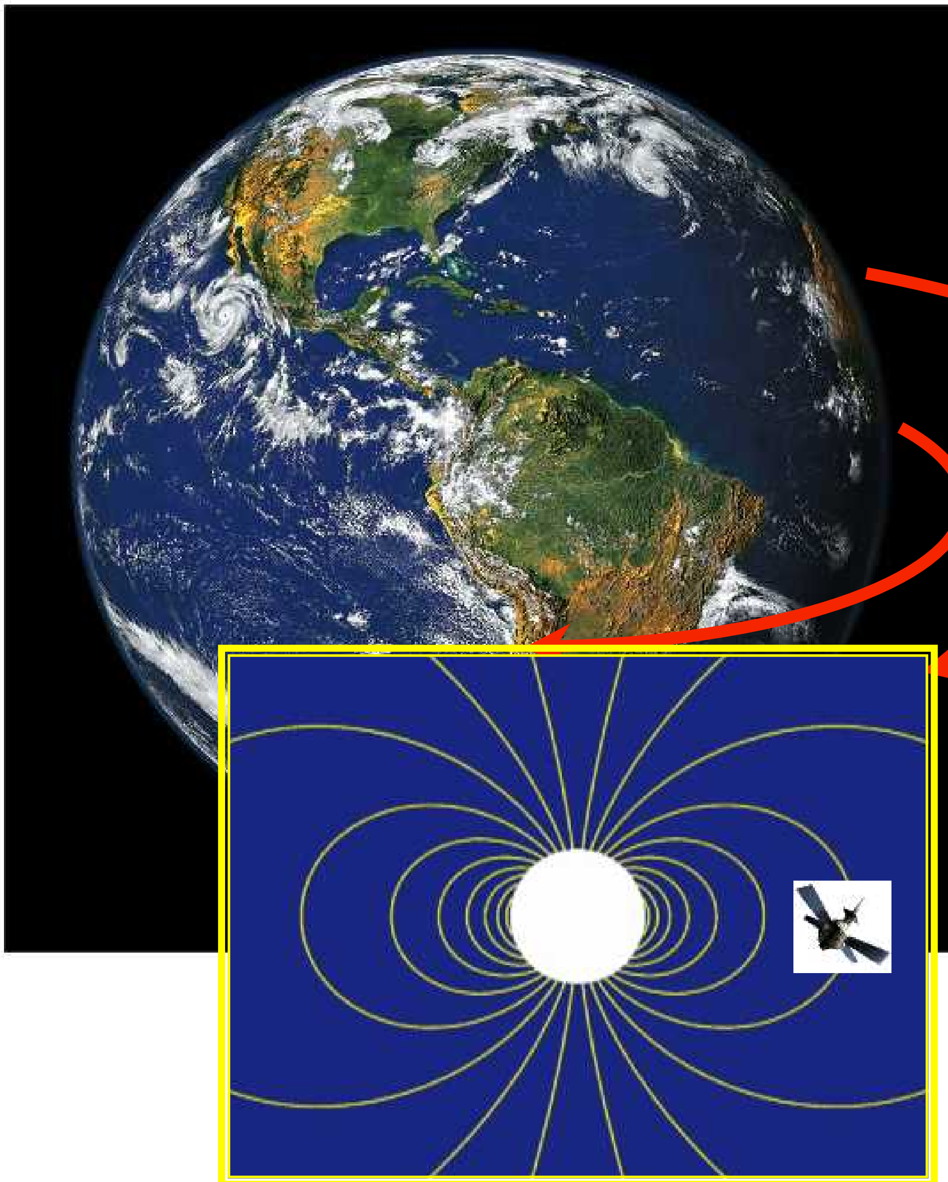
- **Curvature of Space**
- **Warping of Time**
- **Rotational Motion of Space**



$$ds^2 = g_{rr}dr^2 + g_{\theta\theta}\, d\theta^2 + g_{\phi\phi}(d\phi \; - \omega\, dt)^2 \quad - \alpha^2 dt^2$$

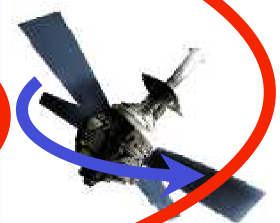
space curvature — space rotation — time warp

Angular Velocity of Space (Frame Dragging)



$\omega = 2G\,\boldsymbol{J} / r^3$

$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$



Gravity Probe B

$\boldsymbol{\Omega} = 1/2 \text{ curl } \mathbf{v}$

Alternative Viewpoint:
"Gravitomagnetism"

A= ω x r GM potential

H= curl A GM field

Ω = 1/2 **H**

Overview

Predictions from the Golden Age of Black-Hole Theory (1963 - 1976)

- Black-Hole Uniqueness (for quiescent hole, M and J => all)
  - ("no hair theorem")
- Cosmic Censorship (no naked singularities)
- Evolution of Horizon in Response to Tidal Gravity of Companion
- Rotational Energy and its Extraction
- The First and Second Laws of Black-Hole Mechanics
- Black-hole Vibrations (normal modes of oscillation)

Discuss these in context of LISA's Gravitational-Wave Observations

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Go Beyond the Golden Age Predictions:

- The nonlinear dynamics of wildly disturbed BHs
- The structure of a black hole's interior

Black-Hole Uniqueness ("No-Hair" Theorem)

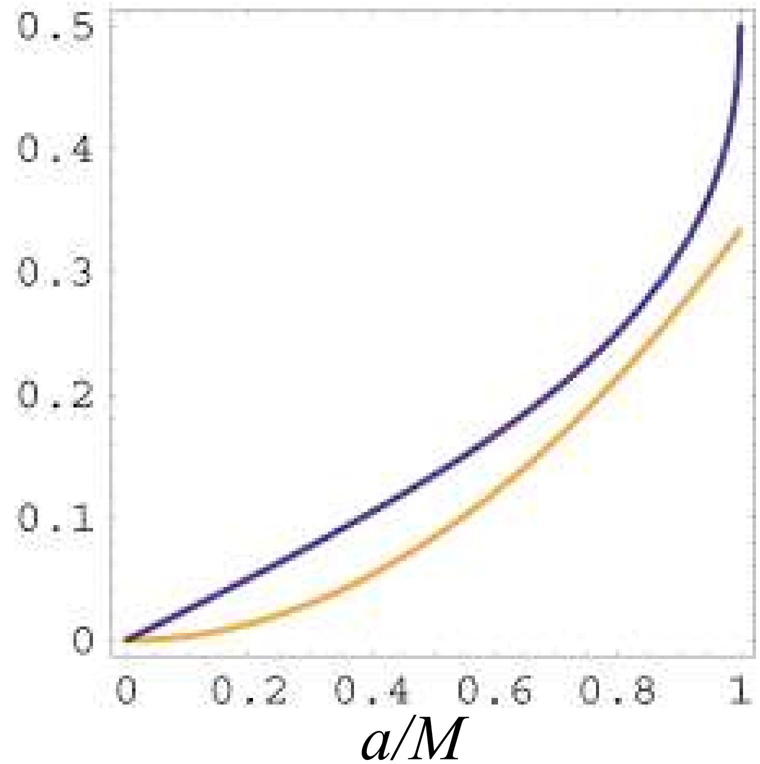
[Israel, Carter, Robinson, …]

- All quiescent astrophysical black holes are described by the Kerr metric, plus weak perturbations due to companions, accretion disks, magnetic fields, …
- Kerr metric is fully determined by two parameters:
  - Hole's mass M (as manifest in Kepler's laws for distant planets)
    - or $GM/c^2 = \boldsymbol{M}$ [M_{sun} = 1.5 km]
  - Hole's spin angular momentum J (as manifest in frame dragging)
    - or $J/Mc = \boldsymbol{a}$
    - $\boldsymbol{a}$ cannot exceed $\boldsymbol{M}$; for $\boldsymbol{a} > \boldsymbol{M}$ the Kerr metric describes a naked singularity
- "A black hole has no hair"

Really: 2 Hairs

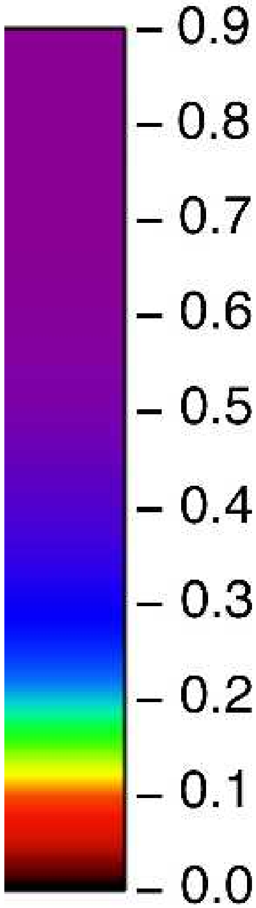
Examples of Black-Hole Properties as functions of *M*, *a*

- Horizon angular velocity (ω at Horizon):
  $\Omega_H = (a/M)(c/M)/(1+\sqrt{1-a^2/M^2})$
  in units of
  $c/M = (0.2\ \text{rad/sec})(10^6\ M_{sun}/M)$

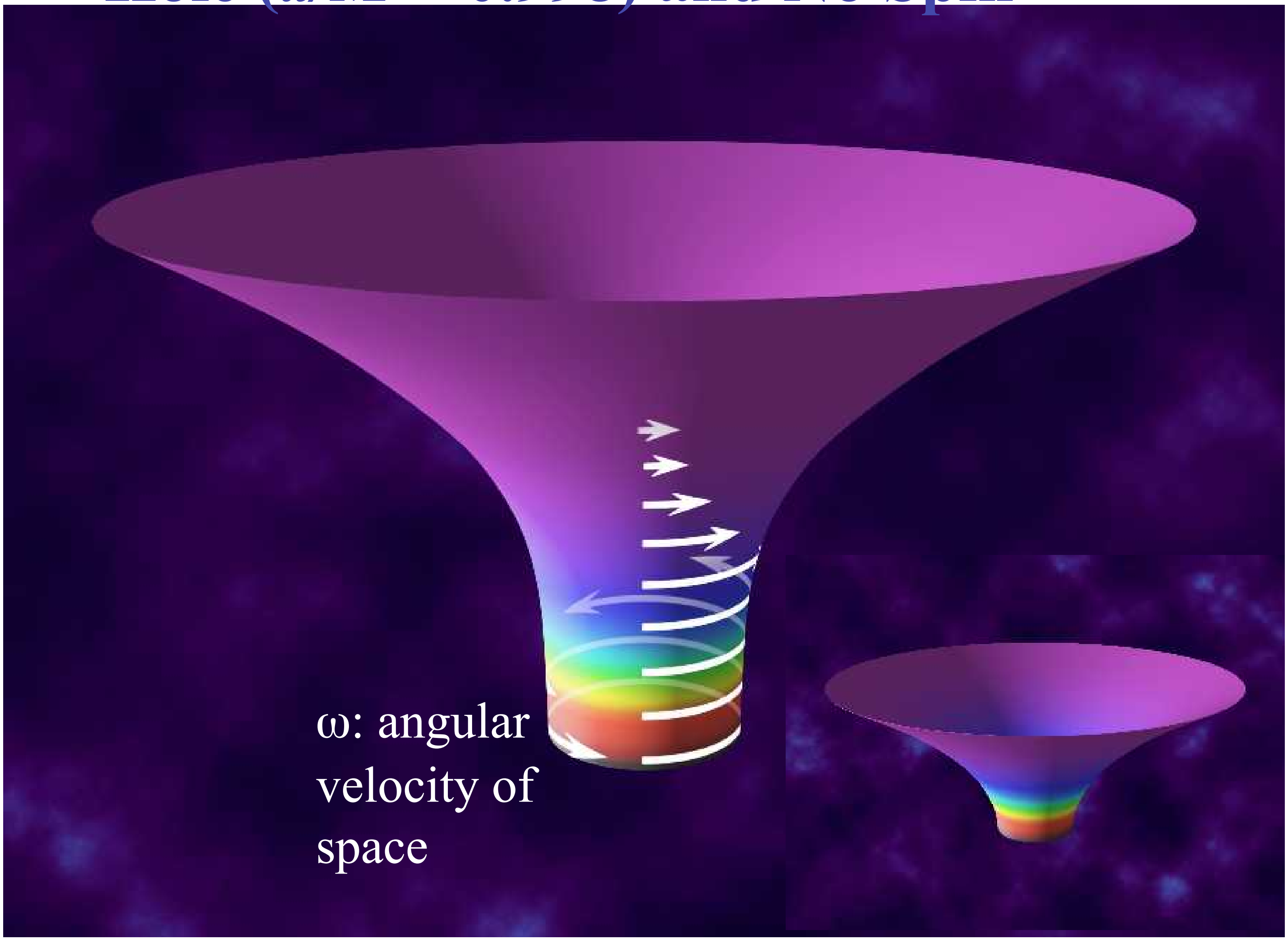


- Mass quadrupole moment,
  . $Q = Ma^2/3$, in units of MM^2

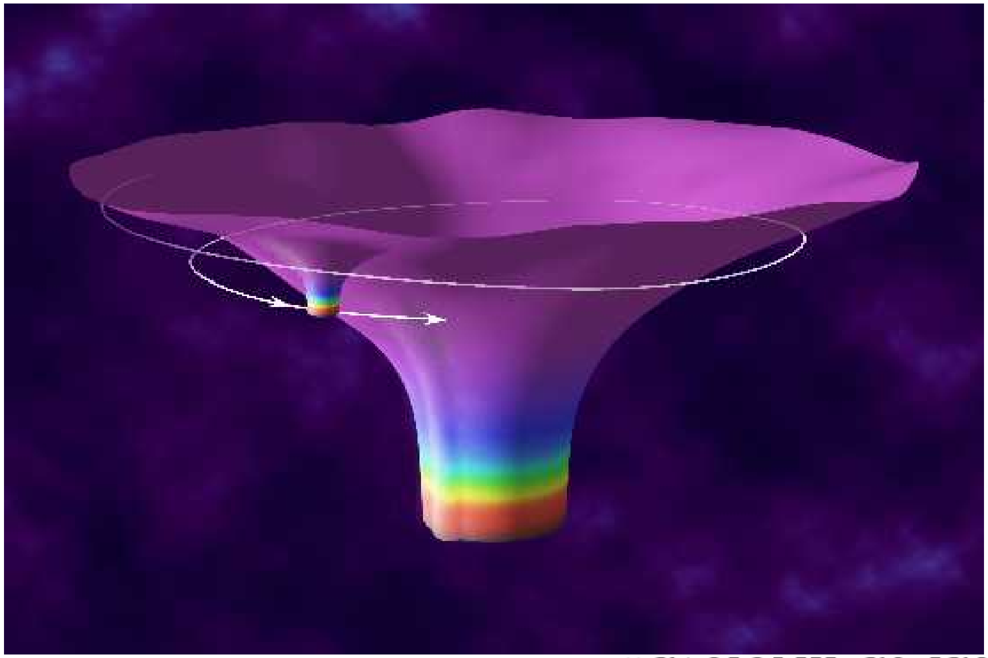
Map of Spacetime Geometry for a Fast-Spinning Hole (a/M = 0.998) and No Spin



α: Rate of Time Flow,

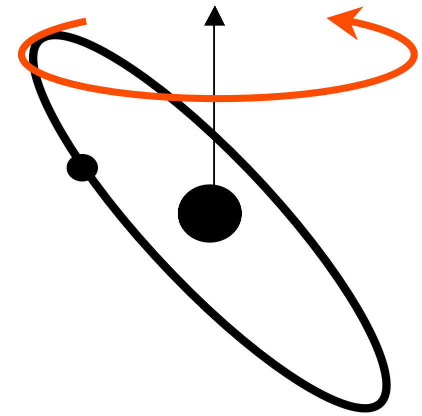


Mapping a Large Hole's Spacetime Curvature via Gravitational Waves: LISA

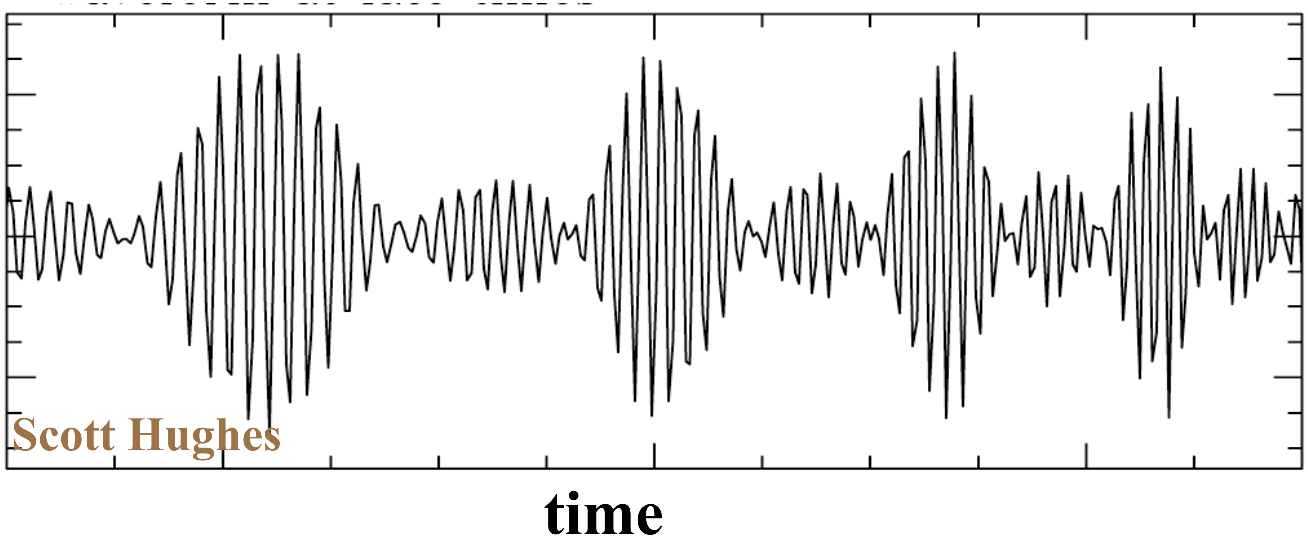


EMRI: Extreme Mass Ratio Inspiral

Map <=> two families of multipole moments

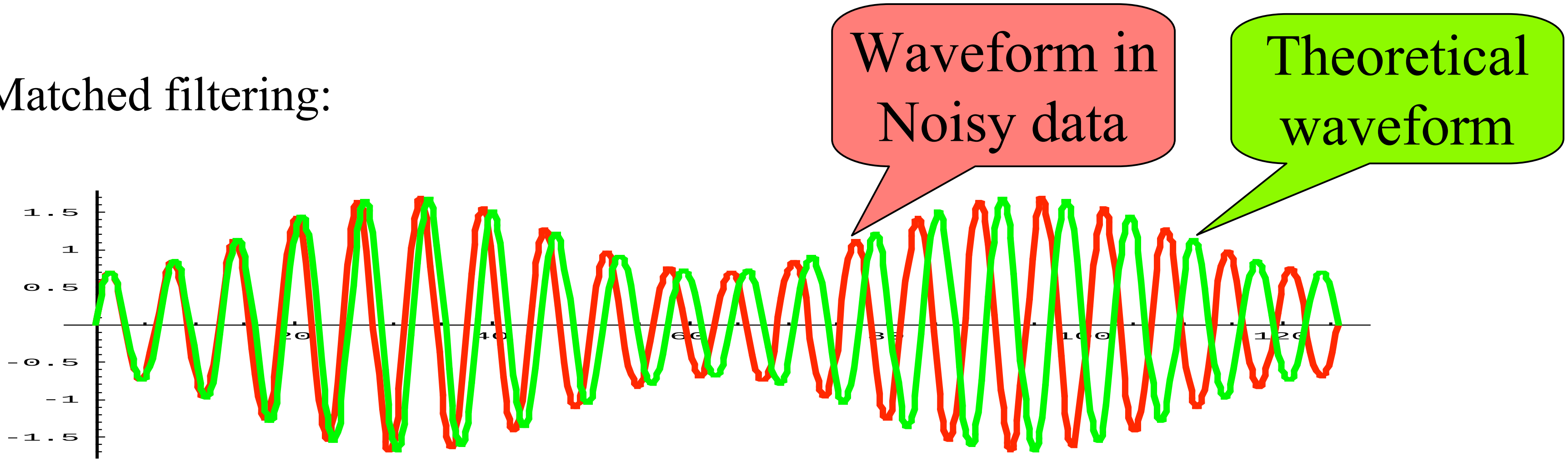


Full map encoded in waveform's phase evolution [Ryan]



Extracting the Map

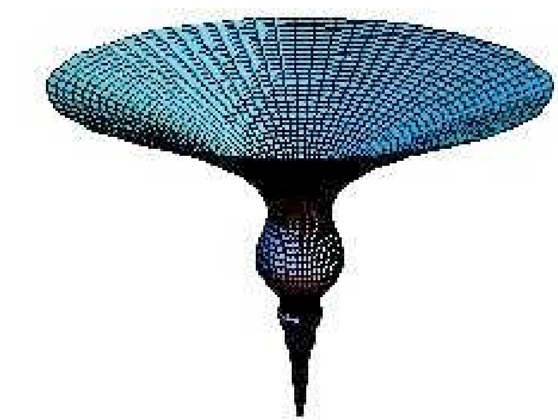
- Matched filtering:



  - If waveform phase **$\phi(t)$** slips by ~ 1 radian, it is obvious in cross correlation
  - LISA's ideal source: ~ 10 M_{sun} hole spiraling into ~10^6 M_{sun} hole:
    - ~ 200,000 cycles of waves in last year, emitted from orbital circumferences ~ 6 x (horizon circumference) and smaller. ***[Phase measureable to a part in a million]***

Cosmic Censorship Conjecture
[Penrose]

- In the modern universe (as opposed to the big bang):
  - Singularities occur only inside black holes
- No Naked Singularities

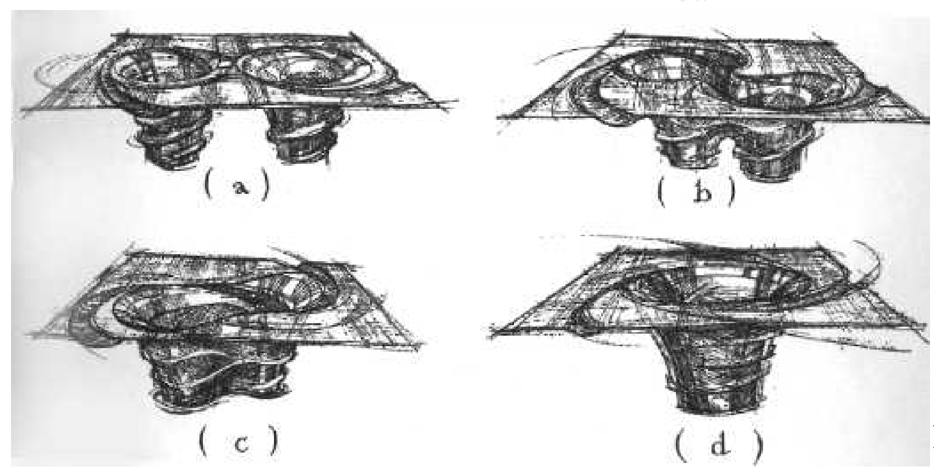


- GW searches for naked singularities:
  - Map the spacetime geometry of massive body

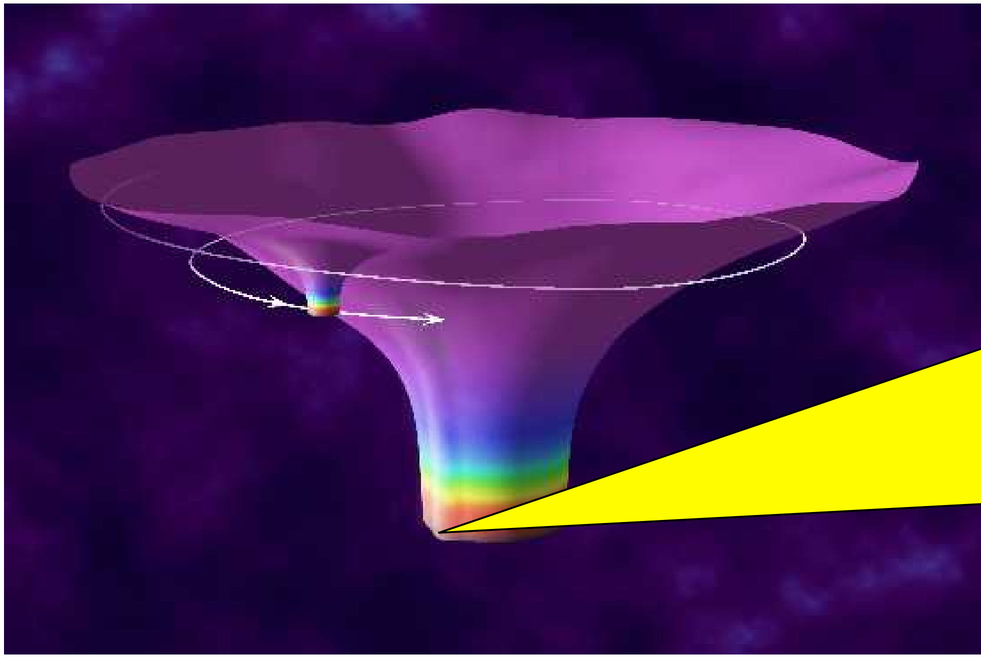
LISA's EMRI observations

  - Collisions of Black Holes: is the final object a black hole?

Will be tested by LISA (and LIGO)



Tides on a Black-Hole Horizon
Horizon Evolution



TIDE

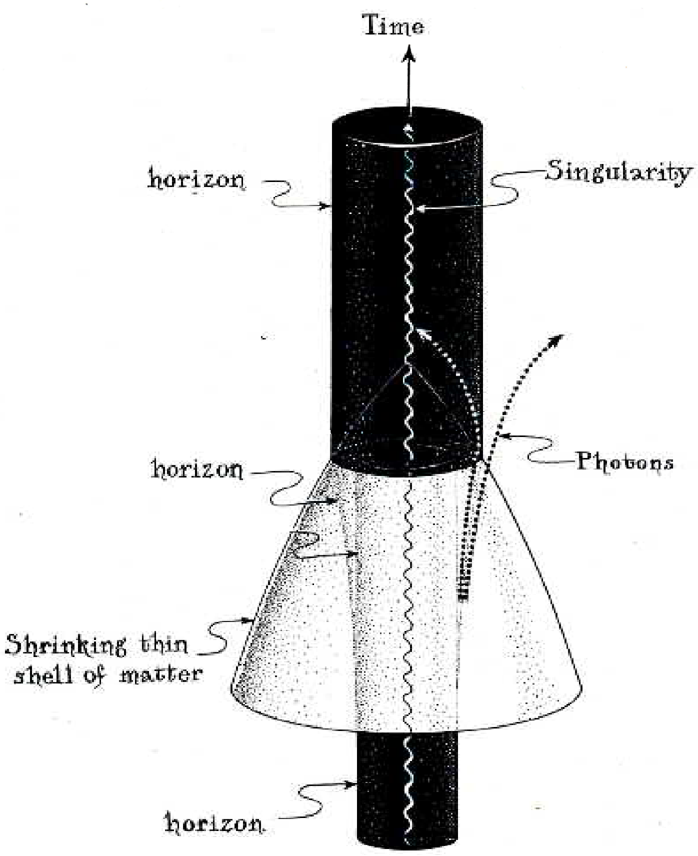
Tidal dissipation in horizon
alters hole's mass and spin
[Hartle and Hawking]

Induced multipole moments:
pull on small hole; alter its orbital energy & angular momentum; alter the GW phase evolution

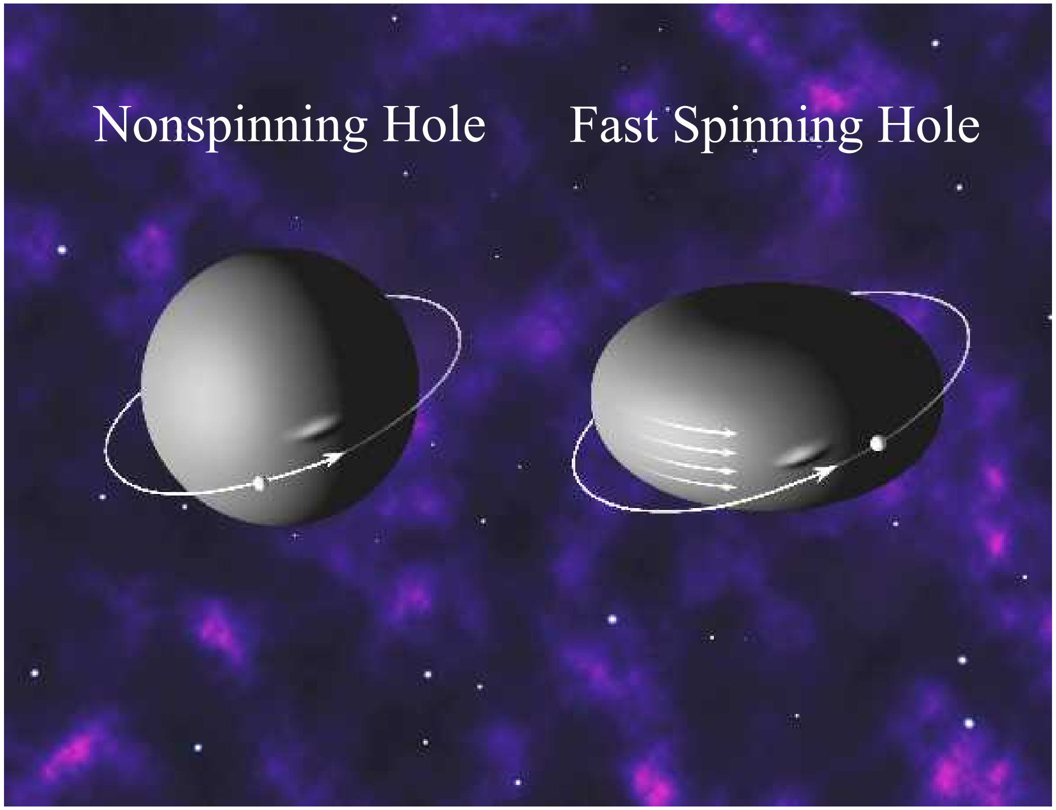
Will be observed in detail by LISA

Horizon's Teleological Evolution

- Collapse of a thin shell onto a black hole

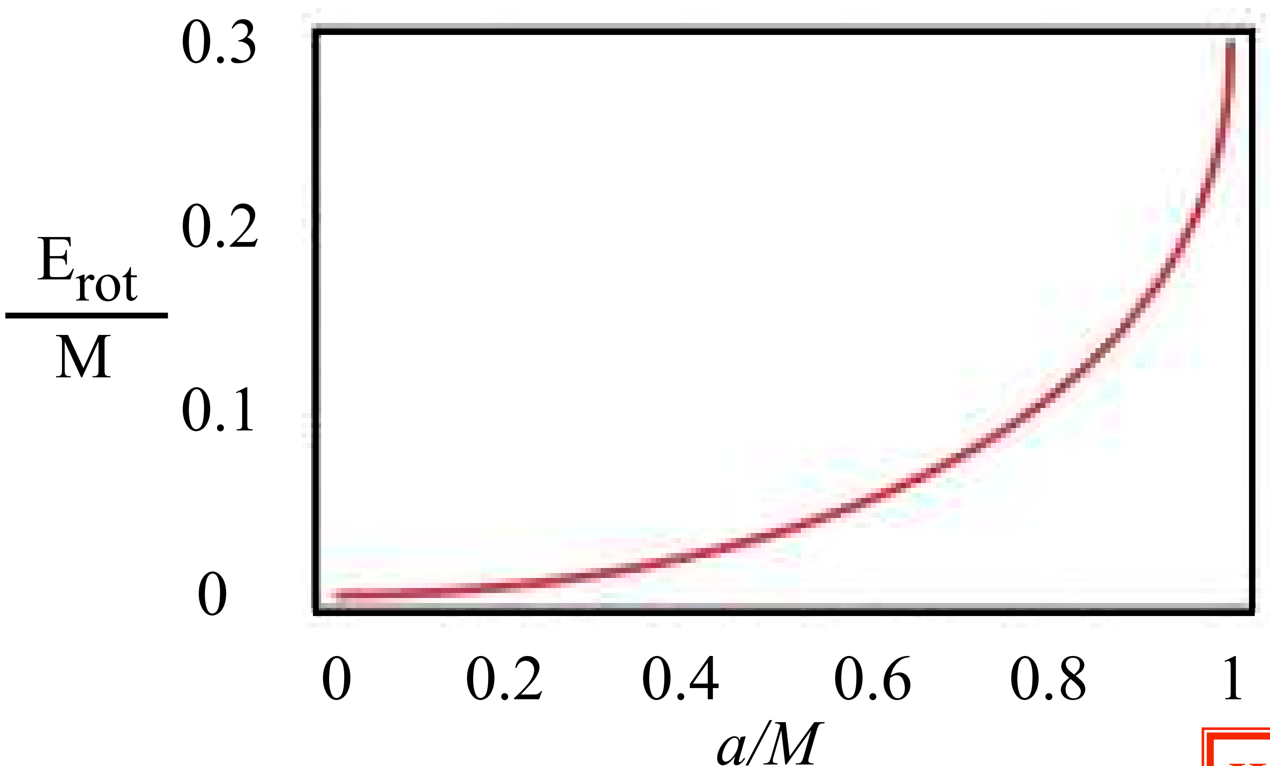


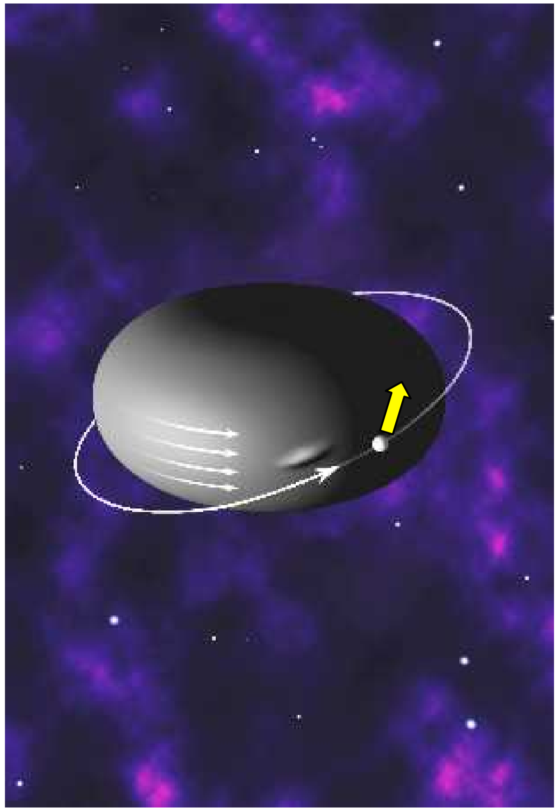
- Tide raised on horizon by small companion



Rotational Energy and Its Extraction

- Rotational energy resides in the whirling space outside the hole, and is extractable





Will be observed in detail by LISA

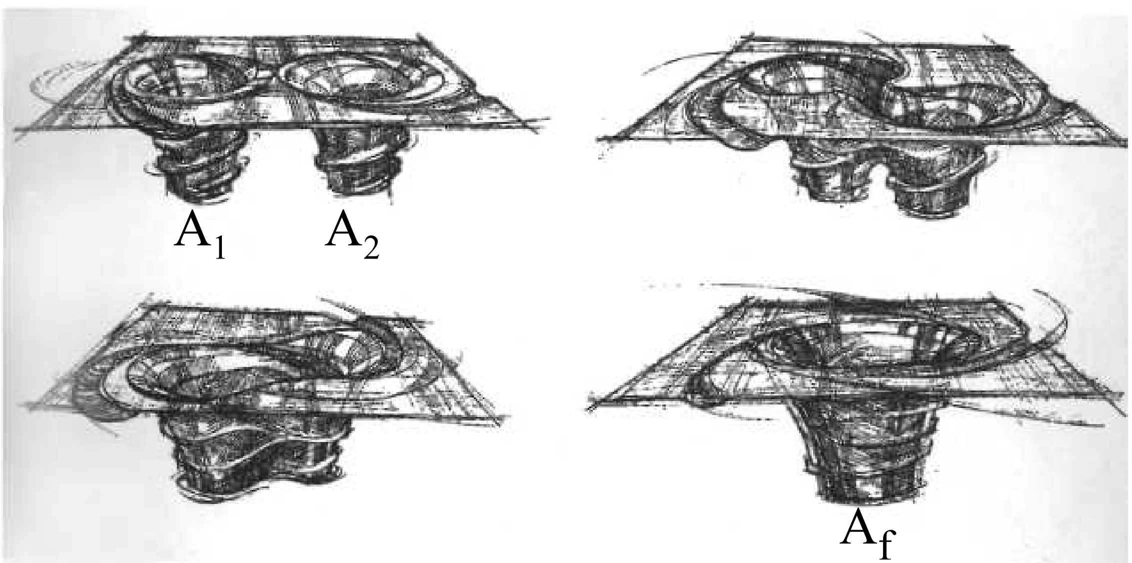
The Laws of Black Hole Mechanics

- First Law -- connecting one quiescent (Kerr) BH state to another
  - $dM = \Omega_H \, dJ + (g_H/8\pi) dA_H$

Surface gravity = lim α a

Horizon area

- Second Law -- for highly dynamical holes as well as quiescent: The horizon area A_H can never decrease
  - Example: Black-Hole Collisions:

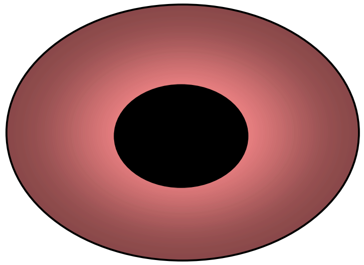


$A_f > A_1 + A_1$

Will be tested by LISA (and LIGO)

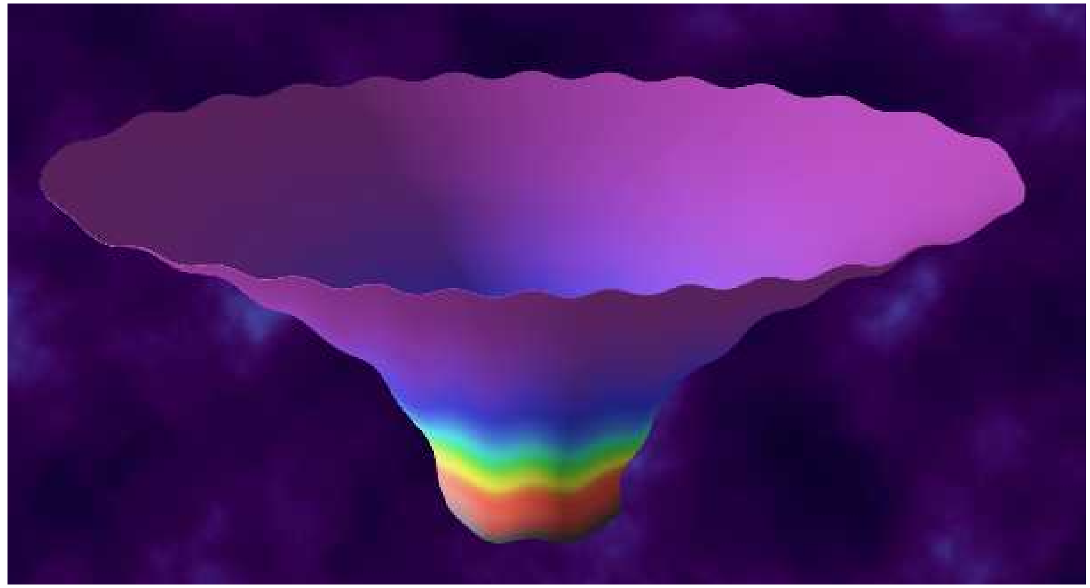
The Laws of Black Hole Thermodynamics

- Stationary Observers outside a black hole see the horizon surrounded by a thermal atmosphere, with temperature $T = T_H/\alpha$; $T_H = (\hbar/2\pi k) g_H \sim 60$ nK (M_{sun} / M)



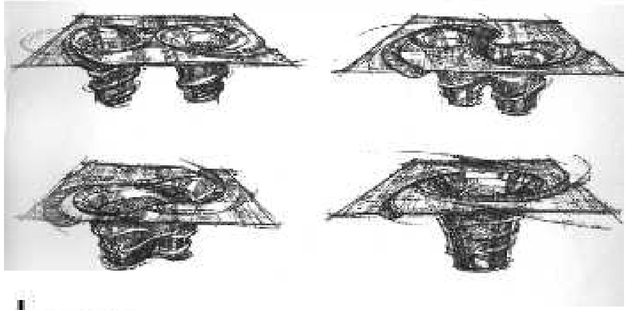
- The hole has an entropy $S_H = k\ A_H / 4L_{Planck}^2 \sim 10^{77}\ k\ (M/Msun)^2$
- First Law: $dM = \Omega_H\ dJ + (g_H/8\pi) dA_H$ <=> $dM = \Omega_H\ dJ + T_H\ dS_H$
- Statistical Mechanics of Entropy: $\exp(dS_H/k)$ = number of quantum mechanically distinct ways that dM and dJ could have been injected into the hole's thermal atmosphere
- Second Law -- A_H cannot decrease <=> S_H cannot decrease in classical general relativity.
  - More generally: Entropy of hole plus its environment cannot decrease

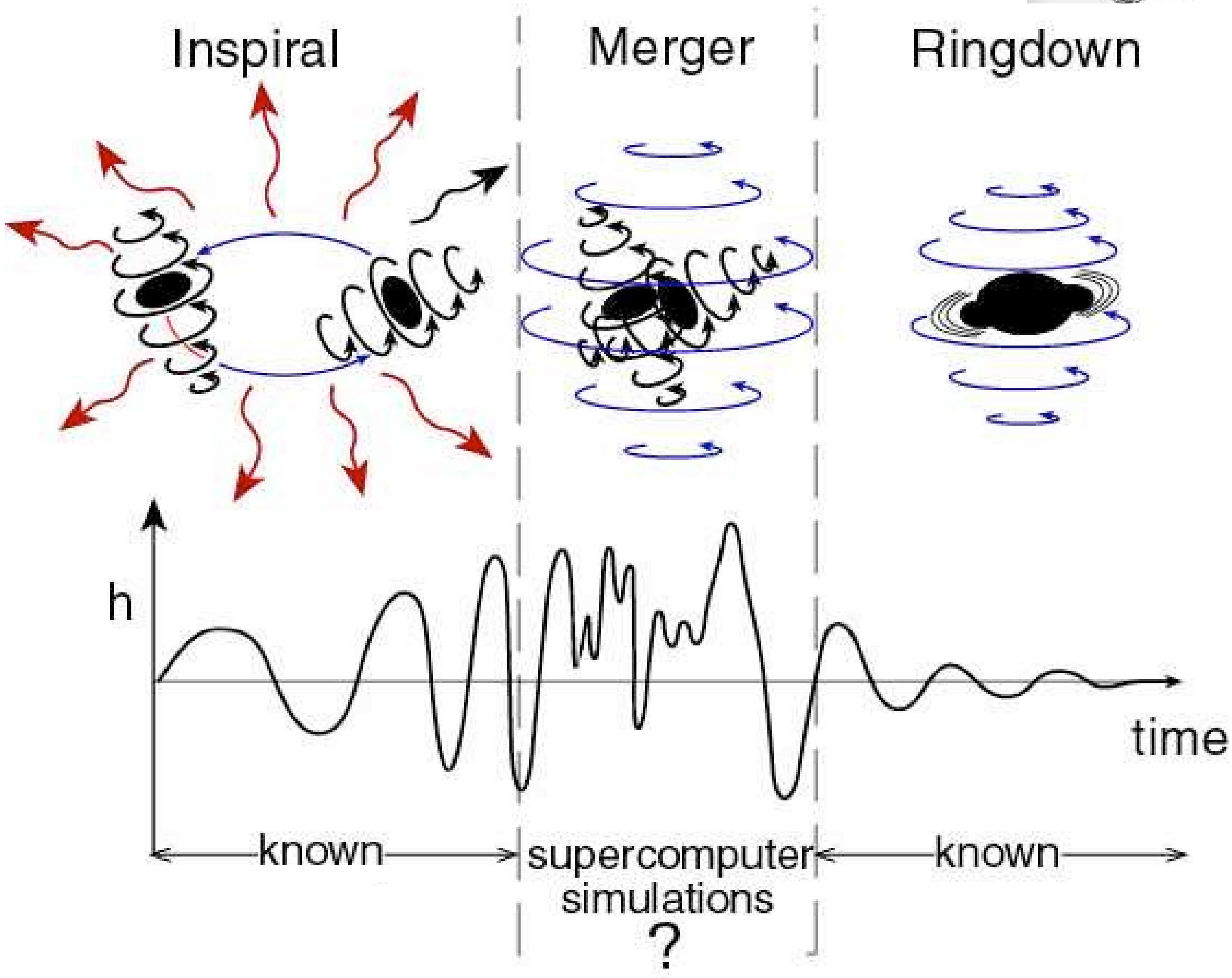
Black Hole Vibrations



- Normal Modes
  - Energy goes down horizon and out to infinity
  - "Hair" radiated away!
- Spectrum of modes:
  - f_{nlm} determined by *M,a*
- n=0, l=2, m=2 is easiest to excite; most slowly damped,
  - $f \approx (c/M)[1 - 0.63(1-a/M)^{0.3}]$
  - $Q = \pi f \tau \approx 2/(1-a/M)^{0.45}$
- Measure f & Q; infer *M* & *a*

Beyond the Golden Age: Black-Hole Collisions



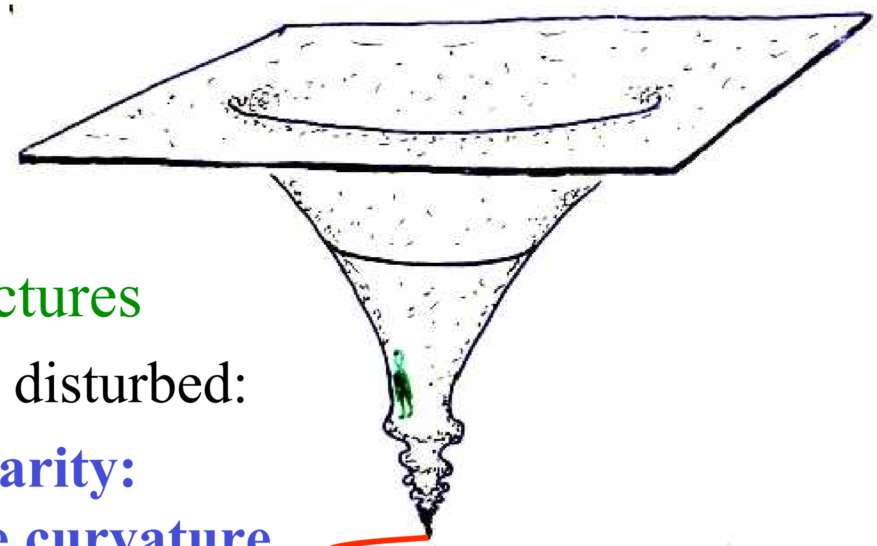


Nonlinear dynamics of warped spacetime

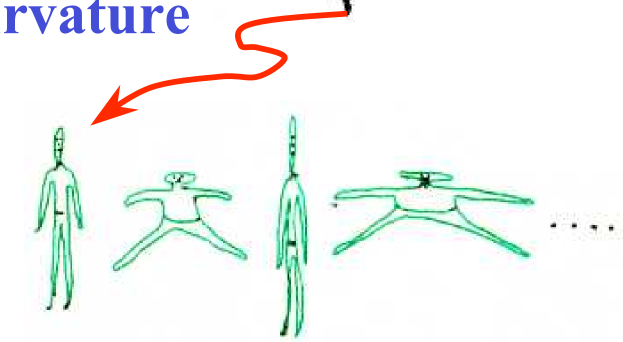
Beyond the Golden Age: Black-Hole Interior

- **Kerr exterior and Horizon**
  - are stable against small perturbations
  - shake off all "hairs" but two: M & a
- **Kerr interior**
  - "Tunnel" into another universe
  - Cauchy horizon at entrance to "tunnel"
    - Is unstable against small perturbations
    - For real black holes: replaced by a singularity

Black-Hole Interior



- ***The Singularity*** - two generic structures
  - Soon after hole is born or strongly disturbed:
    - **BKL spacelike, strong singularity: chaotic, oscillatory spacetime curvature**



  - After hole has settled into quiescent state:
    - **Israel-Poisson weak, null singularity:**
      - **Monotonically diverging tidal forces, but**
      - **You hit the singularity before being crushed**

Conclusion

- Gravitational wave observations (LISA and LIGO et al) will probe the structure and dynamics of black holes' warped spacetime in exquisite detail
  - A "golden age" of black-hole observation, to match the 1960's and 70's "golden age" of black-hole theory

- But observations will be limited to the holes' exterior.
  - In our lifetimes our only probes of black-hole interiors will be via pencil, paper, and supercomputer simulations (numerical relativity)