Weak-Strong Model for the Combined Effect of Beam-Beam Interaction and Electron Cloud

F. Zimmermann
CERN, 1211 Geneva 23, Switzerland

I review a weak-strong few-particle model for the combined effect of electron cloud and beam-beam interaction and present some numerical examples for PEP-II and KEKB.

1. INTRODUCTION

A number of indications suggest an interplay of the electron cloud and the beam-beam interaction. Both introduce a head-tail wake field and both induce a tune shift which varies along the length of the bunch. There is a strong evidence from weak-strong simulations that the two effects enhance each other, as illustrated in Figs. 1-3 [1].

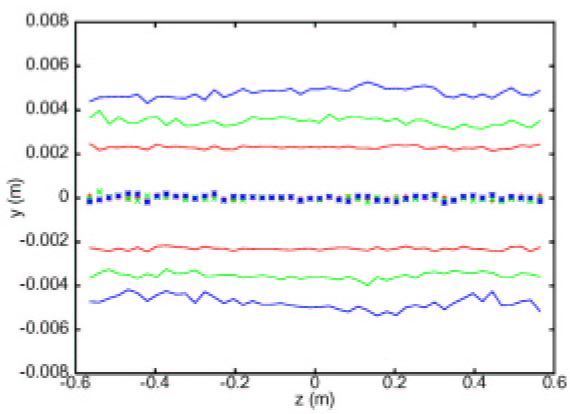

Figure 1: Rms beam size and centroid position along the length of a bunch after 0, 250 and 500 turns in the SPS simulated by the HEADTAIL code [2] for an electron cloud of density ρ=10^{12} m^{-3} [1] (G. Rumolo, 2001).

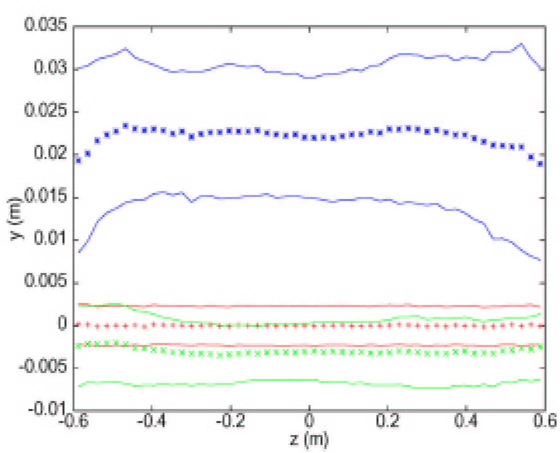

Figure 2: Rms beam size and centroid position along the length of a bunch after 0, 250 and 500 turns in the SPS simulated by the HEADTAIL code [2] for an electron cloud of density ρ=10^{12} m^{-3} and a rotation around the beam center on each turn representing a collision with an effective beam-beam parameter of ξ=-0.037, held constant [1] (G. Rumolo, 2001).

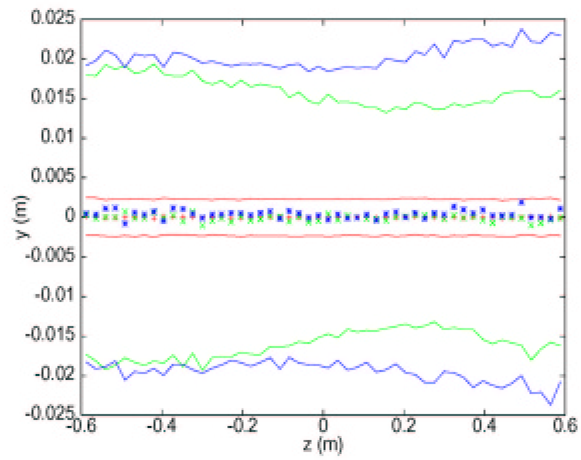

Figure 3: Rms beam size and centroid position along the length of a bunch after 0, 250 and 500 turns in the SPS simulated by the HEADTAIL code [2] for an electron cloud of density ρ=10^{12} m^{-3} and a rotation around the beam center on each turn representing a collision with an effective initial beam-beam parameter of ξ=-0.037, which then varies from turn to turn in accordance with the beam-size evolution [1] (G. Rumolo, 2001).

The threshold for the vertical blow up of the positron beam in KEKB and PEP-II is observed to be lower when both effects act simultaneously as compared to the situation when either only the beam-beam collision or only the electron cloud are present. This can be inferred by comparing (1) the specific luminosity attained for a few bunches and the nominal colliding beams, and (2) the vertical beam sizes measured by the synchrotron light monitor in presence and absence of the electron beam, as a function of positron beam current. In this paper I review a weak-strong analysis developed in 2001, which was presented at the Two-Stream Workshop at KEK [1]. This model was inspired by K. Cornelis [3]. It was initially devised to study the combined effect of space charge and electron cloud for the CERN SPS, but the weak-strong model remains the same, if the beam-beam interaction is considered instead of the space charge [1]. Subsequently, various strong-strong models for the combined effect of electron cloud and beam-beam interaction were considered by K. Ohmi and A. Chao. These models made partly inconsistent predictions [4].

2. MODEL

The specific luminosity for different bunch spacings at KEKB suggests that the combined effect of electron-cloud and beam-beam interaction is more harmful than the two phenomena individually. This synergy can be modelled in a weak-strong approximation by a few-particle model, where the electron cloud is represented both by a constant wake field coupling leading and trailing particles and by a linear tune shift along the bunch, and the beam-beam interaction is modelled by an inverse parabolic tune variation with longitudinal distance from the bunch centre. The model was described in Ref. [2]. Below we briefly review the main features.

In this model a bunch is represented by a few macro-particles. Two primary effects of the electron cloud are represented, namely the electrons first give rise to a transverse wake between leading and trailing particles, and second to a tune shift which increases roughly linear along the bunch, due to the accumulation of electrons at the transverse beam centre during the passage of the bunch (electron pinch). The beam-beam interaction adds to this a further tune variation, which we approximate by an inversely parabolic dependence on the longitudinal position. Both the electron-cloud tune shift and the beam-beam tune shift are referenced to a static coordinate system, thought to be centred at the closed orbit, and not to the centroid position of the macro-particles. The electron-cloud wake is considered to be constant, independent of the distance between driving and test particles. Not included in this model is the wake coupling from the beam-beam interaction [5], and neither are any nonlinear transverse forces.

In a two-particle model, the variation of the tune with longitudinal position does not yield any instability, while the variation of the tune with momentum error induces the head-tail instability (see [6], page 198). As first suggested by K. Cornelis, to observe an instability from the dependence of the tune on longitudinal position, a minimum number of 3 macro-particles is needed. These particles are taken to be equal in charge and they are distributed uniformly in synchrotron phase space:

$$z_1 = \hat{z}\cos\left(\frac{\omega_s s}{c}\right)$$

$$z_2 = \hat{z}\cos\left(\frac{\omega_s s}{c} + \frac{2\pi}{3}\right)$$

$$z_3 = \hat{z}\cos\left(\frac{\omega_s s}{c} + \frac{4\pi}{3}\right)$$

The longitudinal ordering of the three particles changes every 6th of a synchrotron period. Listing the leading particle first, the successive patterns are (1,3,2), (3,1,2), (3,2,1), (2,3,1), (2,1,3), and (1,2,3). The first two of these patterns are illustrated in Fig. 4.

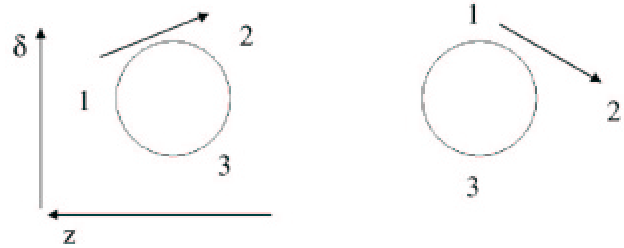

Figure 4: Schematic of three macro-particles in longitudinal phase space during the first and second 6th of a synchrotron period.

The variation of the macro-particle tune with longitudinal position is parametrized as:

$$\omega_\beta(z) = \omega_{\beta,0}\left(1 - a\frac{z}{\hat{z}} - b\frac{z^2}{\hat{z}^2}\right)$$

where a is due to the electron-cloud effect b due to the beam-beam interaction. A positive value of z means the particle is ahead of the bunch centre, and the parameter $\hat{z}$ denotes the length of the bunch. For the B factories, one has both $a>0$ and $b>0$, whereas for the SPS (space charge) or LHC (proton-proton collisions) $a>0$ and $b<0$.

The betatron phase of the jth particle at time $t=s/c$ is obtained by integration (after inserting the solution for the unperturbed motion):

$$\phi_j(s) = \overline{\omega}_\beta - \frac{a\omega_{\beta,0}}{\omega_s}\sin\left(\frac{\omega_s s}{c} + \phi_{j,0}\right) - \frac{b\omega_{\beta,0}}{4\omega_s}\sin\left(\frac{2\omega_s s}{c} + 2\phi_{j,0}\right)$$

where the electron-cloud tune shift introduces a modulation of the betatron phase at the synchrotron frequency and the beam-beam tune shift a modulation at twice this frequency. Adding the driving force from preceding bunches via the electron-cloud wake W_0, the betatron equation of motion for the jth particle becomes

$$y_j'' + \left[\frac{\omega_\beta(z)}{c}\right]^2 y_j = \sum_{n, z_n > z_j} \frac{N_b r_0 W_0}{3\gamma C} y_n$$

where the sum is over all previous bunches, and changes after every 6th of a synchrotron period. We evaluate the motion over the first two sixths of a period. The solutions for the later two thirds are then obtained by simple permutations. We make the usual ansatz of an unperturbed betatron motion, which is multiplied by a slowly varying amplitude $\tilde{y}_n$

$$y_n = \tilde{y}_n \exp[-i\phi_n(s)]$$

Inserting this ansatz into the equation of motion above and dropping small terms, i.e., second derivatives of $\tilde{y}_n$, we get

$$\tilde{y}_n' \approx \frac{iNr_0W_0c}{6\gamma C\omega_\beta} \sum_{m,z_m>z_n} \tilde{y}_m \exp\left[-i\phi_m(s)+i\phi_n(s)\right] \quad (1)$$

To proceed further, we assume that the phase differences between macro-particles are small, or that $|\varphi_m-\varphi_n|<<1$. Note that, as first pointed out by K. Oide, this assumption is not fulfilled, if the electron-cloud or beam-beam tune shifts are large and the synchrotron tune is small, in which case one would need to expand the exponential of a cosine into a series of Bessel functions. Under our simplifying assumption, we may expand the exponential to first order only, and we can then easily integrate over the first 6th of a synchrotron period.

$$\tilde{y}_n\left(\frac{T_sc}{6}\right) \approx \tilde{y}_n(0)$$
$$+ iD\sum_{m,z_m>z_n} \tilde{y}_m \left[\frac{T_sc}{6} - i\int_0^{T_sc/6}\left(\phi_m(s)-\phi_n(s)\right)ds\right]$$

where we have defined a new a parameter

$$D \equiv \frac{Nr_0W_0c}{6\gamma C\omega_\beta}$$

Besides D, we will find it convenient to introduce three further parameters, which completely determine the instability behaviour of our problem, namely

$$C \equiv D\frac{\pi c}{3\omega_s}$$

$$A \equiv D\frac{a\overline{\omega}_\beta c}{\omega_s^2}$$

$$B \equiv D\frac{b\overline{\omega}_\beta c}{8\omega_s^2}$$

The parameter C characterizes the strength of the electron cloud wake field, the parameter A is proportional to the product of the electron-cloud wake field and the electron-cloud induced tune shift (which may themselves be proportional to each other), and the parameter B is proportional to the product of the electron-cloud wake field and the beam-beam induced tune shift. Using A, B and C, the complex particle amplitudes after a 6th period are related to the initial amplitudes via:

$$\tilde{y}_1(\pi/3) \approx \tilde{y}_1(0)$$
$$\tilde{y}_3(\pi/3) \approx \tilde{y}_3(0) + \tilde{y}_1(0)\left[iC - \frac{3}{2}A - \frac{3}{2}B\right]$$
$$\tilde{y}_2(\pi/3) \approx \tilde{y}_2(0) + \tilde{y}_1(0)\left[iC - 3B\right]$$
$$+ \tilde{y}_3(0)\left[iC + \frac{3}{2}A - \frac{3}{2}B\right]$$

Similarly, we can write the solution for the 2nd 6th period as

$$\tilde{y}_3(2\pi/3) \approx \tilde{y}_3(\pi/3)$$
$$\tilde{y}_1(2\pi/3) \approx \tilde{y}_1(\pi/3)$$
$$+ \tilde{y}_3(\pi/3)\left[iC + \frac{3}{2}A + \frac{3}{2}B\right]$$
$$\tilde{y}_2(2\pi/3) \approx \tilde{y}_2(\pi/3) + \tilde{y}_1(\pi/3)\left[iC + 3B\right]$$
$$+ \tilde{y}_3(\pi/3)\left[iC - \frac{3}{2}A + \frac{3}{2}B\right]$$

We can rewrite these solutions in a more compact form as

$$\vec{y}(\pi/3) = M_{\pi/3}\vec{y}(0)$$
$$\vec{y}(2\pi/3) = M_{2\pi/3}\vec{y}(\pi/3)$$

where the two matrices are given by

$$M_{\pi/3} \equiv \begin{pmatrix} 1 & 0 & 0 \\ iC-3B & 1 & iC+\frac{3}{2}(A-B) \\ iC-\frac{3}{2}(A+B) & 0 & 1 \end{pmatrix}$$

$$M_{2\pi/3} \equiv \begin{pmatrix} 1 & 0 & iC+\frac{3}{2}(A+B) \\ iC-\frac{3}{2}(A-B) & 1 & iC+3B \\ 0 & 0 & 1 \end{pmatrix}$$

We recall that the ordering of the particles was (1,3,2), (3,1,2) for the first and second 6th of a synchrotron period, which is followed by (3,2,1), (2,3,1), and, finally, by (2,1,3) and (1,2,3). This change in ordering between thirds of a period is a simple permutation, which can be represented by the permutation matrix:

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

The total transformation over a full synchrotron period is

$$M_{tot} = \left(PM_{2\pi/3}M_{\pi/3}\right)^3 \equiv M_{1/3}^3$$
$$M_{1/3} \equiv PM_{2\pi/3}M_{\pi/3}$$

The stability of this system is determined by the eigenvalues of the matrix $M_{1/3}$.Only keeping terms of first and second order in A, B and C, and neglecting all higher-order cross products, the characteristic polymonial of $M_{1/3}$ becomes

$$p(\lambda) = 1 + \left(3iC + 3iBC + \frac{9}{4}A^2 + \frac{9}{4}B^2 + C^2\right)\lambda$$
$$+ 3iC\lambda^2 - \lambda^3$$

For example, without an electron cloud, one has A=B=C=D=0, and all eigenvalues lie on the unit circle, i.e., there is no instability.

The magnitude of the eigenvalues of $M_{1/3}$ translates into a growth time in units of seconds as

$$\tau_{\text{growth}} = \frac{T_s/3}{\ln|\lambda|}$$

An analogous calculation was repeated for four macro-particles, in order to explore the dependence of the growth time on the particle number. The resulting expressions can be found in Ref. [1].

Concerning the appropriate choice of values for the parameters *A*, *B*, *C*, and *D*, we note that, if the electron density ρ_e is known, the electron cloud wake can be estimated from [7]

$$W_0 = \frac{8\pi\rho_e C}{N_b}$$

and the electron-cloud induced tune shift from [8]

$$\Delta Q_{ec} = \frac{r_p}{2\gamma}\beta C\rho_e$$

This relation may also be used conversely, i.e., to infer the average electron density from the measured tune shift.

3. APPLICATION TO THE SPS

In Ref. [1], we presented the example of the CERN SPS. We considered the cases of the electron-cloud wake alone (*B=A=0*), the electron-cloud wake and the electron-cloud tune shift (*B=0*), and the last two plus the tune shift from the direct space-charge force, using the 3- and 4-particle models (parameters of the latter case are marked by a subindex ‘4’). The SPS parameters and the pertinent growth times are listed in Table 1. The growth rates change by 5-10% due to the beam-beam interaction. Larger changes in the growth rates are found for larger space-charge or beam-beam tune shifts. Also the eigenvector patterns show a larger difference than the eigenvalues.

The phase modulation amplitudes entering in the exponent of Eq. (1) are 1.7 and 2.0, for the electron-cloud and space-charge terms, respectively, and thus larger than 1, so that the subsequent expansion may not be a good approximation.

Table 1: Parameters of the CERN SPS and growth rates predicted for various configurations.

circumference	6900 m
average beta function	40 m
betatron tune	26.6
synchrotron tune	0.0046
beam momentum	26 GeV/c
rms bunch length	0.3 m
bunch population	10^{11}
electron density	10^{12} m-3
electron wake W_0	1.7×10^{6} m^{-2}
electron-cloud tune shift	0.0077
space-charge tune shift	-0.0365
τ for $C=2.4, B=0, A=0$	0.88 ms
τ for $C=2.4, B=0, A=3.8$	0.88 ms
τ for $C=2.4, B=-2.3, A=3.8$	0.91 ms
τ for $C_4=1.8, B_4=0, A_4=2.9$	0.71 ms
τ for $C_4=1.8, B_4=-1.7, A_4=2.9$	0.66 ms

4. APPLICATION TO PEP-II

The parameters for PEP-II and the instability growth times computed for nominal conditions with and without the beam-beam interaction are listed in Table 2. The phase modulation amplitudes entering in the exponent of Eq. (1) are 0.16 and 0.31. Hence, they and their differences are smaller than 1, and the linear expansion we have employed appears justified.

Table 2: Parameters of PEP-II and growth rates predicted for various configurations.

Circumference	2200 m
average beta function	18 m
betatron tune	36.5
synchrotron tune	0.03
beam momentum	3 GeV/c
bunch population	10^{11}
electron density	10^{12} m-3
electron wake W_0	5.5×10^{5} m^{-2}
electron-cloud tune shift	0.009
beam-beam tune shift	0.07
τ for $C=0.23, B=0, A=0$	3.9 ms
τ for $C=0.23, B=0, A=0.07$	3.3 ms
τ for $C=0.23, B=0.06, A=0.07$	3.1 ms
τ for $C_4=0.17, B_4=0, A_4=0.05$	0.9 ms
τ for $C_4=0.17, B_4=0.057, A_4=0.05$	0.5 ms

Figure 5 displays the rise times obtained from the 3- or 4-particle models as a function of the beam-beam tune shift on an exaggerated horizontal scale. The electron-cloud wake and tune shift are taken to be constant, as listed in Table 2. The figure shows that positive tune shifts are more harmful than negative ones. Positive beam-beam tune shifts partially compensate the electron cloud tune shift at the tail of the bunch.

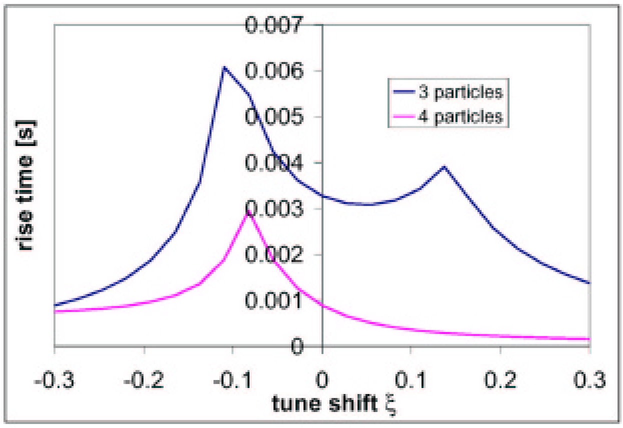

Figure 5: PEP-II growth time as a function of beam-beam parameter ? for a constant wake and tune shift due to the electron cloud, corresponding to ,computed for the 3 and 4-particle models.

It has been criticized that the multi-particle models do not predict a clear instability threshold, unlike the 2-particle model for the conventional head-tail instability. Figure 6 shows the growth rate computed in the 3 and 4-particle models without linear and parabolic tune shift representing electron pinch or beam-beam effect, but only keeping the constant wake coupling head and tail particles.

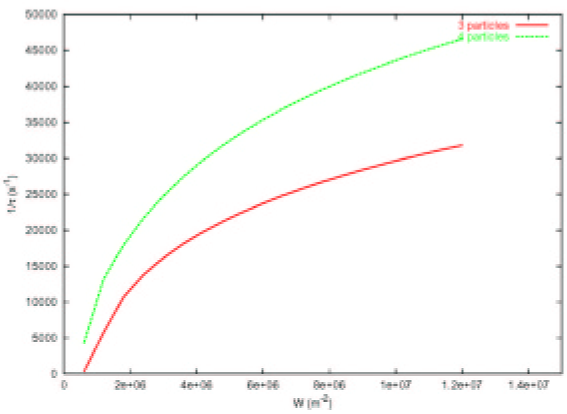

Figure 6: Growth rate in PEP-II of conventional head-tail instability without beam-beam interaction and without electron-cloud tune shift as a function of the strength of the wake-field, computed for the 3 and 4-particle models.

Though the growth rate is never exactly zero, Fig. 6 suggests an apparent threshold at W_0~6×10^5 m^{-2}, which is 40% smaller than the exact threshold of the 2-particle model:

$$W_{thr,2part} \approx \frac{8\gamma C \omega_\beta \omega_s}{\pi r_e c^2 N_b} \approx 1.1\times10^6\,\mathrm{m}^{-2}$$

Therefore, we expect that for practical purposes also the multi-particle model can explain observed 'thresholds'.

5. APPLICATION TO KEKB

Assuming a net electron density of 6×10^{11} m-3, which is suggested by tune-shift measurements [9], we can compute the dependence of the head-tail growth rates on the beam-beam parameter for the KEKB parameters listed in Table 3. In this case, the phase modulation amplitudes entering in the exponent of Eq. (1) are 0.21 and 0.48, which is smaller than 1, so that our linear expansion may apply. The resulting growth times for the 3 and 4-particle models are shown in Fig. 7. As in Fig. 5, there still is a large difference between the cases of 3 and 4 particles, which indicates a slow convergence. Nevertheless, the curves suggest that the growth times are shorter than 1 ms, and that the beam-beam interaction for the B factory (positive ξ) likely acts destabilizing and possibly reduces the rise time by a factor of 2.

Table 3: Parameters of KEKB and growth rates predicted for various configurations.

Circumference	3016 m
average beta function	15 m
betatron tune	43.55
synchrotron tune	0.019
beam momentum	3.5 GeV/c
bunch population	8×10^{10}
electron density	6×10^{11} m-3
electron wake W_0	5.7×10^5 m^{-2}
electron-cloud tune shift	0.006
beam-beam tune shift	0.05
τ for $C=0.30, B=0, A=0$	4.1 ms
τ for $C=0.30, B=0, A=0.08$	3.5 ms
τ for $C=0.30, B=0.09, A=0.08$	3.0 ms
τ for $C_4=0.22, B_4=0, A_4=0.06$	0.75 ms
τ for $C_4=0.22, B_4=0.07, A_4=0.06$	0.49 ms

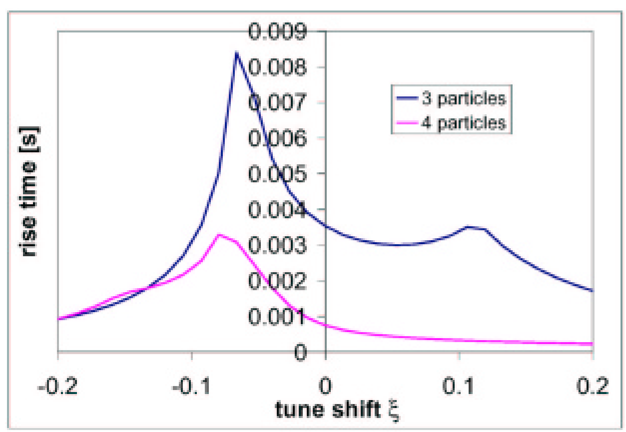

Figure 7: Growth time of the head-tail instability in KEKB driven by electron cloud and beam-beam interaction for a cloud density of ρ=6×10^{10} m^{-3} as a function of beam-beam tune shift, according to the 3 and 4-particle models.

As we did for PEP-II, ignoring the electron-cloud tune shift and the beam-beam tune shift, we can again obtain the conventional growth rate as a function of the wake-field strength. It is illustrated in Fig. 8 for the 3- and 4-particle models. Though there is no mathematical threshold, we infer an apparent threshold around a wake-field strength of 6-

$7x10^5$ m^{-2}, about 20% less than the threshold predicted by the classical 2-particle model:

$$W_{thr,2part} \approx \frac{8\gamma C\omega_\beta\omega_s}{\pi r_e c^2 N_b} \approx 8.4\times10^5 \mathrm{m}^{-2}.$$

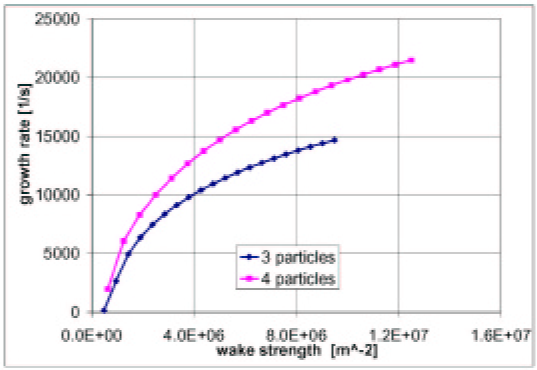

Figure 8: Growth rate of conventional head-tail instability in KEKB without tune shift from electron pinch and without tune shift due to beam-beam interaction as a function of the head-tail wake strength, predicted by 3 and 4-particle models.

6. CONCLUSION

The beam-beam interaction introduces a Gaussian variation of the betatron tune along the bunch. Simulations show that this additional tune variation enhances the electron-cloud instability. We developed an analytical model, where a bunch consists of 3 or 4 macroarticles, and where the electron cloud is represented by a constant wake field and by a linear tune shift along the bunch, and the beam-beam interaction by a parabolic tune shift. The model predicts that the beam-beam interaction acts destabilizing for PEP-II and KEKB and it suggests typical rise times below 1 ms. The agreement between model and simulation or observation improves with an increasing number of macro-particles.

Acknowledgments

I thank G. Rumolo for his fruitful collaboration over many years. I also thank K. Cornelis, K. Ohmi, K. Oide and E. Perevedentsev, for various helpful discussions and suggestions.

References

[1] F. Zimmermann and G. Rumolo, presented at the Two-Stream Instability workshop, KEK, Tsukuba, also published in ‘Contributions of the SL-AP Group to the Twi-Stream Instabilities Workshop’, CERN-SL-2001-067 (2001).

[2] G. Rumolo, F. Zimmermann, “Practical User Guide for HEADTAIL,” CERN-SL-Note-2002-036 AP (2002).

[3] K. Cornelis, private communication (2001).

[4] K. Ohmi and A. Chao, “Combined Phenomena of Beam-Beam and Beam Electron-Cloud effects in Circular e+e- Colliders,” Proc. ECLOUD’02, Geneva, 15-18 April, 2002, CERN-2002-001 (2002).

[5] E.A. Perevedentsev, “Simplified Theory of the Head-Tail Instability of Colliding Bunches,” Proc. PAC99 New York, p. 1521 (1999).

[6] A. Chao, “Physics of Collective Beam Instabilities in High Energy Accelerators,” J. Wiley, New York (1993).

[7] K. Ohmi, F. Zimmermann, “Head-Tail Instability due to Electron Cloud in Positron Storage Rings,” PRL 85, 3821-3824 (2000).

[8] K. Ohmi, F. Zimmermann, “Study of Coherent Tune Shift Caused by Electron Cloud in Positron Storage Rings,” Proc. APAC2001, Beijing, 2001, and CERN-SL-2001-0058 (2001).

[9] F. Zimmermann, “New Simulations of Positron Beam electron-Cloud Build Up and Electron-Beam Coupled Bunch Instabilities for KEKB and Super-KEKB,” KEK, October 2003, unpublished.