Emittance Partitioning in Photoinjectors

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Motivation

- We need very small transverse emittances for a future XFEL – driven by energy diffusion:

$$\frac{\Delta E_{rms}}{E} = \sqrt{\frac{55}{48\sqrt{3}}}\sqrt{\frac{\hbar e B^3}{4\pi\varepsilon_0 m^5 c^6}}\,\gamma L^{1/2}$$

- Induced rms energy spread (100-m wiggler) is 0.015% at 20 GeV (probably okay) and 0.026% at 35 GeV (probably not okay)
- We want 50-keV photons – overlap of electron beam and photon beam puts an upper bound of about 0.15 μm for the transverse emittances
- We want 250 pC to 500 pC of bunch charge (from the needed number of photons), violates the accepted scaling:

$$\varepsilon \approx 1\mu\mathrm{m}\left(\frac{q}{\mathrm{nC}}\right)^{1/2}$$ (for example Braun from Monday)

(which would indicate about 20 pC)

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Motivation continued

- Phase space from a photoinjector is very cold – overall volume is more than sufficient for our needs:

"typical" photoinjector at 0.5 nC:

$\varepsilon_x \sim 0.7$ mm mrad
$\varepsilon_y \sim 0.7$ mm mrad
$\varepsilon_z \sim 1.4$ mm mrad

volume $\sim 0.7\ (\mu m)^3$

our needs:

$\varepsilon_x \sim 0.15$ mm mrad
$\varepsilon_y \sim 0.15$ mm mrad
$\varepsilon_z \sim 100$ mm mrad

volume $\sim 2.3\ (\mu m)^3$

this comes from 0.01% energy spread and 80 fs at 20 GeV

Can we control the phase-space partitioning?

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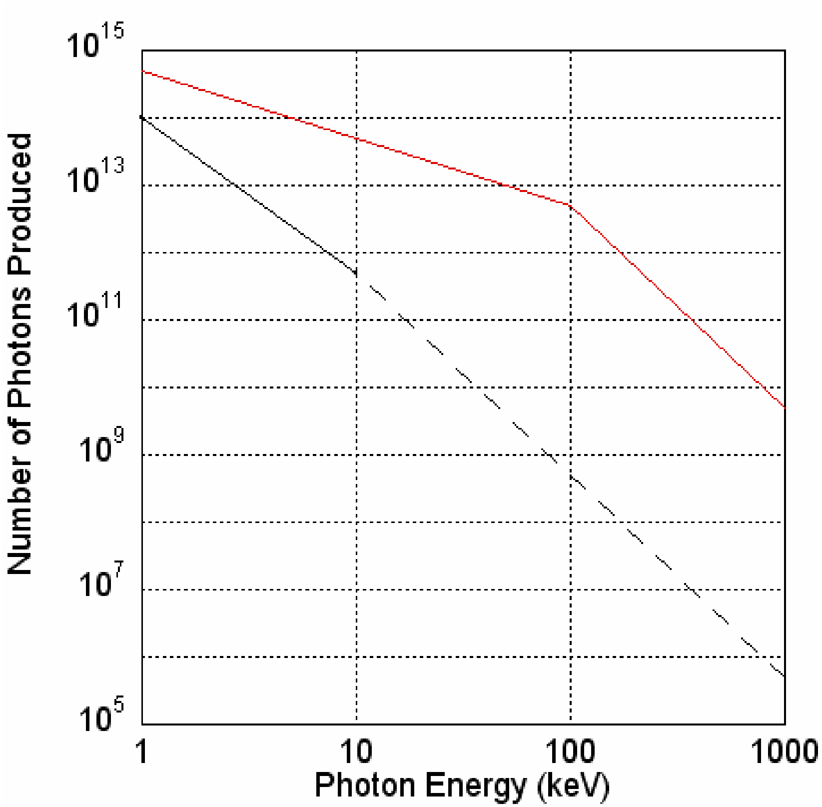
Motivation continued

- We can get huge gains in peak photon flux if we can partition the phase space:

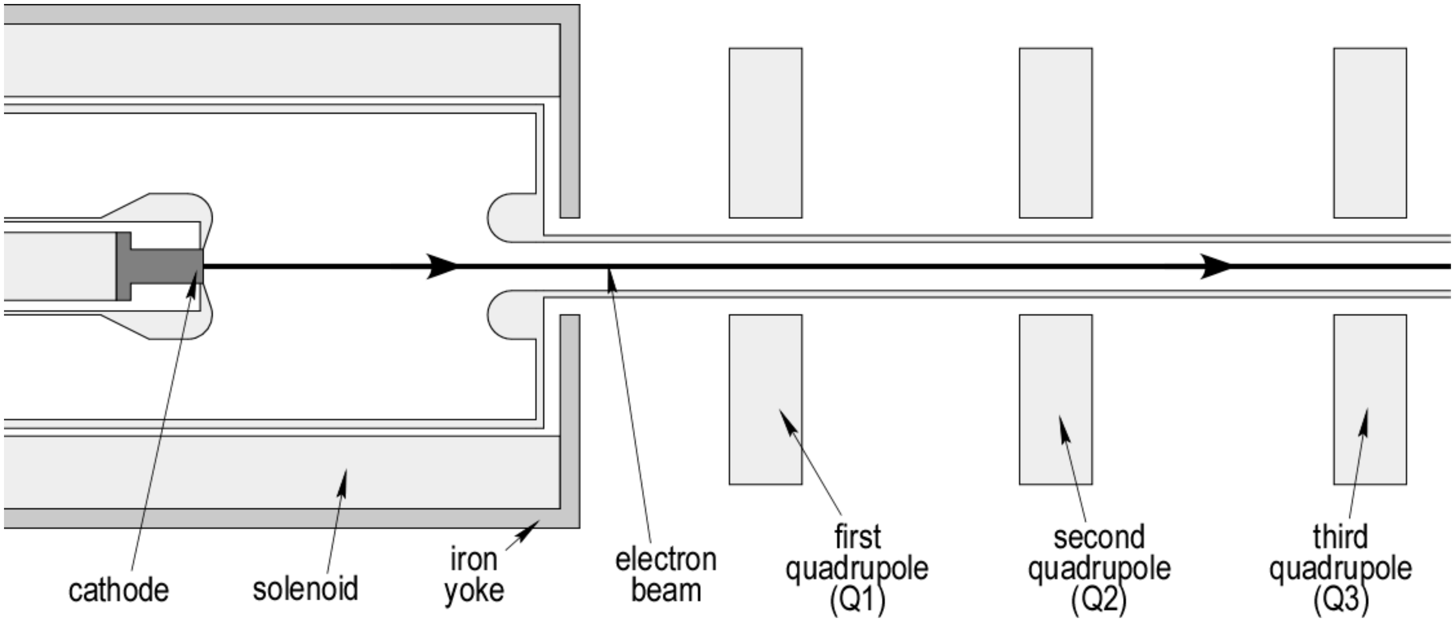


Black line: $$\varepsilon \approx 1\mu\mathrm{m}\left(\frac{q}{\mathrm{nC}}\right)^{1/2}$$

Red line: Optimal partitioning

Dashed line: CSR validity questions

Review of FBT – Correlations plus Three Skew Quadrupoles:



Shove some of the horizontal emittance into the vertical emittance by introducing a correlation at the cathode and removing it with the downstream optics

$$\varepsilon_x = \frac{(\varepsilon_{instrinsic})^2}{2\mathbf{L}} \qquad \varepsilon_y = 2\mathbf{L} \qquad \text{where} \qquad \mathbf{L} = \frac{eB_{cat}R_{cat}^2}{8\gamma\beta cm}$$

Limitations of Current Photoinjector and FBT/EEX Technologies

Photoinjector SOA at 0.5 nC (say ~mm cathode radius and 2 ps drive laser)

$\varepsilon_x \sim 0.7$ mm mrad
$\varepsilon_y \sim 0.7$ mm mrad
$\varepsilon_z \sim 1.4$ mm mrad

Emittance generation plan (KJK – AAC08)
Start with 200 fs laser, bigger cathode:

	"Standard" FBT stage	*"Standard" EEX stage*
$\varepsilon_x \sim 2.1$ mm mrad	$\varepsilon_x \sim 0.14$ mm mrad	$\varepsilon_x \sim 0.14$ mm mrad
$\varepsilon_y \sim 2.1$ mm mrad	$\varepsilon_y \sim 35$ mm mrad	$\varepsilon_y \sim 0.14$ mm mrad
$\varepsilon_z \sim 0.14$ mm mrad	$\varepsilon_z \sim 0.14$ mm mrad	$\varepsilon_z \sim 35$ mm mrad

The problem is that photoinjectors don't scale this way

FBTs and EEXs Arise From Conservation of Certain Quantitites: Eigen-Emittances

- Let ♦ denote the beam second moment matrix

$$J = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

- The eigenvalues of J♦ are called eigen-emittances

- Eigen-emittances are invariant under all linear symplectic transformations, which include all ensemble electron beam evolution in an accelerator

  - however, the eigen-emittances can be *exchanged* among the x-p_x, y-p_y, z-p_z phase planes

- We can control the formation of the eigen-emittances by controlling correlations when the beam is generated (like in a FBT)

- We recover the eigen-emittances as the beam rms emittances when all correlations are removed

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More About Eigen-Emittances

- Dragt's code MaryLie has a procedure, since the 1990's, to calculate eigen-emittances and the matrix needed to recover the eigen-emittances (so we are not blind in this process)
    - Invariance of eigen-emittances also known to Courant (1966). Origin of this goes back to J. Williamson (1936).

- Straightforward to generate the right eigen-emittances, issue with implementing this concept is to ensure that nonlinearities (especially nonlinear correlations) do not interfere with our ability to "unwind" the linear correlations

- Not clear how nonlinear emittance growths affect the partitioning

- The FBT (and FNAL demonstration) is an existence proof that you can in one regime

- Emittance compensation is an existence proof that you can in another regime (well, sort of; mostly nonlinear correlation)

Some Definitions

Definitions:

$$\sigma = \begin{pmatrix} \langle x^2 \rangle & \langle xx' \rangle & \langle xy \rangle & \langle xy' \rangle & \langle x(c\Delta t) \rangle & \left\langle x\frac{(\Delta\beta\gamma)}{\gamma} \right\rangle \\ \langle xx' \rangle & \langle x'^2 \rangle & \langle x'y \rangle & \langle x'y' \rangle & \langle x'(c\Delta t) \rangle & \left\langle x'\frac{(\Delta\beta\gamma)}{\gamma} \right\rangle \\ \langle xy \rangle & \langle x'y \rangle & \langle y^2 \rangle & \langle yy' \rangle & \langle y(c\Delta t) \rangle & \left\langle y\frac{(\Delta\beta\gamma)}{\gamma} \right\rangle \\ \langle xy' \rangle & \langle x'y' \rangle & \langle yy' \rangle & \langle y'^2 \rangle & \langle y'(c\Delta t) \rangle & \left\langle y'\frac{(\Delta\beta\gamma)}{\gamma} \right\rangle \\ \langle x(c\Delta t) \rangle & \langle x'(c\Delta t) \rangle & \langle y(c\Delta t) \rangle & \langle y'(c\Delta t) \rangle & \langle (c\Delta t)^2 \rangle & \left\langle z\frac{(\Delta\beta\gamma)}{\gamma} \right\rangle \\ \left\langle x\frac{(\Delta\beta\gamma)}{\gamma} \right\rangle & \left\langle x'\frac{(\Delta\beta\gamma)}{\gamma} \right\rangle & \left\langle y\frac{(\Delta\beta\gamma)}{\gamma} \right\rangle & \left\langle y'\frac{(\Delta\beta\gamma)}{\gamma} \right\rangle & \left\langle z\frac{(\Delta\beta\gamma)}{\gamma} \right\rangle & \left\langle \left(\frac{(\Delta\beta\gamma)}{\gamma}\right)^2 \right\rangle \end{pmatrix}$$

$$\varepsilon_{n,x} = \beta\gamma\sqrt{\langle x^2 \rangle\langle x'^2 \rangle - \langle xx' \rangle^2} \quad \varepsilon_{n,y} = \beta\gamma\sqrt{\langle y^2 \rangle\langle y'^2 \rangle - \langle yy' \rangle^2} \quad \varepsilon_{n,z} = \beta\gamma\sqrt{\langle (c\Delta t)^2 \rangle \left\langle \left(\frac{\Delta(\beta\gamma)}{\gamma}\right) \right\rangle - \left\langle (c\Delta t)\frac{\Delta(\beta\gamma)}{\gamma} \right\rangle^2}$$

$$\sigma_x^2 = \langle x^2 \rangle \quad \sigma_y^2 = \langle y^2 \rangle \quad \sigma_z^2 = \langle z^2 \rangle \quad \sigma_{x'}^2 = \langle x'^2 \rangle \quad \sigma_{y'}^2 = \langle y'^2 \rangle \quad \sigma_{z'}^2 = \left\langle \left(\frac{\Delta(\beta\gamma)}{\gamma}\right)^2 \right\rangle$$

Slides to Build “Physical Intuition”

Symplectic transformations obeys:

$$J = M^T J M \qquad J = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$$

Symplectic transformations look like:

$$R_{skew} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & a & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ a & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad a = \frac{e}{\gamma\beta mc}\int B' dl$$

And lead to these types of correlations:

$$\sigma_2 = R_{skew}\sigma_1 R_{skew}^T = \begin{pmatrix} \sigma_x^2 & 0 & 0 & a\sigma_x^2 & 0 & 0 \\ 0 & \sigma_{x'}^2 + a^2\sigma_y^2 & a\sigma_y^2 & 0 & 0 & 0 \\ 0 & a\sigma_y^2 & \sigma_y^2 & 0 & 0 & 0 \\ a\sigma_x^2 & 0 & 0 & \sigma_{y'}^2 + a^2\sigma_x^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_z^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{z'}^2 \end{pmatrix}$$

Note the signs

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"Nonsymplectic" correlations look like:

$$\sigma_{axial\ \ field} = \begin{pmatrix} \sigma_x^2 & 0 & 0 & -a\sigma_x^2 & 0 & 0 \\ 0 & \sigma_{x'}^2 + a^2\sigma_y^2 & a\sigma_y^2 & 0 & 0 & 0 \\ 0 & a\sigma_y^2 & \sigma_y^2 & 0 & 0 & 0 \\ -a\sigma_x^2 & 0 & 0 & \sigma_{y'}^2 + a^2\sigma_x^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_z^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{z'}^2 \end{pmatrix}$$

$$a = \frac{e}{2\gamma\beta mc} B_{cath} \left(\frac{R_{cath}}{R_{beam}} \right)^2$$

A transversely deflecting rf cavity looks like a skew quad in x-z:

$$R_{rf} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & a & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ a & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$a = eAd \frac{\pi}{b} \frac{1}{m\gamma\beta c}$$

With transversely deflecting rf cavities and "fancy drifts", we can build x-z FBTs

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More generally for a "non-symplectic" beam correlation we can diddle with the correlations to leave only one (by using symplectic transformations):

$$\sigma_{reduced,2} = \begin{pmatrix} \sigma_1^2 & 0 & \alpha\sigma_1^2 & 0 \\ 0 & \sigma_{1'}^2 & 0 & 0 \\ \alpha\sigma_1^2 & 0 & \sigma_2^2 + \alpha^2\sigma_1^2 & 0 \\ 0 & 0 & 0 & \sigma_{2'}^2 \end{pmatrix} \qquad \sigma_{reduced,1} = \begin{pmatrix} \sigma_1^2 & 0 & 0 & \alpha\sigma_1^2 \\ 0 & \sigma_{1'}^2 & 0 & 0 \\ 0 & 0 & \sigma_2^2 & 0 \\ \alpha\sigma_1^2 & 0 & 0 & \sigma_{2'}^2 + \alpha^2\sigma_1^2 \end{pmatrix}$$

$$\varepsilon_{1,2}^2 = \frac{1}{2}\left\{\left[\sigma_1^2\sigma_{1'}^2 + \sigma_2^2\sigma_{2'}^2 + \alpha^2\sigma_1^2\sigma_{2'}^2\right] \pm \sqrt{\left(\sigma_1^2\sigma_{1'}^2 + \sigma_2^2\sigma_{2'}^2 + \alpha^2\sigma_1^2\sigma_{2'}^2\right)^2 - 4\sigma_1^2\sigma_{1'}^2\sigma_2^2\sigma_{2'}^2}\right\}$$

$$\varepsilon_{1,2}^2 = \frac{1}{2}\left\{\left[\sigma_1^2\sigma_{1'}^2 + \sigma_2^2\sigma_{2'}^2 + \alpha^2\sigma_1^2\sigma_2^2\right] \pm \sqrt{\left(\sigma_1^2\sigma_{1'}^2 + \sigma_2^2\sigma_{2'}^2 + \alpha^2\sigma_1^2\sigma_2^2\right)^2 - 4\sigma_1^2\sigma_{1'}^2\sigma_2^2\sigma_{2'}^2}\right\}$$

which does reduce nicely to KJK's FBT case. But it also tells us how to design arbitrary FBTs with initially non round beams (like for x-z FBTs).

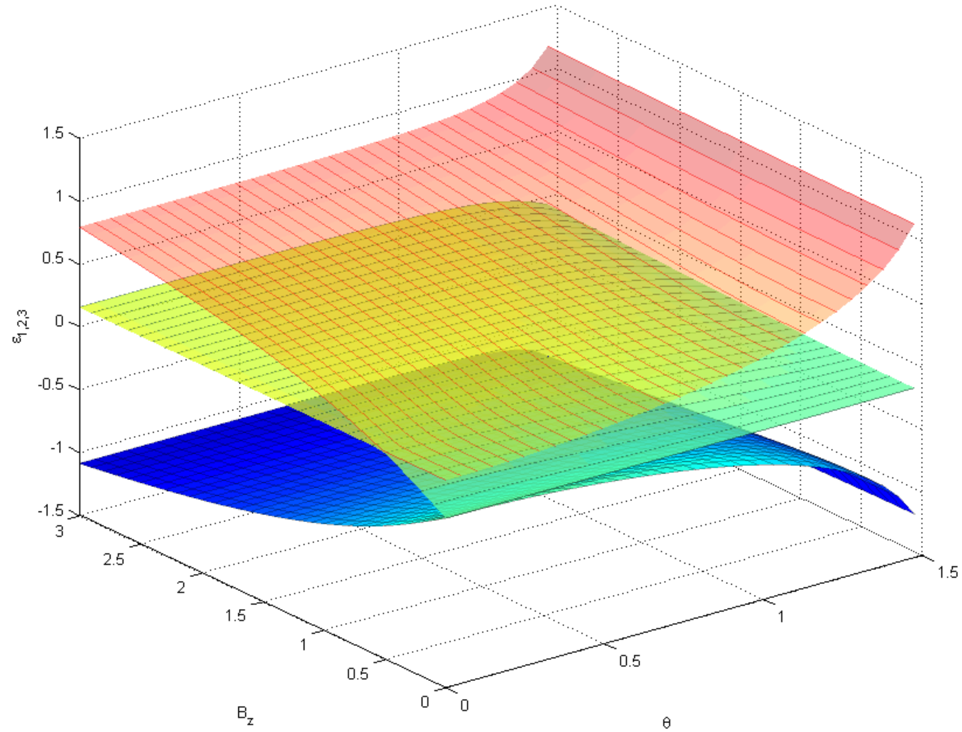
We Know of 3 Correlations that Work

1. If the initial beam has a 25:1 aspect ratio (meets the emittance requirement in one transverse direction, there is the trivial solution of an z-x' correlation where we build an x-z FBT

2. Also, with the same 25:1 aspect ratio, a simple x-z correlation works

3. With an aspect ratio near 2 (elliptical beam), an x-z correlation with axial magnetic field works

Baseline is this last correlation
- next step is to model this design to understand if we can preserve the correlations through acceleration to ~ GeV



How We Find the Eigen-Emittances

$JS = \lambda J \sigma$ Scale for small norm

$M = e^{JS}$ Taylor expansion, eigenvalues will be on unit circle

$N = A^{-1} M A$ Look in Alex Dragt's book to construct *A* (from eigenvalues) where *N* is in normal form – called Williamson form

$E = A \sigma A^T$ Same *A* is used to find eigen-emittances (only strictly if they are not degenerate, but we haven't seen any failures of this algorithm yet)