Noise effects in seeding

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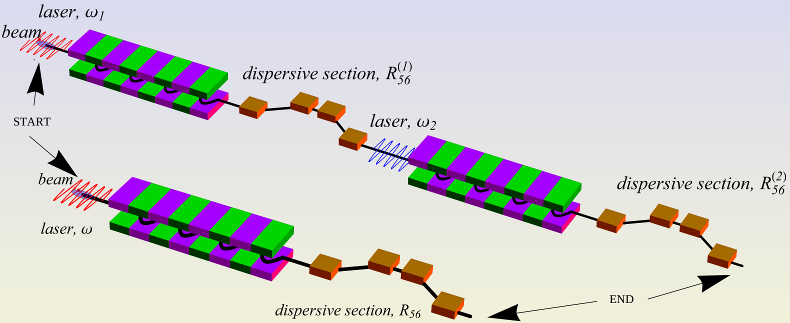
Motivation and introduction

- It is widely accepted that the HGHG seeding amplifies noise as m^2 where m is the harmonic number. If this is also applicable to EEHG, then m is a large number $m = 40 - 100$.
- The actual mechanism of noise amplification in the original papers is not clear. What is exactly the noise, and how does one measure it?
- Z. Huang made an important step (FEL conf. 2006) in developing theoretical approach to the problem
- In this work we will try to establish a framework for theoretical approach to the problem and develop a simple model for noise effects in seeding.

The problem

We assume a shot noise in the beam at the entrance to the seeding device. What is the noise at the exit?



We will use "hats" for variables at the exit, no hats for variables at the entrance.

Radiation form-factor

Radiation at frequency ω of an ensemble of N particles in 1D model is controlled by

$$\mathcal{F}(\omega) = \left| \sum_{j}^{N} e^{ik\hat{z}_j} \right|^2 = \sum_{j,l=1}^{N} e^{ik(\hat{z}_j - \hat{z}_l)},$$

where $k = \omega/c$, and $\hat{z}_j$ is the z coordinate of j-th particle after the seeding device.
Separate the terms with $j = l$

$$N + \sum_{j \neq l} e^{ik(\hat{z}_j - \hat{z}_l)}$$

The first term is the shot noise. The second term is responsible for 1) coherent radiation due to nonuniform distribution of particles in the beam (seeding), 2) radiation due to correlation of positions of different particles.

$$F(k) = \frac{1}{N} \sum_{j \neq l} e^{ik(\hat{z}_j - \hat{z}_l)}$$

Averaged form-factor

We will actually seek the averaged value of F

$$\langle F(k)\rangle = \frac{1}{N}\langle \sum_{j\neq l} e^{ik(\hat{z}_j-\hat{z}_l)}\rangle = \frac{N(N-1)}{N}\langle e^{ik(\hat{z}_1-\hat{z}_2)}\rangle \approx N\langle e^{ik(\hat{z}_1-\hat{z}_2)}\rangle$$

The averaging here should be performed with the help of a two-particle distribution function $\hat{f}_2$. In 1D model, we assume that the particles are characterized by the longitudinal coordinate in the beam $\hat{z}$ and the relative energy deviation $\hat{\eta} = \Delta E/E_0$, $\hat{f}_2(\hat{z}_1, \hat{\eta}_1, \hat{z}_2, \hat{\eta}_2)$.

$$\langle F(k)\rangle = N\int e^{ik(\hat{z}_1-\hat{z}_2)}\hat{f}_2(\hat{z}_1, \hat{\eta}_1, \hat{z}_2, \hat{\eta}_2)\, d\hat{z}_1\, d\hat{z}_2\, d\hat{\eta}_1\, d\hat{\eta}_2$$

Two-particle distribution function

The two-particle distribution function has correlations in it

$$\hat{f}_2(\hat{z}_1, \hat{\eta}_1, \hat{z}_2, \hat{\eta}_2) = \hat{f}_1(\hat{z}_1, \hat{\eta}_1)\hat{f}_1(\hat{z}_2, \hat{\eta}_2) + \hat{g}_2(\hat{z}_1, \hat{\eta}_1, \hat{z}_2, \hat{\eta}_2)$$

The one-particle functions include the effect of density modulation in the beam. The correlation function $\hat{g}_2$ might be responsible for additional effects (noise amplification or suppression).

To find $\hat{g}_2$ one has to solve truncated BBGKY equations (O. Shevchenko, N. Vinokurov). This would be a consistent and adequate approach to the problem, but it is technically complicated. Part of the complexity lies in the retardation effects of the interaction between the particles.

Using initial distribution function

A possible way to avoid using $\hat{f}_2$: work with the variables and the distribution function at the entrance to the system. If one knows transformation from the initial coordinates z_i, η_i to the final ones $\hat{z}_i$, $\hat{\eta}_i$, one can use averaging over the initial coordinates (at the entrance to the seeding device), assuming that the initial distribution function $f_N(z_1, \eta_1, z_2, \eta_2, \ldots, z_N, \eta_N)$ is known.

$$\langle F(k) \rangle = N \int e^{ik[\hat{z}_1(z_1,\eta_1,z_2,\eta_2,\ldots,z_N,\eta_N) - \hat{z}_2(z_1,\eta_1,z_2,\eta_2,\ldots,z_N,\eta_N)]}$$
$$f_N(z_1, \eta_1, z_2, \eta_2, \ldots, z_N, \eta_N) dz_1 dz_2 \ldots dz_N \, d\eta_1 d\eta_2 \ldots d\eta_N$$

It is reasonable to assume that initially there are no correlations in the beam

$$f_N(z_1, \eta_1, z_2, \eta_2, \ldots, z_N, \eta_N) = f_1(z_1, \eta_1) f_1(z_2, \eta_2) \ldots f_1(z_N, \eta_N)$$

Initial distribution function

We will assume a uniform distribution of particles in the bunch with Maxwellian energy distribution,

$$f_1(z,\eta) = \frac{1}{\sqrt{2\pi}\sigma_\eta L} e^{-\eta^2/2\sigma_\eta^2}, \qquad -\frac{1}{2L} < z < \frac{1}{2L},$$

and $f_1 = 0$ otherwise, where L is the length of the bunch.

To discard effects due to the finite length of the beam, we consider the limit $L \to \infty$, $N \to \infty$, but $N/L \to n_0$.

Expressing final variables through initial ones

The final variables are changed because of 1) energy modulation of the beam and 2) due to interaction of the electrons in the modulator. The interaction can be caused by Coulomb forces, or in the process of radiation in the undulator or in bends.

Consider the HGHG seeding. Assume interaction of electrons in the undulator.

The beam is first modulated in energy at wavelength λ_0 and then sent through a chicane with the strength R_{56}.

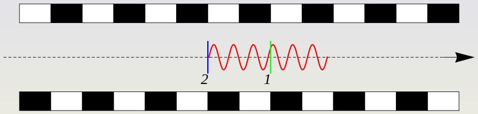
$$\hat{\eta}_i = \eta_i - \Delta\eta \sin(k_0 z_i) + \sum_{j \neq i} h(z_i - z_j)$$

$$\hat{z}_i = z_i + R_{56}\left[\eta_i - \Delta\eta \sin(k_0 z_i) + \sum_{j \neq i} h(z_i - z_j)\right]$$

$\Delta\eta$ is the amplitude of the energy modulation, $k_0 = 2\pi/\lambda_0$. h is the energy exchange function between the two particles in the undulator.

Interaction of particles in the undulator

Consider particles 1 and 2 of the beam traveling though an undulator, 1 is ahead of 2, $\zeta = z_1 - z_2 > 0$. The second particle emits electromagnetic wave (undulator radiation) which goes ahead and interacts with the first particle and changes its relative energy by $h(\zeta)$.



Interaction of particles in the undulator

Assuming a helical undulator, the energy change is (Stupakov&Krinsky, PAC03)

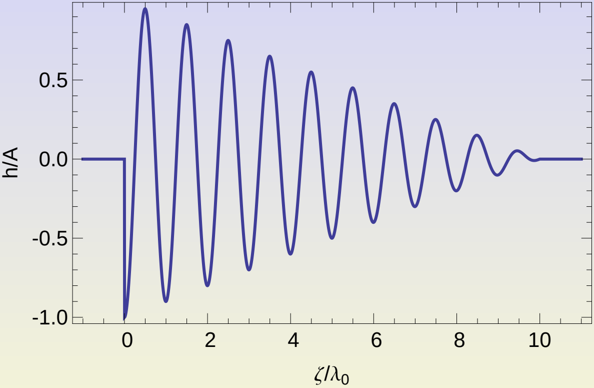
$$h(\zeta) = \begin{cases} -A\left(1 - \frac{\zeta}{\lambda_0 N_u}\right)\cos k_0\zeta, & N_u\lambda_0 > \zeta > 0 \\ 0, & N_u\lambda_0 < \zeta \text{ or } \zeta < 0 \end{cases}$$

where the parameter A is

$$A = 4\pi \frac{r_e L_u}{S\gamma} \frac{K^2}{1 + K^2}$$

with L_u the number of undulator periods, K the undulator parameter, S the transverse area of the beam.

Interaction of particles in the undulator



We now have a mathematical formulation of the noise problem.

Limit of no interaction and small interaction

In the limit of no interaction in the undulator, $h \to 0$, we recover the HGHG seeding with the right bunching factors

$$\langle F(k) \rangle = 2\pi n_0 \sum_m e^{-m^2 k_0^2 R_{56}^2 \sigma_\eta^2} J_m(m k_0 R_{56} \Delta\eta)^2 \delta(k - m k_0)$$

where $n_0 = N/L = I/(I_A r_e)$ the number of particles in the beam per unit length

In the limit of small h, one can use perturbation theory. There are small corrections to $\langle F(k) \rangle$ of the order of

$$\sim n_0 N_u \lambda_0 \, A \, R_{56} m k_0$$

Depending on k, they might have both positive and negative signs.

Suppression of seeding due to interaction

We find suppression of seeding if the following parameter becomes large

$$\Pi = n_0 N_u \lambda_0 m^2 k_0^2 R_{56}^2 A^2 \geq 1$$

Physical meaning of $\sqrt{\Pi} = \sqrt{n_0 N_u \lambda_0} A R_{56} m k_0$: fluctuation of number of particles on the interaction length $\sqrt{n_0 N_u \lambda_0}$ times the interaction energy A times R_{56} compared with $\lambda/2\pi = \lambda_0/2\pi m$.

Typical EEHG parameters: $E = 1.2$ GeV, $I_p = 800$ A, $\lambda_0 = 240$ nm, $N_u = 8$, $K = 1.84$, $\sigma_x = 100$ microns, $R_{56}^{(1)} = 8.2$ mm, $R_{56}^{(1)} = 0.35$ mm. For the first chicane, $\Pi = 3.7$, for the second one $\Pi = 6.8 \cdot 10^{-3}$.

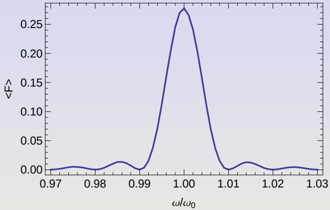
Computer simulations

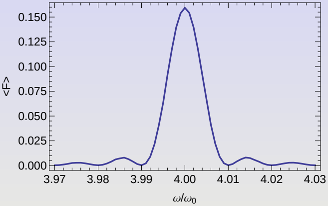
In computer simulations, we randomly distribute N particles on the interval L, add interaction, and numerically compute $\langle F \rangle$.

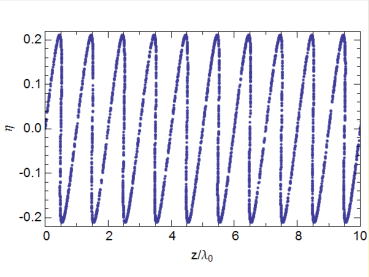
Parameters of the simulations: $N = 3000$, $L = 300\lambda_0$, $\Delta \times R_{56}/\lambda_0 = 0.21$, $\sigma_\eta = 0$.

No interaction

The case of no interaction, $\Pi = 0$

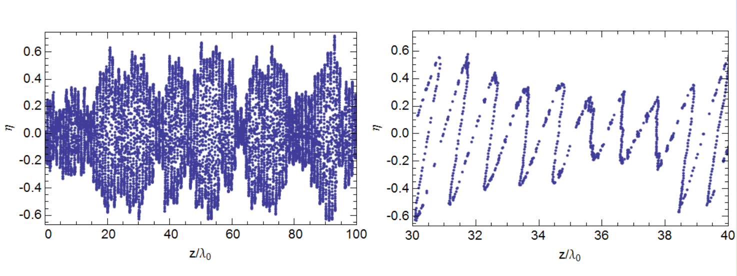






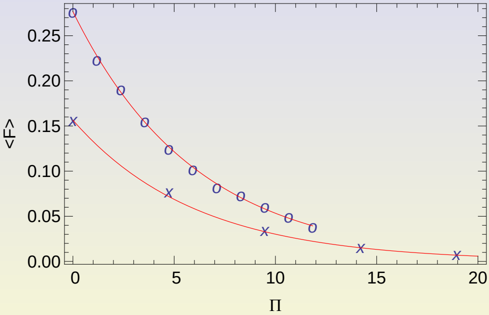
With interaction

$\Pi \approx 12$



Computer simulations

From analytical theory it follows that for $\Pi \gtrsim 1$, the bunching factor decreases. Numerical simulations show the dependence $e^{-\alpha\Pi}$, with $\alpha \approx 0.16$.



Formfactor $\langle F \rangle$ for the 1st and 4th harmonics as a function of Π.

Discussion and conclusions

- A simple 1D model is developed for the analysis of the noise effects due to interaction of particles in the modulator-undulator. Currently the model is applicable for the HGHG seeding, but generalization for EEHG is straightforward.
- In its extreme, the interaction destroys the seeding.
- The proposed model can be extended for 3D interaction.
- In addition to interaction in undulators, the particles also interact in bends. It would be interesting to look at these effects as well.

Thanks to Z. Huang for numerous discussions on the subject.