Beam-Beam Simulations for RHIC and LHC

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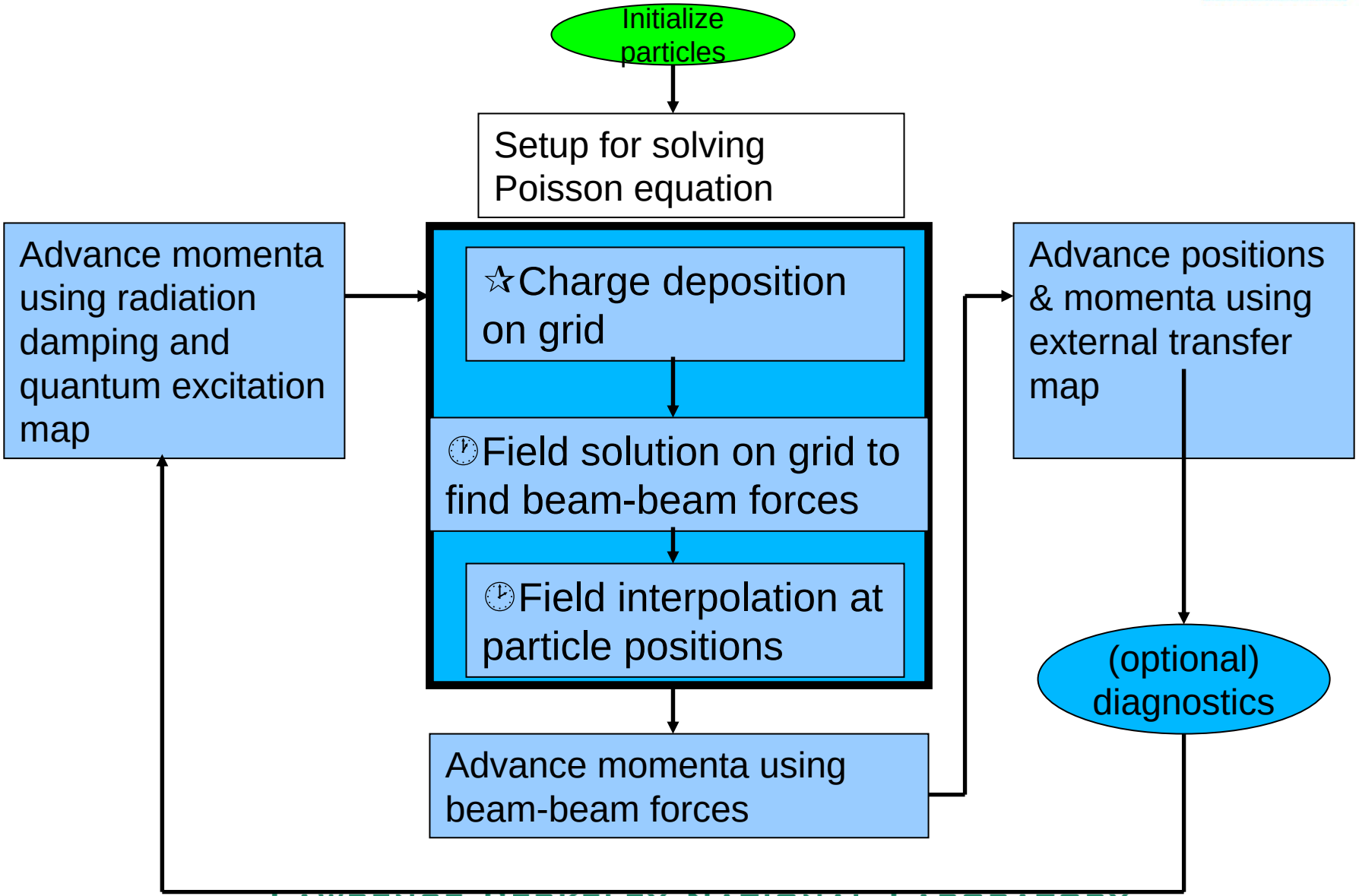
Mini-Workshop on Beam-Beam Compensation
July 2-4, 2007, SLAC, Menlo Park, California

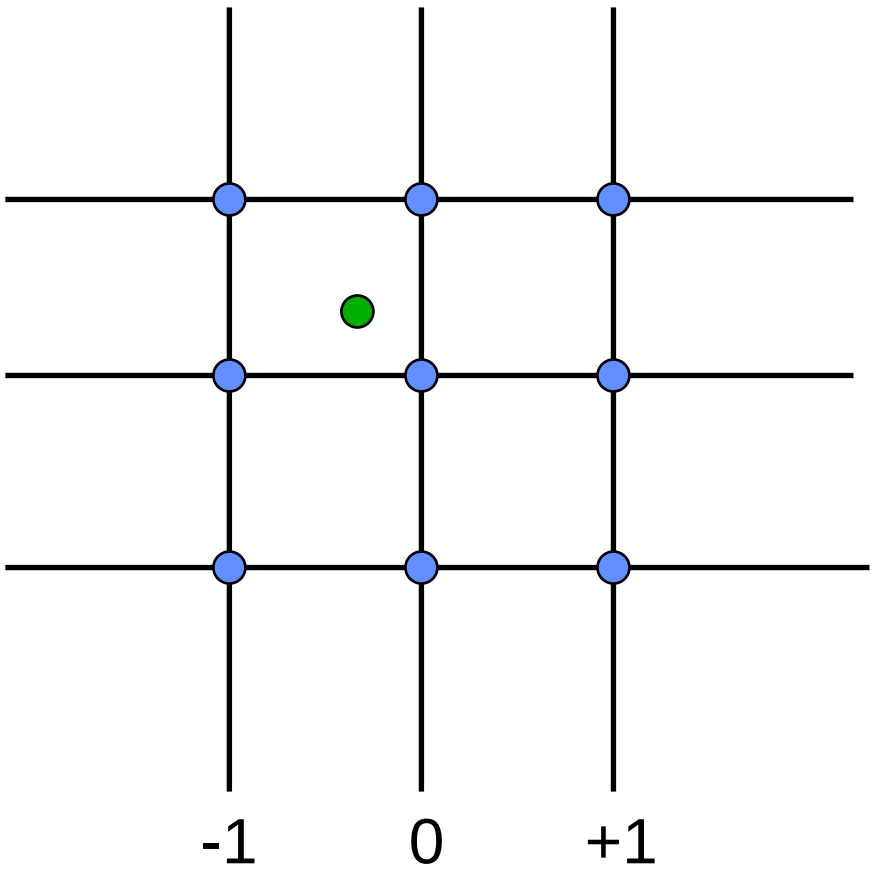
Outline

- **Computational model**
- **Studies of long-range beam-beam effects at RHIC**
- **Tune scan a new working point at RHIC**
- **Simulation mismatch and offset collisions at LHC**

Particle-In-Cell (PIC) Simulation



Quadratic Deposition/Interpolation

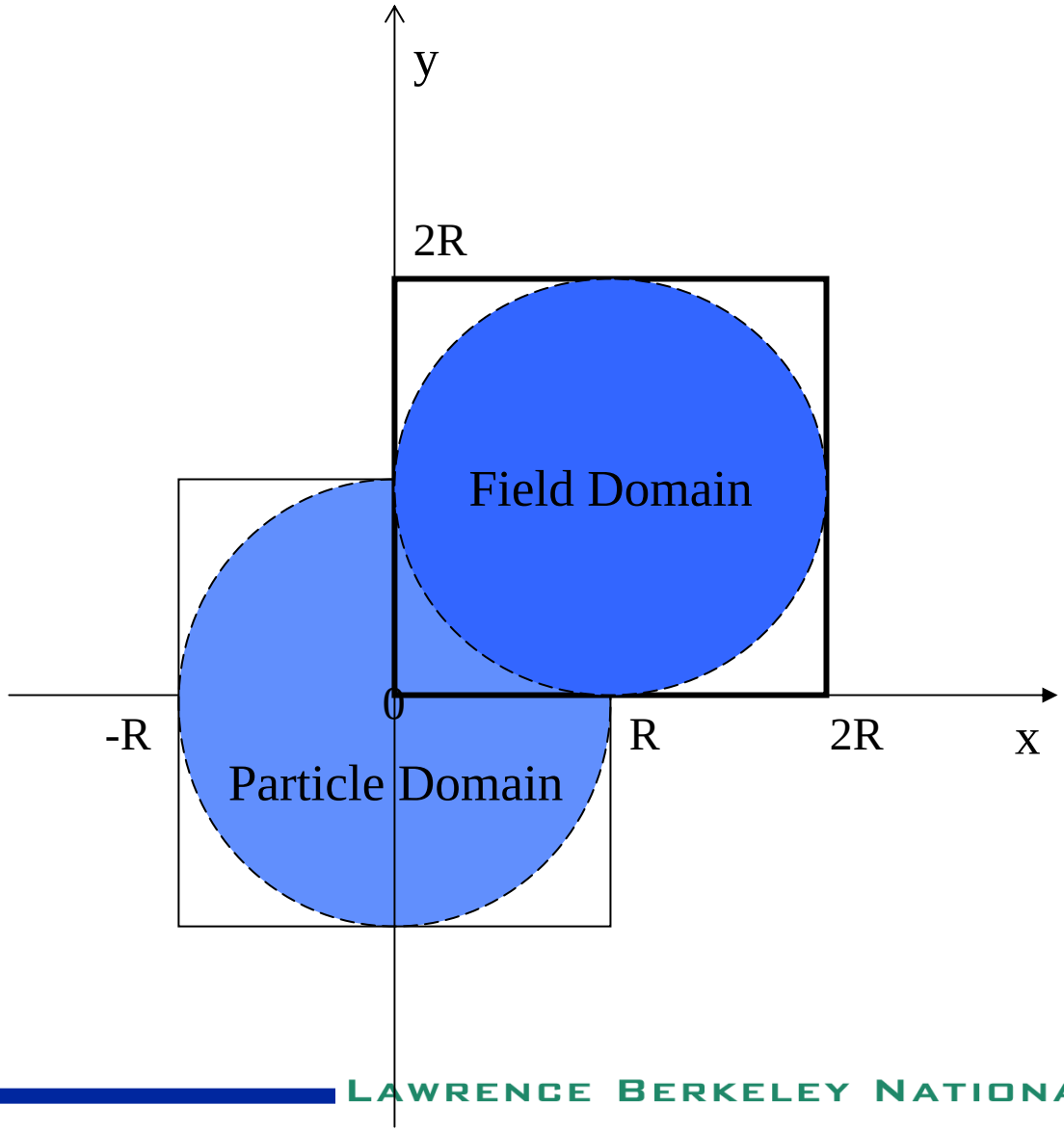


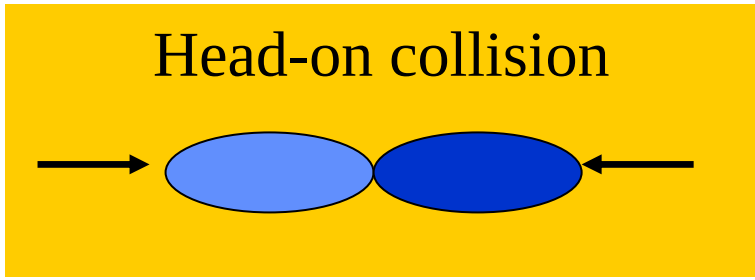
$$w_x^{+1} = \frac{1}{2}\left(\frac{1}{4} - r_x + r_x^2\right)$$

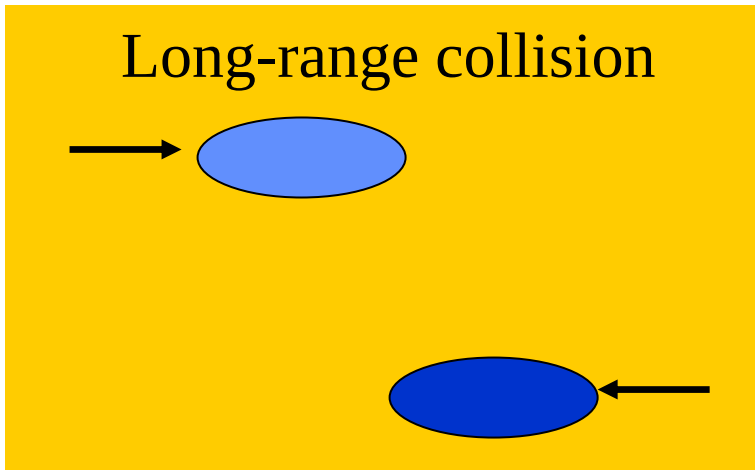
$$w_x^{0} = \frac{3}{4} - r_x^2$$

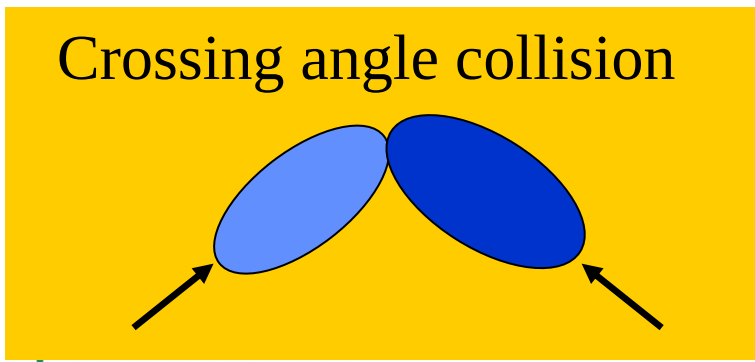
$$w_x^{+1} = \frac{1}{2}\left(\frac{1}{4} + r_x + r_x^2\right)$$

A Schematic Plot of the Geometry of Two Colliding Beams









Green Function Solution of Poisson's Equation

$$\phi(r) = \int G(r, r')\rho(r')dr' \; ; \; r = (x, y)$$

$$\phi(r_i) = h\sum_{i'=1}^{N} G(r_i - r_{i'})\rho(r_{i'})$$

$$\boxed{G(x, y) = -\frac{1}{2}\log(x^2 + y^2)}$$

Direct summation of the convolution scales as N^4 !!!!
N – grid number in each dimension

Green Function Solution of Poisson's Equation (cont'd)

Hockney's Algorithm:- *scales as $(2N)^2 \log(2N)$*

- Ref: Hockney and Easwood, ***Computer Simulation using Particles***, McGraw-Hill Book Company, New York, 1985.

$$\phi_c(r_i) = h\sum_{i'=1}^{2N} G_c(r_i - r_{i'})\rho_c(r_{i'})$$

$$\phi(r_i) = \phi_c(r_i) \quad \text{for i} = 1, \text{N}$$

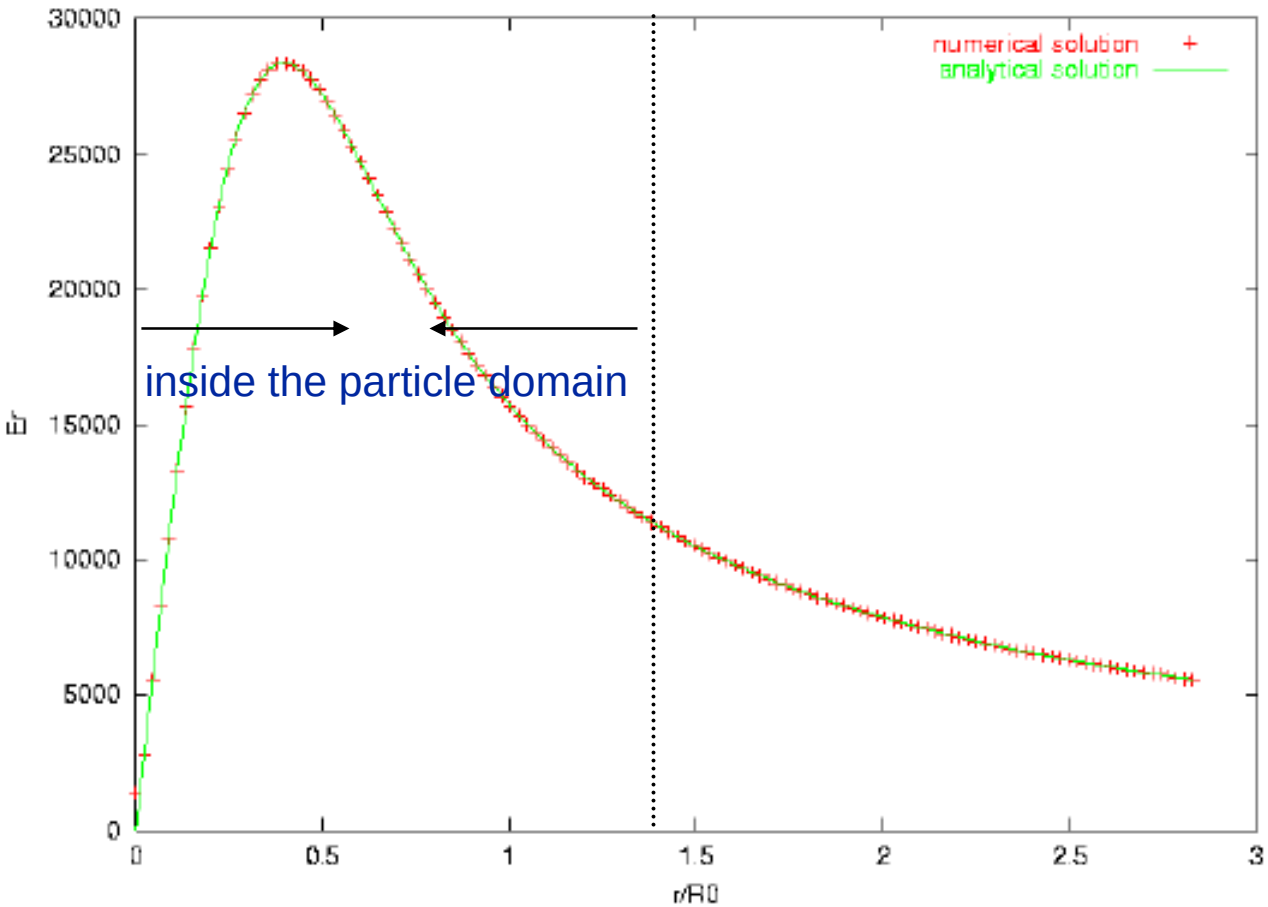
Shifted Green function Algorithm:

$$\phi_F(r) = \int G_s(r,r')\rho(r')dr'$$

$$G_s(r,r') = G(r + r_s, r')$$

Comparison between Numerical Solution and Analytical Solution (Shifted Green Function)

E_x



radius

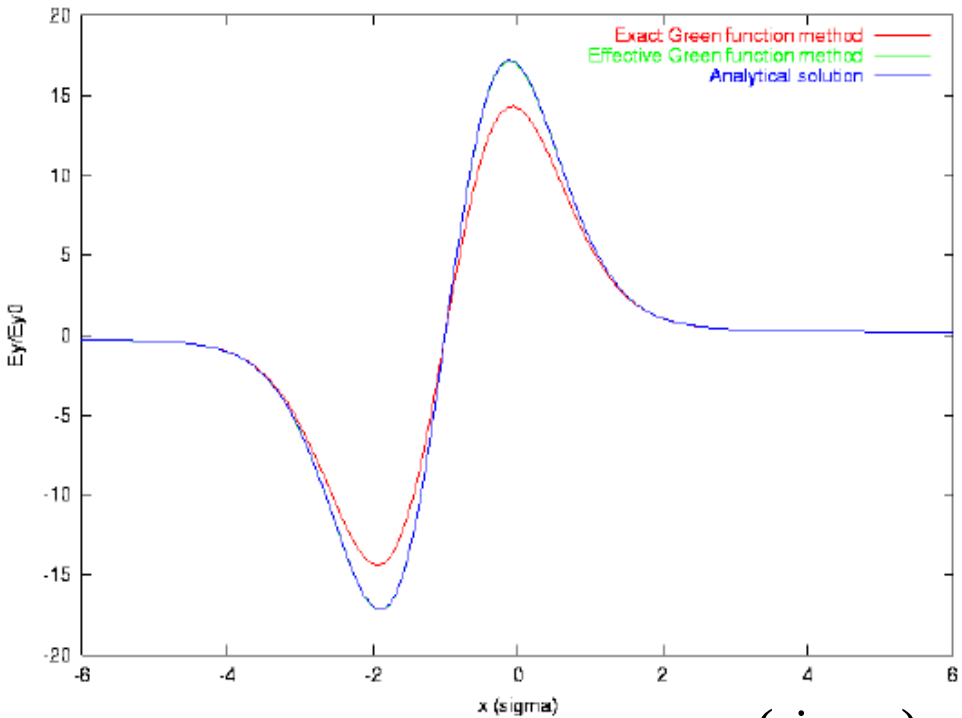
Green Function Solution of Poisson's Equation (Integrated Green Function)

Integrated Green function Algorithm for large aspect ratio:

$$\phi_c(r_i) = \sum_{i'=1}^{2N} G_i(r_i - r_{i'})\rho_c(r_{i'})$$

$$\boxed{G_i(r, r') = \oint G_s(r, r')\, dr'}$$

E_y



x (sigma)

BeamBeam3D:
Parallel Strong-Strong / Strong-Weak Simulation Code

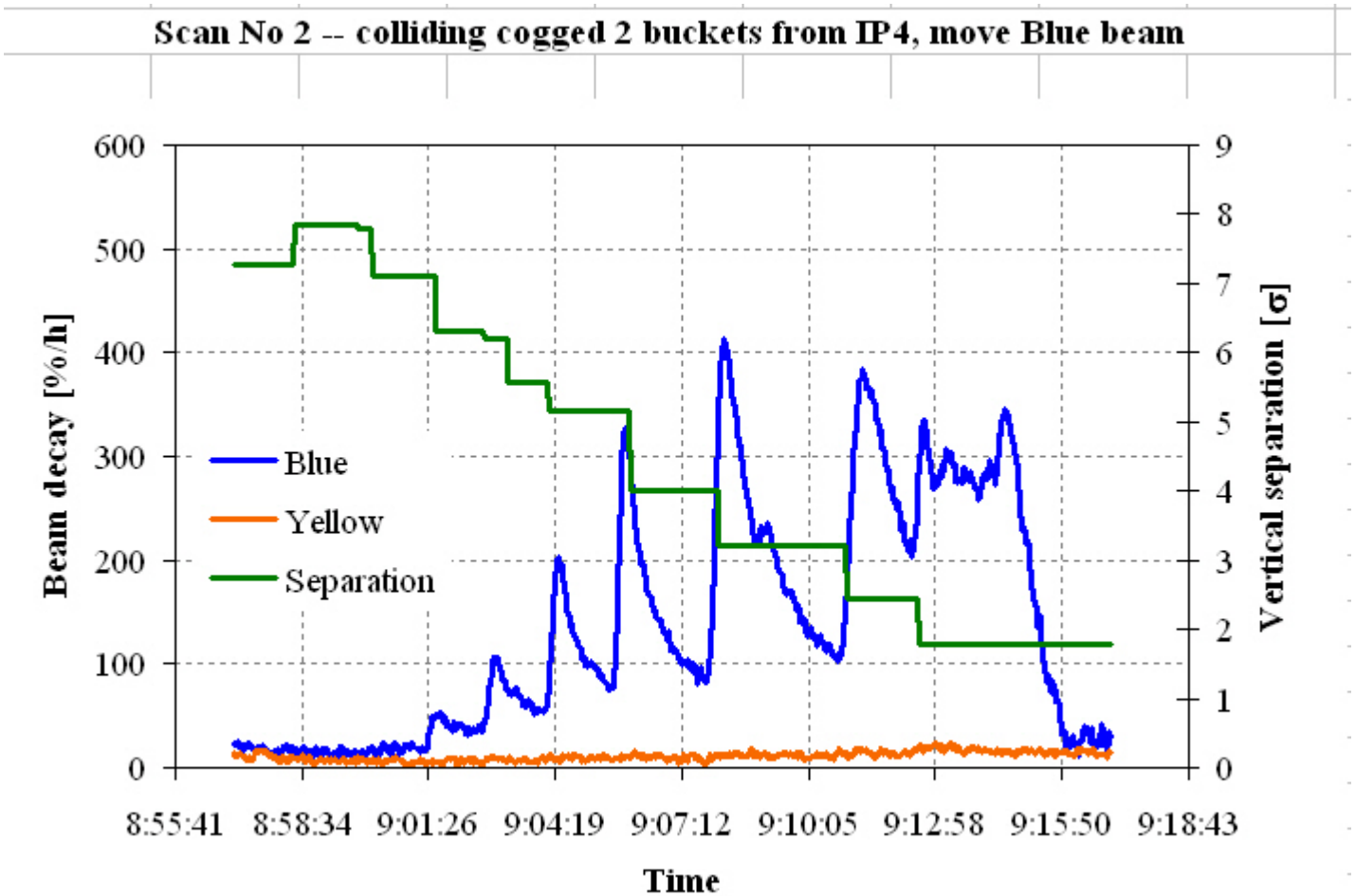
- Multiple physics models:
  - strong-strong (S-S); weak-strong (W-S)
- Multiple-slice model for finite bunch length effects
- New algorithm -- shifted Green function -- efficiently models long-range parasitic collisions
- Parallel particle-based decomposition to achieve perfect load balance
- Lorentz boost to handle crossing angle collisions
- Multi-IP collisions, varying phase adv,…
- Arbitrary closed-orbit separation (static or time-dep)
- Independent beam parameters for the 2 beams

Studies of Long-Range Beam-Beam Effects at RHIC

RHIC Physical Parameters for the Long-Range Beam-Beam Simulations

Beam energy (GeV)	26
Protons per bunch	19e10
β* (m)	20
Rms spot size (mm)	1.6/1.29
Betatron tunes	(0.7xx,0.7xx)
Rms bunch length (m)	3.6
Synchrotron tune	3.7e-4
Momentum spread	1.6e-3
Beam-Beam Parameter	0.0077/0.0095

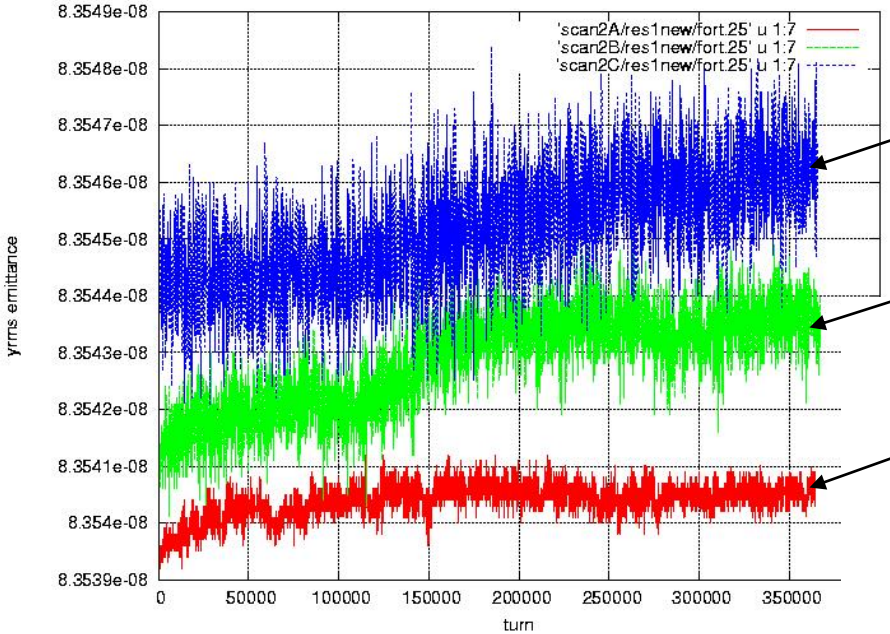
RHIC Long-Range Experiment Scan 2



Tunes (0.7351,0.7223) (0.7282 0.7233)

W. Fischer et. al

RHIC Long-Range Simulation Scan 2 (rms emittance vs. turn)

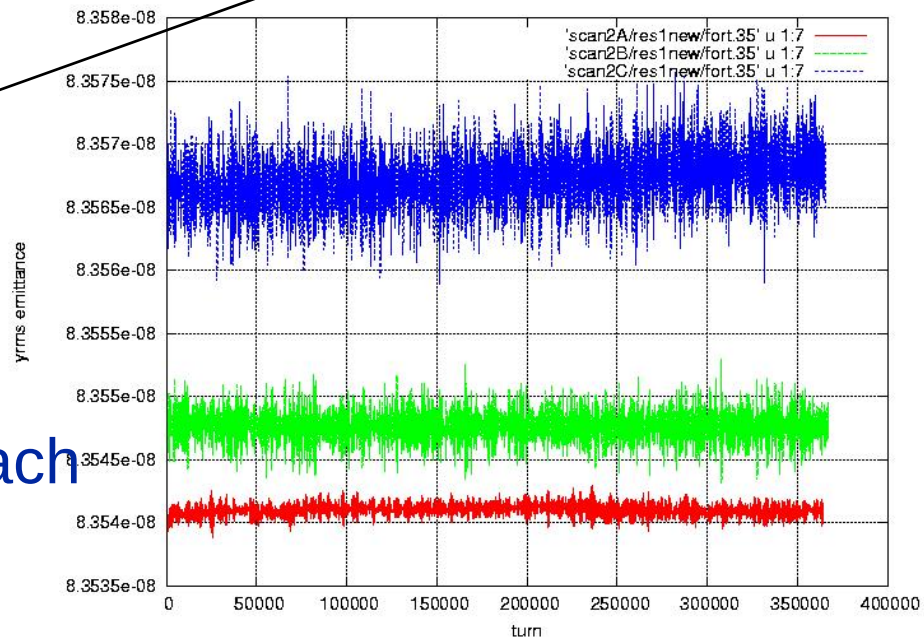


Blue Beam

4.70σ separation

5.54σ

7.15σ



Yellow Beam

One million macroparticles for each beam and 128 x128 grid points

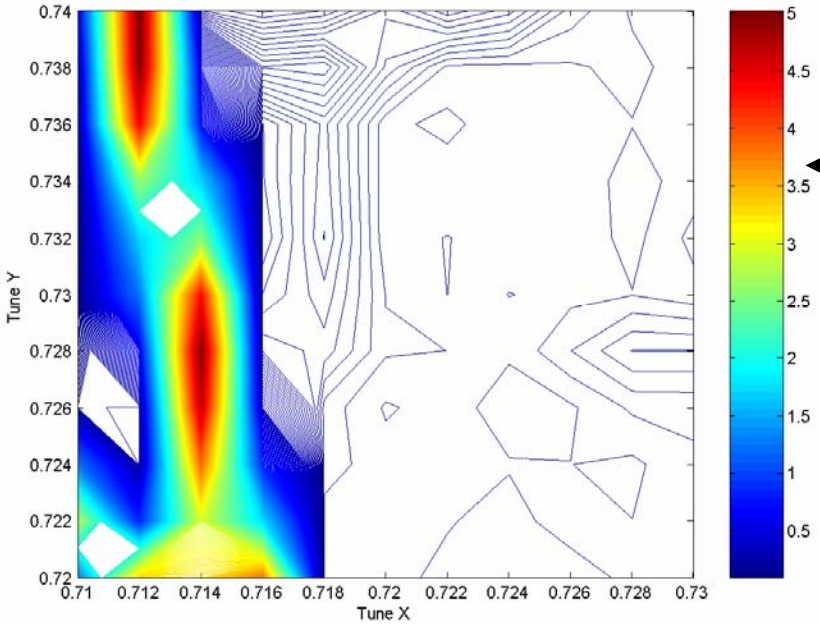
RHIC Physical Parameters for Tune Scan Studies of Long-Range Beam-Beam Effects

Parameter	Unit	Value
proton energy	GeV	100
protons per bunch, N_b	10^{11}	2
emittance $\varepsilon_{N\ x,y\ 95\%}$	mm·mrad	15
beta* (beam1, beam 2)	m	(0.9,1)
rms bunch length	m	0.7
rms momentum devation		0.3e-3
synchrotron tune	10^{-3}	3.7
chromaticity		(2,2)
beam-beam seapartion	sigma	4-6

Computational Model

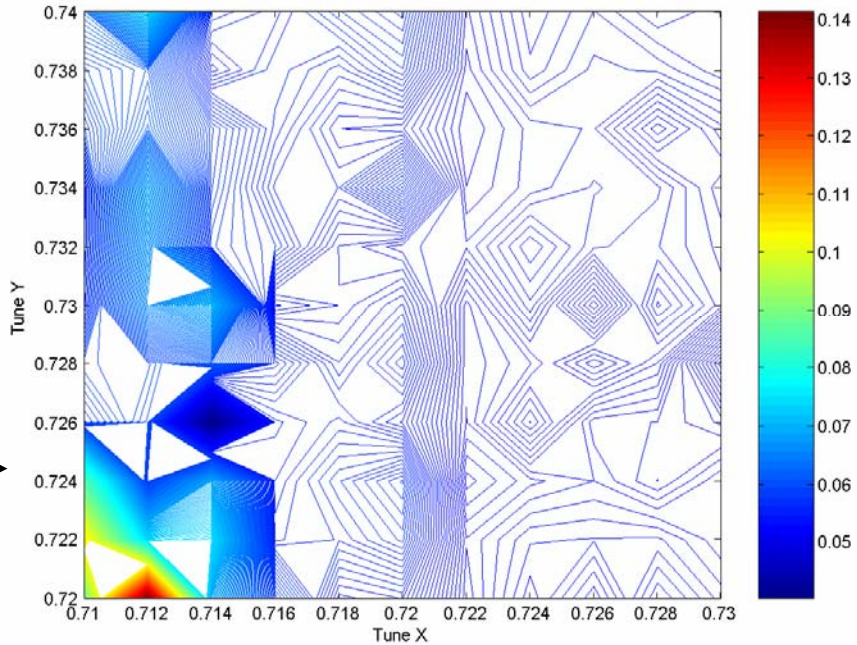
- 4x4 Linear transfer matrix between the center of sextupole magnets
- Thin lens kick from each sextupole magnet around the machine
- One turn kick to include the machine chromaticity effects with dynamic tune modulation
- Self-consistent long-range beam-beam forces

Averaged Emittance Growth with 4 and 6 Sigma Separation (with dynamic tune modulation)



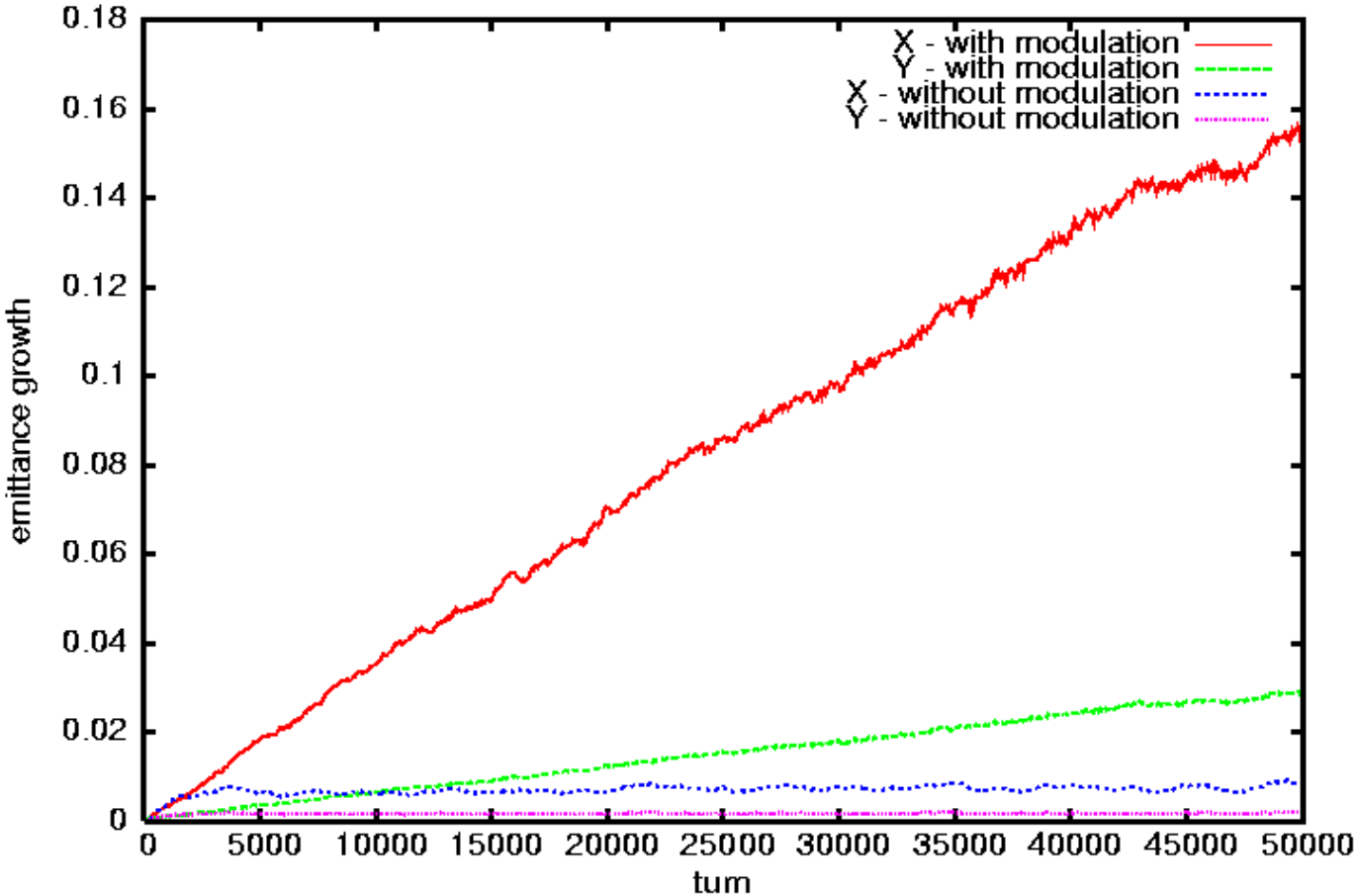
4 sigma

6 sigma



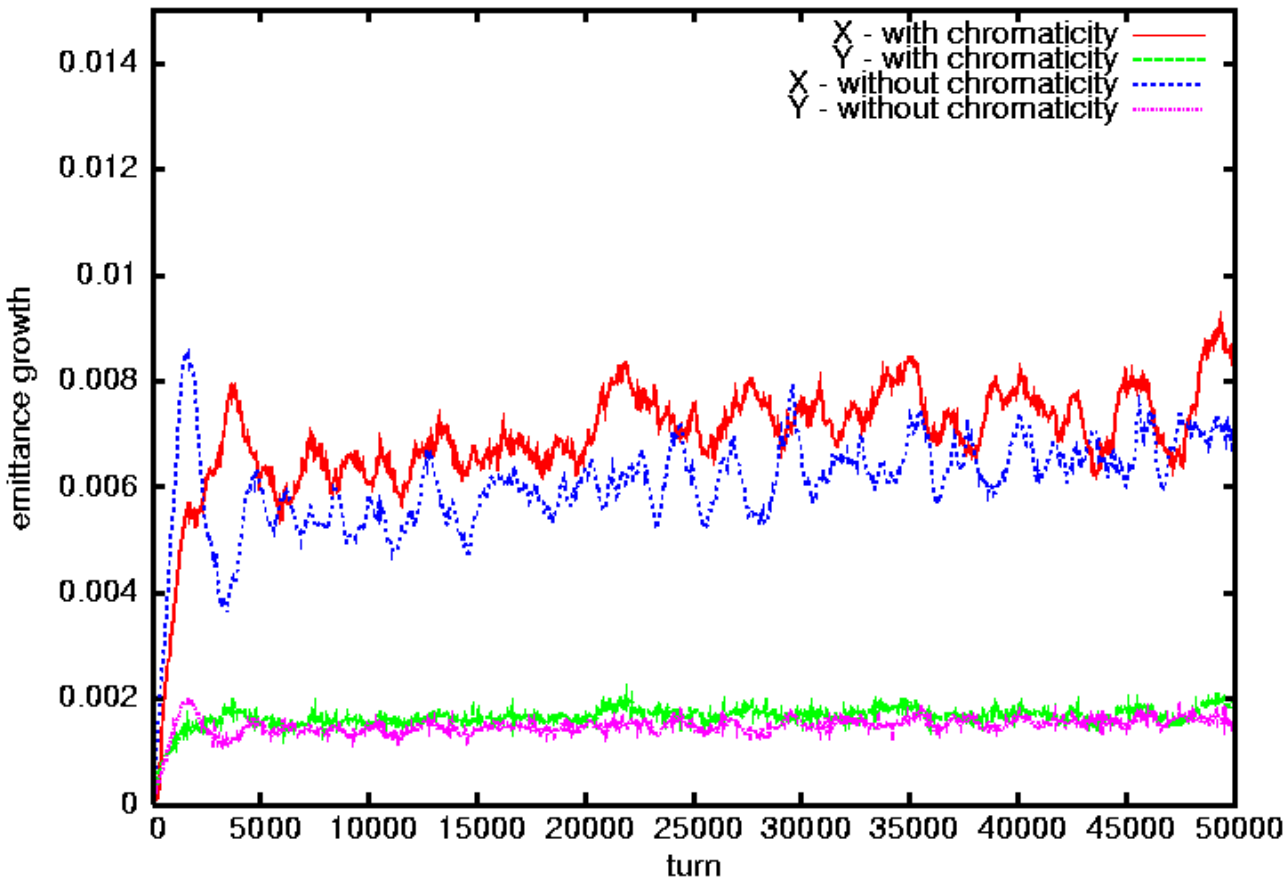
Transverse Emittance Growth Evolution with/without Tune Modulation

(0.714, 0.726) 4 sigma separation



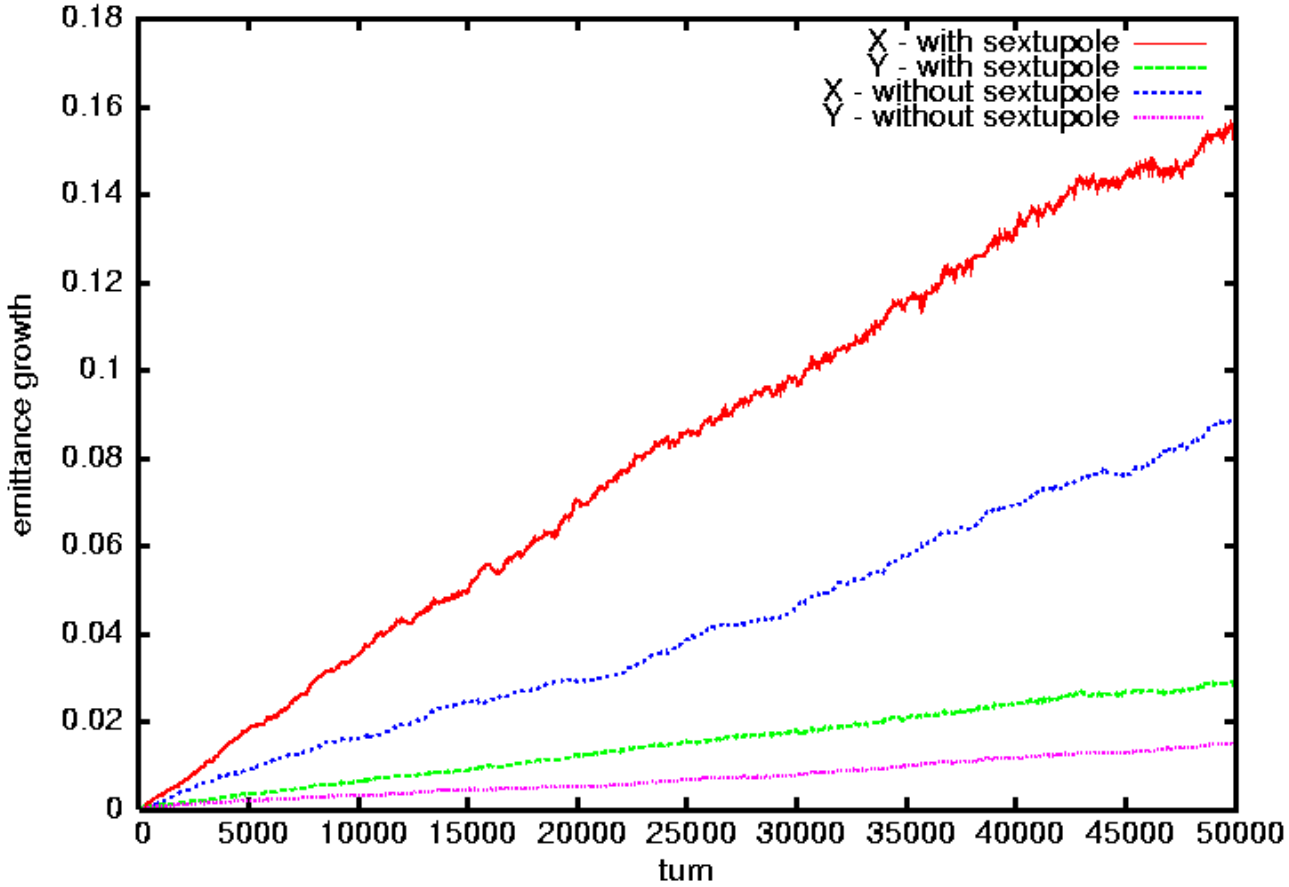
Transverse Emittance Growth Evolution with/without Static Chromaticity

(0.714, 0.726) 4 sigma separation



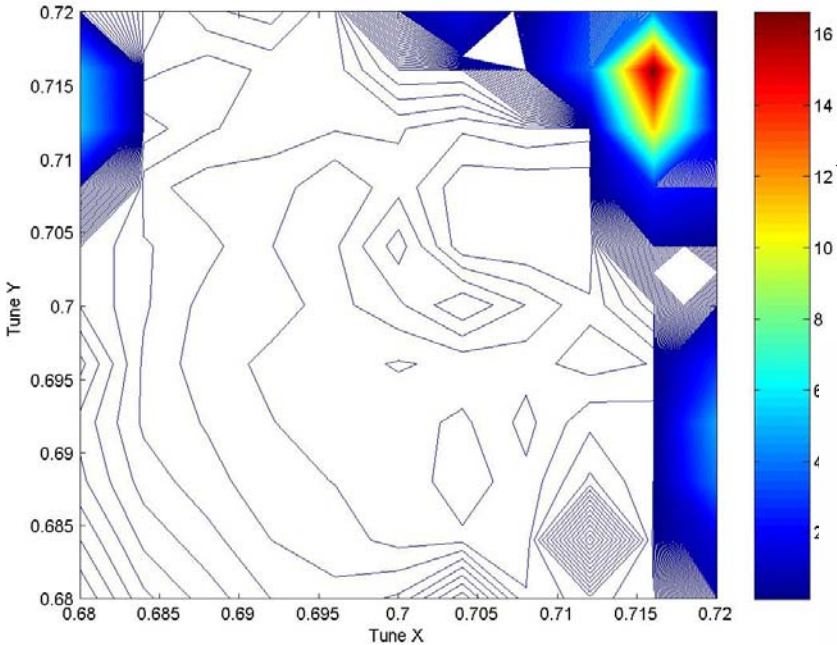
Transverse Emittance Growth Evolution with/without Sextupole Magnets

(0.714, 0.726) 4 sigma separation



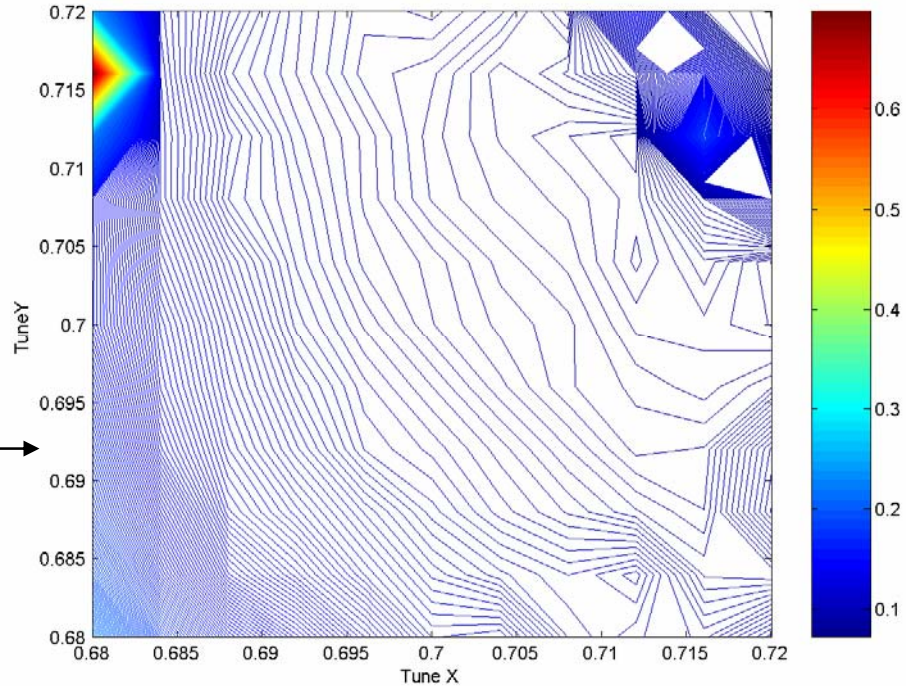
Averaged Emittance Growth with 4 and 6 Sigma Separation

yellow beam fraction tune (0.71,0.69)

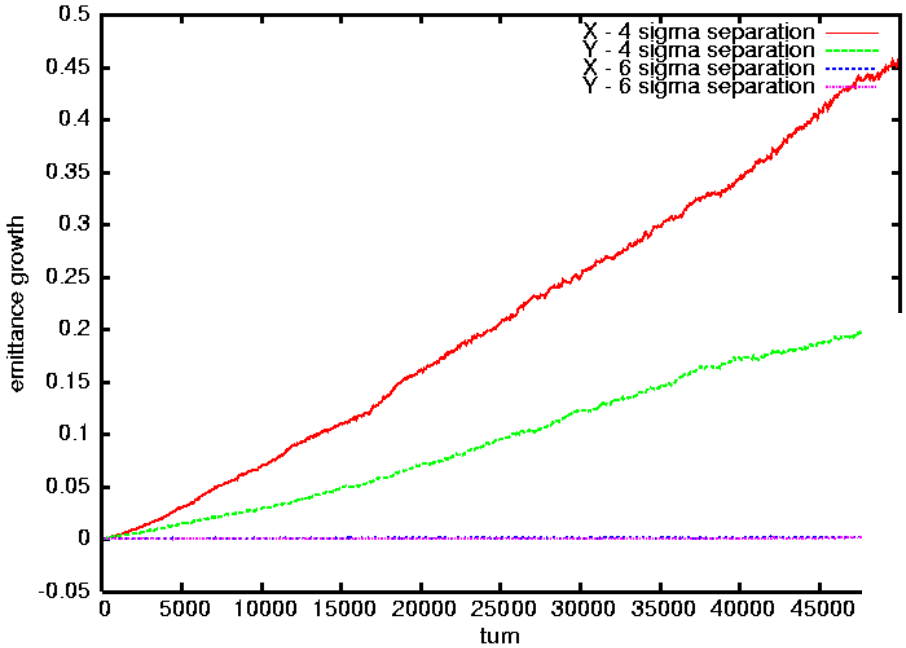


4 sigma

6 sigma



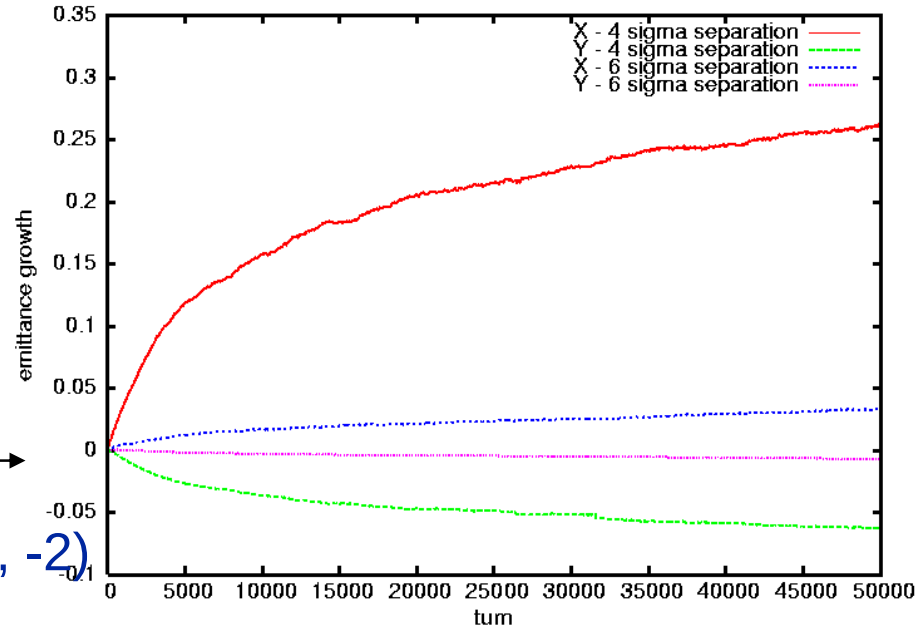
Transverse Emittance Growth Evolution at Two Resonance Islands



7th order resonance (3,4) or (4,3)

(0.716,0.716)

(0.68,0.716)

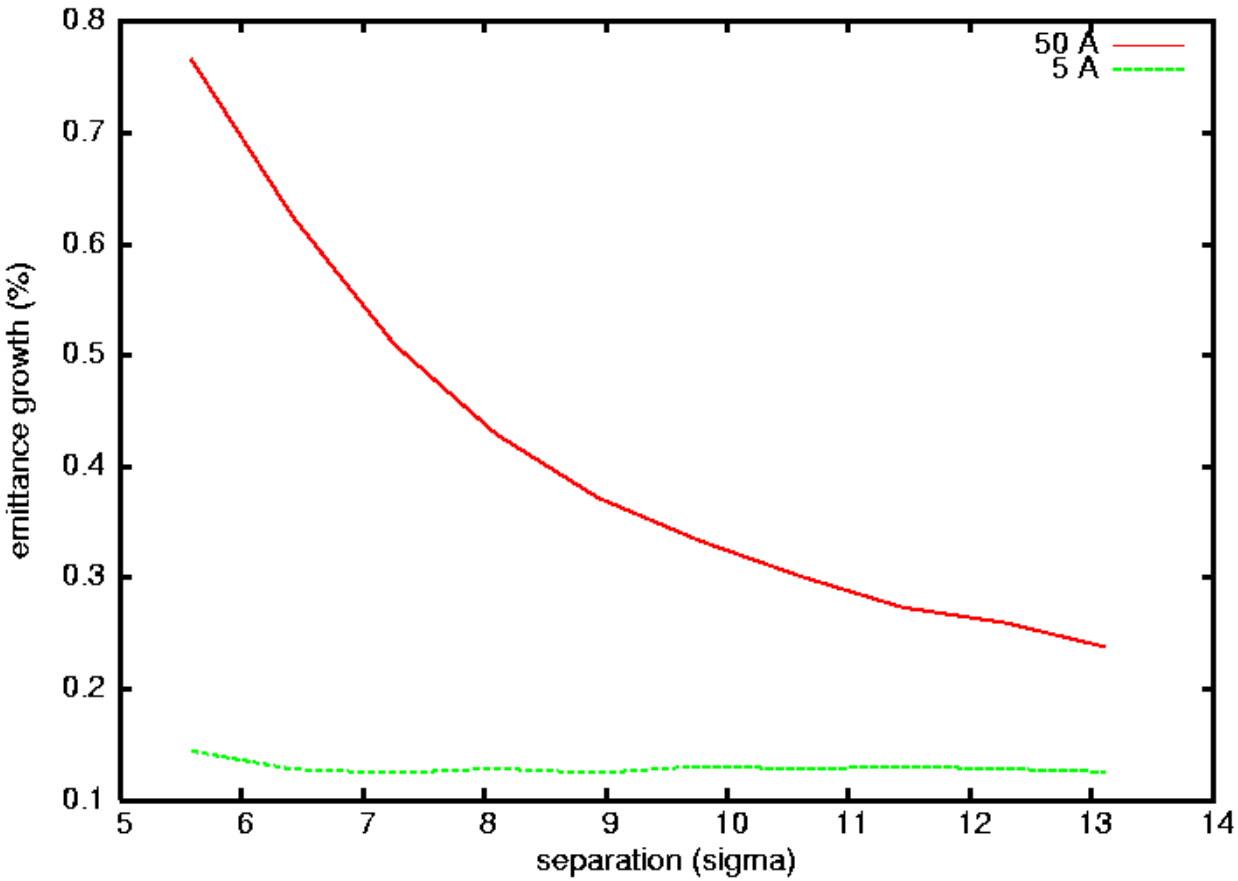


7th order resonance (1,6) or (2,5) or (5, -2)

10th order resonance (5,5)

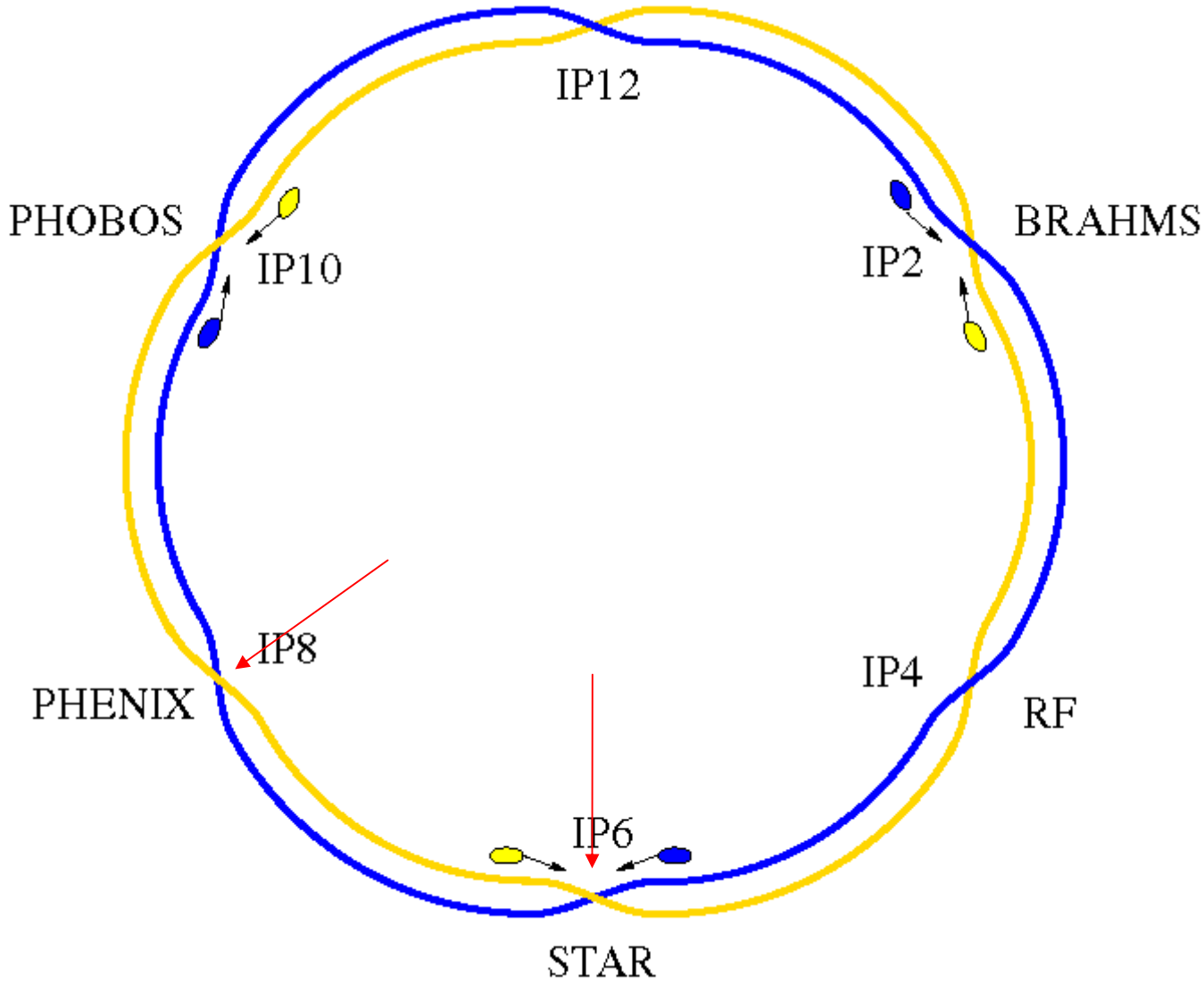
12th order resonance (8,-4)

Blue Gold Beam Emittance Growth vs. Wire Separation (5 A and 50 A current)



Tune Scan Studies of New Working Points at RHIC

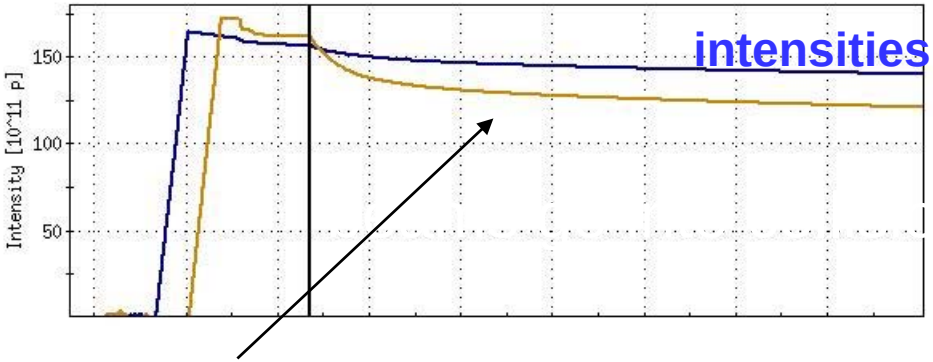
A schematic plot of 3 bunch collisions at RHIC



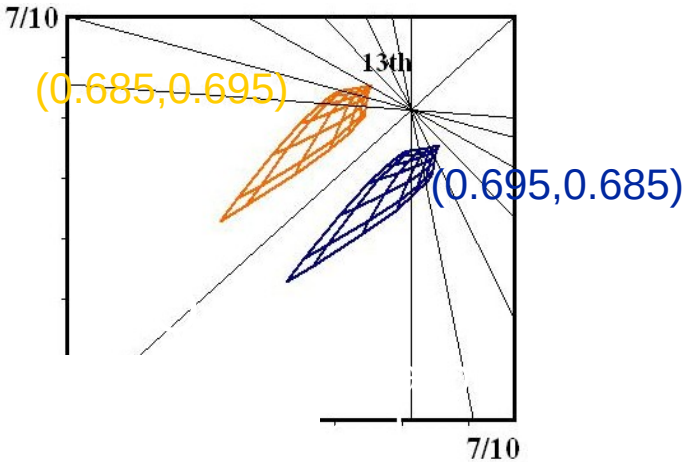
RHIC Physical Parameters for the Tune Scan Simulations

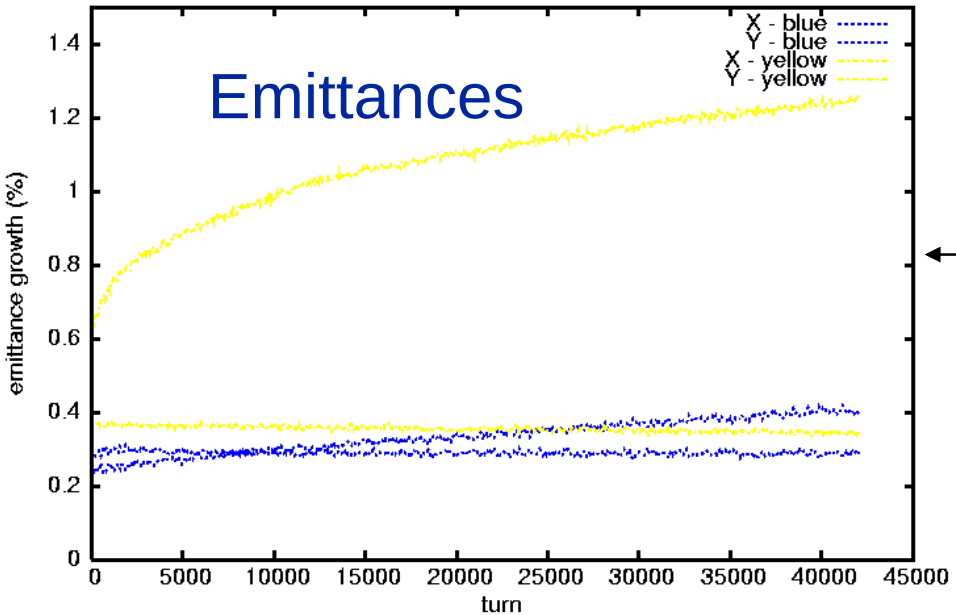
Beam energy (GeV)	100
Protons per bunch	20e10
β* (m)	0.8
Rms spot size (mm)	0.14
Betatron tunes (blue)	(0.695,0.685)
Rms bunch length (m)	0.8
Synchrotron tune	5.5e-4
Momentum spread	0.7e-3
Beam-Beam Parameter	0.00977
Chromaticity	(2.0, 2.0)

Intensity (experiment) and Emittance (simulation) Evolution of Blue and Yellow Proton Beams at RHIC



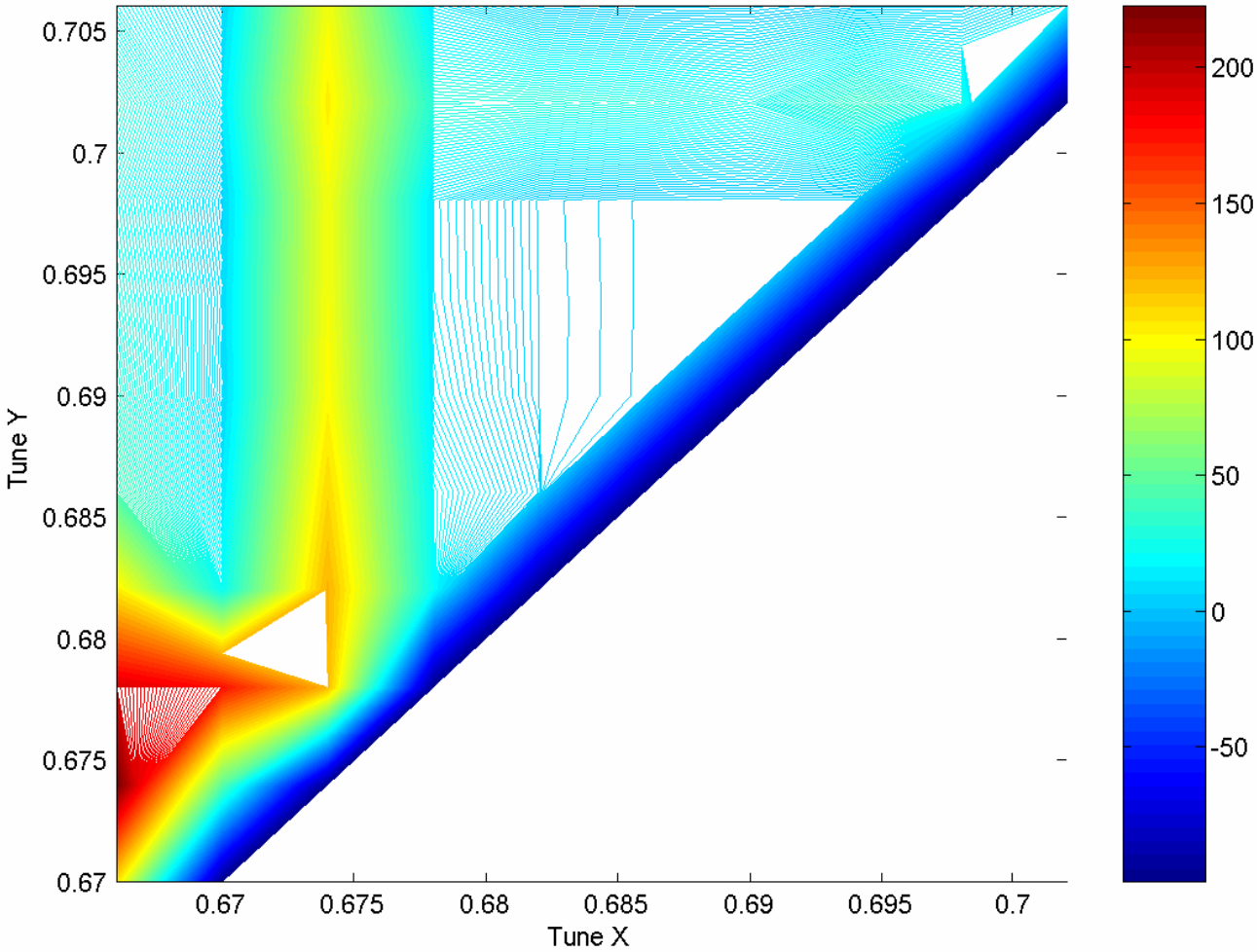
measurements (from W. Fischer, et al.)



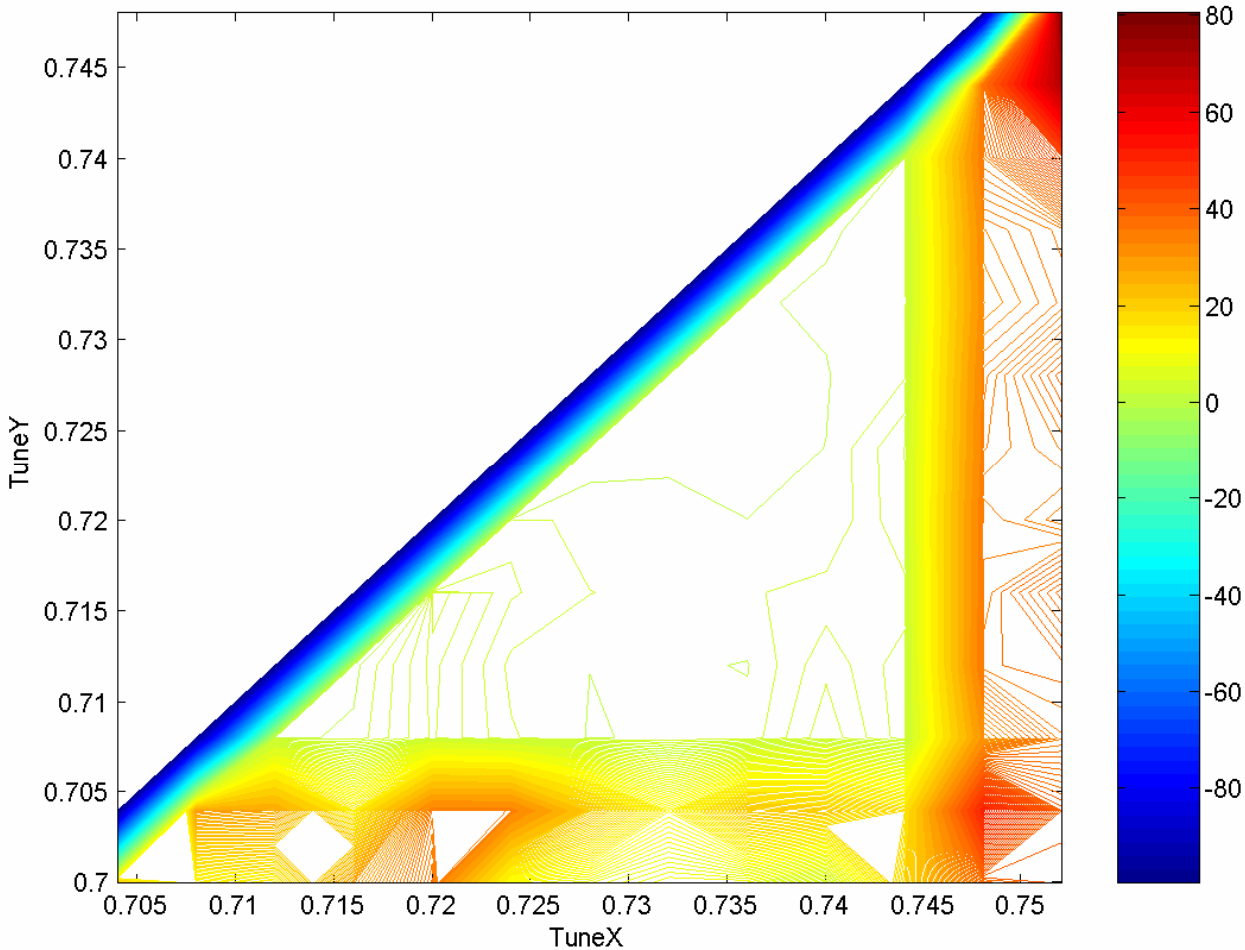


simulation

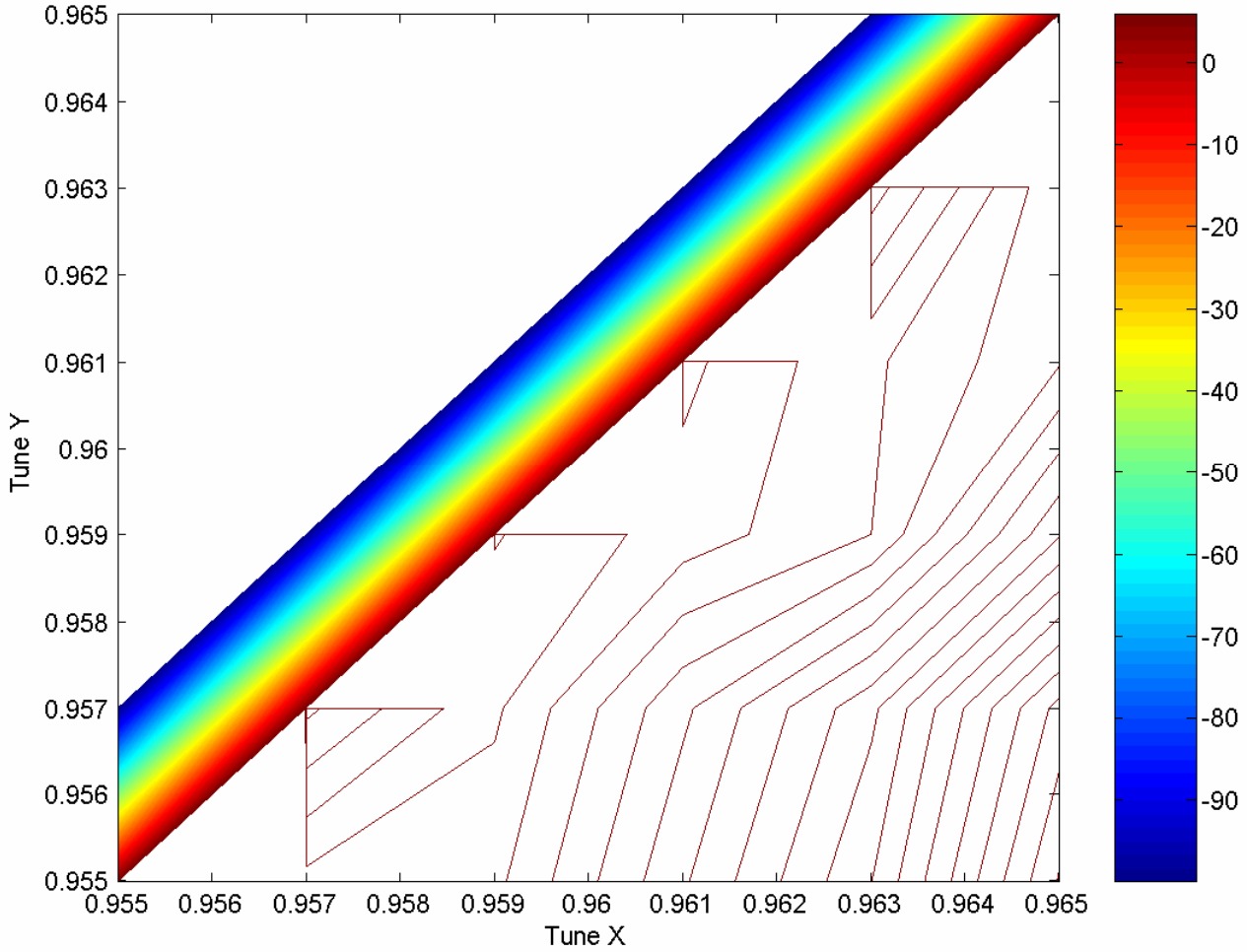
Averaged Emittance Growth vs. Tunes (near half integer, above diagonal)



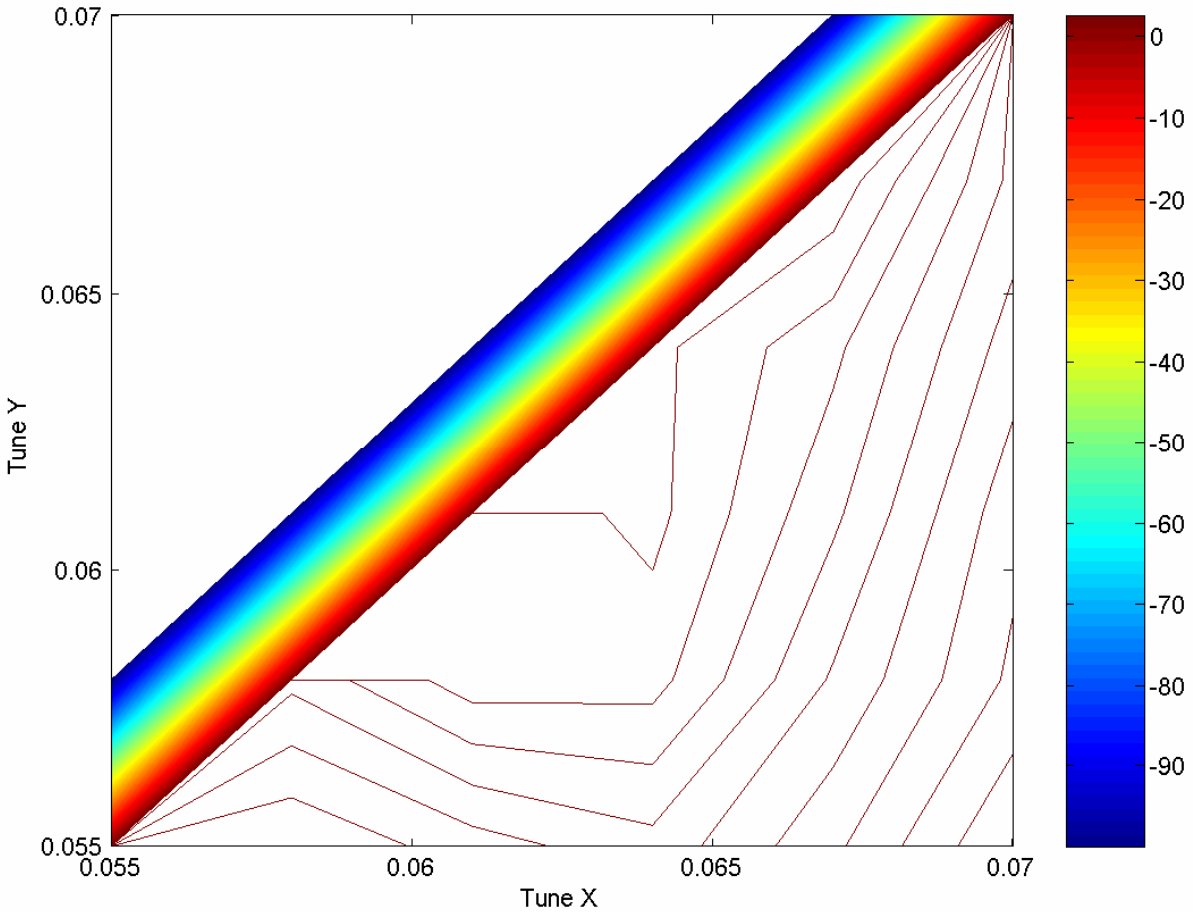
Averaged Emittance Growth vs. Tunes (near half integer, below diagonal)



Averaged Emittance Growth vs. Tunes (below integer, below diagonal)



Averaged Emittance Growth vs. Tunes (above integer, below diagonal)

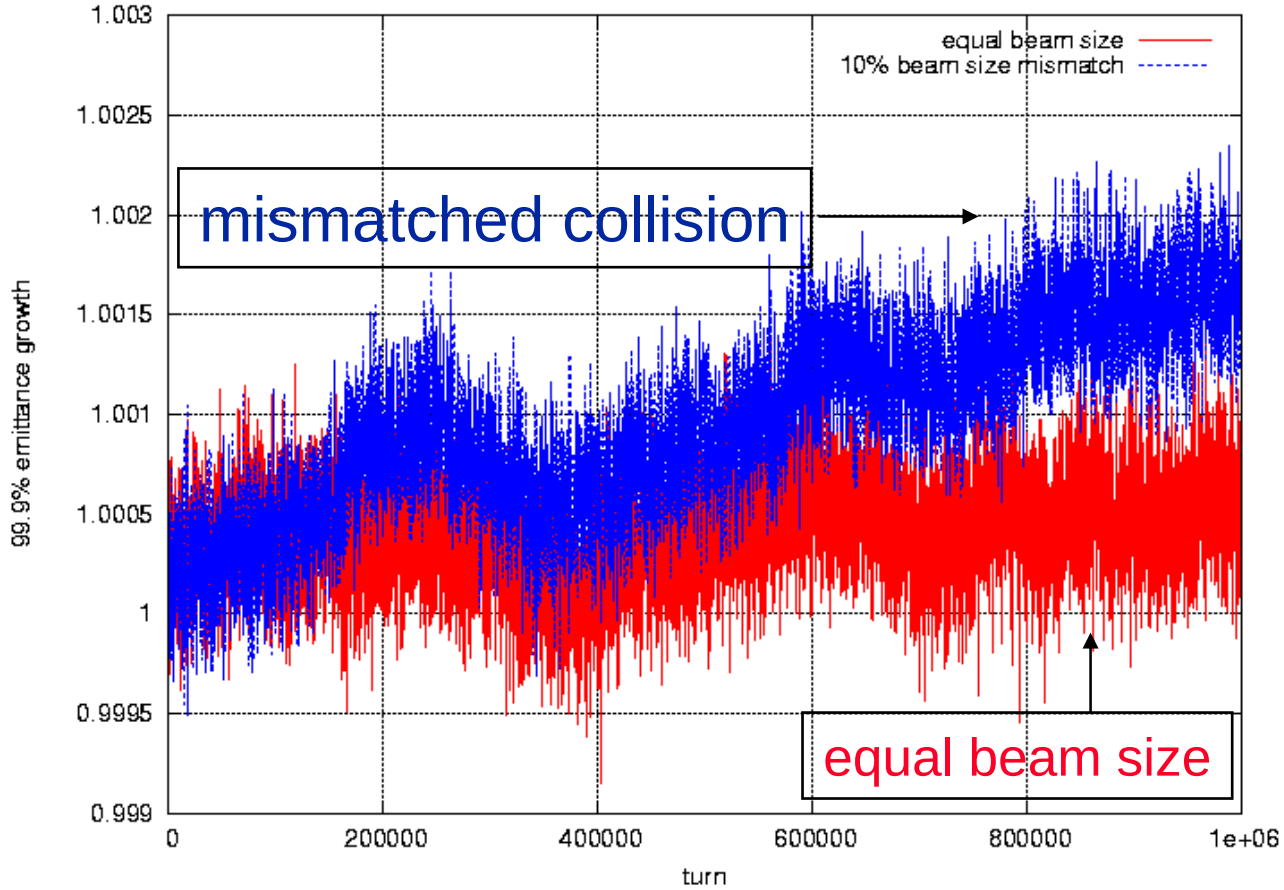


Mismatch and Offset Beam-Beam Collisions at LHC

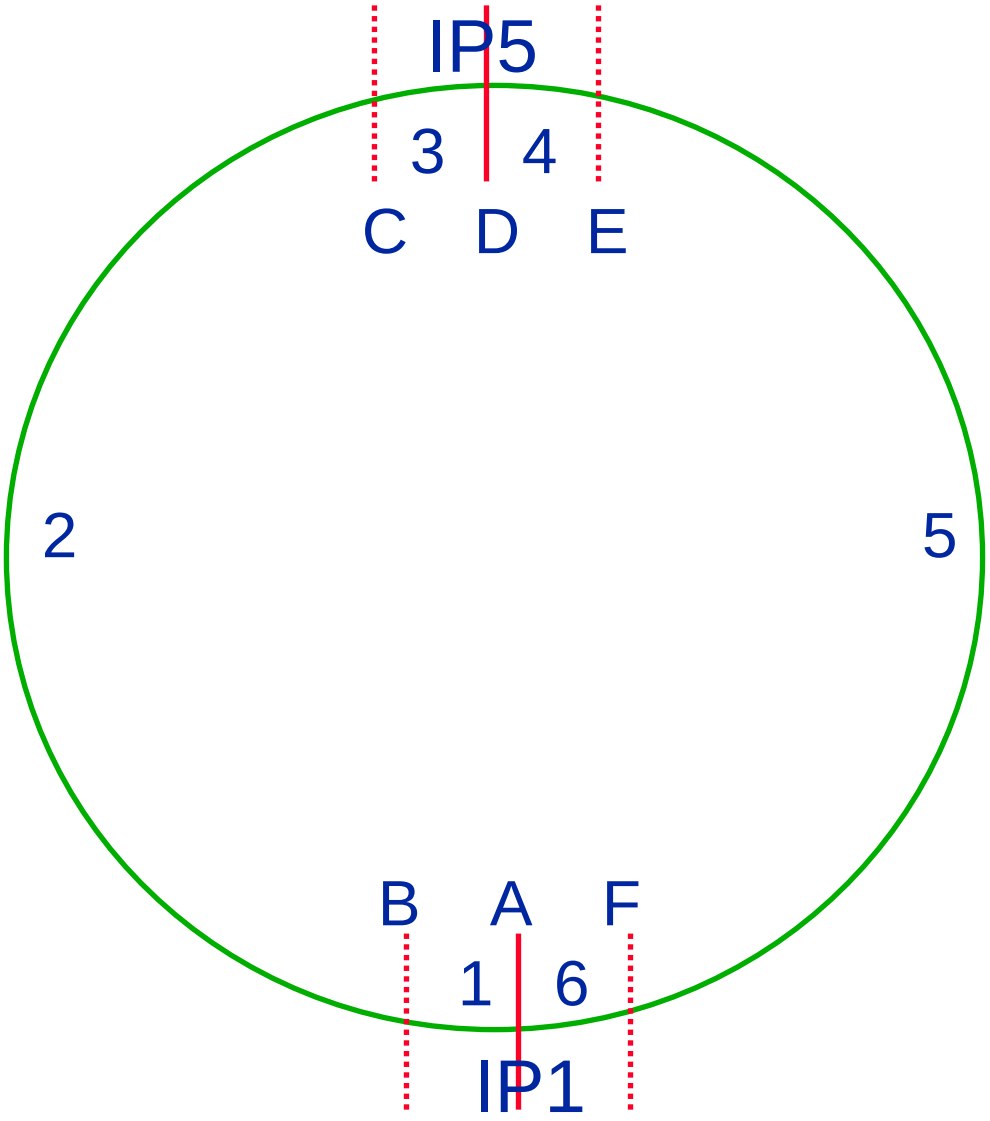
LHC Physical Parameters for the Beam-Beam Simulations

Beam energy (TeV)	7
Protons per bunch	10.5e10
β* (m)	0.5
Rms spot size (mm)	0.016
Betatron tunes	(0.31,0.32)
Rms bunch length (m)	0.077
Synchrotron tune	0.0021
Momentum spread	0.111e-3
Beam-Beam Parameter	0.0034

99.9% Emittance Growth with Equal Beam Size and 10% Mismatched Beam Size



A Schematic Plot of LHC Collision Scheme



One Turn Transfer Map

M = Ma M1 Mb M2 Mc M3 Md M4 Me M5 Mf M6

M = $M6^{-1}$ Mf M6 Ma M1 Mb $M1^{-1}$ M1 M2 M3
$M3^{-1}$ Mc M3 Md M4 Me $M4^{-1}$ M4 M5 M6

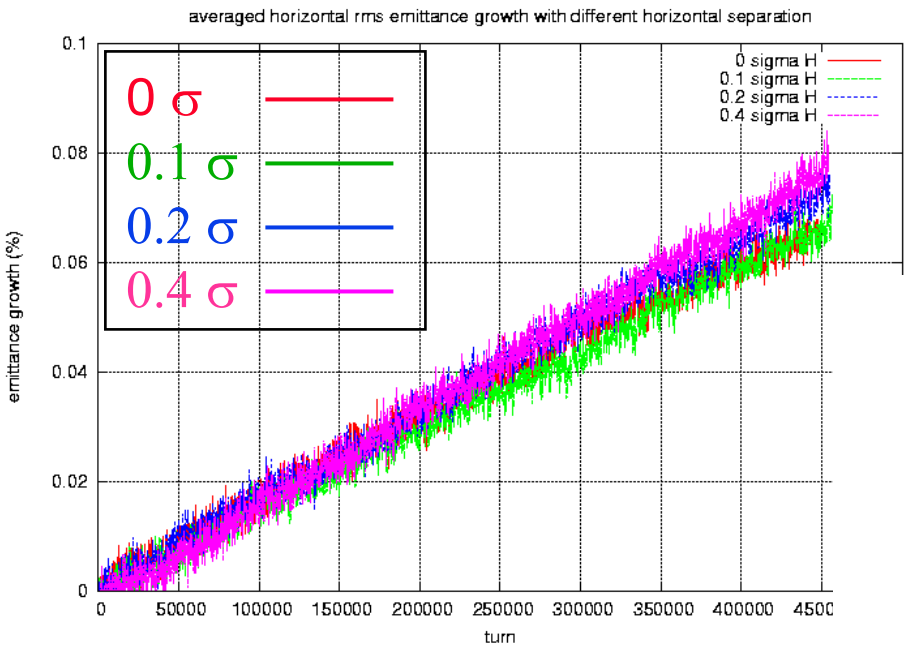
Here, Ma and Md are the transfer maps from head-on beam-beam collisions; Mb,c,e,f are maps from long-range beam-beam collisions; M1-6 are maps between collision points.

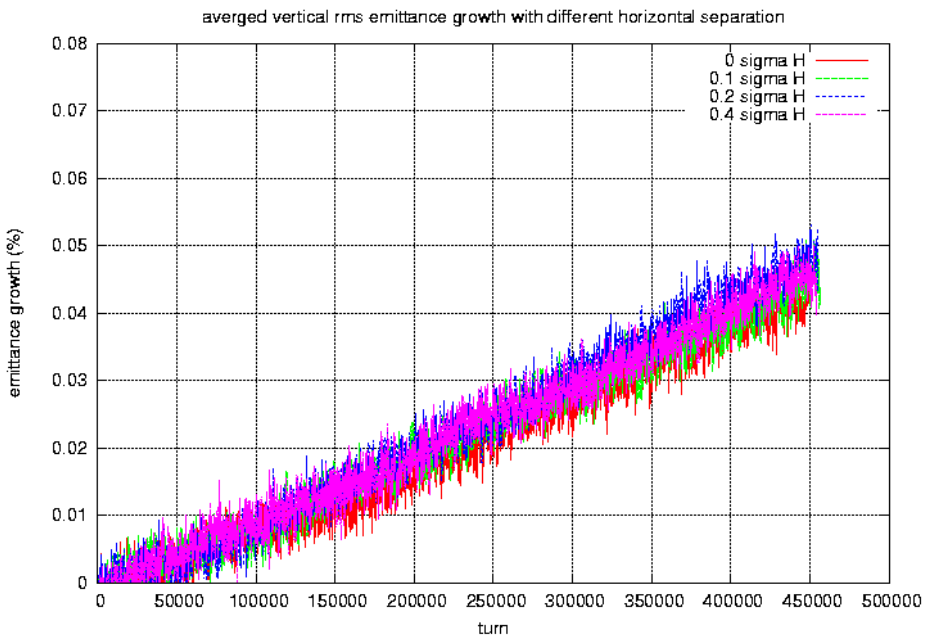
- Linear half ring transfer matrix with phase advanced:

$$\phi_x = 2\pi \times 31.655; \phi_y = 2\pi \times 29.66$$

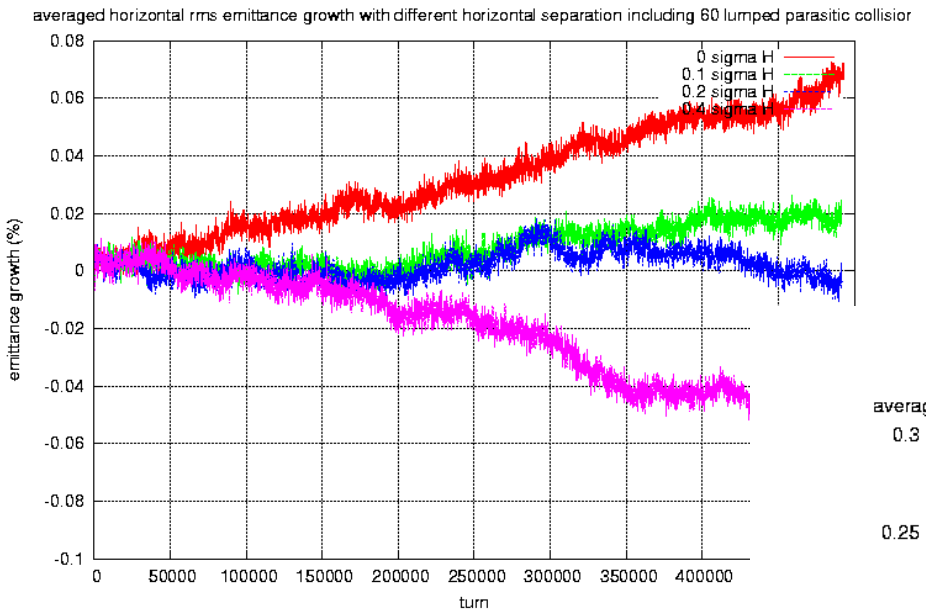
- 90 degree phase advance between long-range collision points and IPs
- 15 parasitic collisions lumped at each long-range collision point with 9.5 σ separation

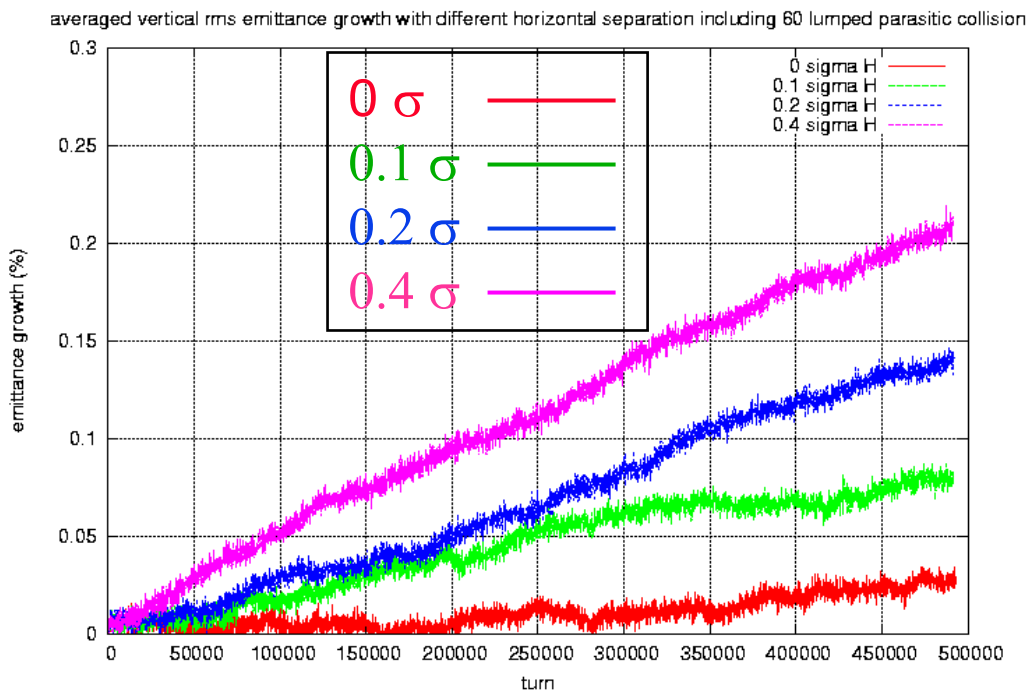
RMS Emittance Growth vs. Horizontal Separation at LHC (No Parasitic Collisions)





RMS Emittance Growth vs. Horizontal Separation at LHC (With 60 lumped Parasitic Collisions)





Summary

- Strong-strong beam-beam simulations qualitatively reproduce the experimental observations at RHIC.
- Emittance growth islands are observed near the 7th and 11th order resonance for the long-range beam-beam interactions at RHIC.
- For 4 sigma separation, significant emittance growth is observed near the resonance within a short period of time. Such emittance growth decreases quickly as the increase of beam-beam separation.
- Working points above integer shows less emittance than the other working points at RHIC.
- Mismatched collisions at LHC causes extra emittance growth.
- Offset collisions including lumped parasitic collisions at LHC show strong vertical emittance growth.