$B \to X_s \gamma$ at NNLO: effect of the photon energy cut

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Fermilab

Loopfest V, SLAC, June '06

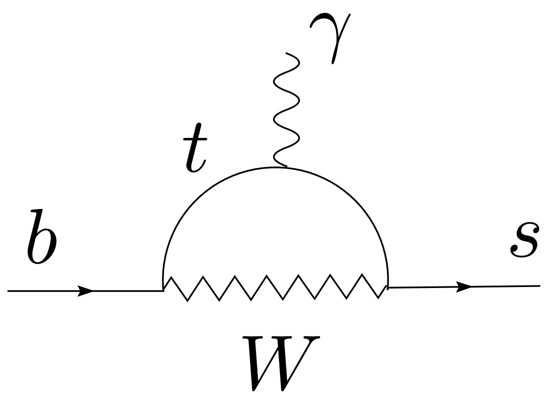
TB and M. Neubert, hep-ph/0512208, hep-ph/0603140 and in preparation

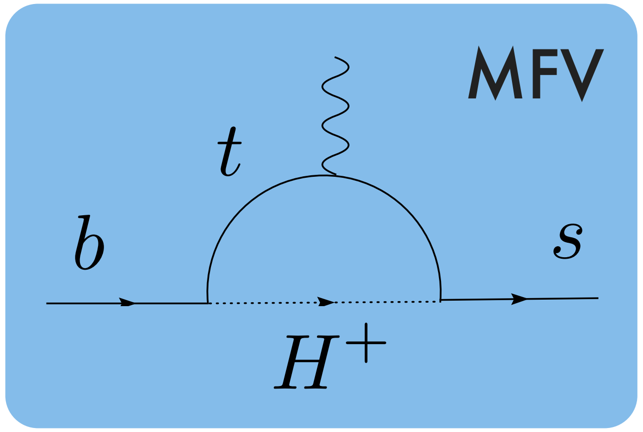
Outline

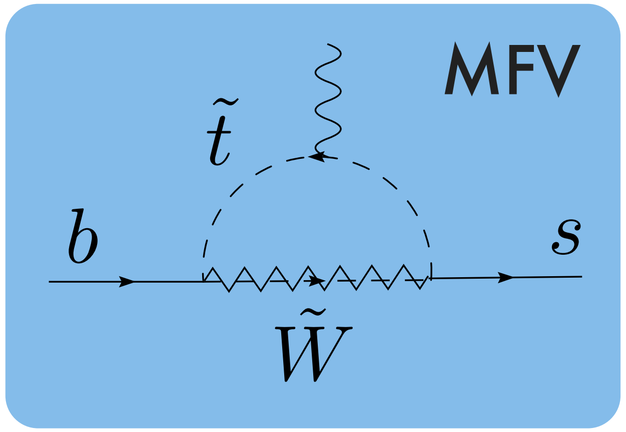
- Introduction
  - Experimental result, theory at NLO
- Status of NNLO calculation
  - Matching, running, matrix elements
- Photon energy cut
  - Factorization and resummation
  - Two-loop calculation of the soft- and jet-function
  - A wonderful formula

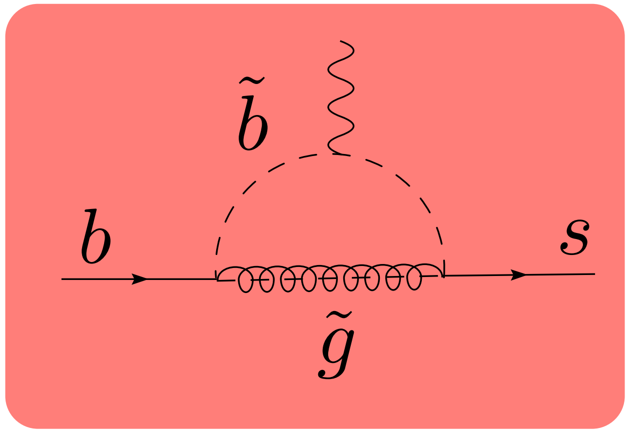
$B \to X_s \gamma$

- The mother of all FCNC processes
  - Suppressed in the SM, but large enough to be well measured.
  - Sensitive probe of New Physics. E.g. MSSM





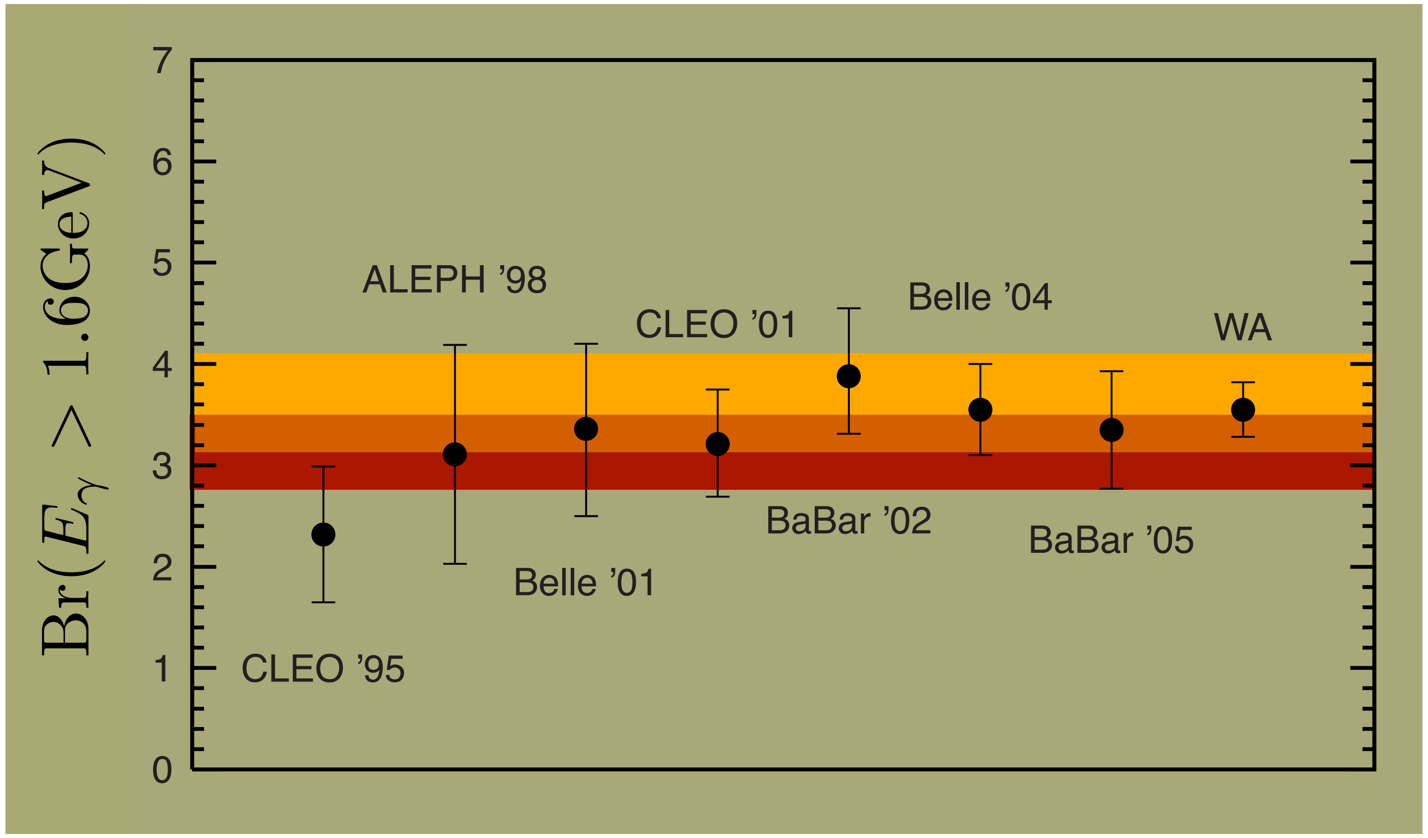




Experiment vs. NLO theory

$$\mathrm{Br}_{\mathrm{exp}}(E_\gamma > 1.6\mathrm{GeV}) = (3.55 \pm 0.24^{+0.09}_{-0.1} \pm 0.03) \times 10^{-4}$$

HFAG ‘06



$m_c/m_b = 0.29 \pm 0.02$ (pole mass)

$m_c/m_b = 0.22 \pm 0.04$ ($\overline{\mathrm{MS}}$)

$m_c(m_c)/m_b^{1\mathrm{S}}$

$$\mathrm{Br}_{\mathrm{SM}}(E_\gamma > 1.6\mathrm{GeV}) = (3.33 \pm 0.29) \times 10^{-4}, \quad m_c/m_b = 0.26 \pm 0.01$$

Haisch ‘06

Experiment?

$$\mathrm{Br}_{\mathrm{exp}}(E_\gamma > 1.6\mathrm{GeV}) = (3.55 \pm 0.24^{+0.09}_{-0.1} \pm 0.03) \times 10^{-4}$$

model uncertainty

$b \to d\gamma$

HFAG '06

- All experiments have $E_\gamma > E_0 \geq 1.8$ GeV
  - HFAG uses *model* shape function to extrapolate results to $E_0 \geq 1.6$ GeV.
    - Use single functional form with two parameters (m_b & μ_π) and
    - model falls off exponentially instead of power-like
- Until recently, experiments used to extrapolate to the total rate.

Extrapolation to lower cut energy E_0

- Standard OPE calculation is unreliable at high E_0. (Note $E_{max}=m_B/2$)
  - Is it reliable at E_0=1.6GeV?
- For current measurements $E_0 \leq 1.9$GeV, model independent calculation is possible! Neubert '04
  - Multi-Scale OPE → *third part of the talk*
  - Compare event fractions: $F(E_0)=\Gamma(E_\gamma>E_0)/\Gamma_{tot}$

	MSOPE @ NLO Neubert '04	HFAG '06 Buchmüller, Flächer '06
$F(E_\gamma>1.8$ GeV)	0.88±0.07	0.933±0.006

PT error dominates

NNLO

- Experimental uncertainty already somewhat smaller than theoretical uncertainty. Need
  - total rate to NNLO and
  - calculation of event fraction to NNLO.

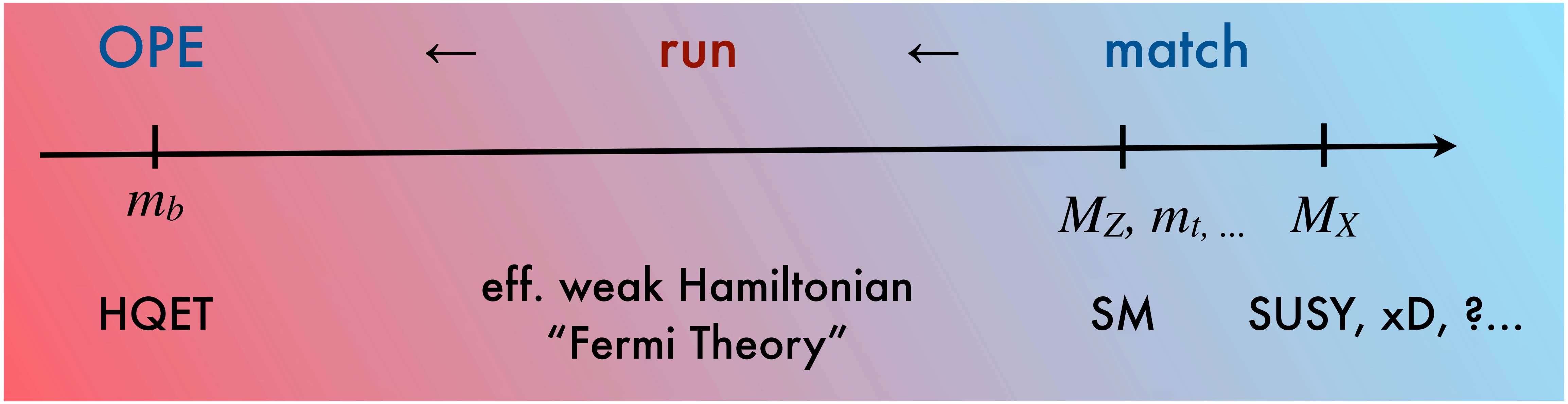
$$\mathrm{Br}(E_\gamma > E_0) = \mathrm{Br}(\mathrm{tot}) \times F(E_0)$$

- With present agreement between theory and experiment large deviation unlikely.
  - Reliable uncertainties crucial to get meaningful bounds on New Physics.

Total $B \to X_s \gamma$ rate to NNLO: quite a loopfest in itself!

Calculation of the total rate

- Three steps



- At lowest order in $1/m_b$ OPE calculation boils down to evaluating the partonic $b \to s\gamma$ matrix elements
- Leading power corrections are known to LO
- NLO EW corrections known

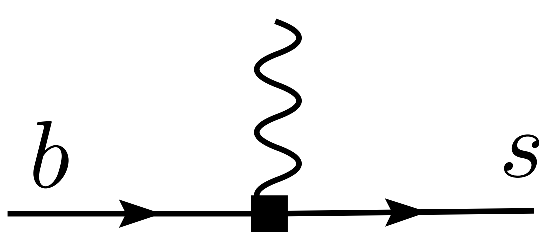
Effective weak Hamiltonian

$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{ps}^* V_{pb} \left[C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3}^{8} C_i Q_i \right],$$

- Small contribution from $|C_{3-6}(m_b)| < 0.07$

LO

Q_7

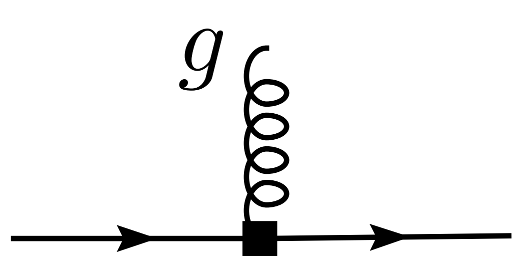


$m_b \bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R$

$C_7(m_b) \approx -0.3$

NLO

Q_8

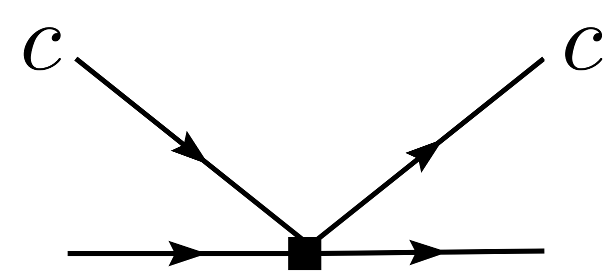


$m_b \bar{s}_L \sigma_{\mu\nu} G^{\mu\nu} b_R$

$C_8(m_b) \approx -0.15$

NLO

$Q_{1,2}$



$\bar{c}_L \gamma_\mu b_L \; \bar{s}_L \gamma_\mu c_L$

$|C_{1,2}(m_b)| \approx 1$

Effective Hamiltonian at NNLO: matching

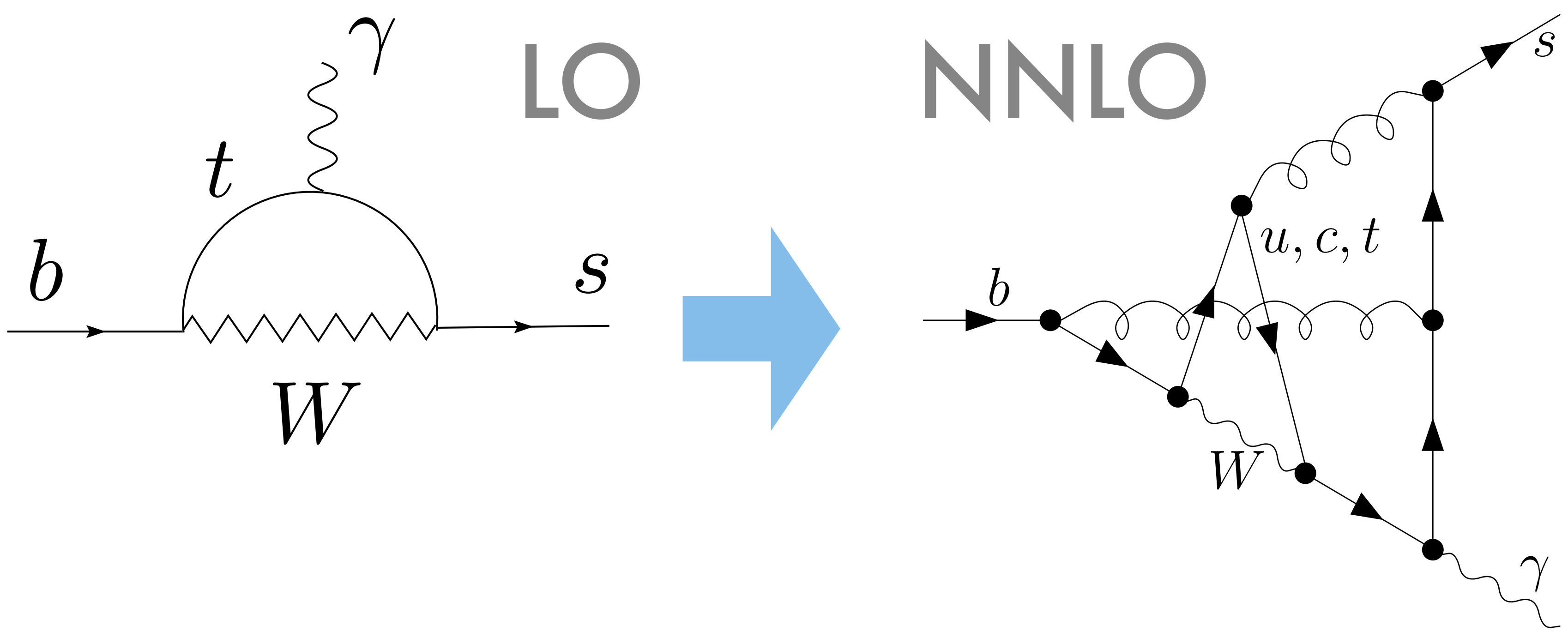
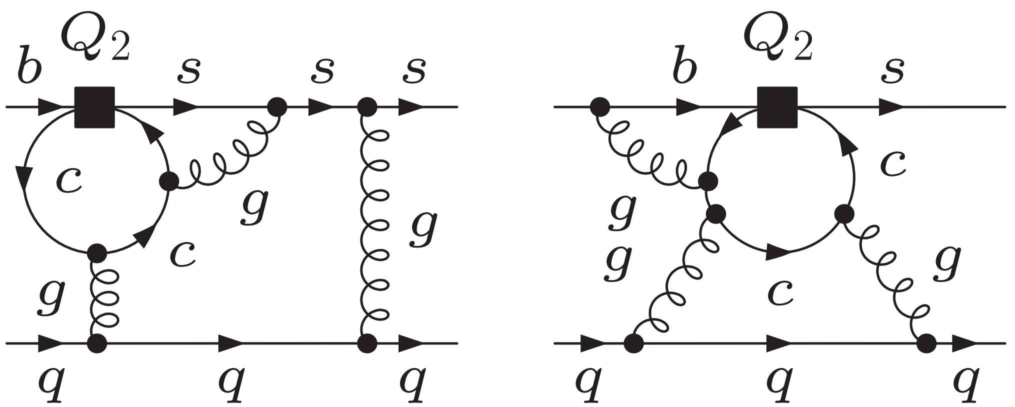


Figure 1: One of the $\mathcal{O}(10^3)$ three-loop diagrams that ~~we~~ have calculated.

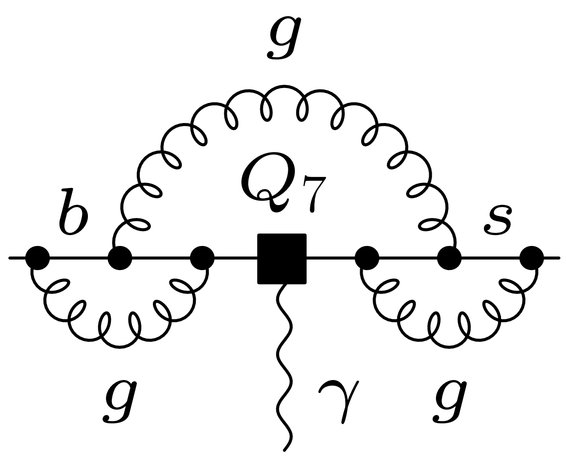
Misiak and Steinhauser '04

- Matching at the weak scale complete.
  - NNLO = three loops because of weak-interaction loop ("penguin")

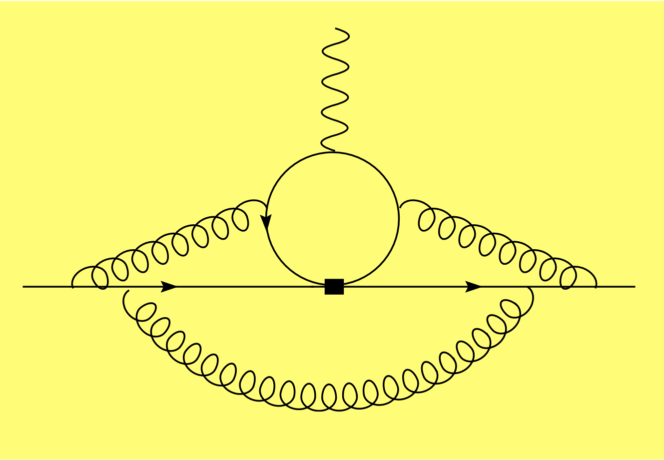
Effective Hamiltonian at NNLO: running



Gorbahn and Haisch '04



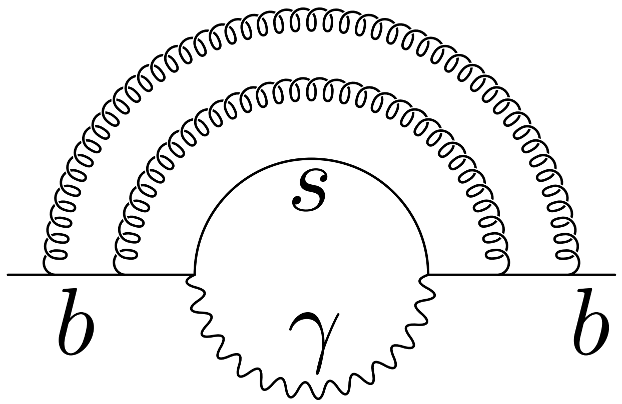
Gorbahn, Haisch, Misiak '05

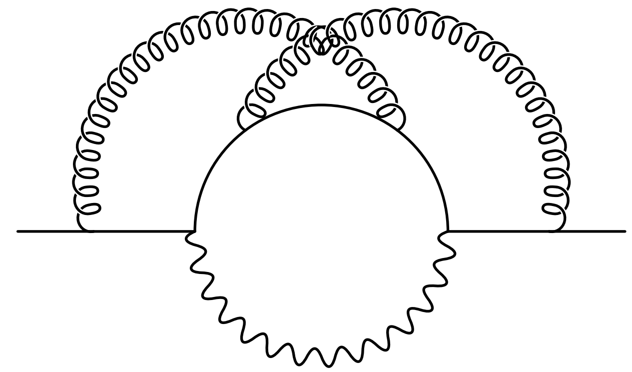
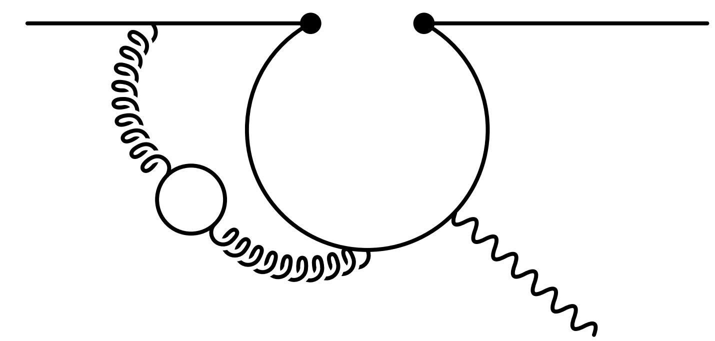


Czakon, Haisch & Misiak, to appear

- Done: Four quark operators, self-mixing of dipole operators.
- *New*: four-loop anomalous dimensions for mixing $Q_{1\text{-}6}$ into Q_7. (>100'000 diagrams)
    - Effect on Br is -2.4%, larger than expected.
- *Soon*: $Q_{1\text{-}6}$ into Q_8. Completes NNLO eff. Hamiltonian!

Matrix elements



Melnikov, Mitov '05
Blokland, Czarnecki, Misiak, Slusarczyk, Tkachov '05

n_f-part: Bieri, Greub, Steinhauser '03

- Done: Matrix element of Q_7.
- Most complicated part of entire calculation are the 3-loop matrix elements of $Q_{1,2}$.
  - n_f-part known.
  - expansion around $m_c/m_b >> 1$ (!) calculated, extrapolation to m_c/m_b. Misiak et al., to appear
    - At NLO this works numerically fairly well, but...

Photon energy cut

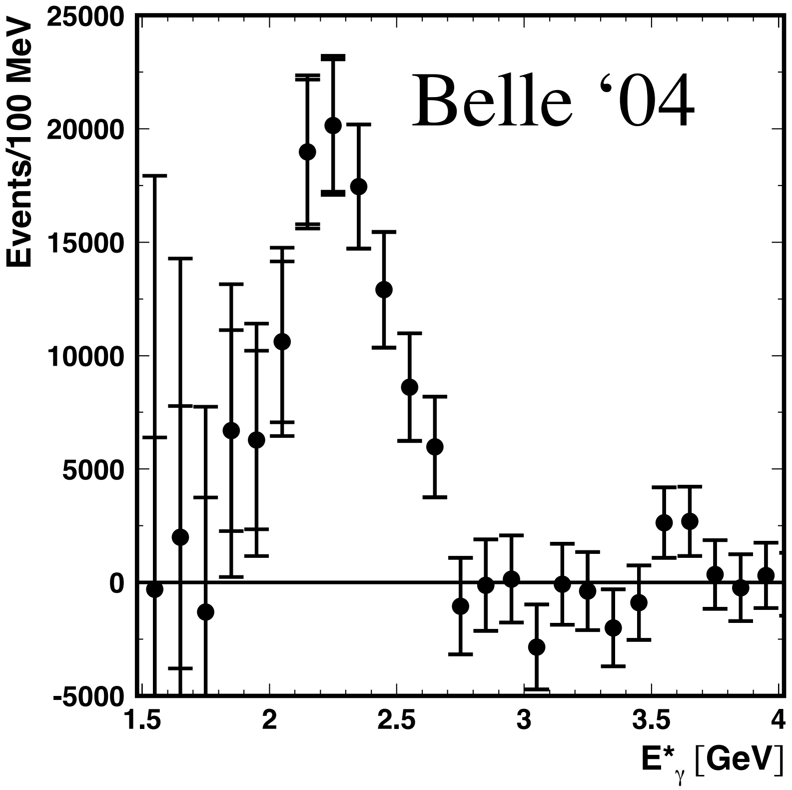
1. The total rate cannot be measured.
2. Even if it could be measured, it could not be calculated.
3. Even if it could be calculated, the result would be incorrect.

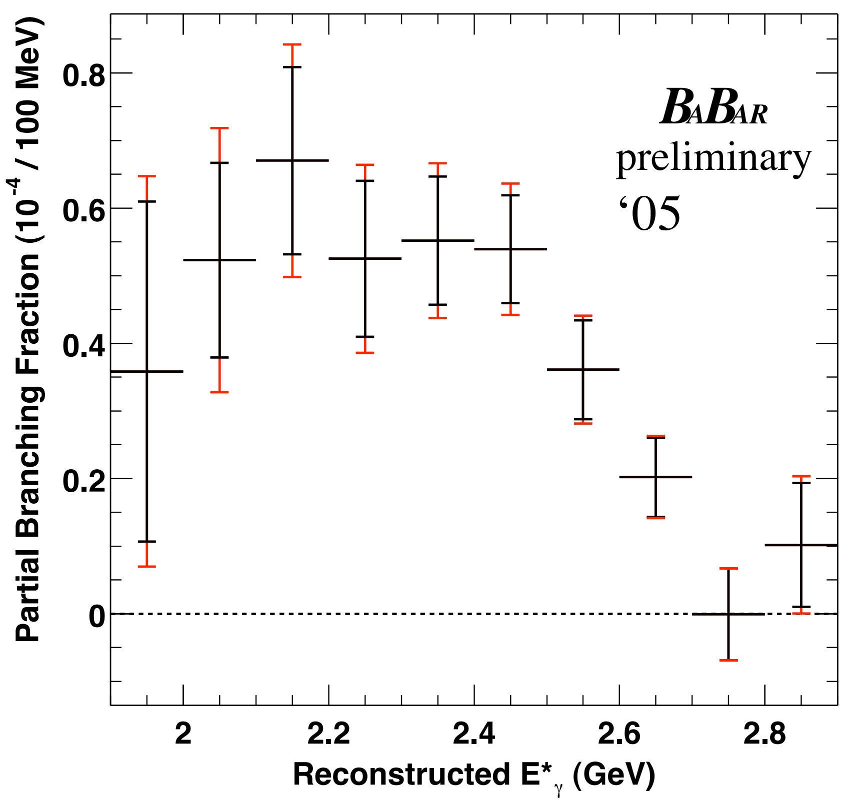
after Gorgias, 483-375 BC

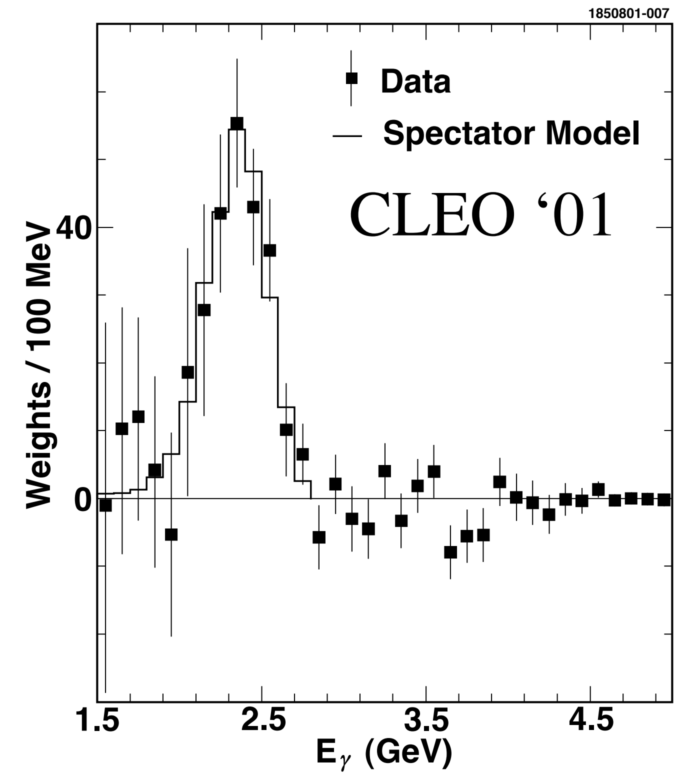
Photon energy cut

- Total rate is not measurable, need to impose cut on photon energy $E_\gamma > E_0$
  - Experimentally very energetic photon is necessary to suppress background.
    - All experiments have $E_0 \geq 1.8\mathrm{GeV}$.
    - Note: $E_\gamma < m_{B/2} \approx 2.6\mathrm{GeV}$
  - Need to cut out charm resonances: decay $\mathrm{B} \to \psi X$ followed by $\psi \to X\gamma$. Achieved by setting $E_0 > 1.5\mathrm{GeV}$.

Photon energy spectrum





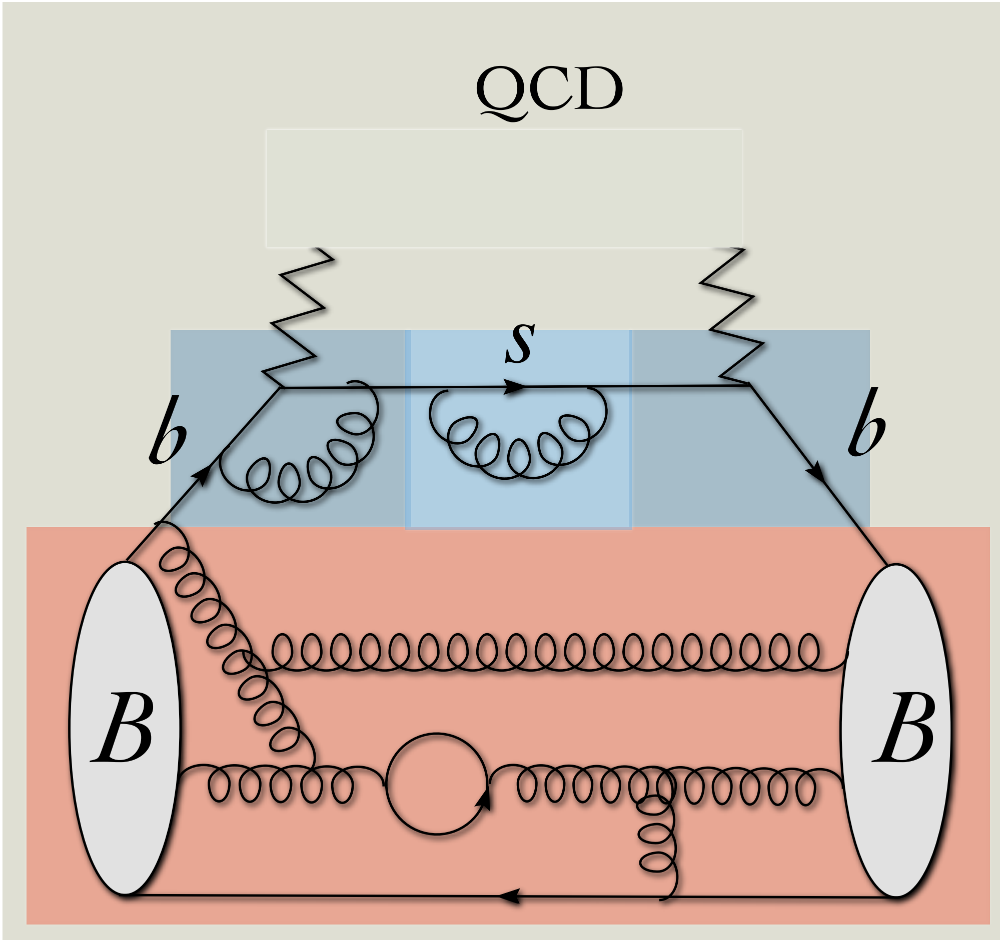


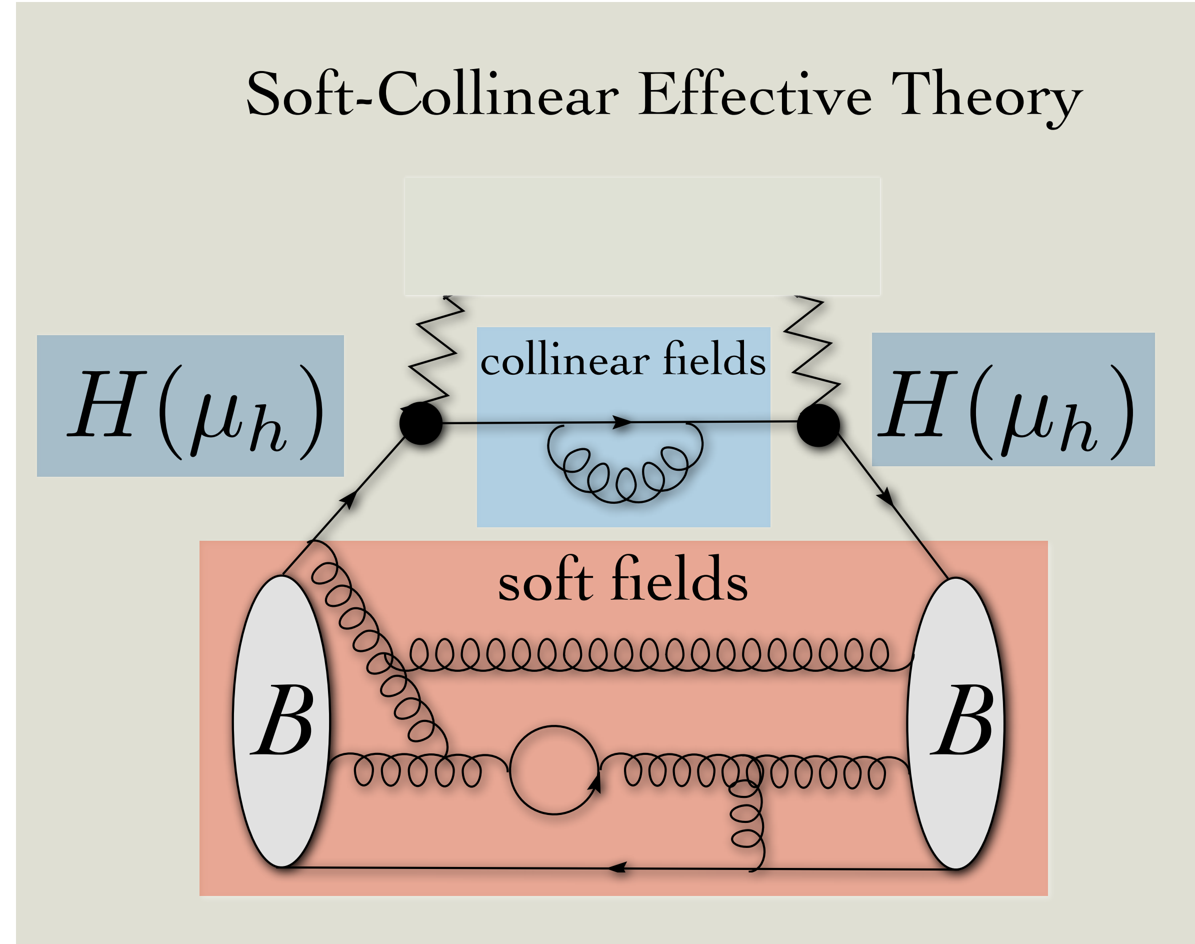
- Belle has E_0=1.8GeV, BaBar E_0=1.9 GeV
- Cut complicates theoretical analysis...

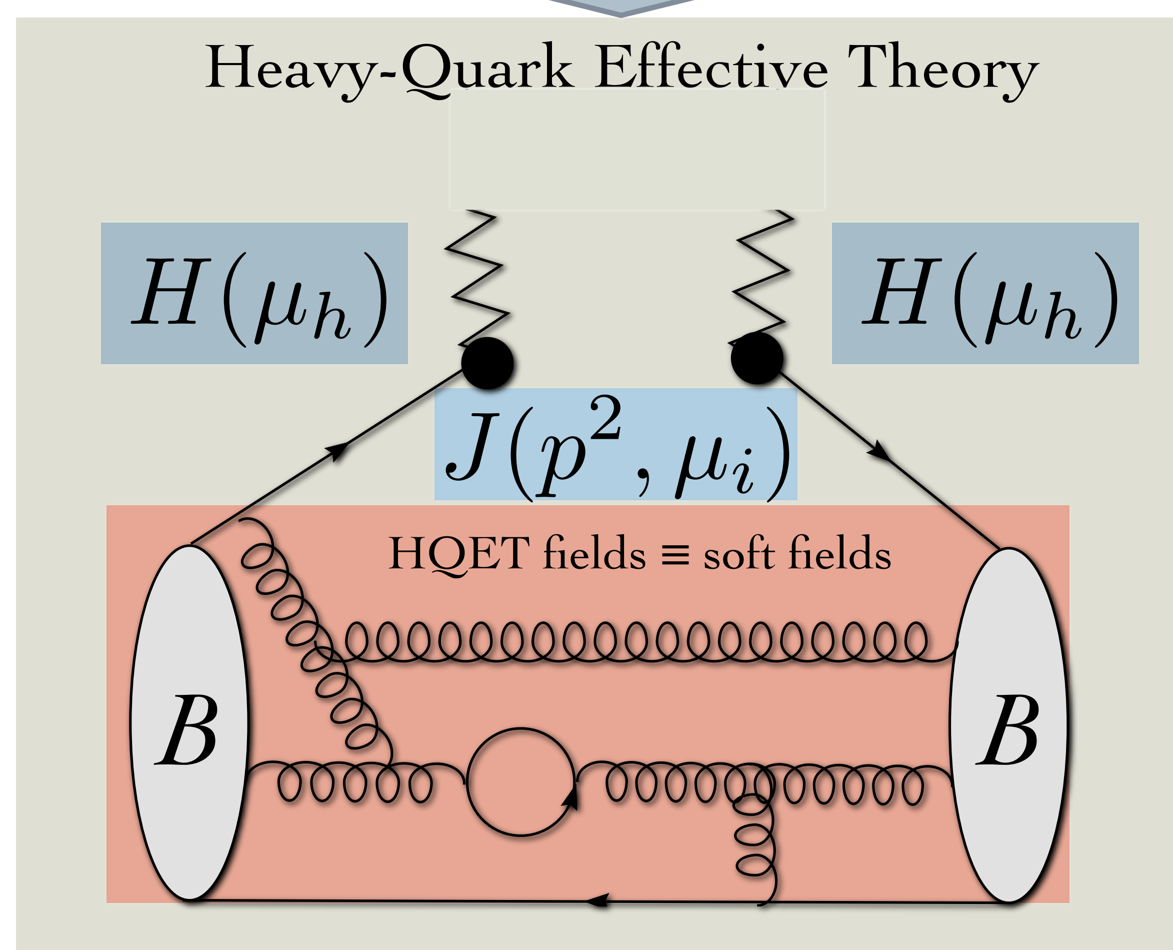
Scales

- With a cut $E_\gamma > E_0$, problem contains three relevant scales
  - Hard scale: m_b
  - Jet scale: $M_X \sim (m_b \Delta)^{1/2}$
  - Soft scale: $\Delta = m_b - 2E_0$
- OPE becomes expansion in Λ/Δ instead of Λ/m_b!

E_0 [GeV]	Δ [GeV]	M_X [GeV]	
0	4.6	4.6	OPE
1.6	1.4	2.5	MSOPE
1.8	1.0	2.1	MSOPE
2.0	0.6	1.7	Shape-function







Factorization theorem

$$\Gamma \sim H^2 J \otimes S$$

hard jet soft

shape function

Korchemsky, Sterman '94

Event fraction

$$F(E_0) = h(m_b, \mu) \int_0^{\Delta} dP \int_0^{P} d\omega \, J(m_b(P - \omega), \mu) \, S(\omega, \mu)$$

- Any choice of scale μ leads to large perturbative log's, either in *h*, *J*, or *S*.
  - Evaluate each part at its characteristic scale and use RG evolution to evolve to a common scale.
  - This resums all (Sudakov) logarithms.
- Same jet-function *J* as in DIS for x→1.

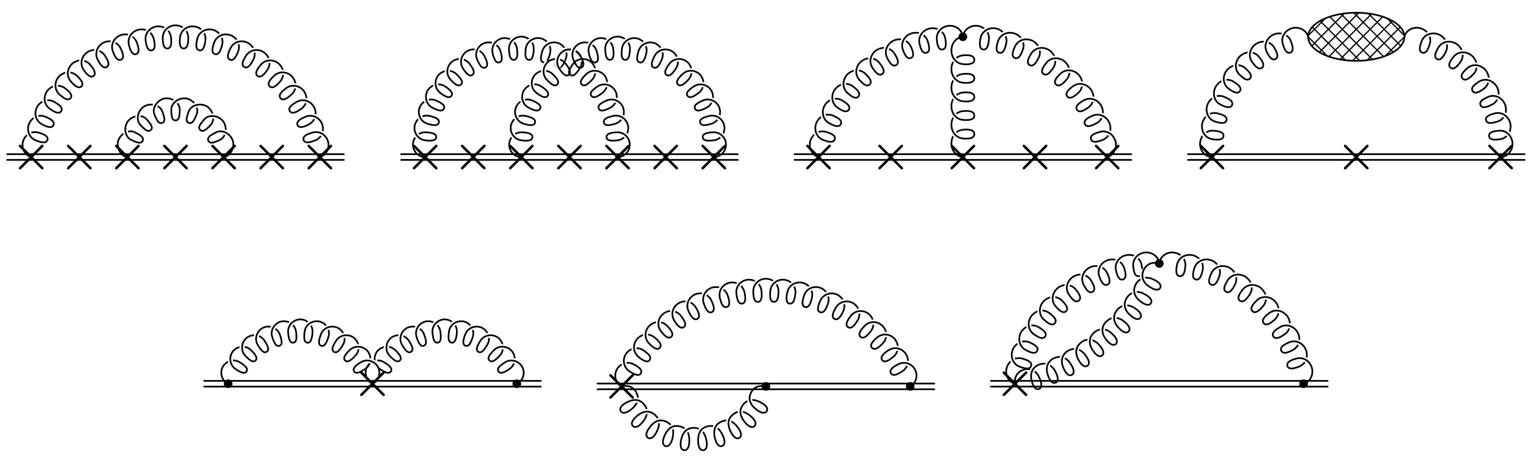
Calculation of the soft function

$$s\left(\ln\frac{\Omega}{\mu},\mu\right) \equiv \int_0^{\Omega} d\omega\, \underbrace{\langle b_v|\,\bar{h}_v\,\delta(\omega + in\cdot D)\,h_v\,|b_v\rangle}_{S_{\text{parton}}(\omega,\mu)} = \frac{1}{2\pi i}\oint_{|\omega|=\Omega} d\omega\,\langle b_v|\,\bar{h}_v\,\frac{1}{\omega + in\cdot D + i0}\,h_v\,|b_v\rangle$$

- Write δ-distribution operator in shape function as discontinuity of propagator

➡ Calculation can be done with standard techniques for loop integrals

Calculation of the soft function



- $\times$-vertex denotes possible insertion of light-cone propagator

- All diagrams are expressed in terms of integrals

$$\int d^d k \, d^d l \, \frac{(-1)^{-a_1-a_2-a_3-b_1-b_2-b_3-c_1-c_2}}{(k^2)^{a_1}\,(l^2)^{a_2}\,[(k-l)^2]^{a_3}\,(v\cdot k)^{b_1}\,(v\cdot l)^{b_2}\,[v\cdot(k+l)]^{b_3}\,(n\cdot k+\omega)^{c_1}\,(n\cdot l+\omega)^{c_2}}$$

- Use integration-by-part relations to reduce to *four master integrals* ("AIR" by Anastasiou and Lazopoulos hep-ph/0404258)

Result for the soft function

$$
\begin{aligned}
s(L,\mu) &= 1 + \frac{\alpha_s(\mu)}{4\pi}\left[c_0^{(1)} + 2\gamma_0 L - \Gamma_0 L^2\right] \\
&\quad + \left(\frac{\alpha_s(\mu)}{4\pi}\right)^2 \left[c_0^{(2)} + \left(2c_0^{(1)}(\gamma_0-\beta_0) + 2\gamma_1 + \frac{2\pi^2}{3}\Gamma_0\gamma_0 + 4\zeta_3\Gamma_0^2\right)L \right. \\
&\quad \left. + \left(2\gamma_0(\gamma_0-\beta_0) - c_0^{(1)}\Gamma_0 - \Gamma_1 - \frac{\pi^2}{3}\Gamma_0^2\right)L^2 + \left(\frac{2}{3}\beta_0 - 2\gamma_0\right)\Gamma_0 L^3 + \frac{\Gamma_0^2}{2}L^4\right]
\end{aligned}
$$

- All $L=\ln(\Omega/\mu)$ terms known from solving RG equation for the shape function → check on the calculation. Neubert ‘05
- Calculation gives constant $c_0^{(2)}$ and checks two loop anomalous dim. γ_1 of shape function Kochemsky, Marchesini ‘93 Neubert ‘04, Gardi ‘04,

$$
\begin{aligned}
c_0^{(2)} &= C_F^2\left(-\frac{4\pi^2}{3} - \frac{3\pi^4}{40} + 32\zeta_3\right) + C_F C_A\left(-\frac{326}{81} - \frac{427\pi^2}{108} + \frac{67\pi^4}{180} - \frac{107}{9}\zeta_3\right) \\
&\quad + C_F T_F n_f\left(-\frac{8}{81} + \frac{5\pi^2}{27} - \frac{20}{9}\zeta_3\right).
\end{aligned}
$$

TB and Neubert, hep-ph/0512208

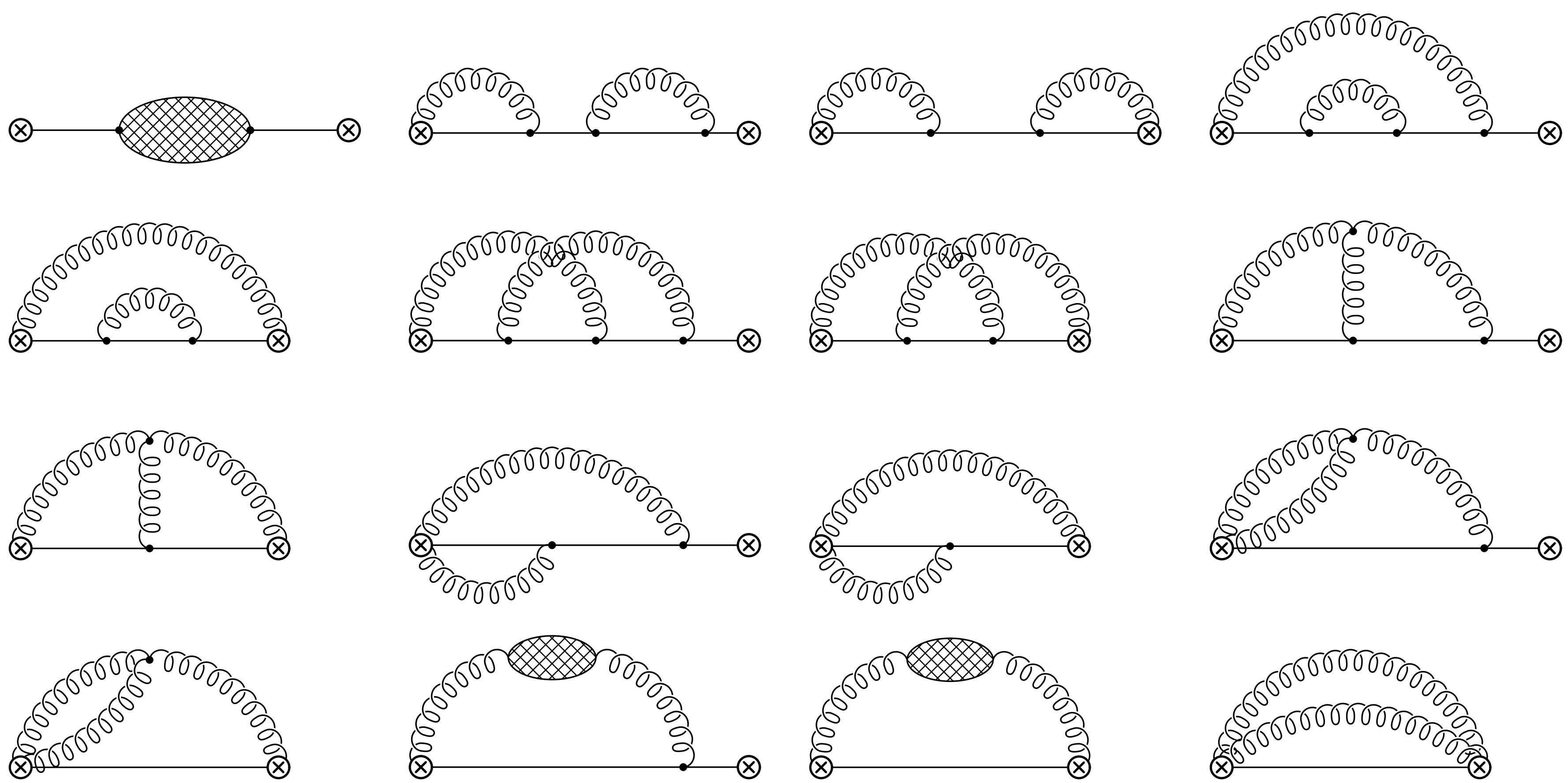
Calculation of the jet-function

$$J(p^2) = \frac{1}{\pi}\mathrm{Im}\left[i\mathcal{J}(p^2)\right] \quad ; \quad \frac{\not{n}}{2}\,\bar{n}\cdot p\,\mathcal{J}(p^2) = \int d^4x e^{-ipx}\langle 0|\,\mathrm{T}\left\{\frac{\not{n}\bar{\not{n}}}{4}W^\dagger(0)\psi(0)\,\overline{\psi}(x)W(x)\frac{\bar{\not{n}}\not{n}}{4}\right\}|0\rangle$$

light-cone vectors n and $\bar{\text{n}}$

QCD propagator with light-like Wilson lines

$$W(x) = \mathbf{P}\,\exp\left(ig\int_{-\infty}^{0} ds\,\bar{n}\cdot A(x+s\bar{n})\right)$$



Result for the jet function

- One difficult master integral

$$\int d^d k \int d^d l \, \frac{1}{k^2 \, l^2 \, (k+p)^2 \, (l+p)^2 \, (k+l+p)^2 \, \bar{n}\cdot k \, \bar{n}\cdot l}$$

  - Use Mellin-Barnes representation, but careful with light-cone propagators
  - Check numerically using sector decomposition

- Result for two-loop constant in J

$$b_0^{(2)} = C_F^2 \left(\frac{205}{8} - \frac{67\pi^2}{6} + \frac{14\pi^4}{15} - 18\zeta_3 \right) + C_F n_f \left(-\frac{4057}{324} + \frac{34\pi^2}{27} + \frac{8}{9}\zeta_3 \right)$$
$$+ C_A C_F \left(\frac{53129}{648} - \frac{208\pi^2}{27} - \frac{17\pi^4}{180} - \frac{206}{9}\zeta_3 \right) .$$

RG evolution of the jet-function

$$\frac{dJ(p^2,\mu)}{d\ln\mu} = -\left[2\Gamma_{\rm cusp}(\alpha_s)\,\ln\frac{p^2}{\mu^2} + 2\gamma^J(\alpha_s)\right] J(p^2,\mu)$$
$$- 2\Gamma_{\rm cusp}(\alpha_s)\int_0^{p^2} dp'^2\,\frac{J(p'^2,\mu) - J(p^2,\mu)}{p^2 - p'^2}$$

Sudakov factor

$$J(p^2,\mu) = \exp\left[-4S(\mu_i,\mu) + 2a_{\gamma^J}(\mu_i,\mu)\right]$$
$$\times\, \widetilde{j}(\partial_\eta,\mu_i)\,\frac{e^{-\gamma_E\eta}}{\Gamma(\eta)}\,\frac{1}{p^2}\left(\frac{p^2}{\mu_i^2}\right)^\eta,$$

$$\eta = 2\int_{\mu_0}^{\mu_i}\frac{d\mu}{\mu}\,\Gamma_{\rm cusp}[\alpha_s(\mu)]$$
$$= 2a_\Gamma(\mu_i,\mu).$$

- Associated jet-function $\widetilde{j}$ is Laplace transform of $J(p^2,\mu_i)$.
- RG evolution of shape function $S(\omega,\mu_i)$ has exactly the same form.

“Wonderful formula”

Neubert ‘05

$$F(E_0) = h(m_b,\mu)\int_0^{\Delta} dP \int_0^{P} d\omega\, J(m_b(P-\omega),\mu)\, S(\omega,\mu)$$

- Plug in solution of RG equations

$$\begin{aligned} F(E_0) \quad = \quad & \left(\frac{m_b}{\mu_h}\right)^{-2a_\Gamma(\mu_h,\mu)} \left(\frac{m_b\Delta}{\mu_i^2}\right)^{2a_\Gamma(\mu_i,\mu)} \left(\frac{\Delta}{\mu_0}\right)^{-2a_\Gamma(\mu_0,\mu)} \\ & \times \exp\left[2S(\mu_h,\mu) - 2S(\mu_i,\mu) + 2S(\mu_0,\mu) - 2a_{\gamma^J}(\mu_h,\mu_i) - 2a_\gamma(\mu_h,\mu_0)\right] \\ & \times h\left(\ln\frac{m_b}{\mu_h},\mu_h\right)\, \tilde{j}\left(\ln\frac{m_b\Delta}{\mu_i^2} + \partial_\eta,\mu_h\right)\, \tilde{s}\left(\ln\frac{\Delta}{\mu_0} + \partial_\eta,\mu_h\right) \frac{e^{-\gamma_E\eta}}{\Gamma(1+\eta)} \end{aligned}$$

h is inferred from Melnikov and Mitov ‘05

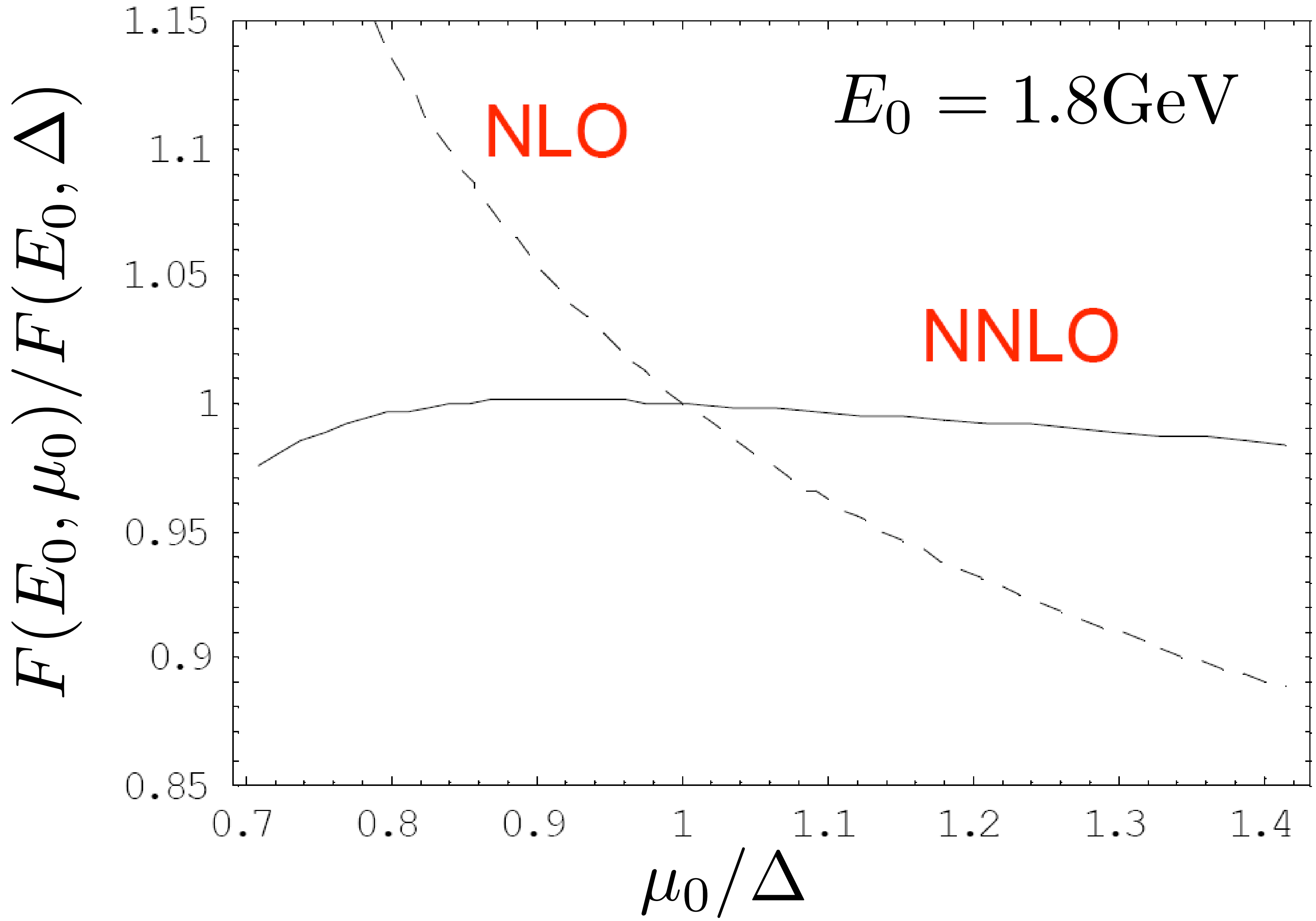
$$\eta = \eta_J + \eta_S = 2a_\Gamma(\mu_i,\mu_0)$$

“Wonderful formula”

$$
\begin{aligned}
F(E_0) \quad = \quad & \left(\frac{m_b}{\mu_h}\right)^{-2a_\Gamma(\mu_h,\mu)} \left(\frac{m_b\Delta}{\mu_i^2}\right)^{2a_\Gamma(\mu_i,\mu)} \left(\frac{\Delta}{\mu_0}\right)^{-2a_\Gamma(\mu_0,\mu)} \\
& \times \exp\left[2S(\mu_h,\mu) - 2S(\mu_i,\mu) + 2S(\mu_0,\mu) - 2a_{\gamma^J}(\mu_h,\mu_i) - 2a_\gamma(\mu_h,\mu_0)\right] \\
& \times h\left(\ln\frac{m_b}{\mu_h}, \mu_h\right) \tilde{j}\left(\ln\frac{m_b\Delta}{\mu_i^2} + \partial_\eta, \mu_h\right) \tilde{s}\left(\ln\frac{\Delta}{\mu_0} + \partial_\eta, \mu_h\right) \frac{e^{-\gamma_E\eta}}{\Gamma(1+\eta)}
\end{aligned}
$$

- All scales separated, no large perturbative logarithms.
- Simple analytic expressions for resummed result
  - no need to go to moment space
  - no Landau pole ambiguities
- Very similar expressions for DIS and Drell-Yan in threshold region. → Matthias’ talk

Reduced scale dependence



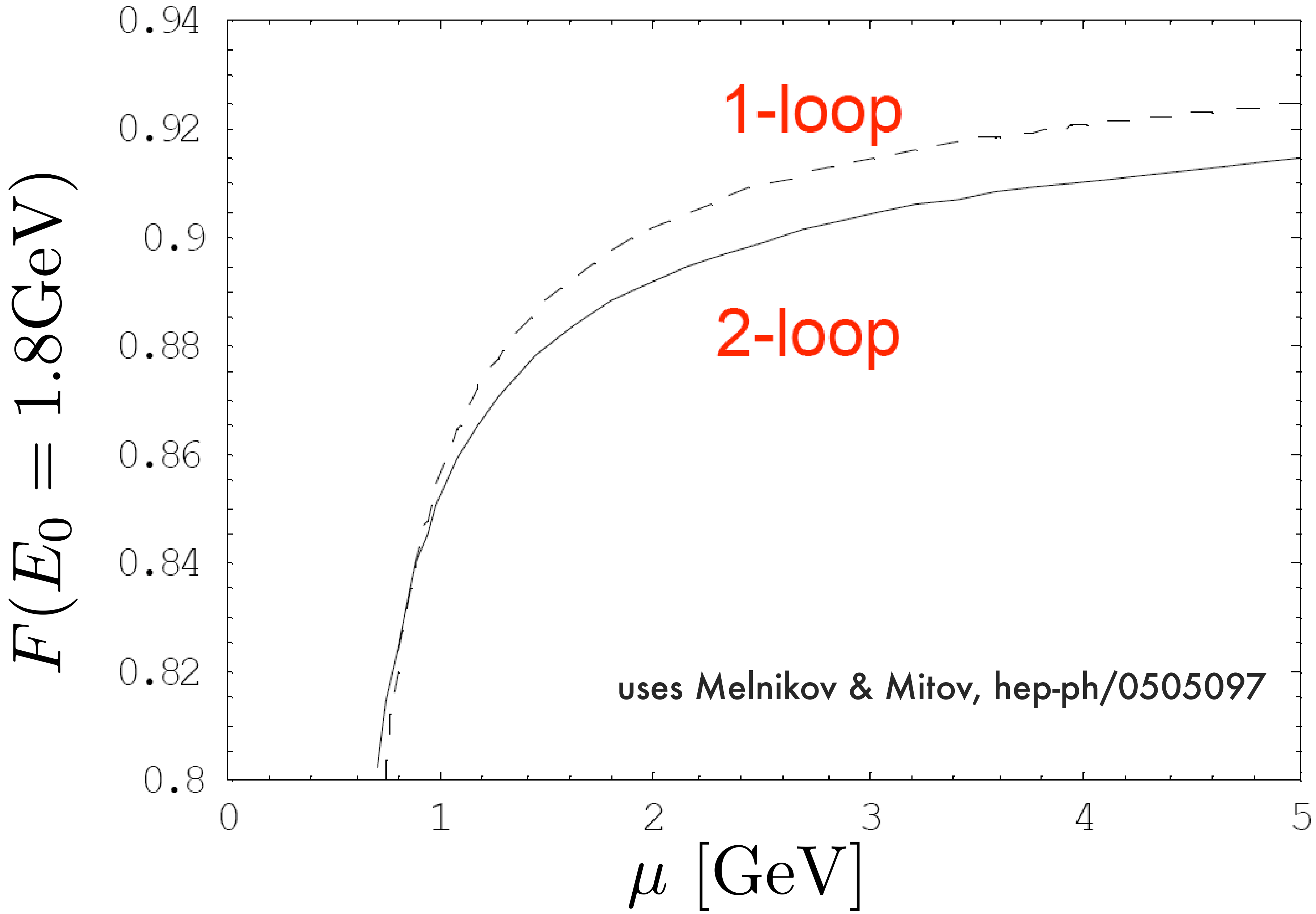
- Result is very stable under variation of the lowest scale $\mu_0 \sim \Delta \approx 1\mathrm{GeV}$!

Summary and Conclusion

- $B \to X_s \gamma$ is an important constraint on New Physics.
- NNLO calculation is progressing fast
  - Calculation of eff. weak Hamiltonian is almost complete.
  - Matrix elements of Q_7 known; approximations for matrix elements of $Q_{1,2}$.
  - Effect of the photon energy cut has been calculated.
- NNLO numbers should follow soon...

extra slides

The need for resummation: fixed order result



- μ=Δ ? μ=(Δm_b)$^{1/2}$, μ=m_b ?

Experimental results

Mode	Reported $\mathcal{B}$	$E_{\rm min}$	$\mathcal{B}$ at $E_{\rm min}$	Modified $\mathcal{B}$ ($E_{\rm min}=1.6$)
CLEO Inc. [264]	$321 \pm 43 \pm 27^{+18}_{-10}$	2.0	$306 \pm 41 \pm 26$	$329 \pm 44 \pm 28 \pm 6 \pm 6$
Belle Semi. [265]	$336 \pm 53 \pm 42^{+50}_{-54}$	2.24	–	$369 \pm 58 \pm 46^{+56}_{-60}$
Belle Inc. [266]	$355 \pm 32^{+30+11}_{-31-7}$	1.8	$351 \pm 32 \pm 29$	$350 \pm 32^{+30}_{-31} \pm 2 \pm 2$
BABAR Semi. [267]	$335 \pm 19^{+56+4}_{-41-9}$	1.9	$327 \pm 18^{+55+4}_{-43-9}$	$349 \pm 20^{+59+4}_{-46-3}$
BABAR Inc. [268]	–	1.9	$367 \pm 29 \pm 34 \pm 29$	$392 \pm 31 \pm 36 \pm 30 \pm 4 \pm 6$

[264] CLEO Collaboration (S. Chen *et al.*), Phys. Rev. Lett. **87**, 251807 (2001).

[265] Belle Collaboration (K. Abe *et al.*), Phys. Lett. B **511**, 151 (2001).

[266] Belle Collaboration (P. Koppunberg *et al.*), Phys. Rev. Lett. **93**, 061803 (2004).

[267] *BABAR* Collaboration (B. Aubert *et al.*), Phys. Rev. D **72**, 052004 (2005).

[268] *BABAR* Collaboration (B. Aubert *et al.*), hep-ex/0507001.