The SPA project and the on-shell scheme (in the MSSM)

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The SPA project - motivation and general idea

MOTIVATION:

- At a future e^+e^- LC measurements with high precision possible
  $\rightarrow$ requires equally accurate theoretical calculations including radiative corrections

- Allows high precision determination of SUSY parameters
  $\rightarrow$ Supersymmetry Parameter Analysis (SPA) - a framework to extract the parameters

GENERAL IDEA:

- The observables (i.e. decay widths, branching ratios, cross-sections) calculated in terms of the $\overline{\mathrm{DR}}$ input parameters - the underlying parameters determined via a global fit

SPA Conventions

- Masses of SUSY particles and Higgs bosons are given as pole masses

- All SUSY parameters in $\mathcal{L}$ are given in the $\overline{\mathrm{DR}}$ scheme (dimensional reduction) at the scale $\hat{M} = 1$ TeV

- All elements in mass matrices, rotation matrices, and corresponding mixing angles at tree-level are given in $\overline{\mathrm{DR}}$ scheme at $\hat{M} = 1$ TeV, except for the Higgs sector where the mixing angle is defined on-shell

- SM input parameters: G_F, α, m_Z, $\alpha_s(m_Z)$ and the fermion masses

- Branching ratios and cross-sections are expressed in terms of pole masses and $\overline{\mathrm{DR}}$ SUSY parameters

Standard Model Parameters – details

- To reach a consistent set of input parameters, all parameters should be transformed to the $\overline{\mathrm{DR}}$ scheme
- SUSY parameters already given in the $\overline{\mathrm{DR}}$ scheme
- To ensure the best precision possible - SM parameters given in different ways

The Standard Model input parameters:

Parameter	SM input	Parameter	SM input
m_e	$5.110 \cdot 10^{-4}$	m_t^{pole}	172.7
m_μ	0.1057	$m_b(m_b)$	4.2
m_τ	1.777	m_Z	91.1876
$m_u(Q)$	$3 \cdot 10^{-3}$	G_F	$1.1664 \cdot 10^{-5}$
$m_d(Q)$	$7 \cdot 10^{-3}$	$1/\alpha$	137.036
$m_s(Q)$	0.12	$\Delta\alpha_{had}^{(5)}$	0.02769
$m_c(m_c)$	1.2	$\alpha_s^{\overline{\mathrm{MS}}}(m_Z)$	0.119

where $Q = 2$ GeV.

Standard Model Parameters – details cont.

- The W-boson pole mass not input – calculated via (Degrassi - Fanchiotti - Sirlin, Pierce et al.)

$$m_W^2 = m_Z^2 \hat{\rho} \left(\frac{1}{2} + \sqrt{\frac{1}{4} - \frac{\alpha^{\overline{\rm DR}}(m_Z)\pi}{\sqrt{2} G_F m_Z^2 \hat{\rho} (1 - \Delta\hat{r})}} \right)$$

$$\Delta\hat{r} = \hat{\rho} \frac{\Pi_{WW}^T(0)}{m_W^2} - \frac{\Pi_{ZZ}^T(m_Z^2)}{m_Z^2} + \delta_{VB}$$

  (where 2–loop + SUSY contributions included)

- Transformation of the input pole masses to $\overline{\rm DR}$ masses necessary
  e.g. for SUSY $\overline{\rm DR}$ mass matrices

- Transformation straightforward

$$m^{\overline{\rm DR}}(Q) = m^{\rm pole} + \delta m^{\rm OS,fin}(Q)$$

- For example top-quark mass: starts from pole mass $m_t^{pole} \to m_t^{\overline{\rm DR}}(m_Z)$
  (with 2-loop gluon shift + 1-loop SUSY)

Standard Model Parameters – details cont.

- First all SM parameters calculated at the scale $Q = m_Z$, then run to $Q = 1\mathrm{TeV}$
- Important: m_b running in 3 steps:
  - The input value is the Standard Model $\overline{\mathrm{MS}}$- value $m_b^{\overline{\mathrm{MS}},SM}(m_b)$

$$m_b^{\overline{\mathrm{MS}},SM}(m_b) \longrightarrow m_b^{\overline{\mathrm{MS}},SM}(m_Z) \qquad \text{using the 4-loop RGE's}$$

  - Change of scheme from $\overline{\mathrm{MS}}$ to $\overline{\mathrm{DR}}$

$$m_b^{\overline{\mathrm{DR}},SM}(m_Z) = m_b^{\overline{\mathrm{MS}},SM}(m_Z)\left(1 - \frac{\alpha_s^{\overline{\mathrm{DR}}}}{3\pi} - \frac{23(\alpha_s^{\overline{\mathrm{DR}}})^2}{72\pi^2} + \frac{4g_2^2}{128\pi^2} - \frac{13g'^2}{1152\pi^2}\right)$$

  - Inclusion of the SUSY corrections + resummation (Carena et al.)

$$m_b^{\overline{\mathrm{DR}}}(M_Z) = \frac{m_{b,\,\mathrm{SM}}^{\overline{\mathrm{DR}}}(M_Z) + \mathrm{Re}\,\Sigma_b'(M_Z)}{1 - \Delta m_b(M_Z)}$$

- Similar steps are followed for the charm quark running mass

Standard Model Parameters – details cont.

- Running the masses from the scale $Q = m_Z$ to $Q = 1\mathrm{TeV}$ using the 2-loop RGE's for the Yukawa couplings (Martin-Vaughn, Yamada, Jack-Jones)

$$Y_f^{\overline{\mathrm{DR}}}(m_Z) = \frac{\sqrt{2} m_f^{\overline{\mathrm{DR}}}(m_Z)}{v_i(m_Z)}, \qquad \text{with} \qquad v^2(m_Z) = 4\frac{m_Z^2 + Re(\Pi_{ZZ}^T(m_Z))}{g'^2(m_Z) + g_2^2(m_Z)}$$

Remaining SM parameters:

- The weak mixing angle is also taken $\overline{\mathrm{DR}}$ i.e.

$$(\cos\theta_W)^{\overline{\mathrm{DR}}} \equiv \hat{c}, \quad \hat{c}^2 \hat{s}^2 = \frac{\pi \alpha^{\overline{\mathrm{DR}}}(m_Z)}{\sqrt{2} m_Z^2 G_\mu (1 - \Delta\hat{r})}$$

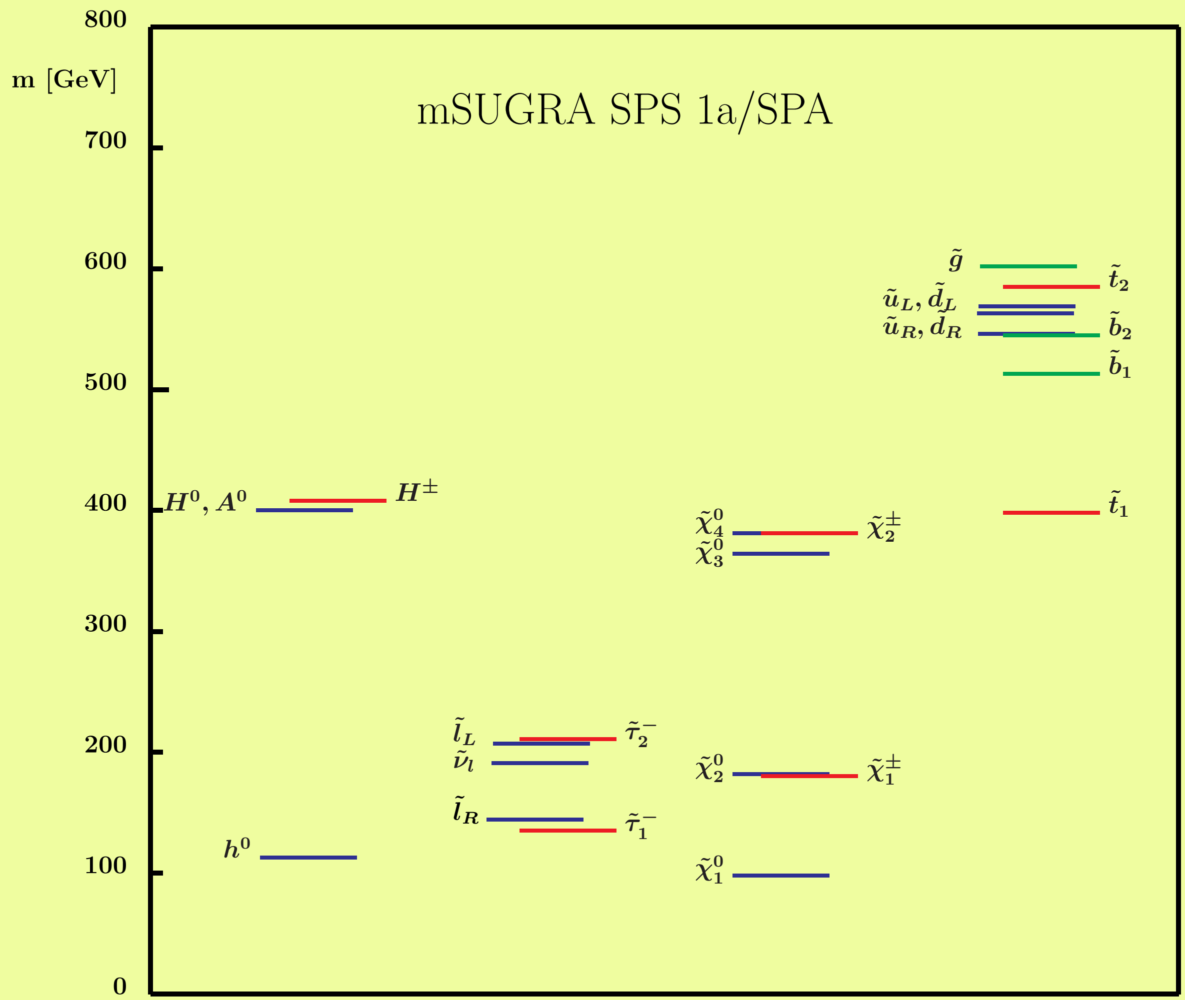
- Fine structure constant transformed from Thomson limit to $\alpha^{\overline{\mathrm{DR}}}(M_Z)$

$$\alpha^{\overline{\mathrm{DR}}}(M_Z) \; = \; \frac{\alpha}{1 - \Delta\alpha_{\mathrm{SM}} - \Delta\alpha_{\mathrm{SUSY}}}$$

$$\Delta\alpha_{\mathrm{SUSY}} \; = \; -\frac{\alpha}{6\pi}\left[\ln\frac{m_{H^+}}{M_Z} \; + \; 4\sum_{i=1}^{2}\ln\frac{m_{\tilde{\chi}_i^+}}{M_Z} + \; \sum_f \sum_{i=1}^{2} N_c Q_f^2 \ln\frac{m_{\tilde{f}_i}}{M_Z}\right]$$

Input example – SPS1a' benchmark point

g'	0.36354	M_1	103.01
g	0.64804	M_2	192.84
g_s	1.08412	M_3	571.44
Y_τ	0.09958	A_τ	-249.8
Y_t	0.88176	A_t	-487.7
Y_b	0.13143	A_b	-766.9
μ	362.35	$\tan\beta$	10.0
$M^2_{L_1}$	$3.7821 \cdot 10^4$	$M^2_{L_3}$	$3.7513 \cdot 10^4$
$M^2_{E_1}$	$1.8399 \cdot 10^4$	$M^2_{E_3}$	$1.7773 \cdot 10^4$
$M^2_{Q_1}$	$28.177 \cdot 10^4$	$M^2_{Q_3}$	$23.416 \cdot 10^4$
$M^2_{U_1}$	$26.198 \cdot 10^4$	$M^2_{U_3}$	$16.734 \cdot 10^4$
$M^2_{D_1}$	$25.972 \cdot 10^4$	$M^2_{D_3}$	$25.682 \cdot 10^4$
$M^2_{H_1}$	$3.2864 \cdot 10^4$	$M^2_{H_2}$	$-11.804 \cdot 10^4$



Why SPA conventions ?

- Quite generally, SUSY parameters depend on the renormalization scheme (on-shell, $\overline{\mathrm{DR}}$, $\overline{\mathrm{MS}}$)

- Choice of the $\overline{\mathrm{DR}}$ scheme motivated by
  - fully consistent scheme (also beyond one-loop)
  - possibility to use it in EW and QCD physics alike
  - simple to calculate with
  - natural scheme for extrapolation to GUT scale
  - no experimental input (no SUSY seen yet) $\longrightarrow$ difficult to motivate the use of another scheme
  - few stringent bounds on SUSY parameter space $\longrightarrow$ need a scheme working for the whole parameter space
- In the future, hopefully, SUSY is seen and parameter space is far more constrained
- That is when other schemes are equally good candidates (perhaps on-shell?)

Why discussing on-shell ?

- After LEP experience – on-shell scheme worth considering (no scale dependence of the parameters)

- Before SPA conventions where agreed on – calculations were done in on-shell (motivated) schemes

- Predictions for SUSY processes at one-loop exist in the on-shell scheme
  e.g. neutralino/chargino decays, sfermion decays, Higgs decays (Eberl,Majerotto,Yamada), (Weber,Eberl,Majerotto), (Guasch,Hollik,Sola), (Arhrib,Benbrik)
  neutralino/chargino, squark and slepton (of the 3rd generation) production processes (Öller,Eberl,Majerotto), (K.K.,Weber,Eberl,Majerotto)

- Short term endeavor – make the use of the existing results by translating the $\overline{\mathrm{DR}}$-input into the required on-shell input

On-shell parameters – sfermion sector

- Sfermion mass matrix as an example of on-shell mass matrix definition
- Tree-level mass matrix in terms of the fundamental parameters

$$\mathcal{M}^2_{\tilde{f}} = \begin{pmatrix} m^2_{\tilde{f}_L} & a_f\, m_f \\ a_f\, m_f & m^2_{\tilde{f}_R} \end{pmatrix}$$

where

$$\begin{aligned} m^2_{\tilde{f}_L} &= M^2_{\{\tilde{Q},\,\tilde{L}\}} + (I_f^{3L} - e_f\, s_W^2)\cos 2\beta\, m_Z^2 + m_f^2\,, \\ m^2_{\tilde{f}_R} &= M^2_{\{\tilde{U},\,\tilde{D},\,\tilde{E}\}} + e_f\, s_W^2 \cos 2\beta\, m_Z^2 + m_f^2\,, \\ a_f &= A_f - \mu\,(\tan\beta)^{-2I_f^{3L}}\,. \end{aligned}$$

- $M^2_{\{\tilde{Q},\,\tilde{L}\}}$ is common to 2 matrices
  $\rightarrow$ one has to consider 2 matrices at the same time – for up-type and for down-type sfermions
  $\rightarrow m_{\tilde{t}_1}, m_{\tilde{t}_2}, \theta_{\tilde{t}}, m_{\tilde{b}_1}, m_{\tilde{b}_2}, \theta_{\tilde{b}}$ not independent (the same problem reappears in the on-shell scheme)

On-shell parameters – sfermion sector cont.

- $\overline{\mathrm{DR}}$ approach – pole masses determined from the vertex functional

$$\Gamma(p^2) = p^2 - M^2_{\mathrm{tree}} + \begin{pmatrix} \hat{\Sigma}_{11}(p^2) & \hat{\Sigma}_{12}(p^2) \\ \hat{\Sigma}_{21}(p^2) & \hat{\Sigma}_{22}(p^2) \end{pmatrix}$$

by requiring

$$\det(\Gamma(p^2)) = 0 \quad \text{where up to 1-loop} \quad s_n^{\mathrm{pole}} = M_n^2 - \hat{\Sigma}_{nn}$$

- OS approach – pole masses input $\longrightarrow$ used to fix the counterterms

$$\Gamma(p^2) = p^2 - M^2_{\mathrm{OS,tree}} + \begin{pmatrix} \hat{\Sigma}^{\mathrm{OS}}_{11}(p^2) & \hat{\Sigma}^{\mathrm{OS}}_{12}(p^2) \\ \hat{\Sigma}^{\mathrm{OS}}_{21}(p^2) & \hat{\Sigma}^{\mathrm{OS}}_{22}(p^2) \end{pmatrix} \stackrel{!}{=} p^2 - M^2_{\mathrm{OS,tree}}$$

  – valid only in case mass matrix elements free parameters of the theory

  – in the sfermion sector this is not the case

  – one has to sacrifice something...

On-shell parameters – sfermion sector cont.

- We require that the on-shell mass matrix $(\mathcal{M}^2_{\tilde{f}})^{\mathrm{OS}}$ has the same form as the $\overline{\mathrm{DR}}$ matrix in terms of the SUSY parameters

- Furthermore, the pole masses are to be obtained by simple diagonalization i.e.

$$(\mathcal{M}^2_{\tilde{f}})^{\mathrm{OS}} = (\mathbf{R}^{\tilde{f}})^T \begin{pmatrix} m^2_{\tilde{f}_1} & 0 \\ 0 & m^2_{\tilde{f}_2} \end{pmatrix} \mathbf{R}^{\tilde{f}}$$

- The conditions fix the counterterms for the matrix elements as

$$\delta(\mathcal{M}^2_{\tilde{f}})_{ij} \;=\; \frac{1}{2} \sum_{l,\,n=1}^{2} R^{\tilde{f}}_{ni} R^{\tilde{f}}_{lj} \, \widetilde{\mathrm{Re}} \left[\Pi^{\tilde{f}}_{nl}(m^2_{\tilde{f}_n}) + \Pi^{\tilde{f}}_{nl}(m^2_{\tilde{f}_l}) \right] .$$

- The mass matrix contains 3 free parameters (e.g. in the stop sector $M^2_{\tilde{Q}}$, $M^2_{\tilde{U}}$, A_t)

- One can use $\delta(\mathcal{M}^2_{\tilde{f}})_{ij}$ to define the counterterms of the free parameters

On-shell parameters – sfermion sector cont.

- The common $M^2_{\tilde{Q}}$ parameter has two ways of fixing the counterterms either from the stop or from the sbottom sector – conventionally the sbottom sector is chosen

$$\begin{aligned}\delta M^2_{\tilde{Q}} &= \delta(\mathcal{M}^2_{\tilde{b}})_{11} - 2m_b\delta m_b - \delta m_Z^2 \cos 2\beta \left(I_b^{3L} - e_b \sin^2\theta_W\right) \\ &\quad - m_Z^2 \left(\delta\cos 2\beta \left(I_b^{3L} - e_b \sin^2\theta_W\right) - \cos 2\beta\, e_b\, \delta\sin^2\theta_W\right) .\end{aligned}$$

- The parameter $M^2_{\tilde{Q}}$ is no longer the same in the stop and sbottom sector
- In the stop sector we have

$$M^2_{\tilde{Q}} \to M^2_{\tilde{Q}} + \Delta M$$

where naturally the finite shift ΔM is given by

$$\begin{aligned}\Delta M &= \delta M^2_{\tilde{Q},\tilde{L}} + 2m_t\delta m_t + \delta m_Z^2 \cos 2\beta \left(I_t^{3L} - e_t \sin^2\theta_W\right) \\ &\quad + m_Z^2 \left(\delta\cos 2\beta \left(I_t^{3L} - e_t \sin^2\theta_W\right) - \cos 2\beta\, e_t\, \delta\sin^2\theta_W\right) - \delta(\mathcal{M}^2_{\tilde{t}})_{11} ,\end{aligned}$$

On-shell parameters – sfermion sector cont.

- The diagrams shows the on-shell vs. $\overline{\mathrm{DR}}$ treatment of the mass matrix renormalization at one loop

$$
\begin{array}{ccc}
\overline{\mathrm{DR}}\ \text{input} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}_{\overline{\mathrm{DR}}} & \xrightarrow{\text{diag}} & \begin{pmatrix} m_{1} & 0 \\ 0 & m_{2} \end{pmatrix}_{\overline{\mathrm{DR}}} \\
\delta(\mathcal{M}^2_{\tilde{f}})_{ij} \Big\downarrow & & \Big\downarrow \hat{\Sigma}_{nn} \\
\begin{pmatrix} M_{11}(+\Delta M) & M_{12} \\ M_{21} & M_{22} \end{pmatrix}_{\mathrm{OS}} & \xrightarrow{\text{diag}} & \begin{pmatrix} M_{1} & 0 \\ 0 & M_{2} \end{pmatrix}_{\mathrm{OS}} = \text{Pole masses}
\end{array}
$$

- Alternative way of including the finite shift – shift the masses, parameters stay the same in all sectors

Neutralino & Chargino sector

- Similar situation in the neutralino and chargino sector
  - 3 free parameters M, M' and μ and 6 masses and 3 rotation matrices!

$$X = \begin{pmatrix} M & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & \mu \end{pmatrix}$$

M' fixed in neutralino sector

M, μ fixed in chargino sector

$$Y = \begin{pmatrix} M' & 0 & -m_Z s_W \cos\beta & m_Z s_W \sin\beta \\ 0 & M & m_Z c_W \cos\beta & -m_Z c_W \sin\beta \\ -m_Z s_W \cos\beta & m_Z c_W \cos\beta & 0 & -\mu \\ m_Z s_W \sin\beta & -m_Z c_W \sin\beta & -\mu & 0 \end{pmatrix}$$

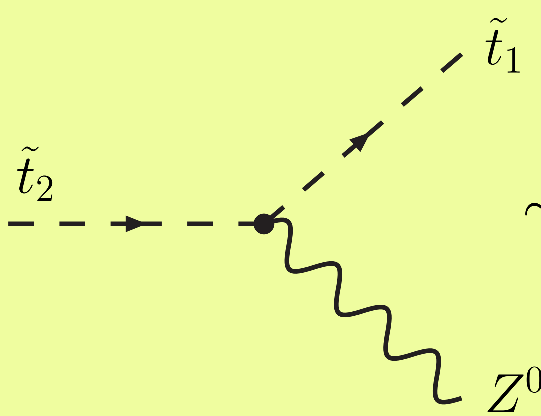
- Finite shifts also added to zero matrix elements!

Other on-shell parameters – $\theta_{\tilde{f}}$

- Mixing parameters fixed similarly to CKM matrix in the SM
  $\longrightarrow$ suffers from similar problems as the CKM matrix fixing (gauge invariance)
  (Denner, Sack), (Gambino,Grassi,Madricardo)
- Counterterm prescription

$$\delta\theta_{\tilde{f}} = \frac{1}{4}\left(\delta Z_{12}^{\tilde{f}} - \delta Z_{21}^{\tilde{f}}\right) = \frac{\mathrm{Re}\left(\Pi_{12}^{\tilde{f}}(m_{\tilde{f}_2}^2) + \Pi_{21}^{\tilde{f}}(m_{\tilde{f}_1}^2)\right)}{2\left(m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2\right)}$$

- Motivated by abstract relations
  $\longrightarrow$ different fixing necessary in a rigorous on-shell scheme – using some experimental input (e.g. $\tilde{t}_2 \rightarrow \tilde{t}_1 Z^0$)



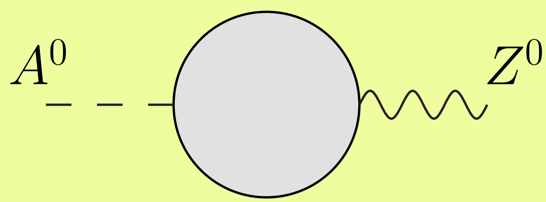
$\sim \sin 2\theta_{\tilde{f}}$

Other on-shell parameters – $\tan\beta$

- $\tan\beta$ – auxiliary parameter in the Higgs sector

$$\tan\beta = \frac{v_2}{v_1}$$

- At the moment 2 common ways of fixing the counterterm $\delta\tan\beta$
  (Chankowski,Pokorski,Rosiek), (Freitas, Stöckinger)
  - $\overline{\mathrm{DR}}$ prescription – easy to work with, but generally gauge dependent
  - requiring no $A^0 - Z^0$ mixing at $p^2 = m^2_{A^0}$ which also results in a gauge dependent counterterm



$$0 = \mathrm{Im}\hat{\Sigma}_{A^0Z^0}(m^2_{A^0}) \rightarrow \frac{\delta\tan\beta}{\tan\beta} = \frac{1}{m_Z\sin 2\beta}\Sigma_{A^0Z^0}(m^2_{A^0})$$

- For the on-shell scheme one possibility left
  – using the decays $H^+ \rightarrow \tau^+\nu_\tau$ or $A^0 \rightarrow \tau^+\tau^-$

Other on-shell parameters – $A_{b,\tau}$

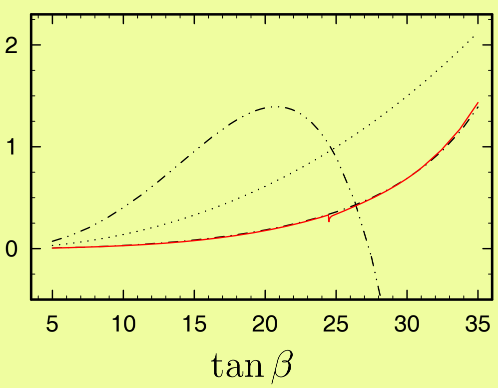
- $A_{b,\tau}$ determined in the sbottom/stau mass matrix renormalization
  – the counterterm

$$\delta A_{b,\tau} = \frac{\delta(\mathcal{M}^2_{\tilde{b},\tilde{\tau}})_{12}}{m_{b,\tau}} - \frac{(\mathcal{M}^2_{\tilde{b},\tilde{\tau}})_{12}}{m_{b,\tau}} \frac{\delta m_{b,\tau}}{m_{b,\tau}} + \delta\mu \tan\beta + \mu\,\delta\tan\beta\,.$$

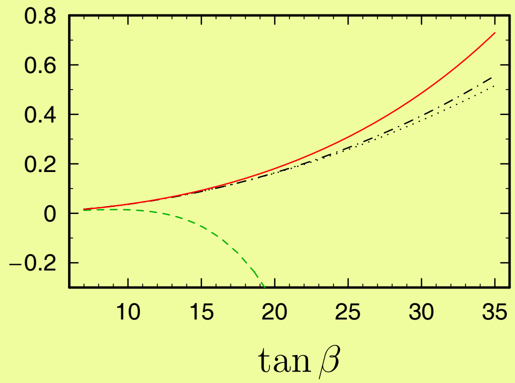
- The counterterm is huge for large $\tan\beta$ – the sources are
  $\longrightarrow$ the element $\delta(\mathcal{M}^2_{\tilde{b},\tilde{\tau}})_{12}$
  $\longrightarrow$ the QCD corrections contained in the on-shell counterterm δm_b

- This fixing is relevant in processes with $A_{b,\tau}$ in the tree-level coupling e.g. $\tilde{b}_2 \to \tilde{b}_1 A^0$
  $\longrightarrow$ causes the perturbation expansion to fail (Weber,Eberl,Majerotto)

$\Gamma(\tilde{b}_2 \to \tilde{b}_1 A^0)$



$\Gamma(A^0 \to \tilde{\tau}_1 \bar{\tilde{\tau}}_2 + c.c.)$



Summary & Outlook

- Consistent framework and conventions for a SUSY Parameter Analysis ("SPA project") was presented
- $\overline{\mathrm{DR}}$ scheme proposed by the SPA project as
  - simple to apply + good stability
  - natural extrapolation to GUT scale
- On-shell "motivated" scheme discussed
  - mass renormalization problematic – too few free parameters
  - other parameters suffer from instability in some regions of the phase space
- Short term outlook: Make old calculations SPA-compliant by transforming between the schemes.
- Long(er) term outlook: Use $\overline{\mathrm{DR}}$ scheme for calculations and redo the old ones
- Very long term outlook: After some SUSY experimental input – possible revival of the on-shell scheme