Three-loop time-like splitting functions in Mellin space and NNLO fragmentation

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DESY

Work in progress with: M. Cacciari; Lance Dixon; S-O. Moch

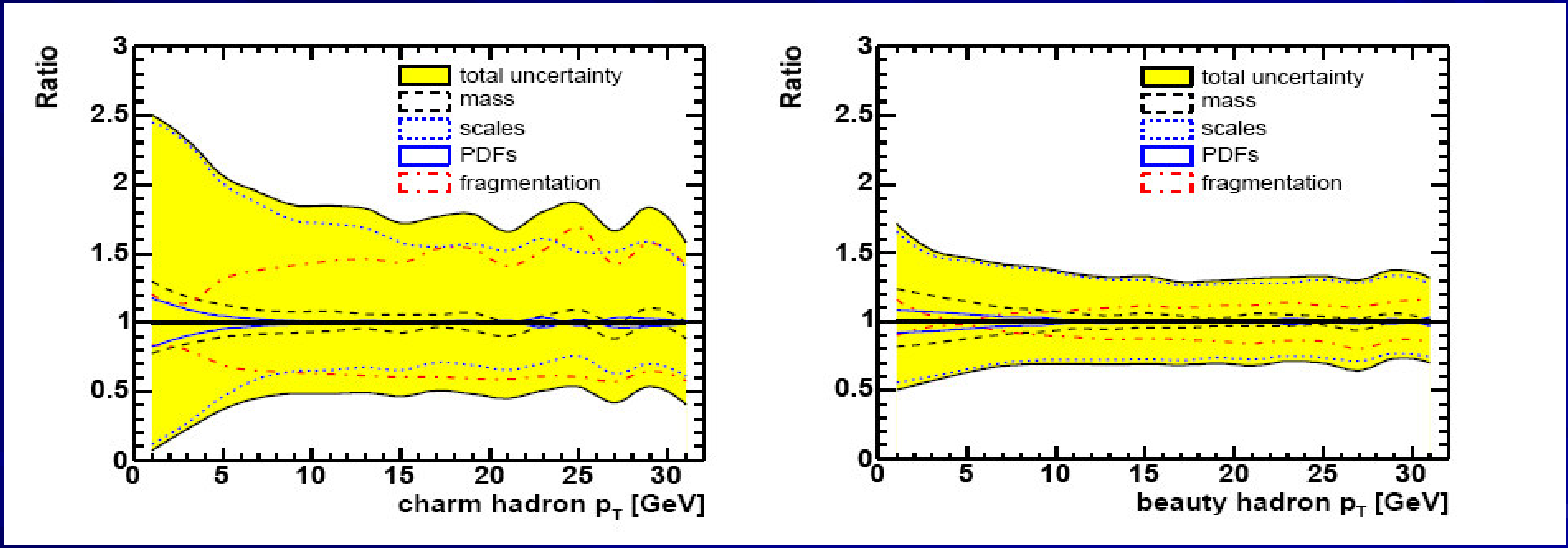
Also based on: hep-ph/0604160 (with Sven Moch)
hep-ph/0604053 (with Sven Moch and Andreas Vogt)
hep-ph/0410205
hep-ph/0404143 (with Kirill Melnikov)

Motivation

Heavy flavor production at the LHC:

- huge number of b-quarks: 100,000/sec
- great experimental studies of b-quark P_T - spectra possible:
  16M selected B-events/year at CMS alone.

Here are the theoretical uncertainties at LHC (HERA-LHC working group hep-ph/0601164):



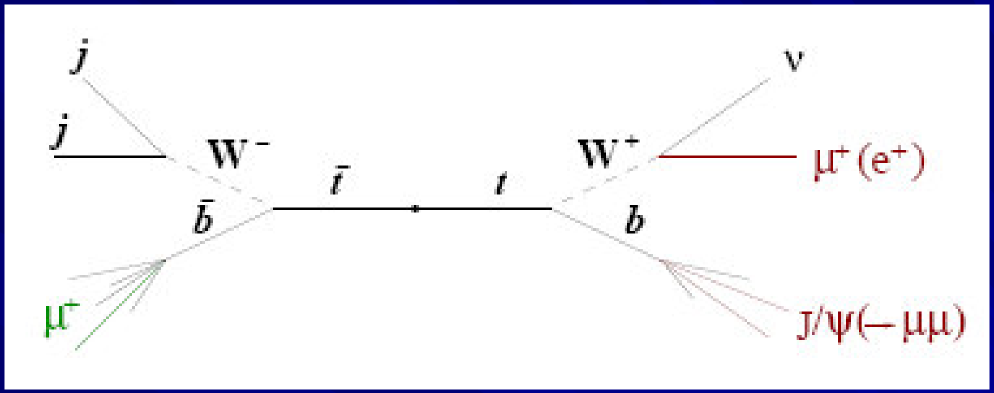
Heavy flavor fragmentation is a dominant uncertainty!

Motivation

b-quark P_T - spectra are not the only interesting physics sensitive to non-perturbative fragmentation.

Top mass measurement from b-fragmentation:

$$\Delta m_{top} \approx 1\mathrm{GeV}$$



- Method proposed by A. Kharchilava, hep-ph/9912320.
- Further studies with MC's:
  Corcella, Mangano and Seymour: hep-ph/0004179
- Based on HERWIG 6.0 and 6.1
- Detailed study from CMS (2006): … only after the first year or so the dominant uncertainty will be systematical which in turn is dominated by theory… (N)NLO will be needed…

The largest irreducible uncertainty is from b-fragmentation.

A set of fragmentation functions at NNLO is needed.

Motivation: summary

What is needed to get Fragmentation Functions at NNLO?

Predicted for hadron colliders → $$\frac{d\sigma_B}{dp_T} = \frac{d\sigma_b}{dp_T} \otimes D^{DGLAP} \otimes D^{ini} \otimes D^{\text{n.p.}}_{b\to B}$$

Measured at e^+e^- → $$\frac{d\sigma_B}{dE} = \frac{d\sigma_b}{dE} \otimes D^{DGLAP} \otimes D^{ini} \otimes D^{\text{n.p.}}_{b\to B}$$

DGLAP evolution: three-loop time like splitting functions Non-singlets available

Perturbative fragmentation functions at NNLO All components known

e^+e^- coefficient functions at two loops Known and checked

Fit to the LEP data of b-spectra In progress …

Perturbative components

Derivation of the 3-loop (NNLO) time-like splitting functions.

Idea: extract them from the $1/\epsilon$ pole of any collinearly sensitive observable.

Two approaches:

1) use specific process (like e^+e^-):
   - more complicated to evaluate
   - can produce the coefficient functions too (the terms of ϵ^0)
   - added bonus: precise determination of the strong coupling from Longitudinal fragmentation.

2) process-independent evaluation in a fictitious process (with L. Dixon):
   - simpler to compute, but
   - no additional benefit beyond the splitting functions
   - related approaches used previously at two loops:

     Kosower, Uwer (2001)

     Melnikov, A.M.; A.M. (2004)

   - what happens at 3-loops? Very hard IBP reductions; very slow with Laporta. Seems unfeasible in momentum (z-) space.

Perturbative components

Is there a way to speed up the reductions? Yes, work in Mellin space! A.M.(2005)

How? Idea: perform the Mellin integration before the phase space integrations.

Basically, the effect is:
$$\frac{1}{\left(p^2+\ldots f(z)\ldots\right)^C} \rightarrow \frac{1}{\left(p^2+\ldots\right)^{C+N}}$$

Therefore, one needs to perform IBP reduction over simpler propagators but one of the powers is an abstract number. Easy to generalize to several variables …

Working in Mellin space is more effective than in z-space since one works in the natural “co-ordinates” for the IBP reduction.

Properties:

- much faster reductions,
- smaller number of master integrals,
- masters satisfy difference equations in the Mellin variable N
- purely algebraic extraction of the dependence on the kinematics.

Perturbative components

Calculations in Mellin space (cont.):

1) Unlike the DIS calculations also performed in Mellin space, our method does not rely on OPE or the Optical Theorem.

   Therefore, it is suitable for not-completely inclusive processes that require separate treatment of all physical cuts!

2) The N- and z-space calculations are physically completely equivalent.

   However, the N-space approach produces insight about the analytical structure of the Feynman integrals, and moreover, of the solutions to the recurrence relations like the IBP identities.

Examples:

Solutions in z- and N-spaces in d dimensions:	z-space:	N-space
	$F(\dots\epsilon\dots;f(z))$	$F(\dots\epsilon,N\dots;1)$

Perturbative components

Application in Mellin space at two loops: the NNLO coefficient functions in e^+e^-

Calculate the energy spectrum of massless quarks and gluons at two loops:

A.M., S-O Moch (2006)

$$\frac{d\sigma^{(T,L,A)}}{dz} \quad ; \quad e^+e^- \to q/g + X \quad ; \quad 0 \le z \le 1$$

Evaluated in expansion around d=4:

$$\frac{d\sigma}{dz} = \cdots + \alpha_S^2 \left(\cdots + P/\epsilon + A\epsilon^0 + B\epsilon^1 + \ldots \right)$$

During the evaluation we encounter:

- 6 N-dependent Real-Real Masters → Expressed in harmonic sums with
- 5 N-dependent Real-Virtual Masters → the help of the difference equations.

The dependence on the kinematics is extracted algebraically. Very few integrals have to be evaluated and they are pure numbers independent on kinematics. They were derived previously in a different context:

Gehrmann-De Ridder, Gehrmann, Heinrich (2003)

Working in Mellin space minimizes the evaluation of Feynman integrals !

Perturbative components

The 3-loop time-like splitting functions (needed for NNLO evolution)

Idea: all non-singlet functions can be extracted from the $1/\epsilon$ poles at three loops in e^+e^-

$$\frac{d\sigma}{dz} = \cdots + \alpha_S^3 \left(\cdots + P^{(2)}/\epsilon + A\epsilon^0 + \ldots \right)$$

But VERY hard to calculate directly at present. The only distribution know to three loops at present are the three-loops calculations in DIS: Moch, Vermaseren Vogt (2004,2005)

$$\frac{d\sigma}{dx} = \delta(1-x) + \cdots + \alpha_S^3 \left(\cdots + P^{(2),S}/\epsilon + A + \ldots \right)$$

The structure of the two cross-sections is similar; is there a relation between the two?

Previously discussed by

Gribov, Lipatov (1972)
Curci, Furmanski, Petronzio (1980)
Stratmann, Vogelsang (1997)
Dokshitzer, Marchesini and Salam (2006)

We propose an analytical continuation DIS → e^+e^- A.M., S-O Moch, A. Vogt (2006)

Perturbative components

The 3-loop time like splitting functions (cont.)

Such continuation requires matching of:

- amplitudes (exact)
- scaling variables $\left(x \to 1/z \ ; \ Q^2 \to \ - \ Q^2\right)$
- proper phase-space modifications (multiplicative factor of $z^{1-2\epsilon}$)
- matching the analytical continuation of branch cuts (harder)

The latest point would have been trivial if the DIS result was known on a per cut basis. But we know only their sum (evaluated with the Optical Theorem).

The analytical continuation we propose is able to exactly predict the e^+e^- partonic cross-section at orders α_S (NLO) and α_S^2 (NNLO) for each known power of ϵ .

What happens when we apply It to 3 loops? Produces a term which does not agree with the sum rules. It is of the form $\sim C_F^3 p_{qq}(z) \ln^2(z)\pi^2$

We can identify the term that causes this discrepancy on physical grounds and correct it by hand. That restores the sum-rules, but is this all?

Perturbative components

The 3-loop time like splitting functions (cont.)

The corrected 3-loop non-singlet prediction coincides with another one due to Dokshitzer, Marchesini and Salam for the difference between the space- and time-like functions.

Although the 3-loop splitting functions are very complicated, their difference is quite compact:

$$\frac{P^{(2),T} - P^{(2),S}}{2} = \left[\ln(z).\left(P^{(1),T} + P^{(1),S}\right)\right] \otimes P^{(0)} + \left[P^{(1),T} + P^{(1),S}\right] \otimes \left[\ln(z).P^{(0)}\right]$$

In fact it is even possible to extend the arguments of DSM to get the difference between space and time-like non-singlet splitting functions even at 4 loops!

Work is underway:

- on the singlet components,
- independent checks/derivations,
- extend the results to 3-loop coefficient functions (Longitudinal fragmentation)

Towards b-fragmentation at NNLO

To properly predict b-quark fragmentation one has to:

- predict the spectrum of the massive b-quark at FO (NNLO)
- resum the large mass logs to NNLL with DGLAP
- fit LEP data to extract the non-perturbative fragmentation function.

$$e^+e^- : \quad \frac{d\sigma_B}{dE} = \frac{d\sigma_b}{dE} \otimes D^{ini} \otimes D^{DGLAP} \otimes D^{\mathrm{n.p.}}_{b\to B} \; + \; O(m^2)$$

Up to terms ~ O(m) one can predict the massive b-spectrum from purely massless calculations:

- massless coefficient functions

Rijken and van Neerven (1996)
A.M. and Sven Moch (2006)

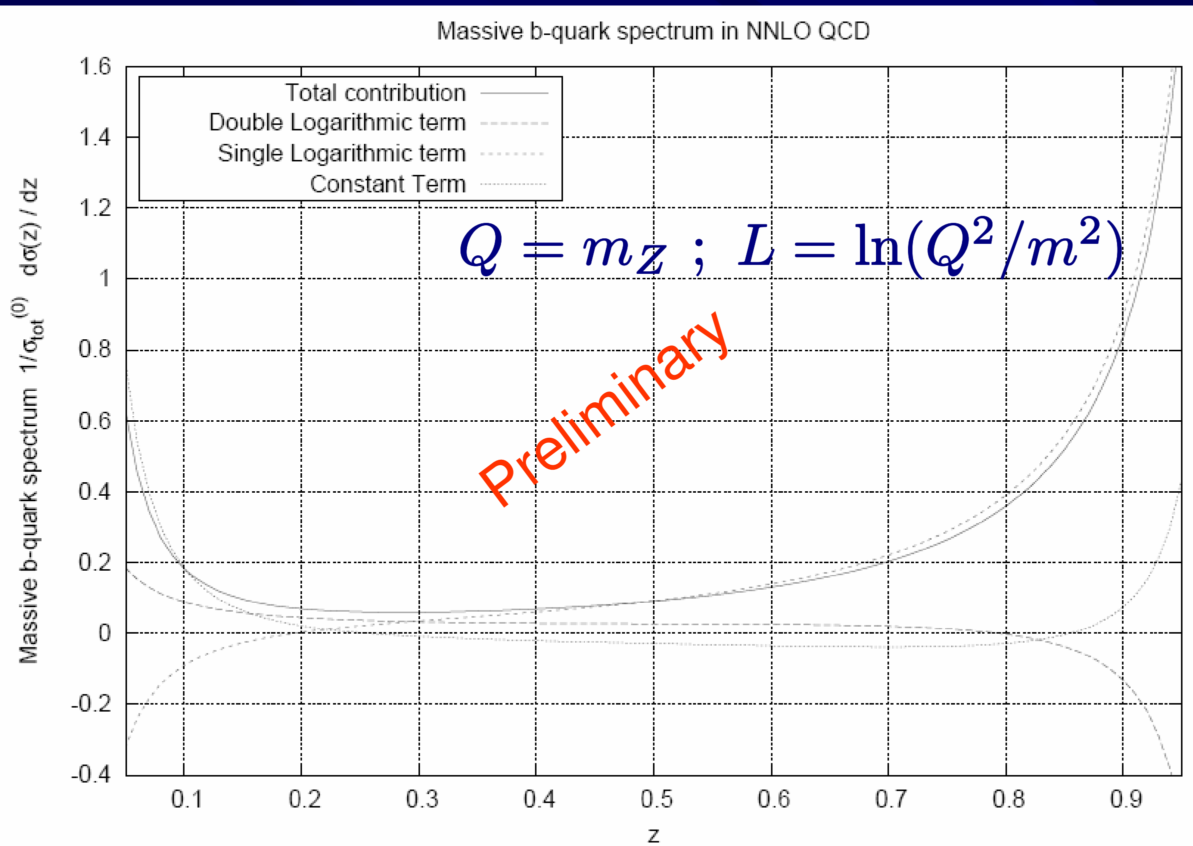
- Perturbative Fragmentation Function: Mele-Nason (1991 at NLO)

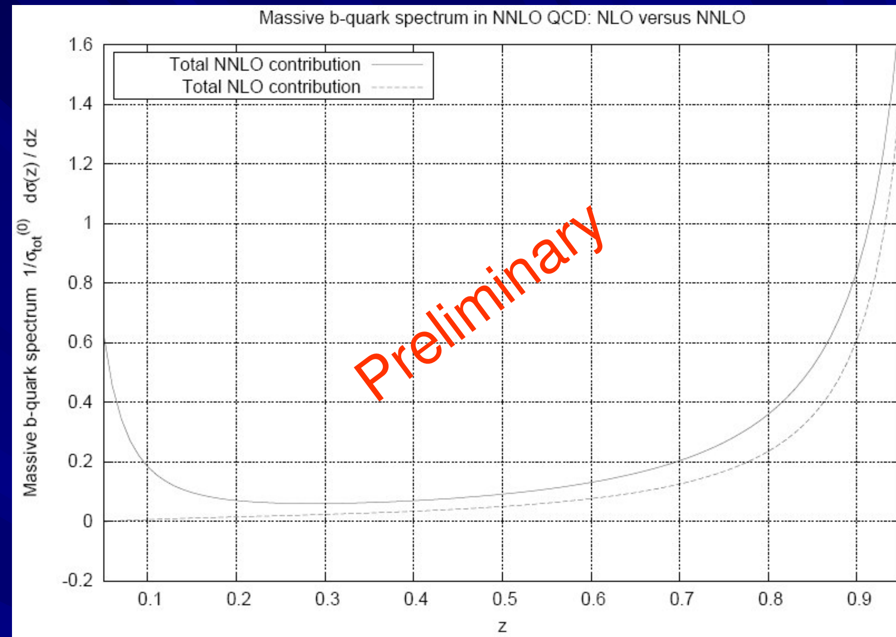
Kirill Melnikov, A.M.; A.M. (2004) at NNLO

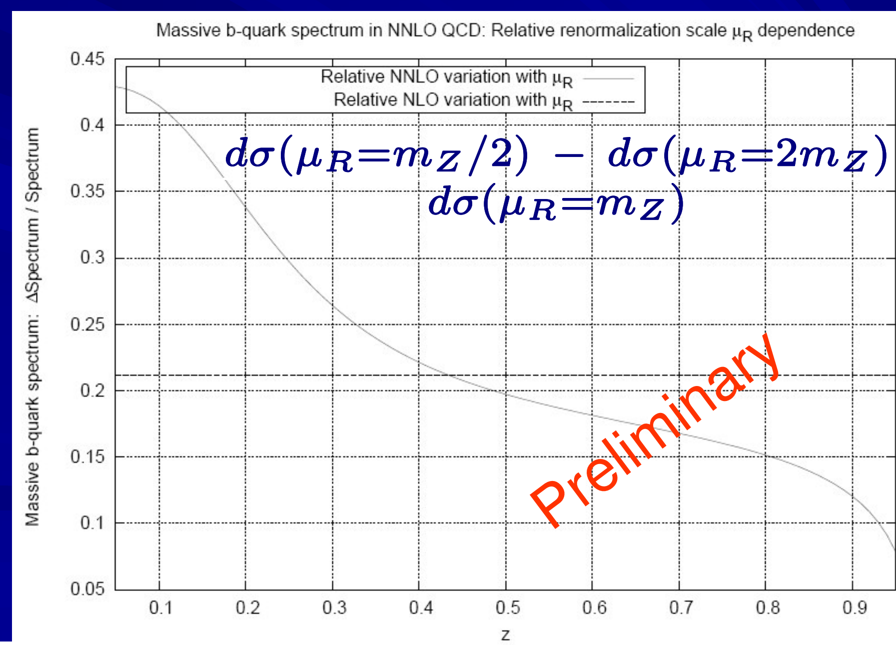
This formalism has so far been only applied to NLO and for the logarithmic terms at NNLO. Here, it is applied for a first time to predict the “constant contribution”.

Some preliminary plots:

Towards b-fragmentation at NNLO







The soft-gluon, soft "singlet" contribution enters for the first time at this order. Will improve with resummation. Similar behavior seen in other processes (muon decay) Arbuzov, Czarnecki, Gaponenko (2002), Arbuzov, Melnikov (2002); Anastasiou, Melnikov, Petriello (2005).

Not checked yet vs the numerical results for the same observable Nason, Oleari (1999). The O(m) terms?

Summary

- There have been exciting developments towards b-fragmentation at NNLO
- Two-loop coefficient functions in electron positron annihilation,
- 3-loop non-singlet time-like splitting functions,
- The perturbative heavy quark Fragmentation Function at two loops.

- Presented first application to massive b-spectrum at NNLO from massless calculations. Interesting new features!

- Expect in the following months results on extracted b-fragmentation function at NNLO!

Applications (especially LHC)

- P_T - distribution of b-quarks,
- Extension to charm is also possible.
- Precise top mass measurement from J/Ψ in top decay.