Feasibility of diffraction radiation for a non-invasive diagnostics of the SLAC electron beam.

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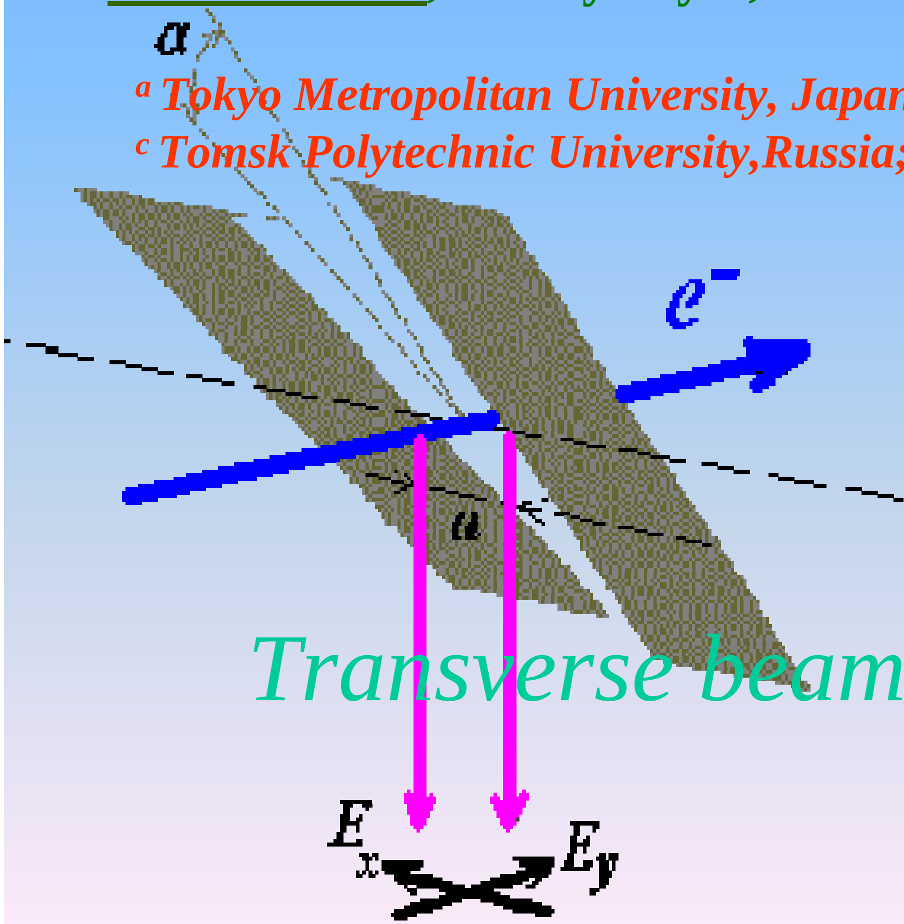
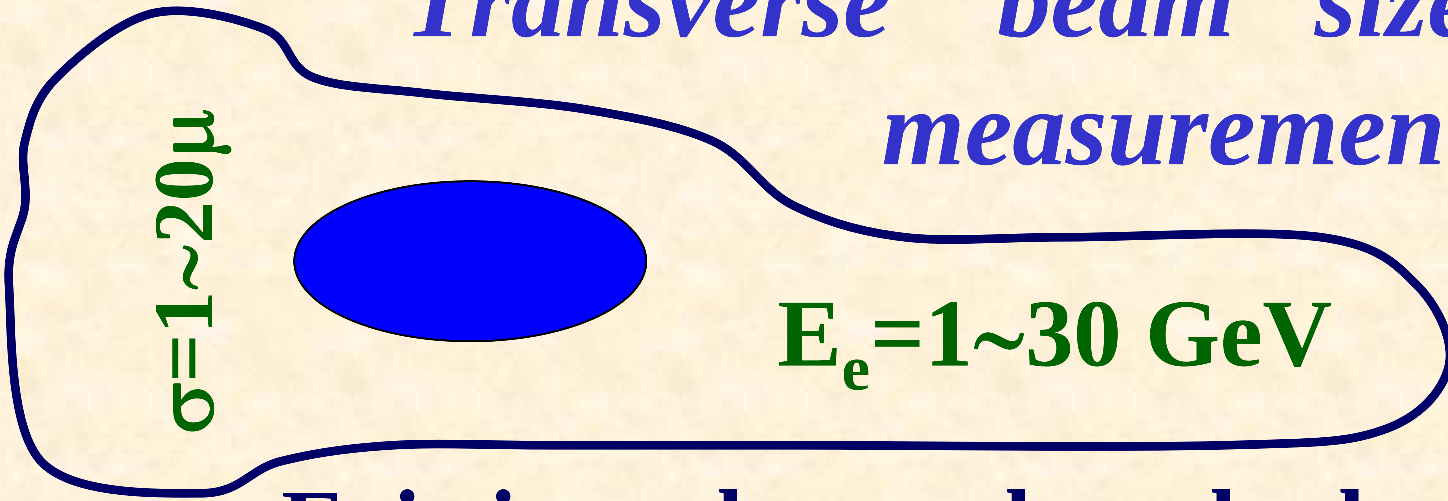
[a] Tokyo Metropolitan University, Japan; [b] KEK, Japan; [e] SLAC, USA
[c] Tomsk Polytechnic University, Russia; [d] UCLA, USA;



Transverse beam size measurement

ATF TPU SLAC collaboration

Transverse beam size measurement



Existing advanced methods

SR - interferometer

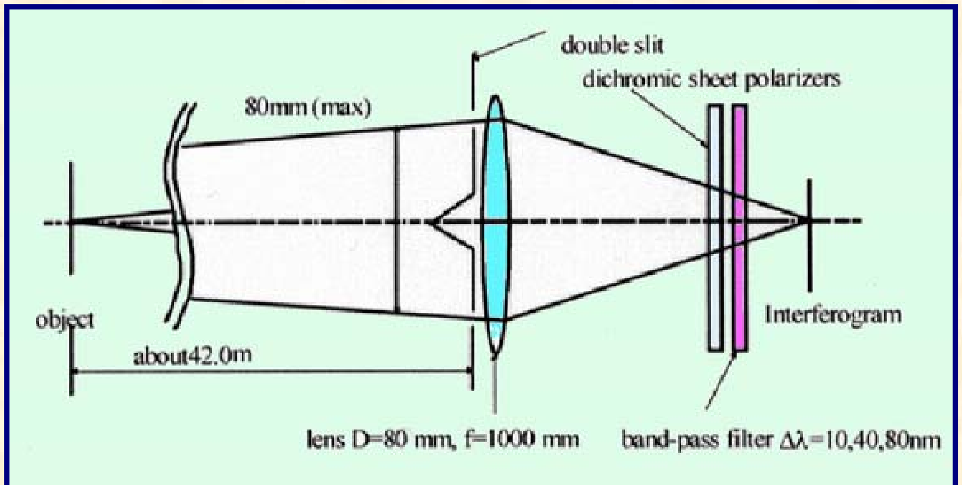


Figure 4: Set up of SR-interferometer

SakaiaY. Yamamoto, et.al., Review of Scientific instruments, 71,3 (2000)

Laser wire scanner

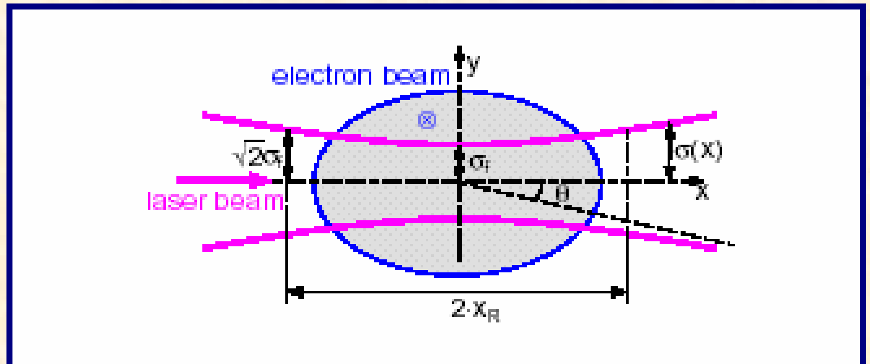


Figure 1: Scheme of a gaussian laser beam focused to its diffraction limit.

H. Sakai, et.al., Phys.Rev.ST Accel.Beams 4:022801,2001.

Transition Radiation Monitor

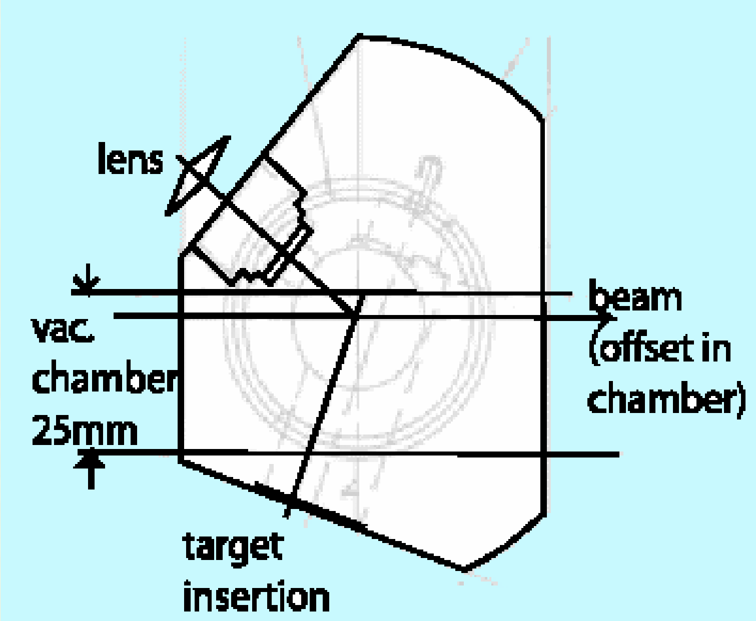


Figure 3: High-resolution optical transition radiation monitor tested at ATF/KEK. The monitor is displaced when the target is inserted in order to bring the beam close to the lens.

M. Ross, et.al., 2001 IEEE Particle Accelerator Conference, Chicago, IL, 2001.

Laser interferometer

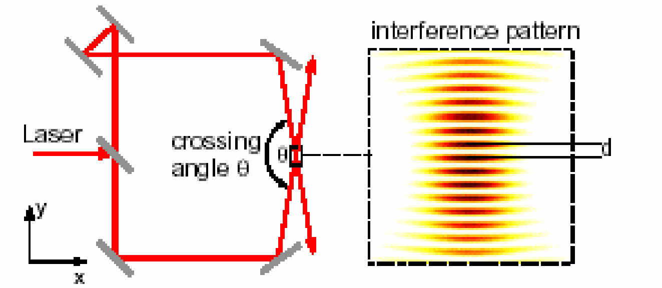


Figure 2: Schema of the generation of an interference pattern using a split laser beam. d is the fringe spacing.

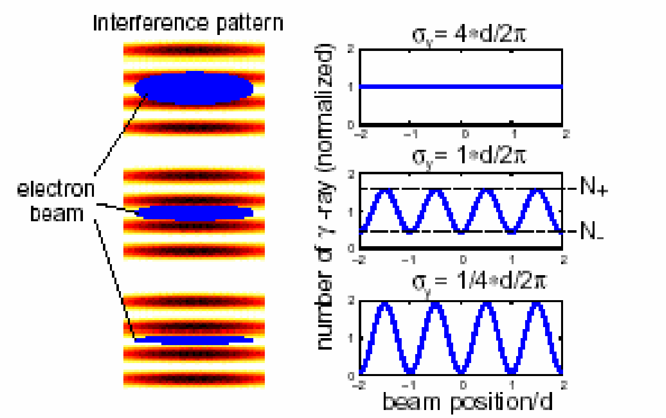


Figure 3: Modulation of Compton scattered photons as a function of the vertical electron beam position for different beam sizes (top large, center medium, bottom small)

H. Sakai, et.al., Phys.Rev.ST Accel.Beams 4:022801,2001.

What about a non-invasive single bunch diagnostics?

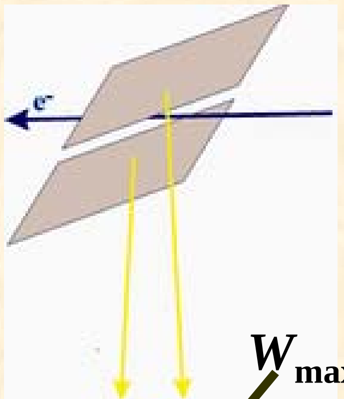
	Non invasive	Single bunch measurement
SR - interferometer	yes	no
Laser wire scanner	yes	no
Transition Radiation Monitor	no	yes
Laser interferometer	yes	no
?	yes	yes

Non-invasive diagnostics based on the Optical Diffraction Radiation

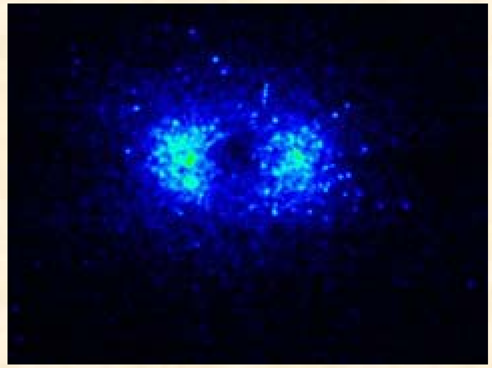
Short prehistory

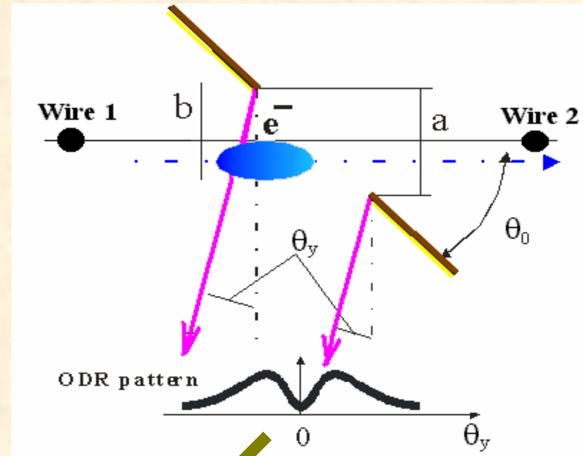
Start: **KEK ATF 2000**

Flat slit target

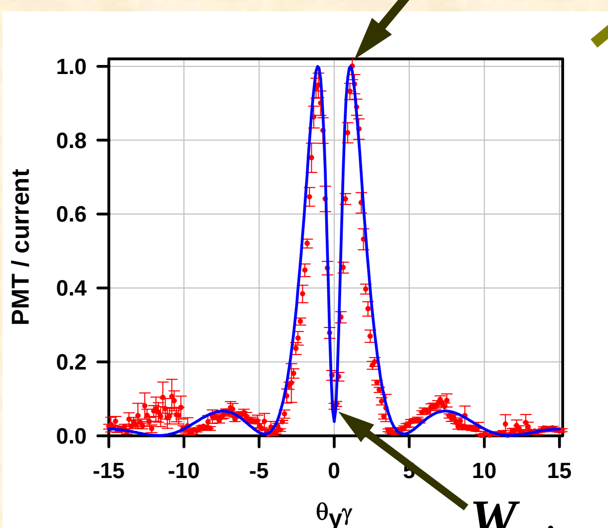


2003





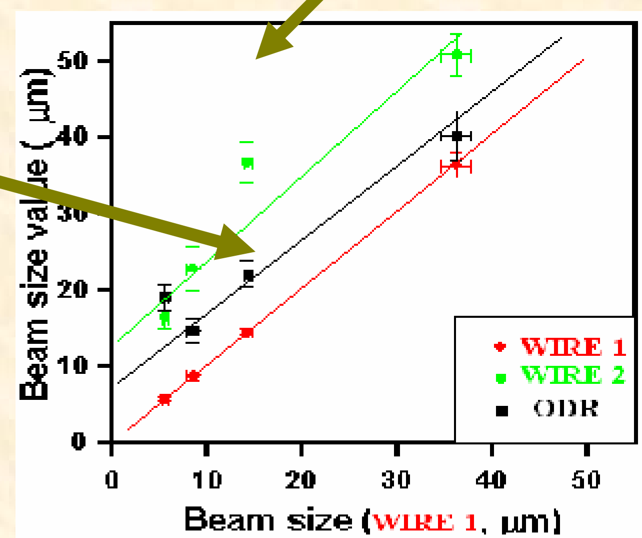
W **max**



W **min**

Measured ODR angular distribution

$$\frac{W_{\min}}{W_{\max}} = f\left(\sigma_e\right)$$



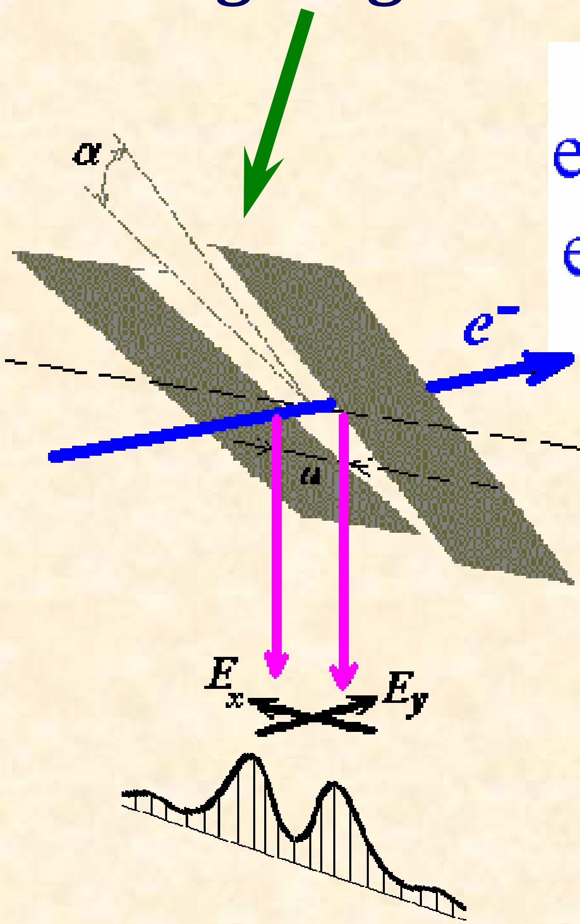
Sensitivity limit was reached.

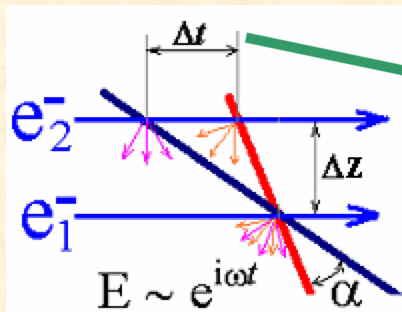
For E_e≈30 GeV the sensitivity decrease catastrophically.

P. Karataev, S. Araki et.al., PRL 93, 244802 (2004)

ODR method modification

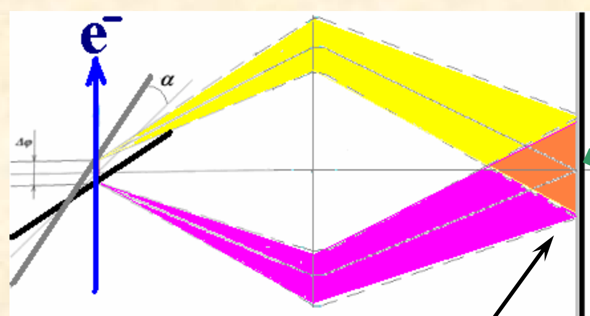
Beam size measurement technique using ODR from crossing target was developed (*G. Naumenko, KEK report, Nov. 2003*)





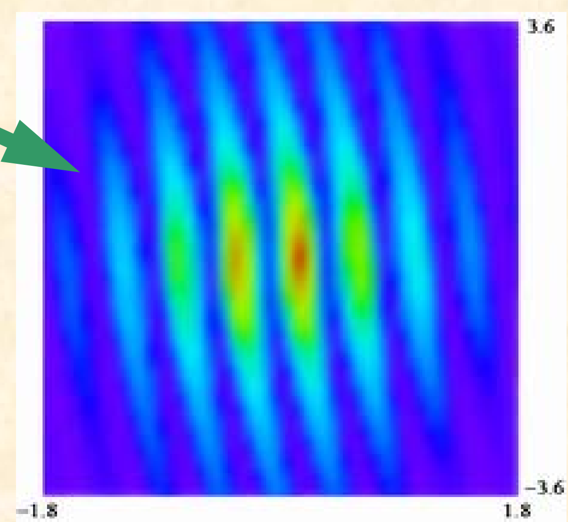
Phase shift: $\Delta\varphi = i\cdot\omega\cdot\Delta t =$

$= i\cdot 4\pi\cdot\alpha\cdot\Delta z/\lambda$



Interference pattern

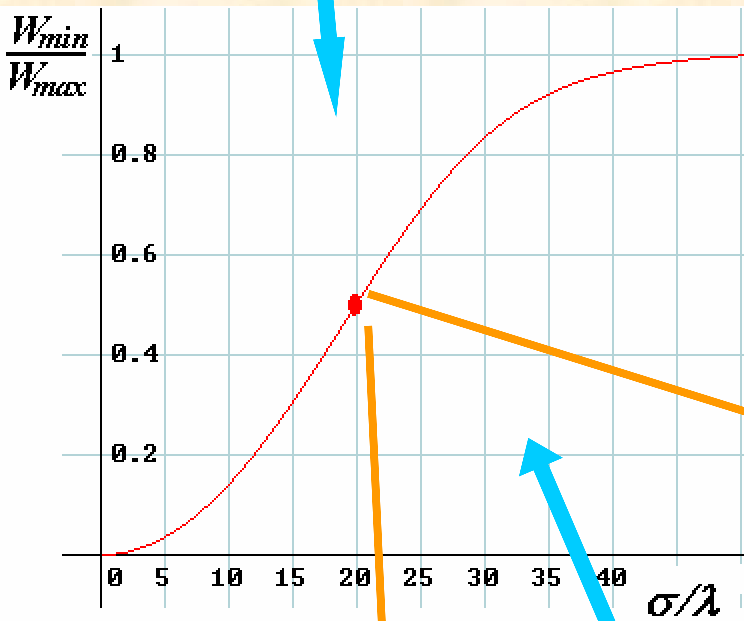
Calculated:



Beam size

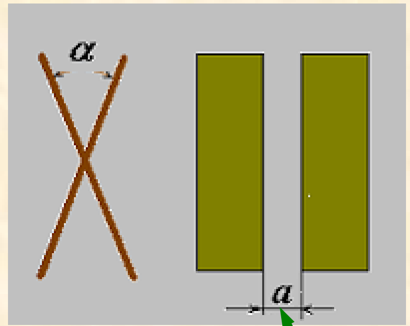
Example:

α=5.6 mrad



For λ=0.5mcm σ=10mcm

No dependence on the Lorenz-factor in far field zone



$$W_{\max} \propto e^{-\pi \frac{a}{\gamma\lambda}}$$

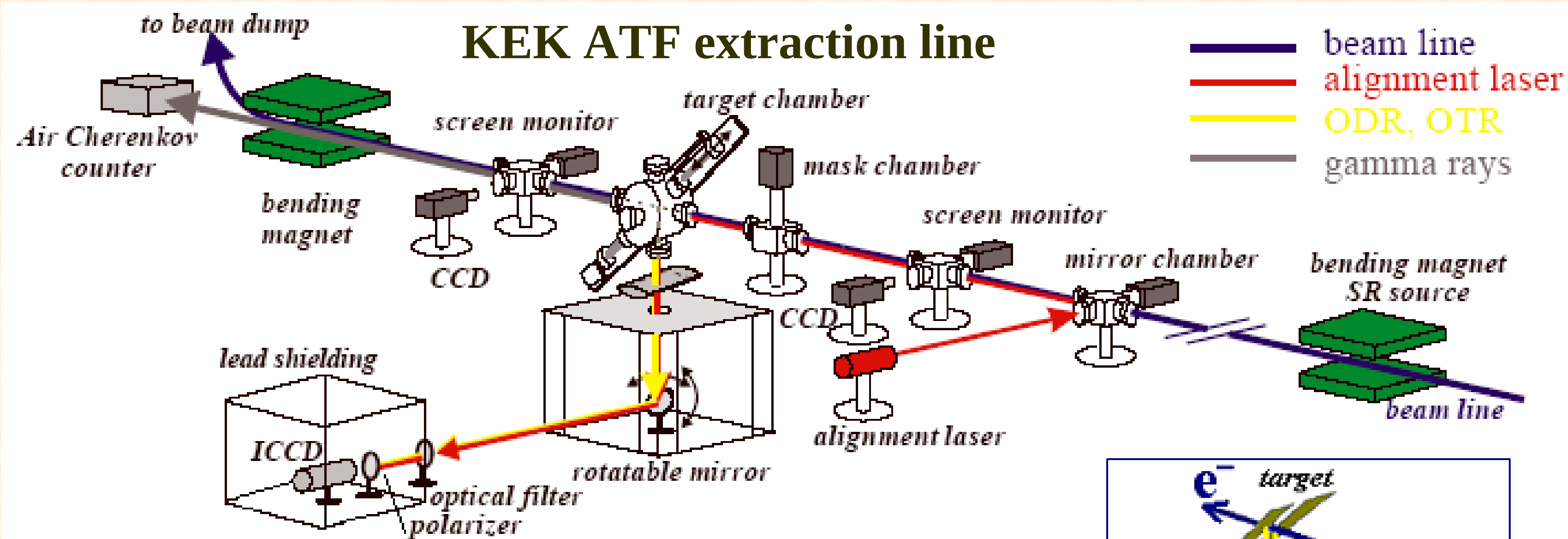
γ is the Lorenz-factor

$$if \quad a \ll \gamma\lambda, \quad W_{\max} \approx W_{OTR}$$

For λ=0.5mcm and γ=60000
γλ=3cm. $a \ll \gamma\lambda$ **is possible**

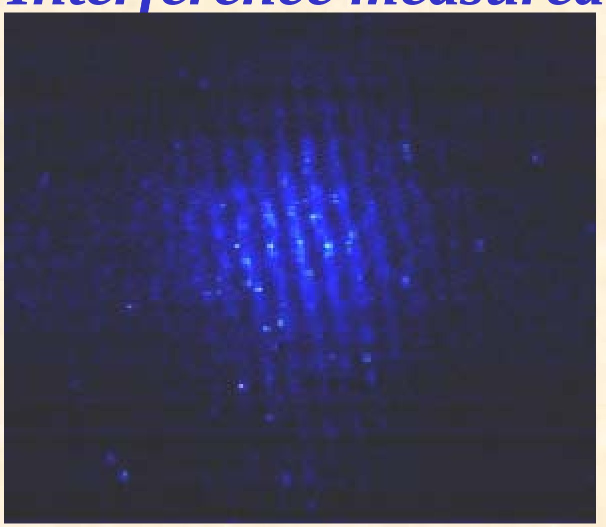
Beam size effect is of the order of OTR intensity, which was measured using CCD from a single bunch.

Test of ODR interference from the crossed target



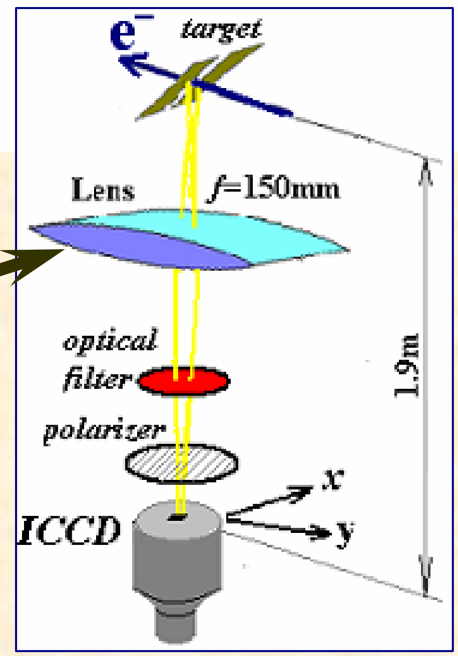
Target parameters: α = 6.2mrad, a = 420μm

Interference measured



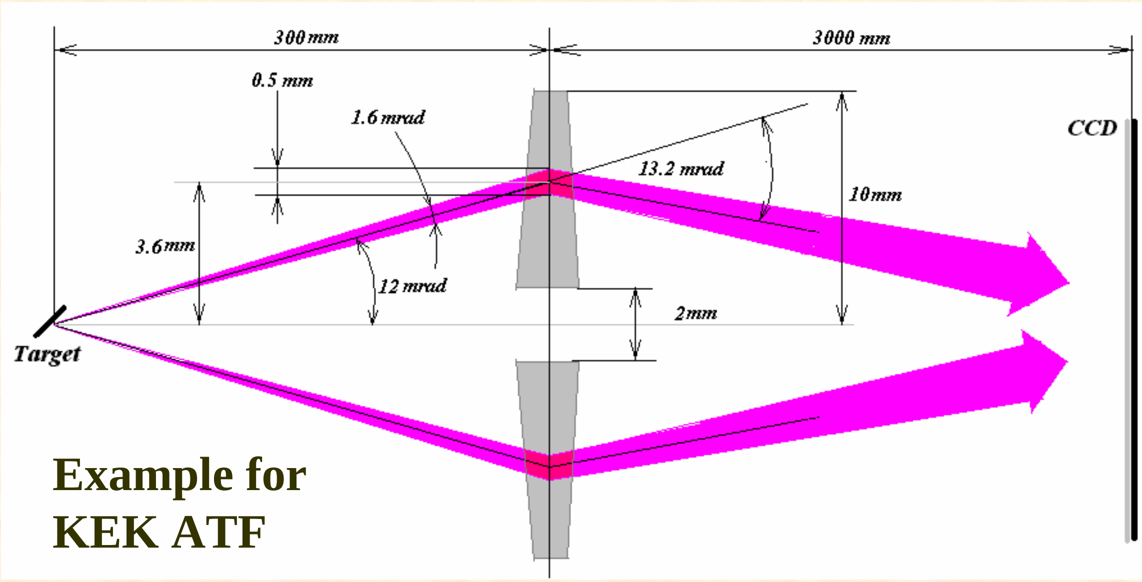
Only for interference test.

Not usable for beam size measurement

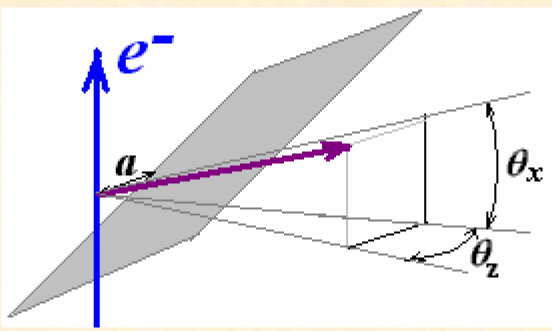


Scheme

Angular pattern bringing together using wending bi-prism



Theory (see apenix 1) is based on the well known expression for ODR from semi-surface in system of specular reflection direction,



$$E_z\left(s^{\pm},a,\theta_x,\theta_z\right)=\frac{i\cdot s^{\pm}\cdot\alpha\cdot e^{-\pi a\left(\sqrt{1+\theta_x^{2}}+i\cdot s^{\pm}\cdot\theta_z\right)}}{4\pi^2\left(\sqrt{1+\theta_x^{2}}-i\cdot s^{\pm}\cdot\theta_z\right)}$$

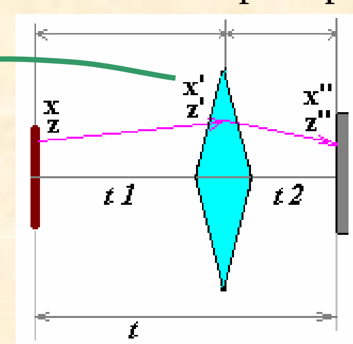
where $s^{+}=1,\quad s^{-}=-1;$ for left and right semi-surfaces

α is the fine structure constant, a is the impact-parameter

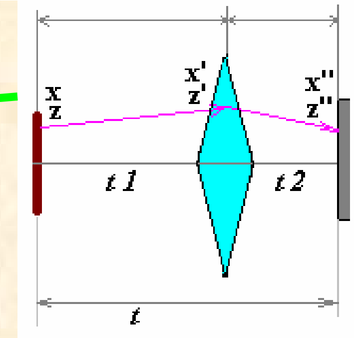
and on a phase shift on bi-prism:

$$\Delta\varphi_{prism}\left(k,x'\right)=-\frac{2\pi}{\lambda}k\cdot\left|x'\right|$$

Where k is the wending angle



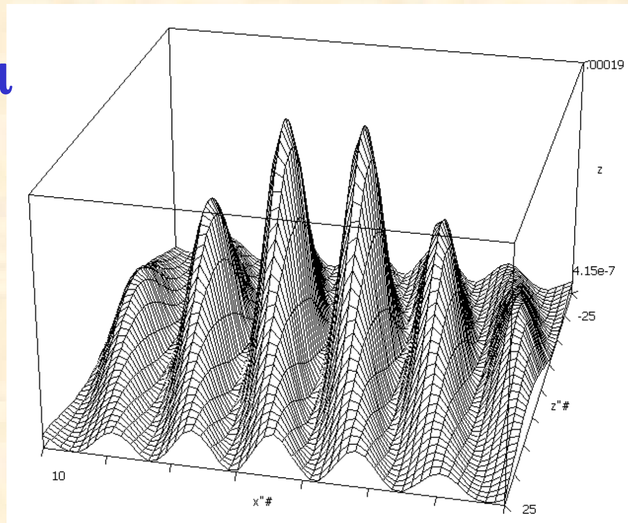
Example for KEK ATF extracted beam



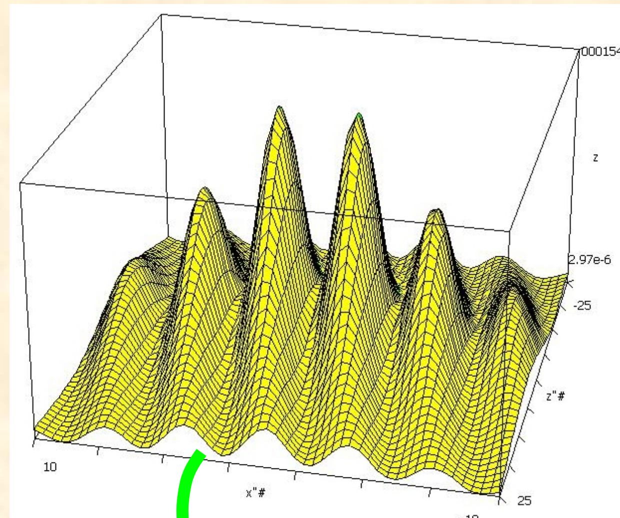
γ=2500, α=5.6mrad, t1=0.25m, t2=2.5m, λ=0.5μ

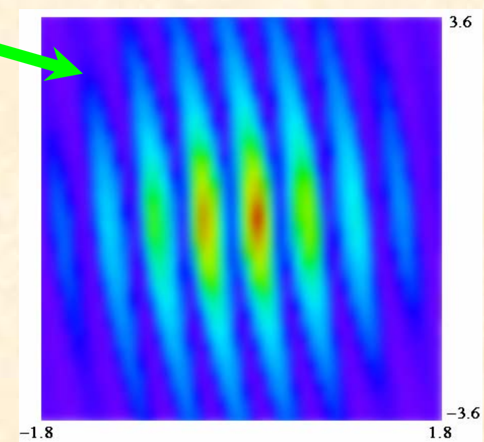
Interference pattern after the integration over the prism surface and over a Gaussian electron beam profile:

σ_e= 2μ

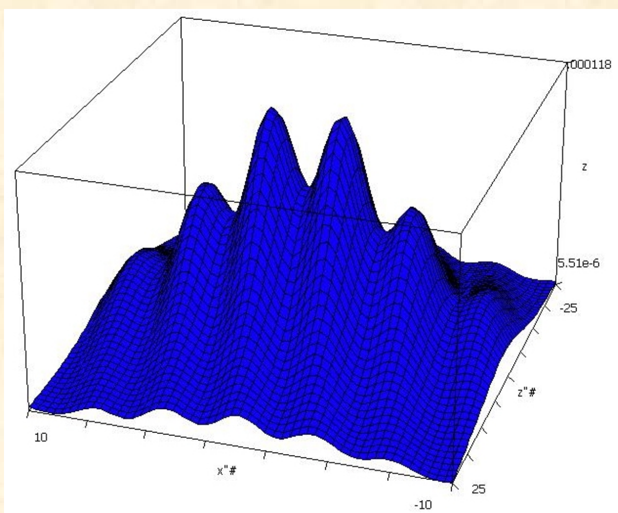


σ_e= 6μ



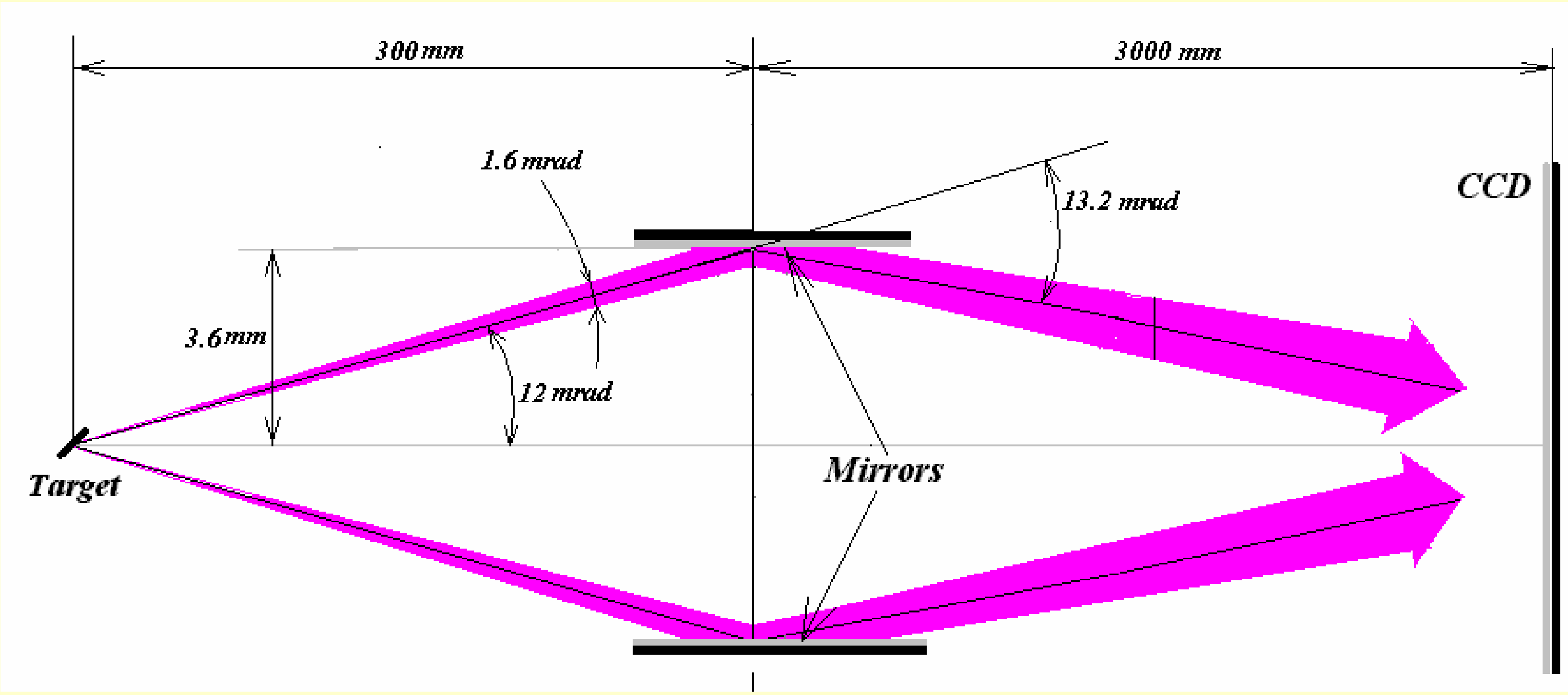


σ_e= 10μ



The single bunch beam size measurement on KEK ATF using bi-prism with a tuning wending angle is planed this year.

The same optical scheme may be realized also using mirrors



This scheme is more simple for understanding but it is more complicate for manufacture of a tuning system.

Some features for SLAC FFTB

E_e= 28.5 GeV,

γ = 57 000;

$\sigma_e \approx 5\mu$;

Beam divergence:

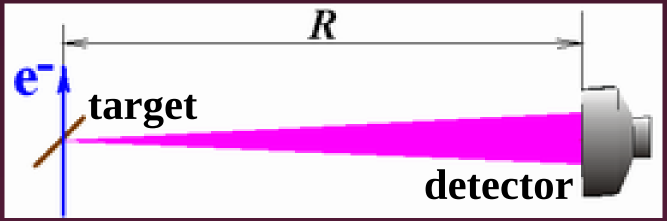
***horizontal* $\approx 5/\gamma$**

***vertical* $\approx 1/\gamma$**

Bunch population: $1\text{-}3\times10^{10}$

1. Near field zone effect

V.A. Verzilov, Phys. Let. A 273 (2000) 135-140



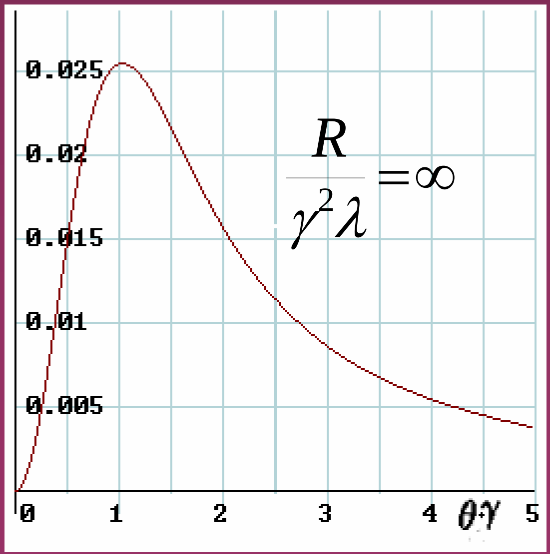
$$\frac{R}{\gamma^2 \lambda} < 1$$

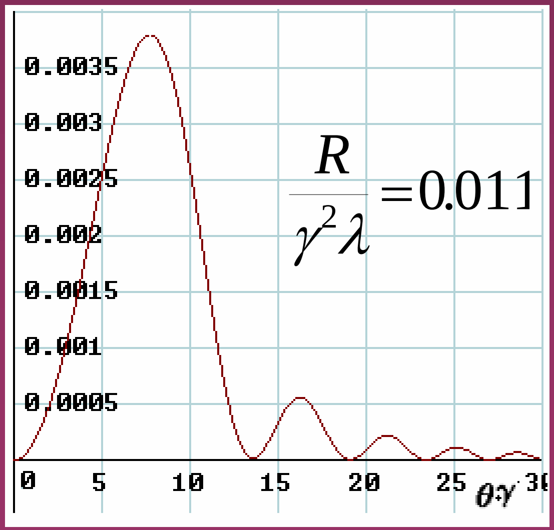
For SLAC FFTB $\gamma^2 \lambda \approx 1800m$

$$\frac{R}{\gamma^2\lambda} \xrightarrow{\text{FFTB}} \frac{20m}{57000^2 \cdot 0.5 \cdot 10^{-6}} \approx 0.011$$

Effect is peculiar to radiation angular distribution.
It shows itself for ODR as well as for OTR.

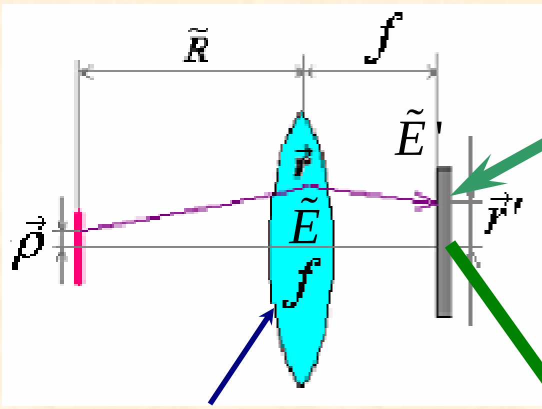
Deformation of OTR angular distribution





Near field zone effect resolution

Example for OTR



Focus length

Angular distribution image

$$\tilde{E}' = \int_0^b \tilde{E}(r)\cdot r\cdot J_1\left(-2\pi\cdot R\cdot r\cdot r'\right)dr,$$

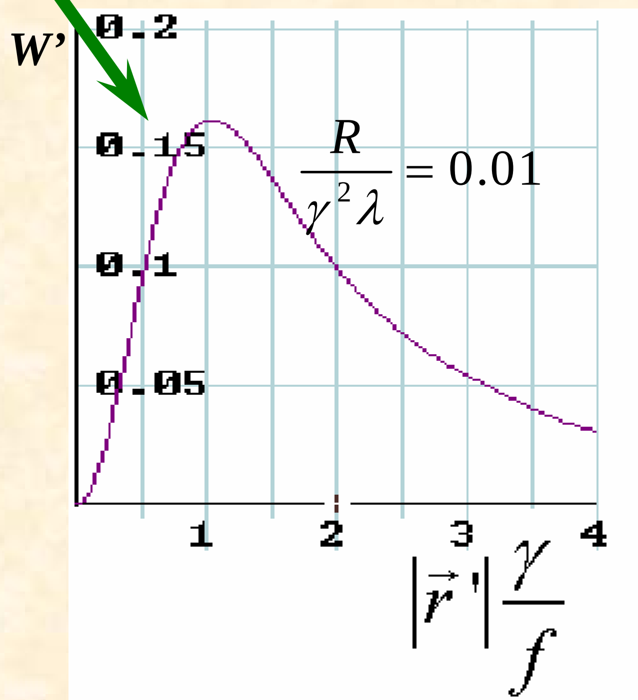
Bessel function

here

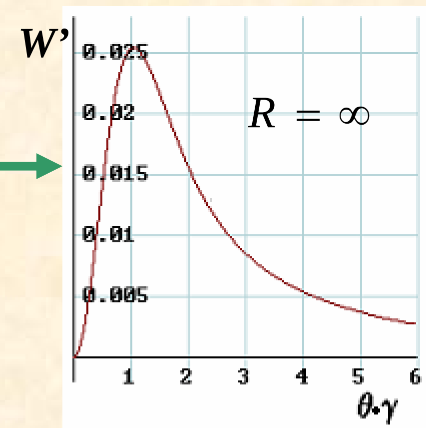
$$r' = \frac{\gamma\cdot|\vec{r}\,'|}{f}$$

$$W' = \left|\tilde{E}'\right|^2$$

Near field zone



Far field zone without lens



2. Electron beam divergence influence

ODR divergence

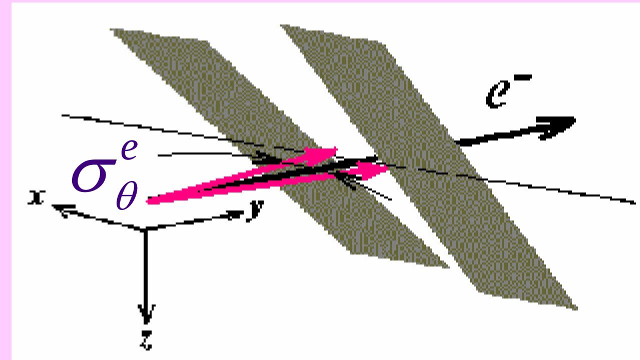
Condition for ODR angular distribution application:

$$\sigma_{\theta}^{e^-} << \sigma_{ODR} \approx \frac{1}{\gamma}$$

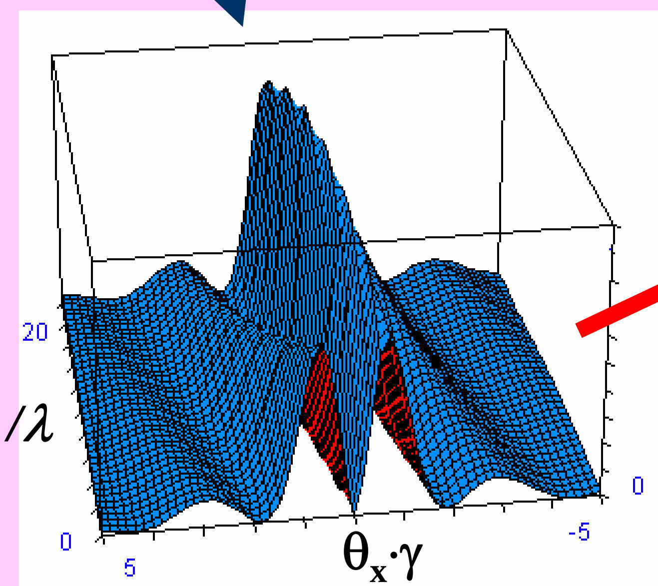
Electron beam divergence:

	KEK ATF	SLAC FFTB
horizontal	≈0.05 /γ	≈ 5 /γ
vertical	≈ 0.007/γ	≈ 1 /γ

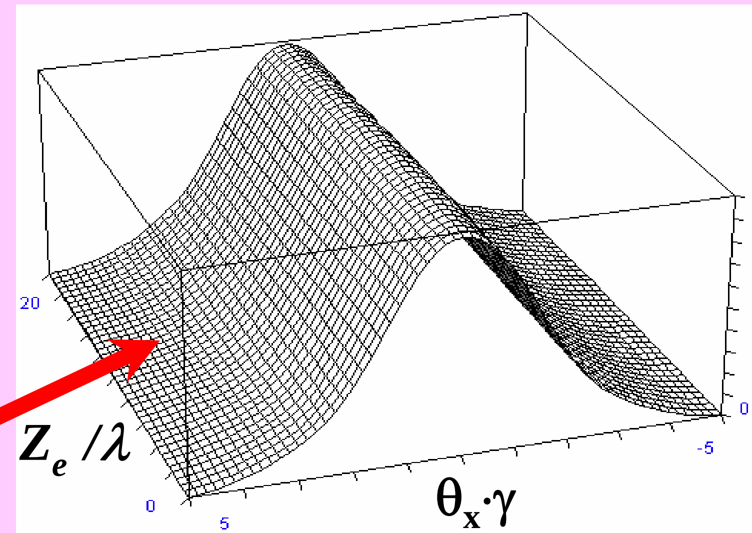
Interference pattern destruction by beam divergence



σ_θ^e □ $\frac{1}{\gamma}$



$$\sigma_\theta^e = \frac{1}{\gamma}$$



No dependence on electron position

No dependence on beam size

Solution of beam divergence problem

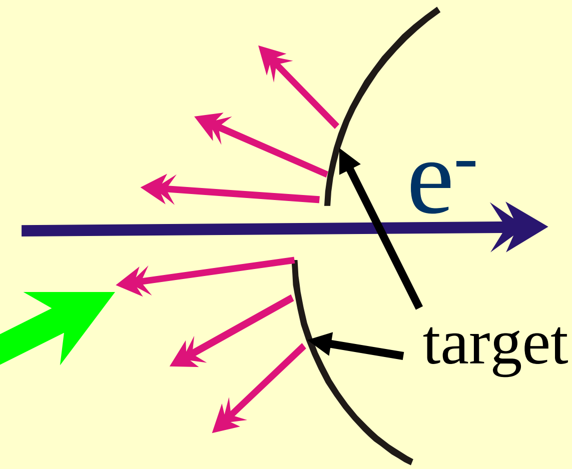
$$\sigma_{\theta}^{e^-} \ll \sigma_{DR}$$

Condition for ODR method application

Not controlled

May be increased

We may increase ODR divergence by using of parabolic segments in crossed target:



Near field model for bent target

$$W_{\perp} = \left| \int_{x'} \int_{z'} \tilde{e}_{\perp} \, dz' dx' \right|^2$$

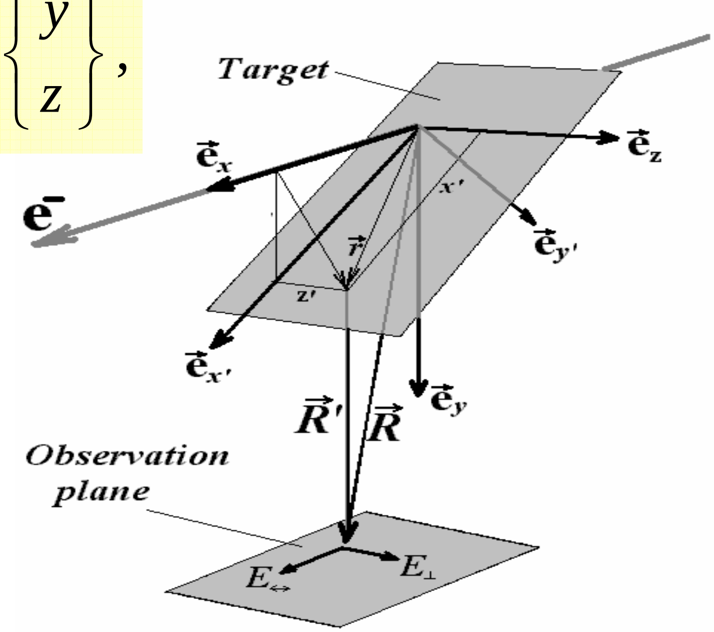
where:

$$\begin{Bmatrix} \tilde{e}_{\leftrightarrow} \\ \tilde{e}_{\perp} \end{Bmatrix} = \frac{\sqrt{\alpha}}{\pi\sqrt{y^2+z^2}} K_1\left(\frac{2\pi}{\gamma\lambda}\sqrt{y^2+z^2}\right) \cdot e^{\varphi} \cdot \begin{Bmatrix} y \\ z \end{Bmatrix},$$

where

$$\varphi = \frac{2\pi}{\lambda}\left(\left|\vec{R}'\right| + x/\beta\right)$$

modified Bessel function of the first order .



To turn to the target system:

$$\begin{cases} x = x'\cos\theta_r - y'\sin\theta_r \\ y = x'\sin\theta_r + y'\cos\theta_r \\ z = z' \end{cases}$$

$$y' = f(x', z')$$

Target surface profile:

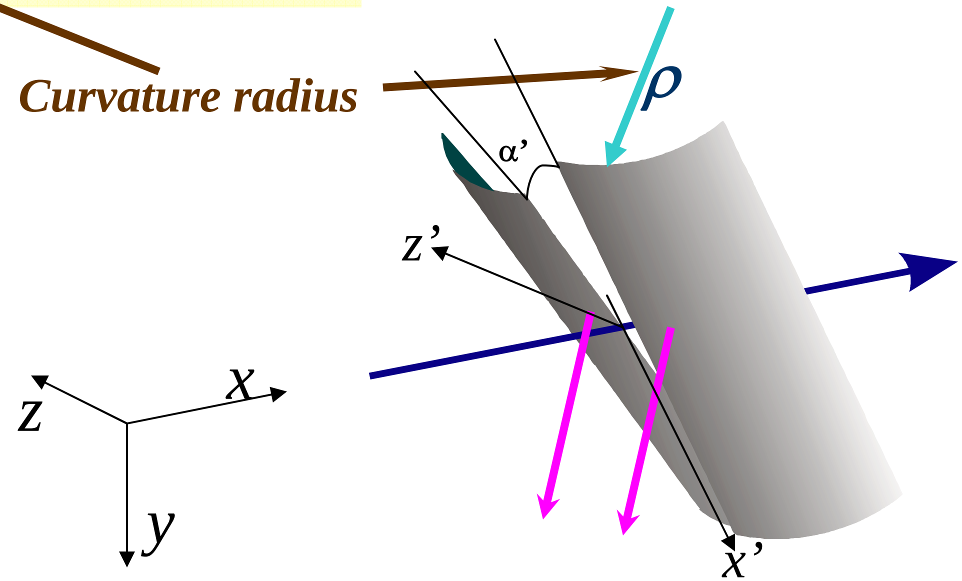
Target surface profile:

angle between half's

$$y' = -\frac{z'^2}{2\rho} + sign(z')\cdot\frac{\alpha}{2}x'_e$$

electron position

Curvature radius



$$W_\perp(\theta_z, x_e) = \left| \int_{x'=-b/2}^{b/2} \left(\int_{z'=-b/2}^{-a/2} \tilde{e}_\perp dz' + \int_{z'=a/2}^{b/2} \tilde{e}_\perp dz' \right) dx' \right|^2$$

DR angular distribution for a Gaussian transverse beam profile

$$W_{\perp}(\theta_z,\sigma_e') = \frac{1}{\sqrt{2\pi\sigma_e'^2}}\int W_{\perp}(\theta_z,x_e')\,e^{-\frac{x_e'^2}{2\pi\sigma_e'^2}}dx_e'$$

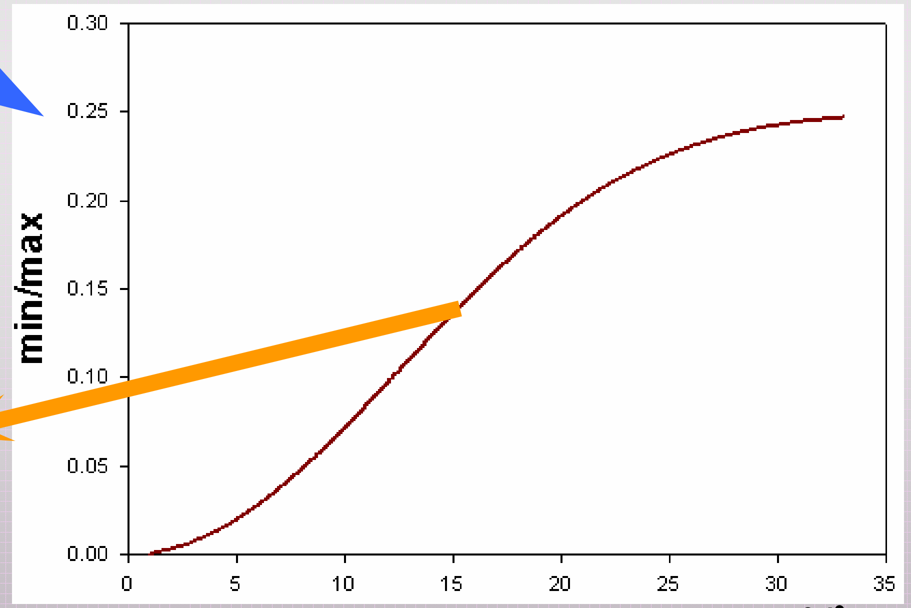
Example for

λ=0.5μ, γ=60000

ρ=0.01·γ²λ≈18m

α=10mrad

min → $W_{\perp}(\theta_z\gamma=0,\sigma_e') \big/ W_{\perp}(\theta_z\gamma=30,\sigma_e')$ ← ***max***



Beam size

$\sigma_e \approx (15/\sqrt{2})\cdot\lambda \approx 11\cdot\lambda = 5.5\mu$

may be measured

However, for low emittance beams ($\sigma_\theta^{e^-} << \dfrac{1}{\gamma}$) we need not a bent target.

Conclusion

• *Beam size ODR effect of this method is of the first order in contrast to the effect of the second order for the method based on a flat slit target. A radiation intensity beam size effect comprises 20~60% of OTR intensity. Single bunch measurement using CCD is possible near well as OTR measurement.*

• *Interference pattern from a crossed slit target were observed.*

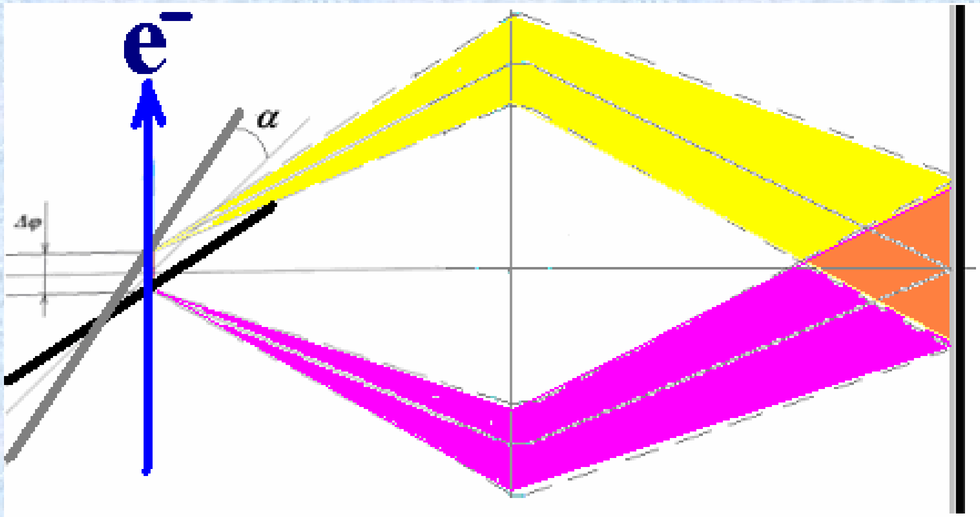
•*The near field effect problem may be resolved using optical system.*

•*The single bunch beam size measurement on KEK ATF using bi-prism with a tuning wending angle is planed this year.*

Thank you

Apendix 1

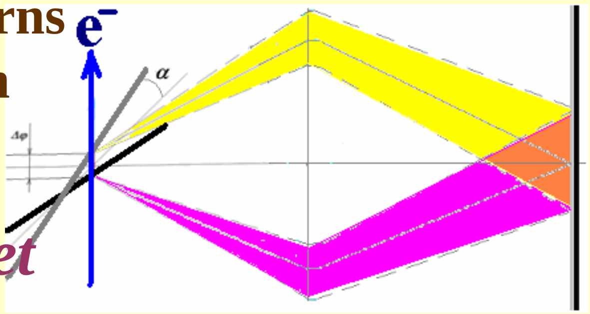
Angular pattern bringing together



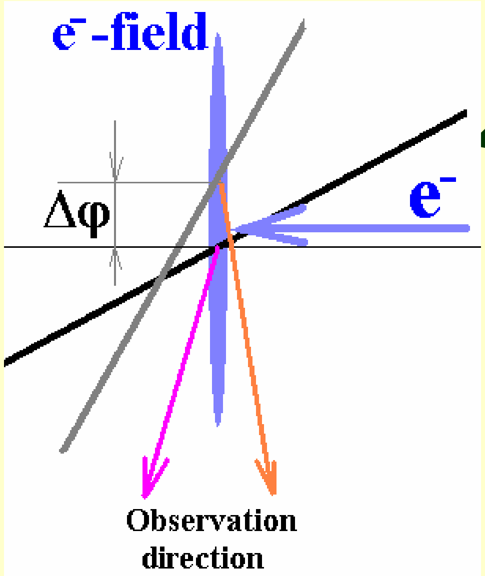
Usage an optical lens

Crossing target beam size measurement technique was developed for an interference of the angular distribution patterns from the both target planes (*see KEK report at Nov. 2003*)

Using a cylindrical optical lens in this plane the patterns may be bringing together only if the radiation spot on the target is focused on the detector plane.



We obtain the image of the spot from the target



In this case the phase difference $\Delta\varphi$ in the observation direction does not depend on the electron position.

So the minimum and maximum intensity position in the spot image does not depend on an electron distribution in the beam.

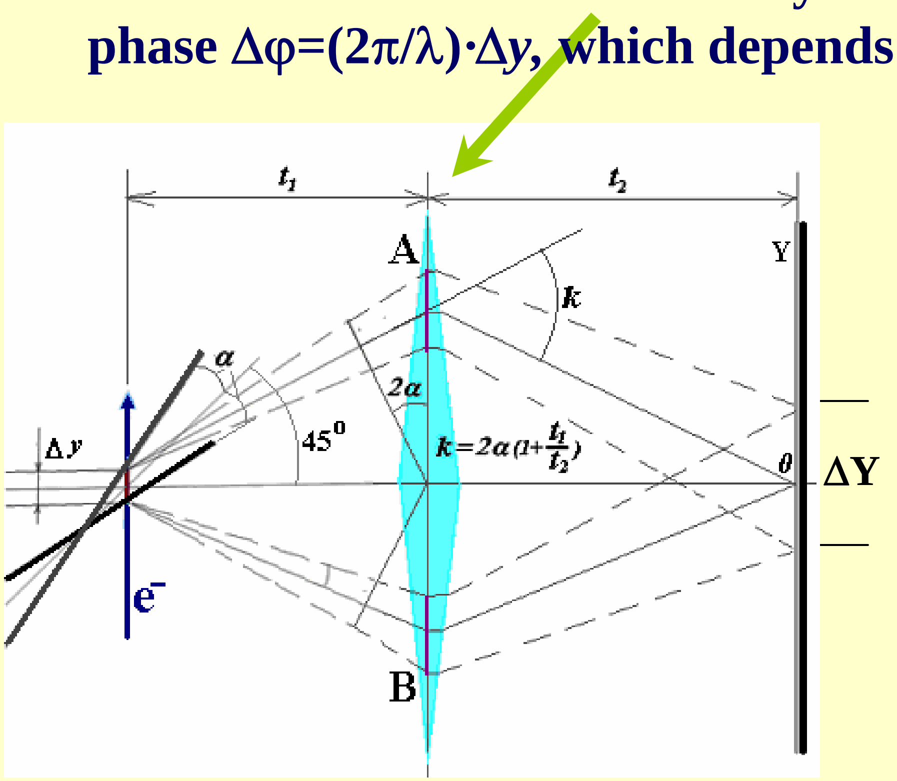
How to bring together the angular distribution patterns?

Wending prism application for bringing together the angular distribution patterns

(Fast introduction)

Wending prism provides the undistorted radiation turning.

Radiation beams A and B are symmetric and they differ only by the phase $\Delta\varphi=(2\pi/\lambda)\cdot\Delta y$, which depends on the electron position.



If Δy<<ΔY, the interference picture depends only on the phase difference, that is, only on the electron position.

For example $\Delta\varphi=2\pi\cdot n$ results an intensity minimum and $\Delta\varphi=\pi+2\pi\cdot n$ results a maximum in the point Y=0 for the perpendicular electromagnetic field component.

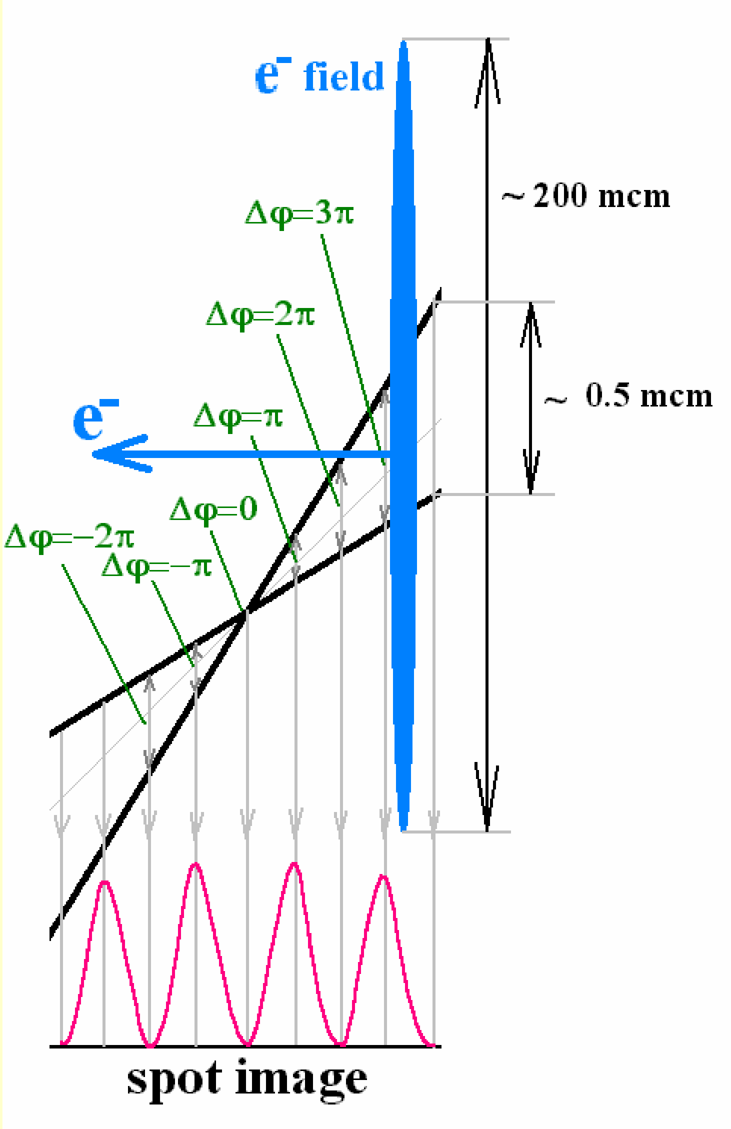
That is, a beam size may be measured

What is difference between an optical scheme with a lens and one with a prism?

Lens

If we bring together radiation beams from both target planes using an optical lens, we obtain on a CCD the image of radiation spot on the target.

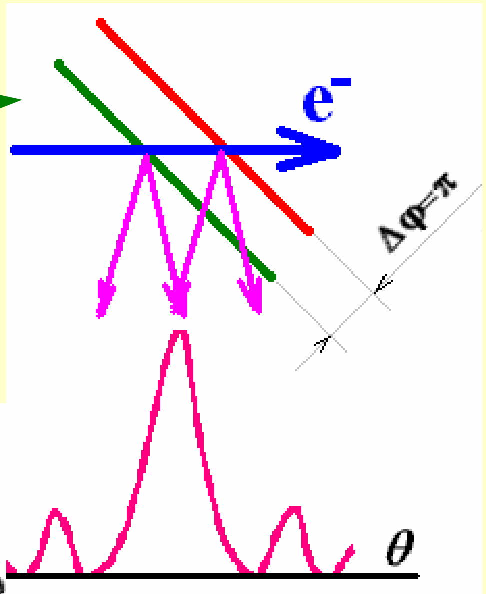
As the transversal size of an electron field is much larger, than the distance between the target semi-planes, the interference picture depends on a target position, but not depends on the electron position.

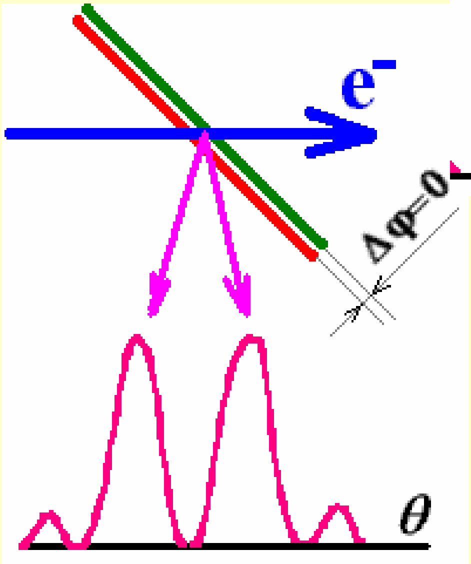


Prism

Here we **don't focus** *a radiation. We have to deal with an angular distribution like in the experiment with the flat slit target.*

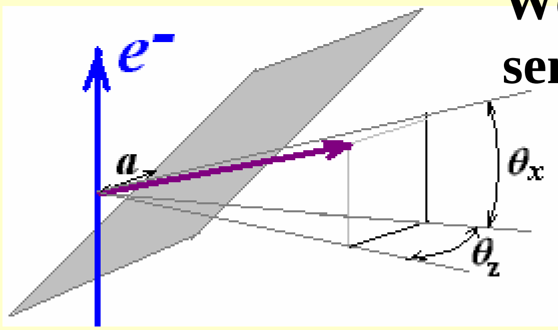
We use effect like one from a bad manufactured flat slit target, where the interference picture depends on the longitude distance between semi-planes:





However for a crossing target the phase shift $\Delta\varphi$ depends on an electron position

Detail analysis



We start from the well known expression for ODR from semi-surface in system of specular reflection direction.

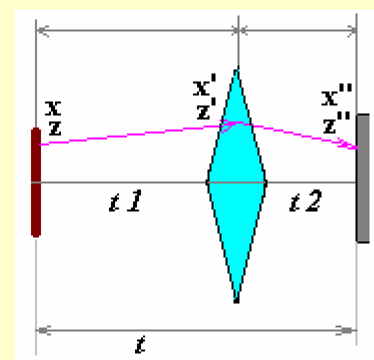
$$E_z\left(s^{\pm},a,\theta_x,\theta_z\right)=\frac{i\cdot s^{\pm}\cdot\alpha\cdot e^{-\pi a\left(\sqrt{1+\theta_x^{\,2}}+i\cdot s^{\pm}\cdot\theta_z\right)}}{4\pi^2\left(\sqrt{1+\theta_x^{\,2}}-i\cdot s^{\pm}\cdot\theta_z\right)}$$

where $s^{+}=1,\quad s^{-}=-1;$ for left and right semi-surfaces

α is the fine structure constant, a is the impact-parameter

Phase shift on a bi-prism:

$$\Delta\varphi_{prism}\left(k,x'\right)=-\frac{2\pi}{\lambda}k\cdot\left|x'\right|$$ Where k is the wending angle



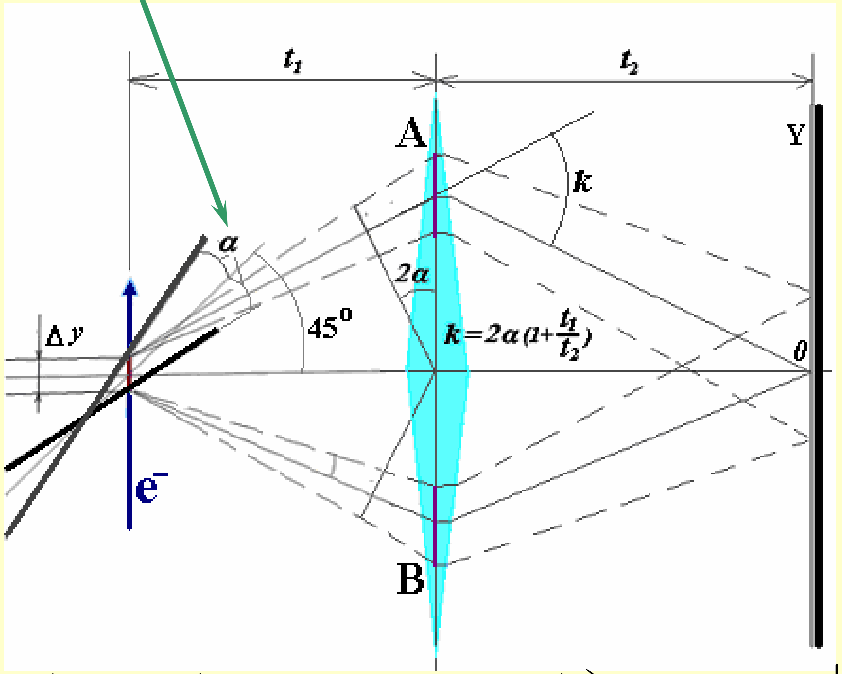
Radiation field from semi-surface of crossed target just downstream to the bi-prism:

$$E_z^{'}\left(s^{\pm},a,\frac{x_e}{\gamma\lambda},\alpha\gamma,\frac{t_1}{\gamma^2\lambda}\right)=E_z\left(s^{\pm},a,\theta_x^{'}-2\cdot s^{\pm}\alpha\gamma,\theta_z^{'}\right)\cdot e^{i\cdot 4\pi\cdot s^{\pm}\cdot\frac{y_e}{\lambda}\alpha}\cdot e^{i\cdot\varphi'}$$

where $\theta_x^{'}=\frac{x'}{t_1}\gamma,\quad \theta_z^{'}=\frac{z'}{t_1}\gamma,$

$$\varphi'=\frac{2\pi}{\lambda}2\alpha\cdot|x'|+\Delta\varphi_{prism}(k,x');$$

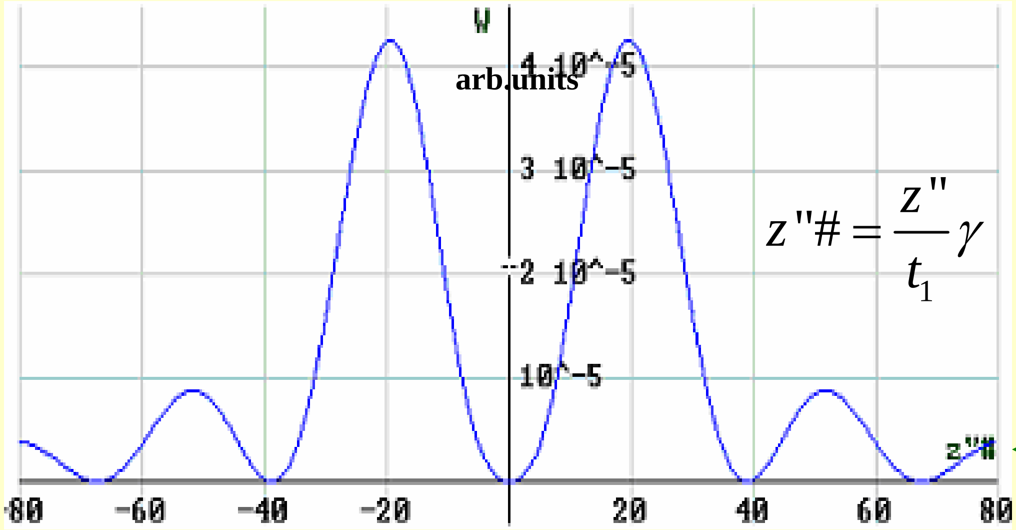
Y_e is the electron position



and a final radiation intensity from the crossed target:

$$W\left(a,\frac{x_e}{\gamma\lambda},\alpha\gamma,\frac{t_1}{\gamma^2\lambda},\frac{t_2}{\gamma^2\lambda}\right)=\left|\iint\left(E_z^{'}\left(1,a,\frac{x_e}{\gamma\lambda},\alpha\gamma,\frac{t_1}{\gamma^2\lambda}\right)+E_z^{'}\left(-1,a,\frac{x_e}{\gamma\lambda},\alpha\gamma,\frac{t_1}{\gamma^2\lambda}\right)\right)\cdot e^{i\cdot\varphi''}dz'dx'\right|^2$$

where $\varphi''=\frac{\pi}{\lambda}\cdot t_2\cdot\left(\left(\frac{x''-x'}{t_2}\right)^2+\left(\frac{z''-z'}{t_2}\right)^2\right)$

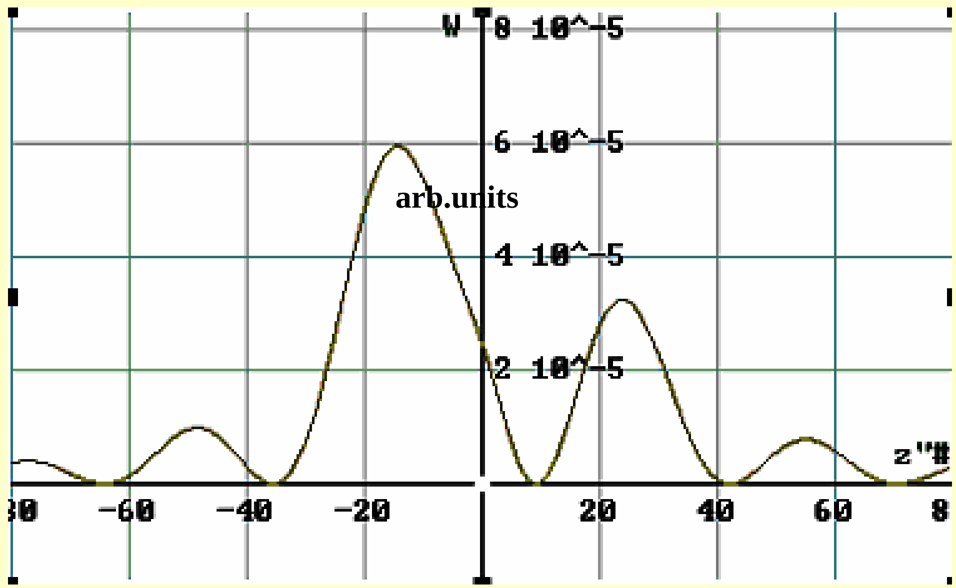


Angular distribution image perpendicular to the target slit.

α=0.003, λ=0.5mcm, x=0.

An electron moves through the target semi-plane cross point.

An electron moves at the distance 4mcm from the target semi-plane cross point



The foregoing allows to use this optical scheme for beam size measurement by the comparison of radiation intensity in minimum and maximum of distribution.

Apendix 2

Near field model for bent target

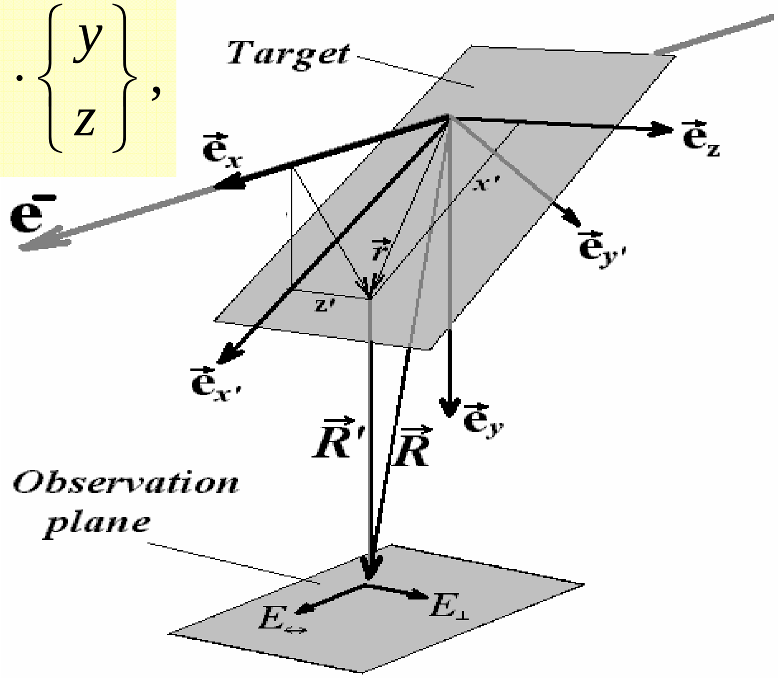
Near field model for bent target

$$\left\{\begin{matrix}\tilde{e}_{\leftrightarrow}\\ \tilde{e}_{\perp}\end{matrix}\right\} = \frac{\sqrt{\alpha}}{\pi\sqrt{y^2+z^2}} K_1\left(\frac{2\pi}{\gamma\lambda}\sqrt{y^2+z^2}\right)\cdot e^{\varphi}\cdot\left\{\begin{matrix}y\\ z\end{matrix}\right\},$$

where

$$\varphi = \frac{2\pi}{\lambda}\left(\left|\vec{R}'\right| + x/\beta\right)$$

modified Bessel function of the first order .



To turn to the target system:

$$\begin{cases} x = x'\cos\theta_r - y'\sin\theta_r \\ y = x'\sin\theta_r + y'\cos\theta_r \\ z = z' \end{cases}$$

Target surface profile:

$$y' = f(x', z')$$

Integral is assumed to be taken over the target surface

$$\left\{\begin{matrix}E_{\leftrightarrow}\\ E_{\perp}\end{matrix}\right\} = \int_{z'}\int_{x'}\left\{\begin{matrix}\tilde{e}_{\leftrightarrow}\\ \tilde{e}_{\perp}\end{matrix}\right\}dx'\,dz'$$

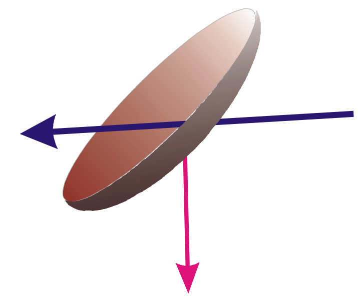
DR Intensity:

$$W_{\leftrightarrow} = \left|E_{\leftrightarrow}\right|^2$$

$$W_{\perp} = \left|E_{\perp}\right|^2$$

Illustration of radiation properties from bent target for simple target geometry

Near field model may be simplified considerable for case of axial symmetry and parabolic target in far field approach:



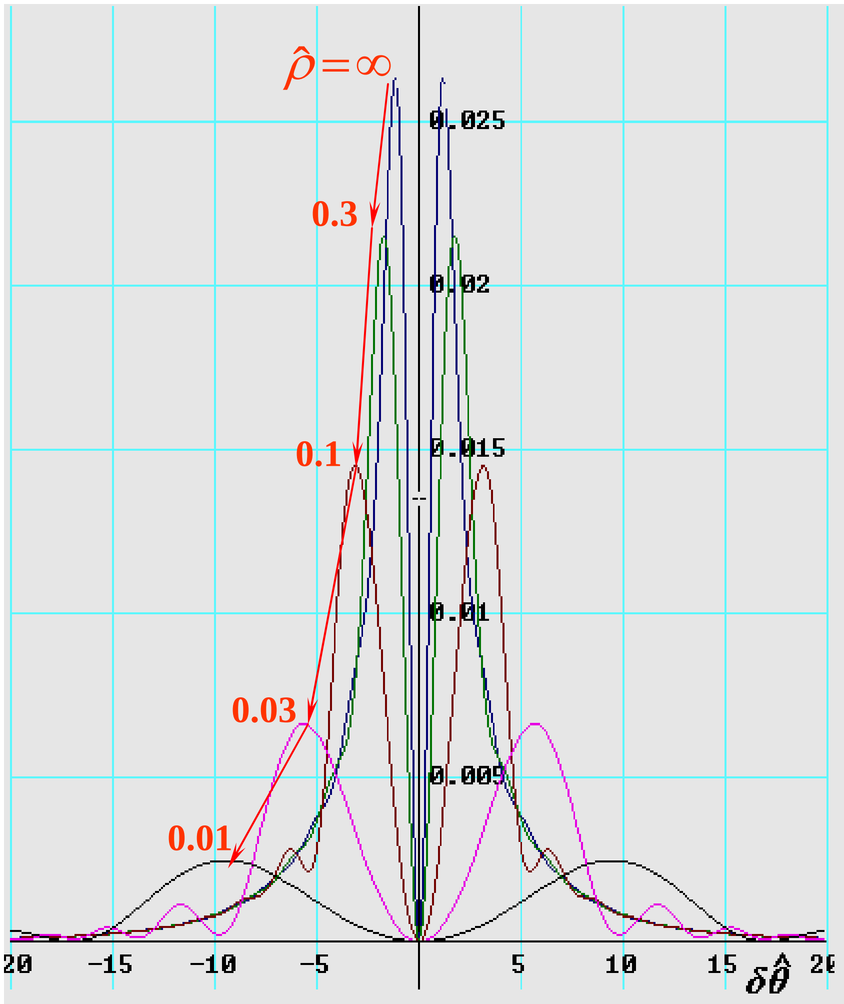
$$W\left(\hat{\rho},\delta\hat{\theta}\right)=\left|\int_{r=0}^{\hat{r}_t}\sqrt{\alpha}\,2\pi r\cdot K_1\left(\frac{2\pi}{\gamma\lambda}r\right)e^{i\sqrt{2}\pi r^2/\hat{\rho}}J_1\left(2\pi r\cdot\delta\hat{\theta}\right)dr\right|^2,$$

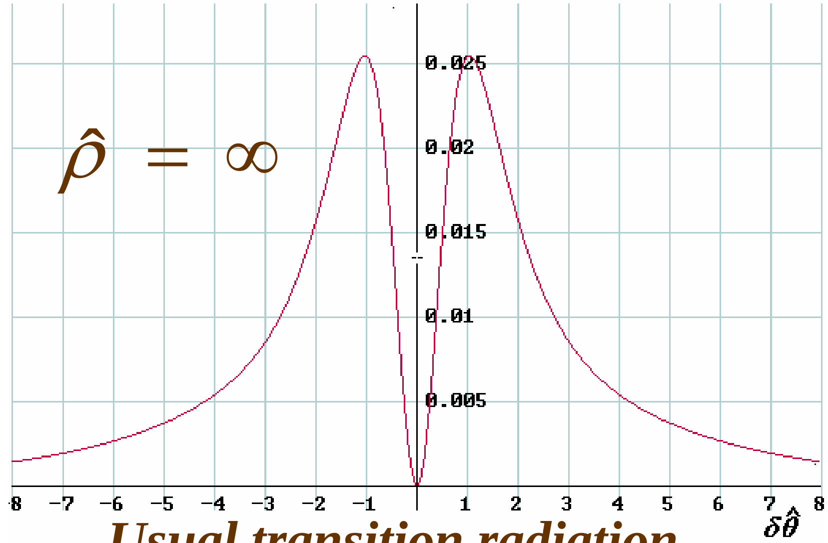
where

$\hat{\rho}=\rho/\gamma^2\lambda,$ $\hat{r}_t=r_t/\gamma\lambda,$ $\delta\hat{\theta}=\delta\theta\cdot\gamma,$ ρ *is a target curvature radius,*

$\boldsymbol{r_t}$ *is a radius of the target projection on the plane, normal to the observation direction,*
$\boldsymbol{\delta\theta}$ *is the observation angle in respect to the direction of specular reflection,*
$\boldsymbol{J_1}$ *is a first order Bessel function.*

Convex target





Usual transition radiation

Example:

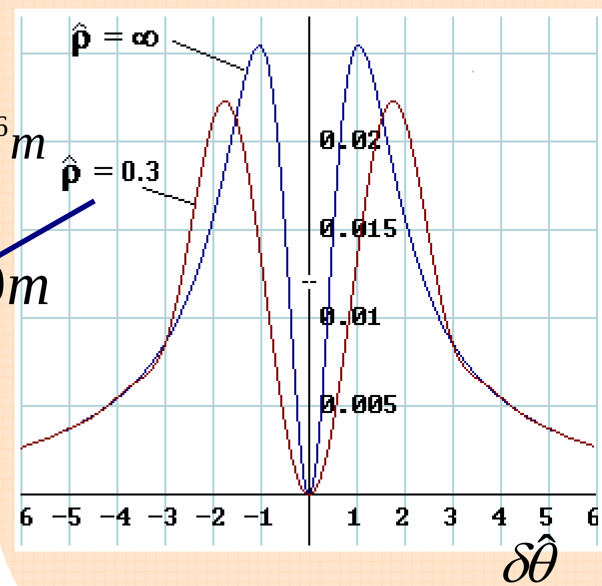
Japanese flat target

$\hat{\rho}=\rho/\gamma^2\lambda$

$\gamma=2500$

$\lambda=0.5\cdot10^{6}m$

$\rho = 0.9m$



Concave target results defocusing effect as well as convex one.

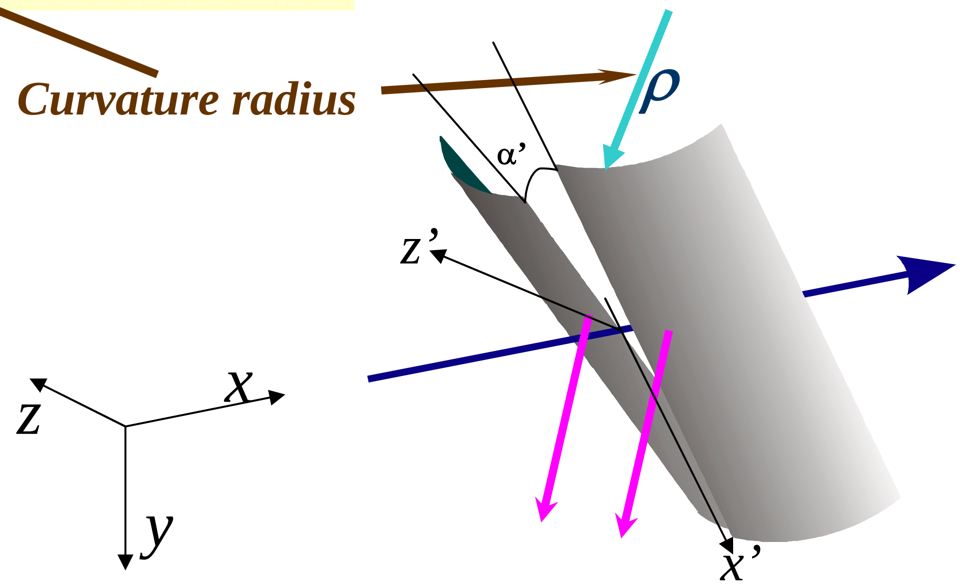
We choose next target surface profile:

angle between half's

$$y' = -\frac{z'^2}{2\rho} + sign(z') \cdot \frac{\alpha'}{2} x'_e$$

electron position

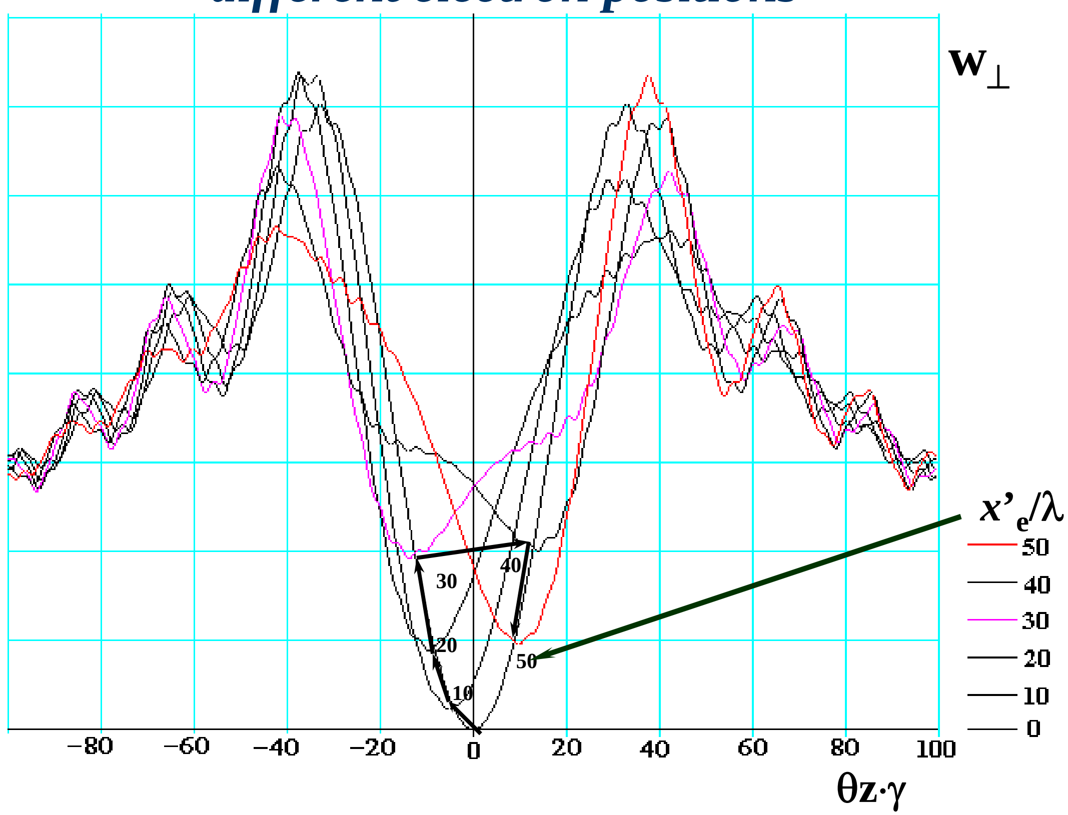
Curvature radius



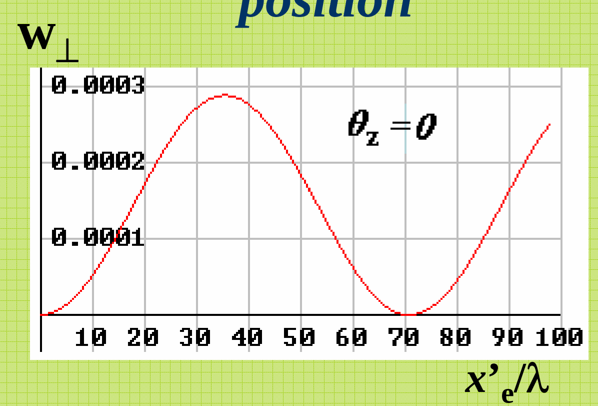
Sample for the bent slit target with slit width $a=0.01\gamma\lambda$, $\rho = \gamma^2\lambda/1000$, $\alpha'=0.01$ in far field approach

$$W_{\perp} = \left| \int_{x'=-b/2}^{b/2} \left(\int_{z'=-b/2}^{-a/2} \tilde{e}_{\perp} dz' + \int_{z'=a/2}^{b/2} \tilde{e}_{\perp} dz' \right) dx' \right|^2$$

Horizontal angular distribution for the different electron positions



Intensity in the center of angular distribution as a function of electron position



DR angular distribution for Gaussian transverse beam profile

$$W_{\perp}(\theta_z, \sigma_e') = \frac{1}{\sqrt{2\pi\sigma_e'^2}} \int W_{\perp}(\theta_z, x_e') e^{-\frac{x_e'^2}{2\pi\sigma_e'^2}} dx_e'$$

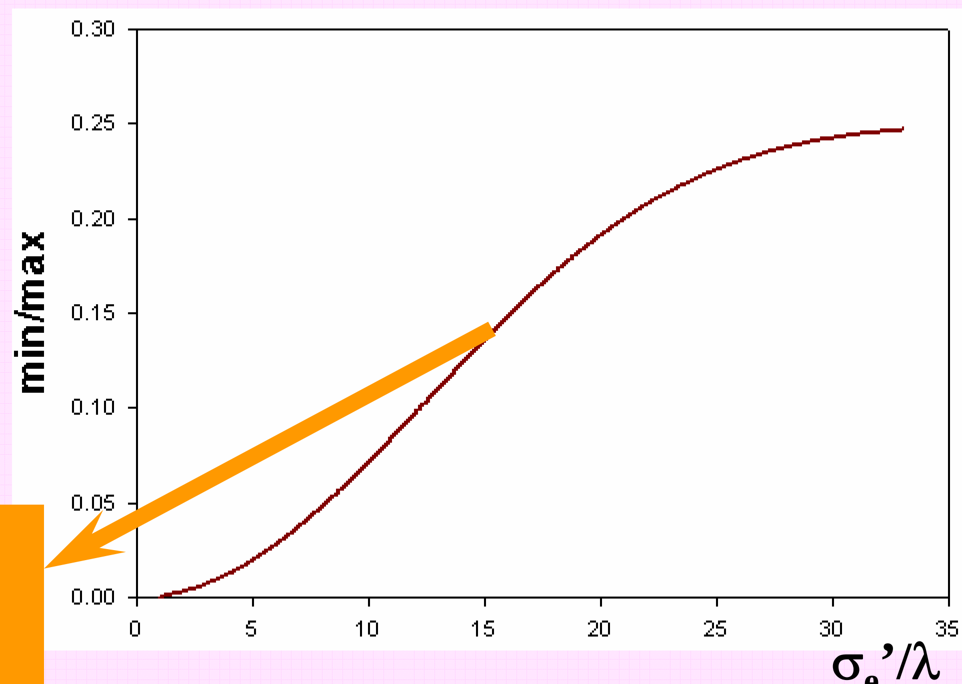
Example for

λ=0.5μ, γ=60000

$\rho = 0.01\cdot\gamma^2\lambda \approx 18$m

α=10mrad

min → $W_{\perp}(\theta_z\gamma = 0, \sigma_e') \Big/ W_{\perp}(\theta_z\gamma = 30, \sigma_e')$ ← *max*



Beam size
$\sigma_e \approx (15/\sqrt{2})\cdot\lambda \approx 11\lambda$
may be measured

Main method advantages

- Radiation intensity is comparable to the OTR intensity. This allows us to look forward to the single bunch measurements using CCD.

- Very wake dependence on the Lorenz-factor allows to use this method for high energy electron beams.

- There is reserve (target crossing angle and wavelength) for using of this method for sub-micron beams.