Dynamics of the Universe

- *Friedmann Equations*
- *Equation of State*
- *Solutions for the scale factor for baseline model*
- *The Inverse Problem*
- *Studying a possible scalar field*

Newtonian Swindle

- *Consider small sphere with radius ~ a*

$$\frac{a'^2}{2} - \frac{4\pi G\rho a^2}{3} = const$$

- *Introduce*

$$\Omega_m = \frac{8\pi G\rho_0}{3H_0^2}$$

so that $$a'^2 = H_0^2[\Omega_m a^{-1} + \ldots]$$

- Exterior mass (Birkhoff)?
- Pressure (OK)?
- Constant (Ω_k=-k/$H_0^2 R_0^2$, Milne Universe,=0 if flat)

GR Approach

- *Field Equations*

$$G^{\alpha\beta} = 8\pi G(\rho + P)u^{\alpha}u^{\beta} + P g^{\alpha\beta}$$

- *Recover first Friedmann equation.*
- *Also use ThI* $d(\rho a^3) = -P d a^3$

to get second Friedmann equation

$$a'' = -4\pi G(P + \rho/3)a$$

Radiation Fluid

$$P \propto a^{-4} \propto T^4$$

$$a'^2 = H_0^2[\Omega_m a^{-1} + \Omega_r a^{-2} + ...]$$

Vacuum Energy

$$\rho = const = -P$$

$$a'^2 = H_0^2[\Omega_m a^{-1} + \Omega_r a^{-2} + \Omega_v a^2 + \Omega_k + ...]$$

Differential Equation for a(t)

Quintessence P=w ρ

ΛCDM Model

- *Suppose that the universe is flat and there is just cold matter with* Ω_m*=0.27 and vacuum energy with P=-*ρ.
- *Solving the Friedmann equation with WMAP parameters gives our baseline model*
- *We can then go ahead and compute the age, distances, volumes etc*

$$a' = H_0\left(\frac{\Omega_m}{a} + (1-\Omega_m)a^2\right)^{1/2}$$

$$a = \left(\frac{\Omega_m}{1-\Omega_m}\right)^{1/3} \sinh^{2/3}\left(\frac{(1-\Omega_m)^{1/2} H_0 t}{2}\right)$$

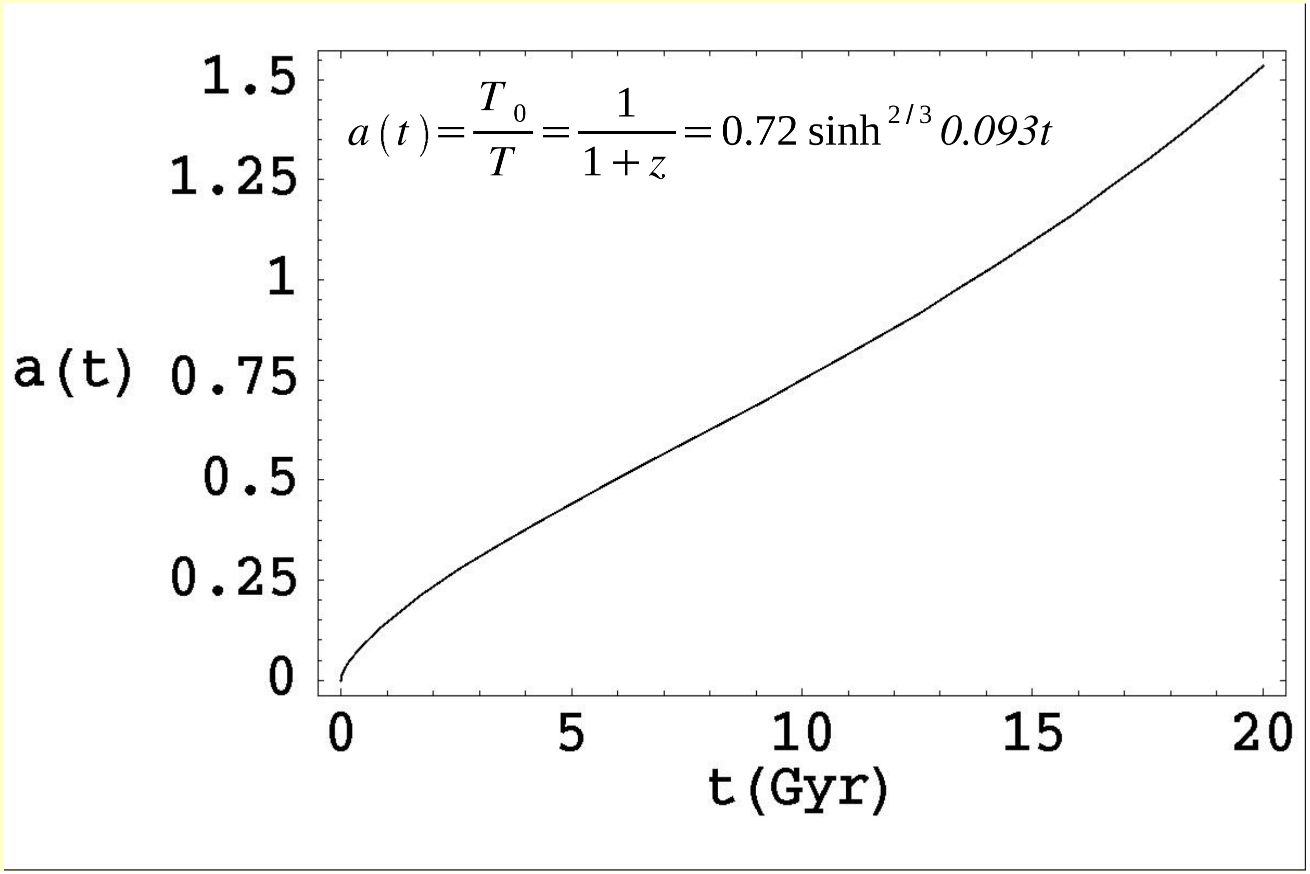
Baseline Model (after WMAP)

- *Flat; k=0; dark energy+matter*
- H_0=71 km s^{-1} Mpc^{-1} [1 Mpc=3.09 10^{22} m]

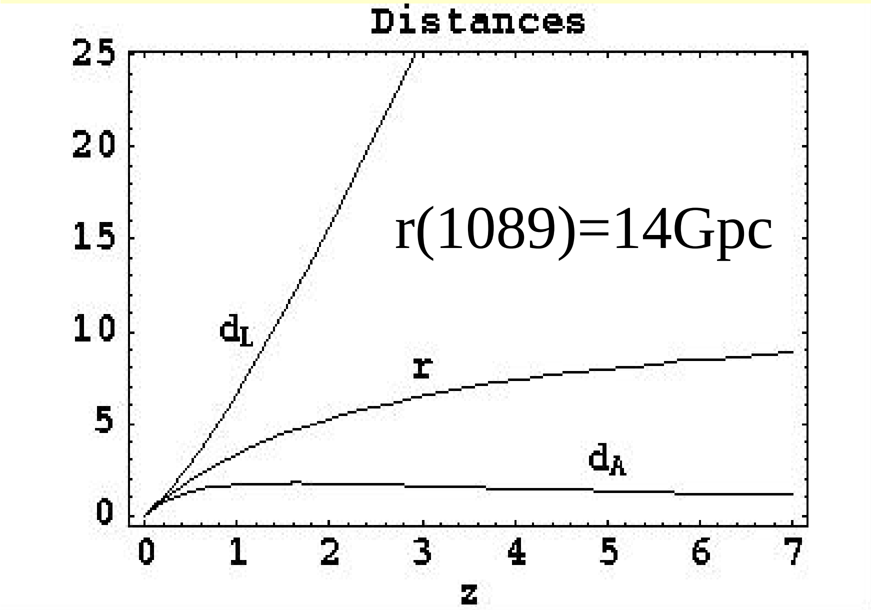
  =$(13.8\ \text{Gyr})^{-1}$ =$(4.23\ \text{Gpc})^{-1}$.
- t_0=13.7 Gyr
- $\Omega_0=\rho/\rho_c=0.27$,
  - $\Omega_b=0.044$
  - $\rho_c=3H_0^2/8\pi G$

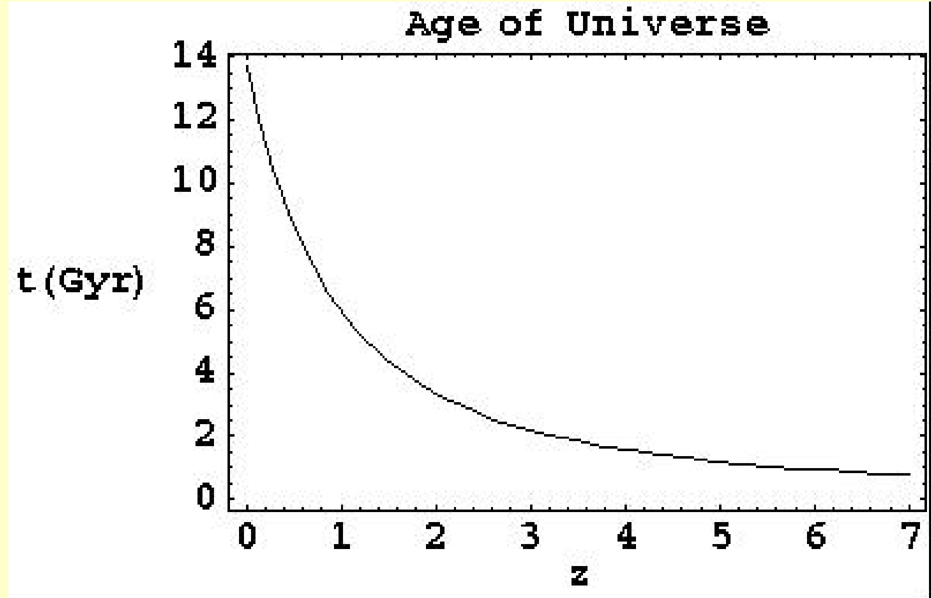
  $=9.4\ 10^{-27}$ kg m^{-3}

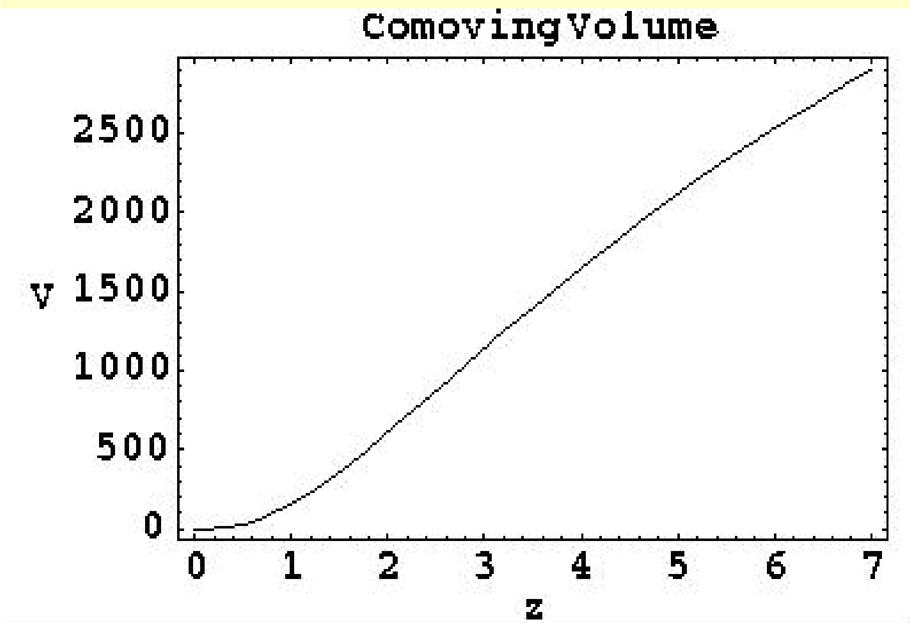
  $=1.4\ 10^{11} M_o\ Mpc^{-3}$
- T_0=2.725 K



Baseline Model







$$r = \int dt / a$$

$$V = 4\pi r^3 / 3$$

Measuring the scale factor

- *Suppose that some mission like SNAP is able to obtain accurate luminosity distances as a function of redshift* $z=1/a-1$
- *This implies that we know* $r(a)=a\, d_L(a)$
- *We can then solve for* $t(a)$ *using* $dt=a\, dr$
- *Inverting we obtain* $a(t)$ *without resorting to a dynamical model*
- *We can then compute* $H(t)$, $q(t)$, $j(t)$, *in principle.*

Density Pressure and Fields

- *Assuming a flat universe (though this is unnecessary),*

$$\rho = \frac{3H^2}{8\pi}; \qquad P = \frac{(1-j)H^2}{4\pi}; \qquad \frac{dP}{dt} = \frac{(1-j)H^3}{4\pi}$$

- *Now assume that the dark energy is contributed by a scalar field. Suppose further that we can ignore spatial gradients and the only other contribution is from cold matter. Then*

$$V(\phi) = \frac{(2-q)H^2}{8\pi} - \frac{\rho_m}{2}$$

$$\phi'^2 = \frac{(1+q)H^2}{4\pi} - \rho_m$$

Dynamics of the Field

- *Next make the slow roll approximation,* $\phi'' = 0$

$$V' = -3H\phi'$$

- *Substituting,*

$$j = -2 - 3q + \frac{12\pi\rho_m}{H^2} = -2 - 3q + \frac{9H_0^2\Omega_m}{2H^2a^3}$$

- *Substituting* $q_0 = 0.6 \Rightarrow j_0 = 1$

Eschatology

- *In order to predict the future you would need a good understanding of* $V(\phi)$

Dynamics Summary

- *Most solutions of Friedmann equations are now irrelevant*
- *Straightforward to compute framework for interpreting observations given a model of dark matter and energy*
- *Possible to infer behavior of putative scalar field given excellent (relative) distances*
- *Strong motivation for efforts like SNAP*