INFLATION

SSI 2003
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MOST INFLUENTIAL IDEA IN COSMOLOGY

NOW BEING TESTED

Additional Reading:

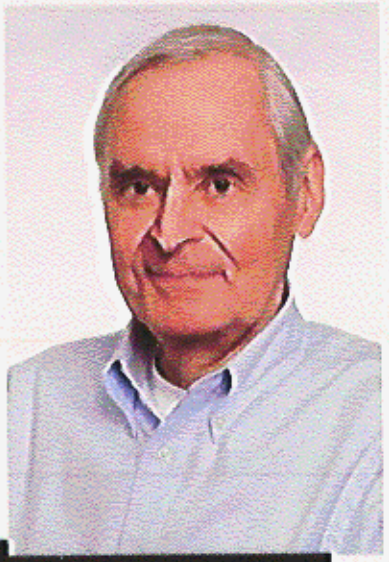
- Kolb&Turner The Early Universe Ch 8
- Liddle&Lyth Cosmological Inflation... Ch 3&7
- astro-ph/ 9607066, 9704062, 0006321, 0212281, 0302225, 0304370

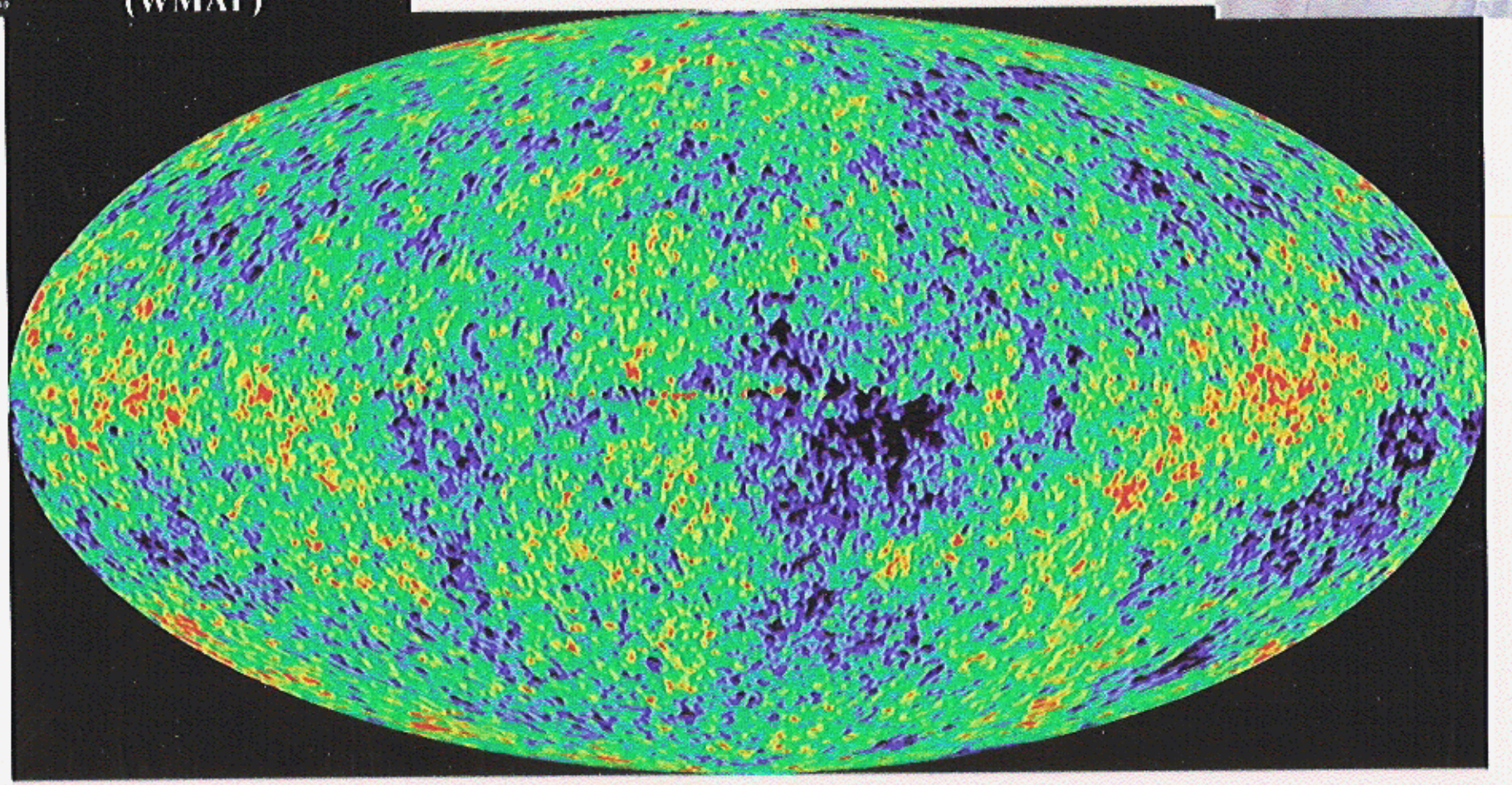
INFLATION SCORECARD

PREDICTIONS	WMAP NOW	GRADE	GOAL
FLAT UNIVERSE			
★ $\Omega_0 = 1.000$	1.02 ± 0.02 $\Omega_0 = 1.03 \pm 0.03$	++*	± 0.001
DENSITY PERTS FROM QM FLUC			
★ ADIABATIC	≥3 ACOUSTIC PEAKS	+	≥7
★ NEARLY SCALE-INVARIANT $(n-1) \sim \mathcal{O}(\pm 0.1)$	0.93 ± 0.03 $n = 1.05 \pm 0.09$	+	± 0.001
★ NEARLY POWER-LAW $dn/d\ln k \sim \pm 10^{-3}$	-0.03 ± 0.02 $dn/d\ln k = -0.02 \pm 0.04$	✓	$\pm 10^{-3}$
★ GAUSSIAN	NO EVIDENCE AGAINST	✓	
→ **CDM**	"HAS MUCH OF THE TRUTH"	++	
GRAV WAVES FROM QM METRIC FLUC			
★ $T/S \gtrsim 10^{-3}$ (??)	$T/S \lesssim \mathcal{O}(1)$		$10^{-3}/10^{-4}$
★ NEARLY SCALE INVARIANT	$n_T = -\frac{1}{5} T/S$	0.71 (95% cl)	± 0.03

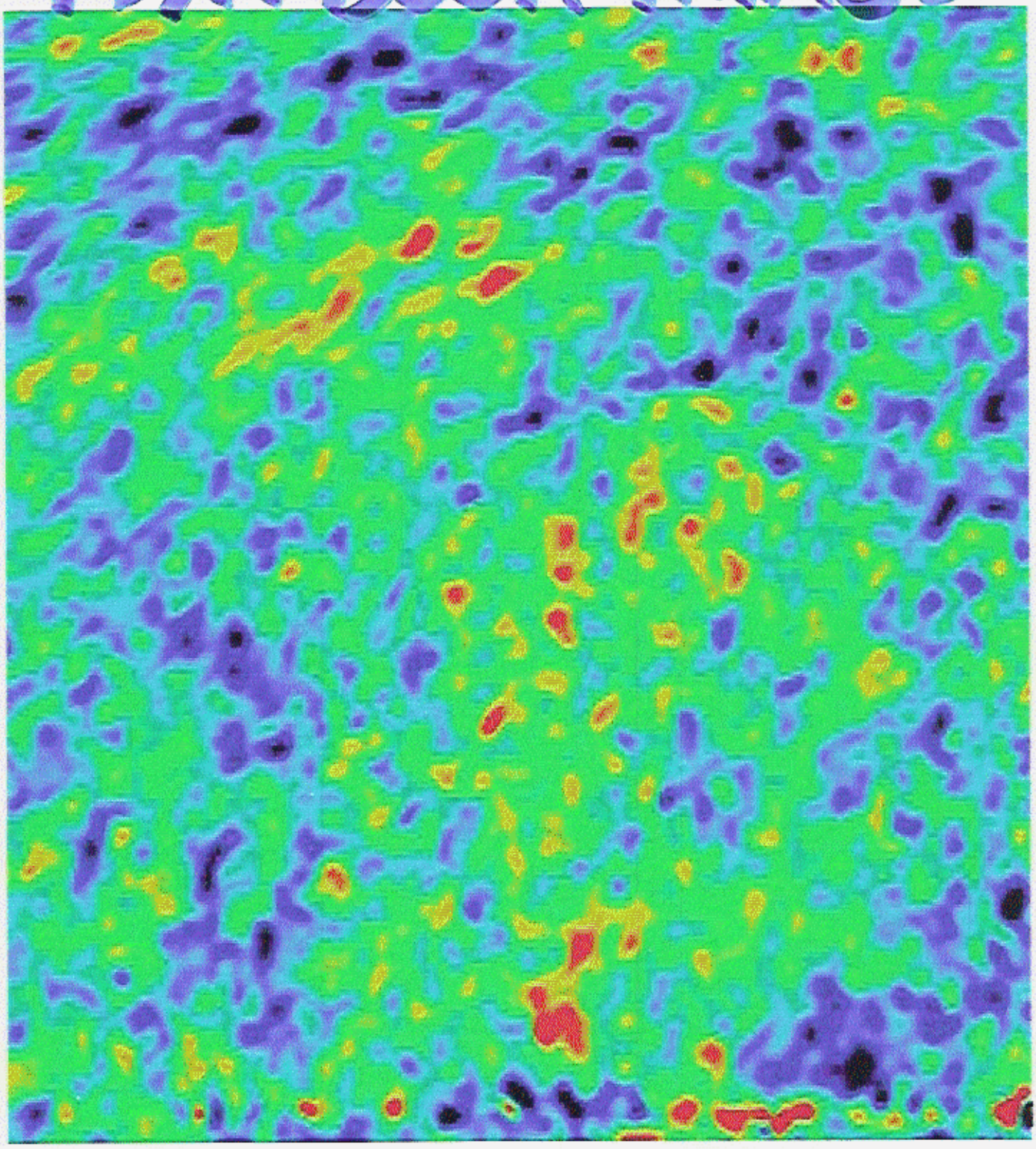
* FOR DOING IT THE HARD WAY

MAP OF THE UNIVERSE AT A SIMPLER TIME (400,000 YRS)





TEXT BOOK IMAGE



"QUANTUM FUZZ"

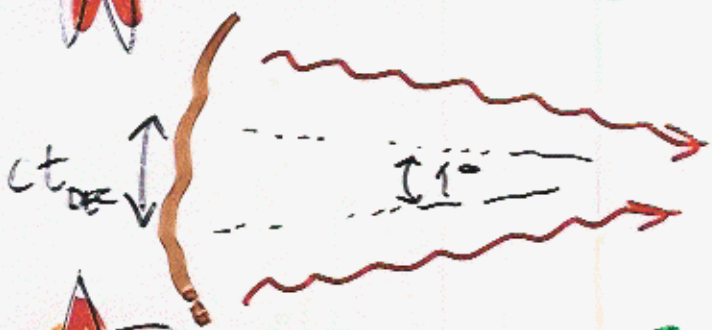

STATUS OF INFLATION:
EXCELLENT!

INFLATION

MOTIVATION:

STD COSMOLOGY IS VERY SUCCESSFUL, BUT...
HANDFUL OF SHORTCOMINGS:

- "HORIZON PROBLEM": WHY SO SMOOTH



- ORIGIN OF INHOMOGENEITY
  WHERE 'SEEDS' FOR STRUCTURE COME FROM

- FLATNESS: WHY IS $\Omega_0 \sim \mathcal{O}(1)$

$\Omega \sim 1$ is unstable

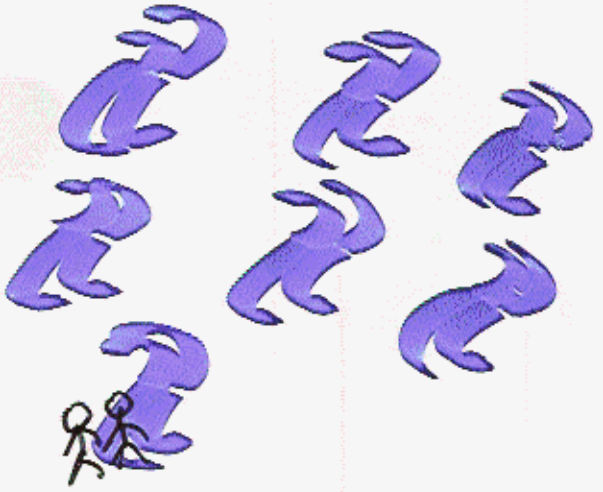
- MONOPOLE PROBLEM: OVERPRODUCTION
  (HORIZON PROBLEM AGAIN)

SPACE-TIME GEOMETRY

our initial geometry

smooth, small ripples

15 Gyr



generic initial geometry

10^{-43} sec

Black holes, anisotropy, ...

"A MESS"

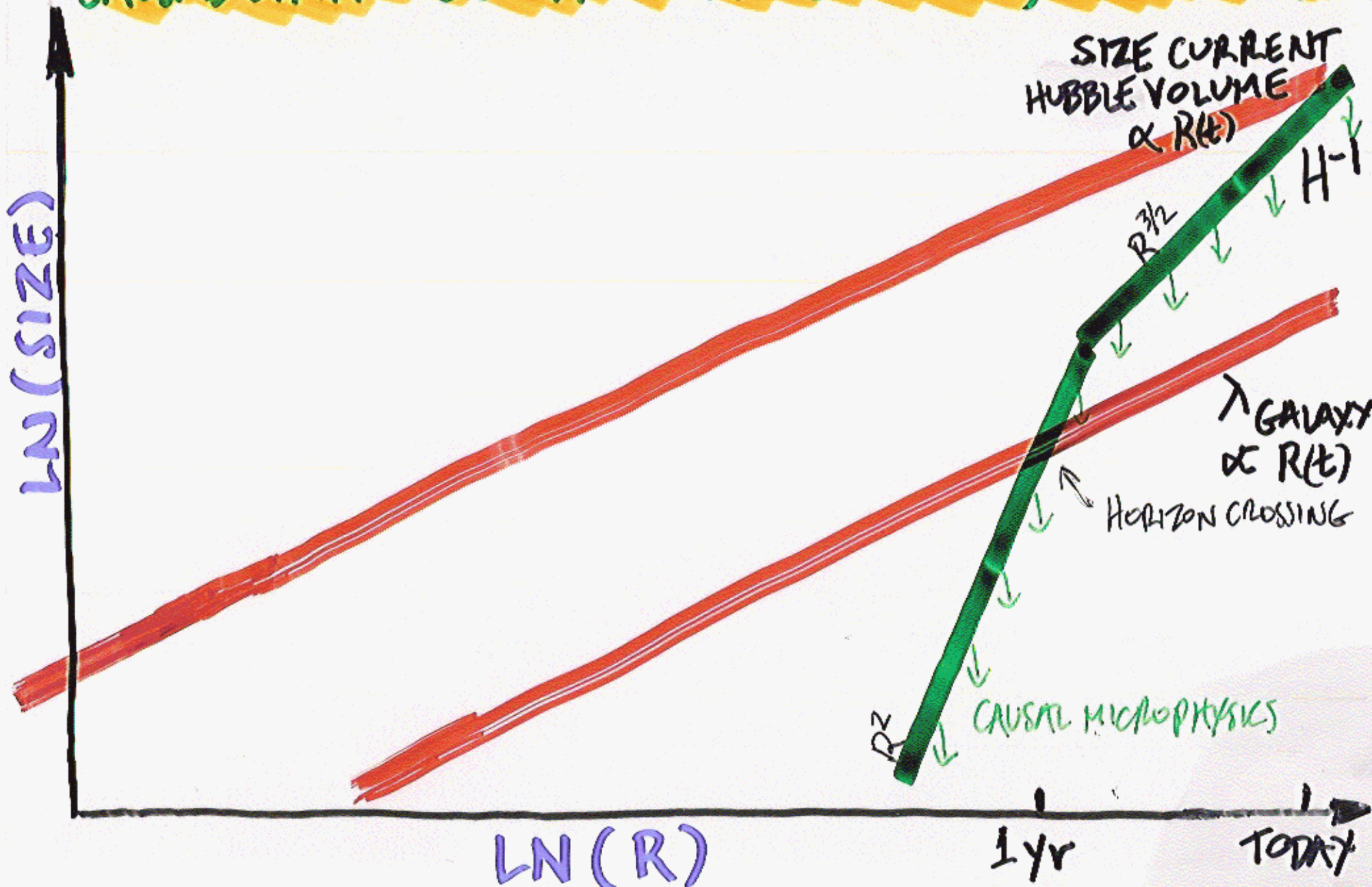
Collins & Hawking '73

NOT LOGICAL INCONSISTENCY!

DILEMMA OF INITIAL DATA (WHY SO SPECIAL?)

THE HORIZON PROBLEMS

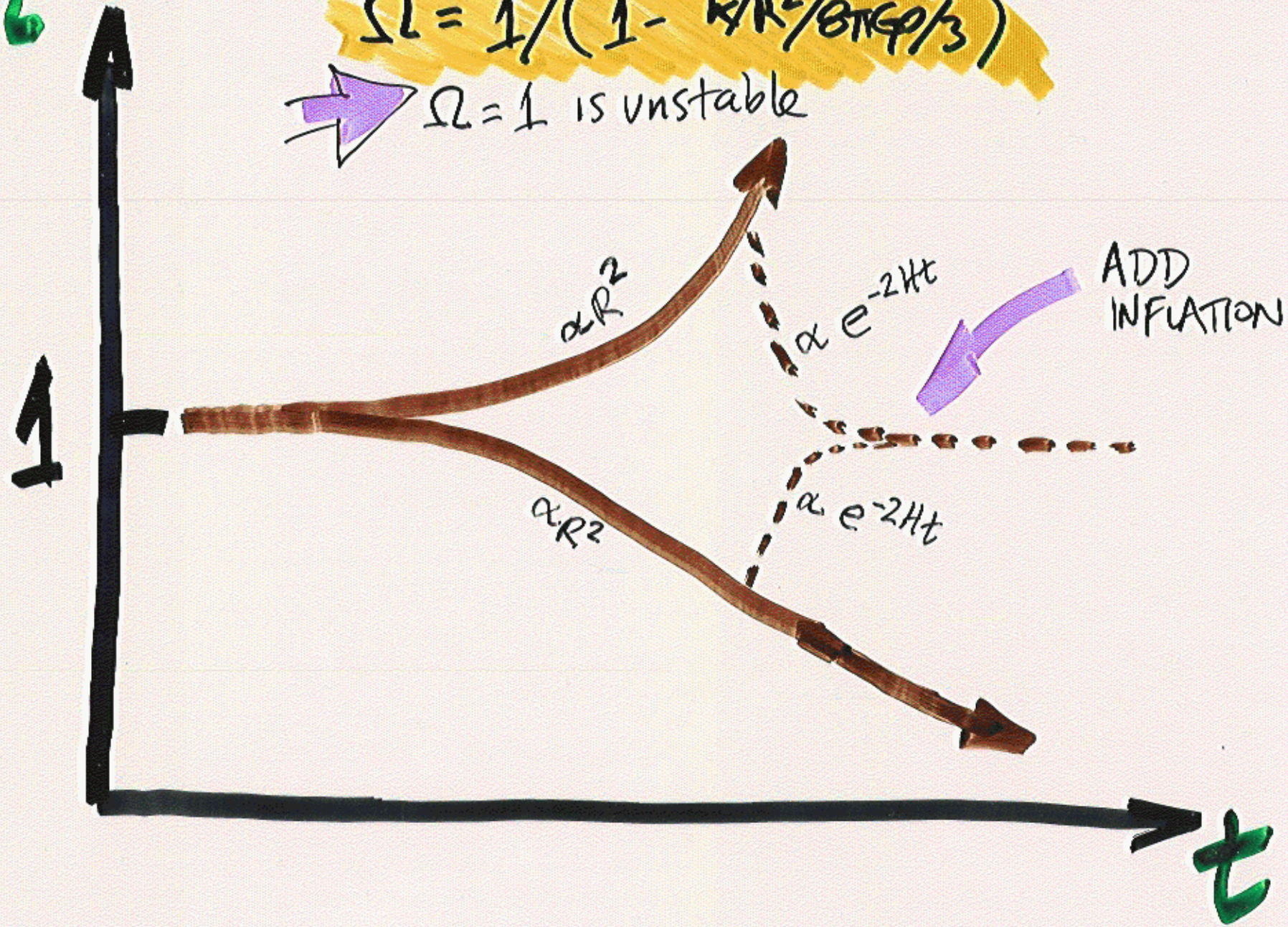
"CAUSAL LIMIT": $ct \sim H^{-1} \sim R^n$ (RD: $n=2$; MD: $n=3/2$)



FLATNESS PROBLEM

$\Omega = 1/(1 - k/R^2/8\pi G\rho/3)$

$\Omega = 1$ is unstable



Promise of INFLATION:

- TO LESSEN DEPENDENCE OF PRESENT STATE UPON INITIAL STATE
- "GRANDER" BIG BANG MODEL -- FROM 10^{-2} sec → 10^{-32} sec

Underlying Physics

- SPECULATIVE --- BUT WELL DEFINED (Classical Field Theory motivated by grand unification, superstrings, supersymmetry)

SOLVING HORIZON PROBLEM

ACCELERATED EXPANSION

AKA SUPERLUMINAL EXPANSION

$$R \propto t^n \qquad n > 1$$

$$\Rightarrow \ddot{R} \propto n(n-1)t^{n-2} > 0$$

$$\Rightarrow H^{-1} \sim t \sim R^{1/n} \sim R^{<1}$$

e.g. $H = \text{const}$, $R \sim e^{Ht}$

ENTROPY PRODUCTION

$$S_{HOR} \sim (H^{-1})^3 T^3$$ ENTROPY IN HORIZON-VOL

$$S_{Ho} \sim 10^{88}$$ ENTROPY IN OBS. UNIVERSE

$$N_{HOR} \sim (m_{pl}/T)^3 \sim \frac{10^{57}}{(T/1\,GeV)^3}$$

" RESTATEMENT OF HORIZON PROBLEM

NEED TO INCREASE BY LARGE FACTOR !

Resolving the Puzzles

Horizon Problem:

"50 e-folds of inflation suffices"

$$N \equiv \ln(R_f/R_i) = \int H dt > 50$$

Flatness Problem:

same >50 e-folds will keep $|\Omega_0 - 1|$ small

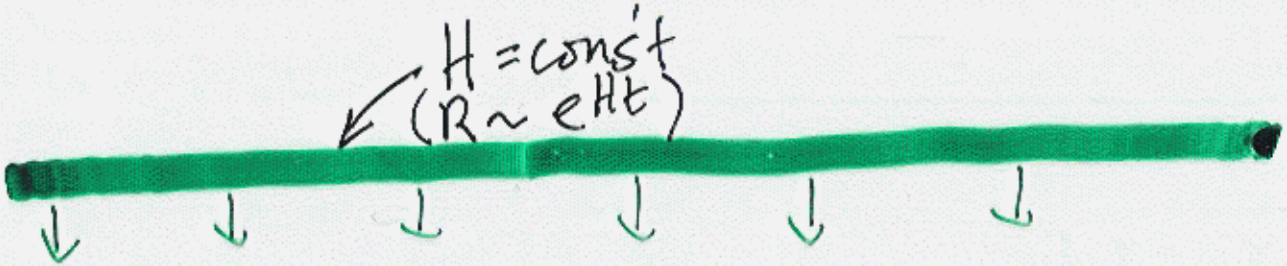
Entropy Increase:

$S \sim R^3 T^3$

$T_f \sim T_i$

$R_f > e^{50} R_i$

$$S_f > e^{150} S_i \approx 10^{65} S_i$$

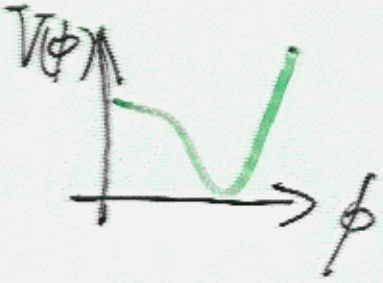

H = const
(R ~ eHt)

SCALAR FIELD DYNAMICS

ALL MODELS OF INFLATION CAN BE DESCRIBED BY CLASSICAL EVOLUTION OF A SCALAR FIELD

$$\mathcal{L} = R^3\left(\tfrac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi)\right)$$

$$\rightarrow R^3\left(\tfrac{1}{2}\dot{\phi}^2 - V\right)$$



EOM: $\frac{d}{dt}\frac{\delta}{\delta\dot{\phi}}\mathcal{L} - \frac{\delta}{\partial\phi}\mathcal{L} = 0$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

↑ Hubble friction

BALL ROLLING DOWN HILL W/ FRICTION

Friedmann Equation:

$$H^2 \equiv (\dot{R}/R)^2 = \frac{8\pi G}{3}\rho$$

$$\rho_\phi = \tfrac{1}{2}\dot{\phi}^2 + V(\phi)$$

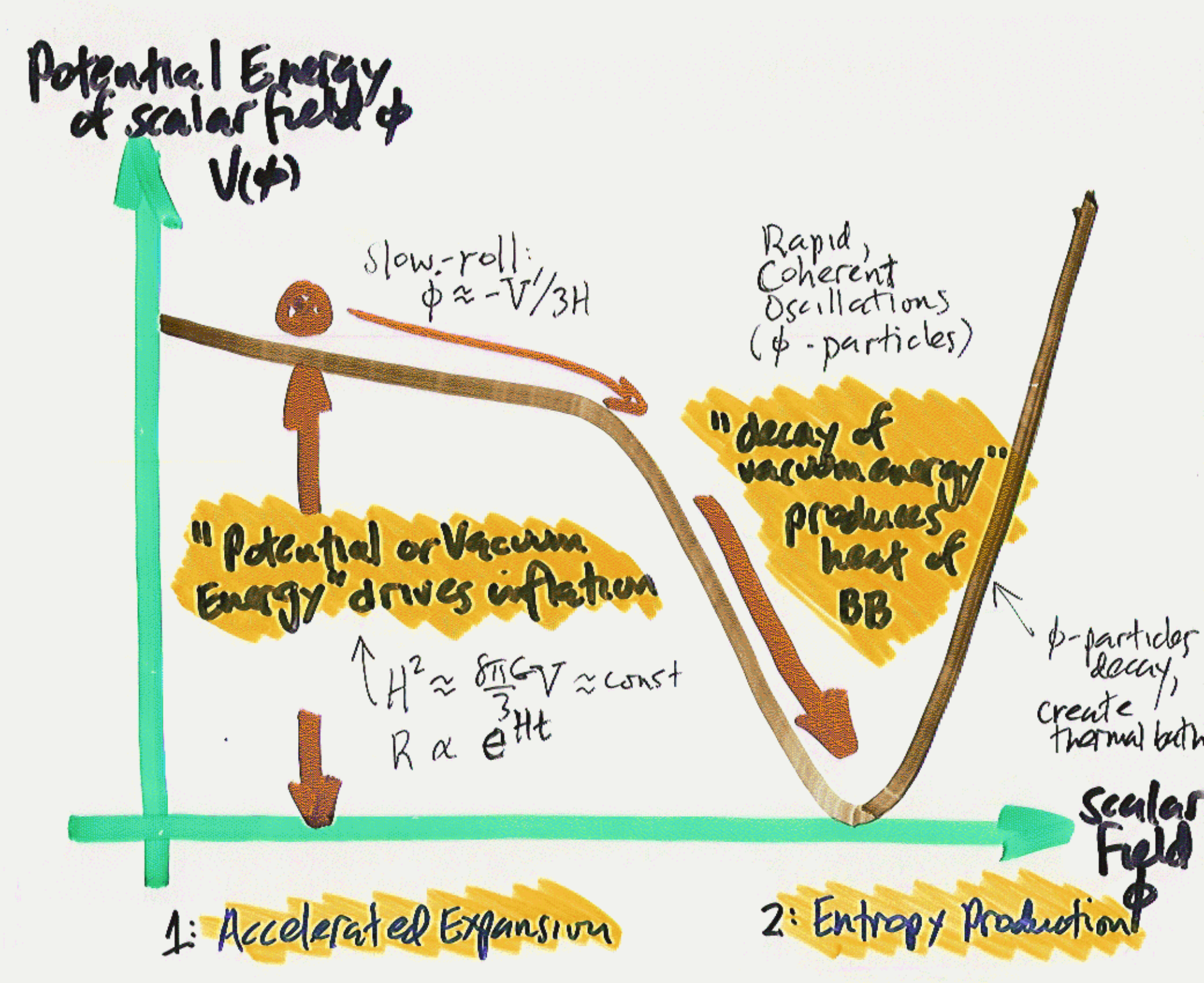
$$H^2 = \frac{8\pi G}{3}\left(\tfrac{1}{2}\dot{\phi}^2 + V\right)$$

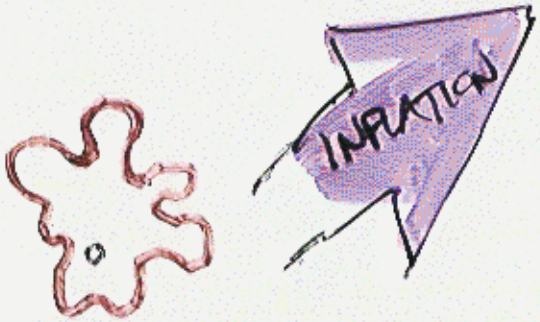
$$\approx \frac{8\pi G}{3}V \approx \text{const}$$

DURING "SLOW ROLL"

$$\Rightarrow R \sim e^{Ht}$$

EXPONENTIAL GROWTH


Potential Energy of scalar field ϕ
V(ϕ)
Slow-roll:
$\dot{\phi} \approx -V'/3H$
Rapid, Coherent Oscillations (ϕ-particles)
"decay of vacuum energy" produces heat of BB
"Potential or Vacuum Energy" drives inflation
$H^2 \approx \frac{8\pi}{3} G V \approx \text{const}$
$R \propto e^{Ht}$
ϕ-particles decay, create thermal bath
Scalar Field ϕ
1: Accelerated Expansion
2: Entropy Production



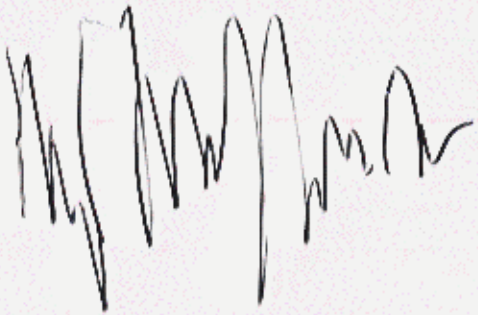
TINY (<<< 1cm) BIT OF UNIVERSE IS FLAT & SMOOTH (but too small to contain all we see today)

ALL THAT WE CAN SEE TODAY

(STILL SMOOTH & FLAT)

FLUCTUATIONS: MICRO TO MACRO

Bardeen-Steinhardt-MST; Guth-Pi; Hawking; Starobinskii 1982



$\ll 10^{-8}$ cm

ENORMOUS STRETCHING DUE TO INFLATION

$\gg$ LIGHT YRS

QUANTUM FLUCTUATIONS ON SUBATOMIC SCALE

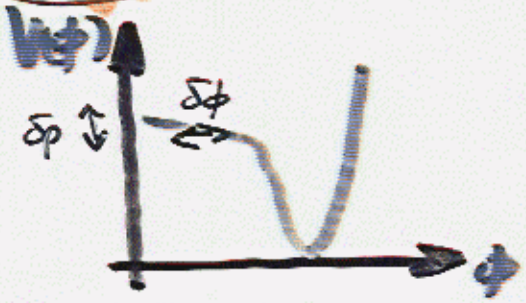
"LUMPY" DISTRIBUTION OF MATTER ON MACRO SCALE

QM Fluctuations in the "Inflaton" ϕ

$$\Delta\phi \simeq H/2\pi$$

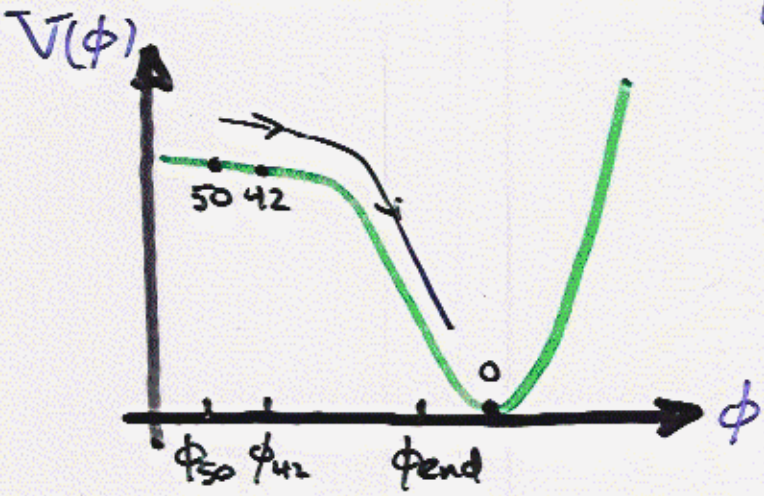
Energy Density Perturbations

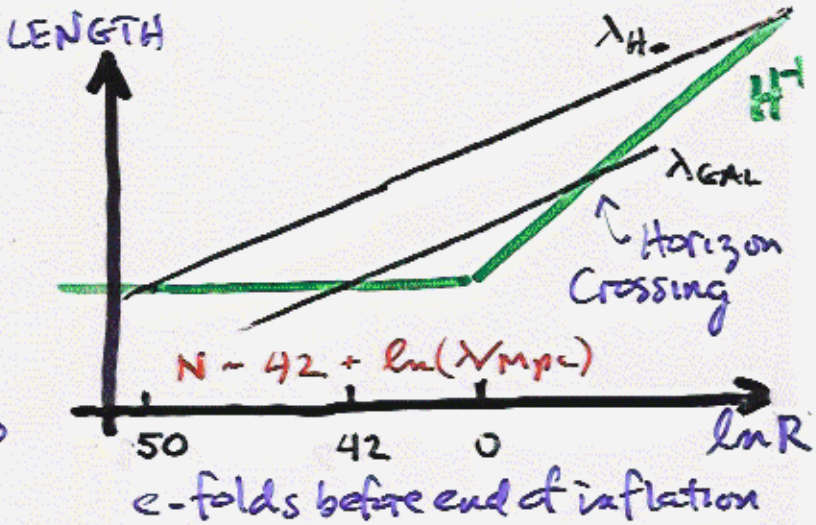
$$\delta\rho_\phi \simeq V'\Delta\phi$$



Density perturbations after reheating

QUANTUM FLUCTUATIONS





"INTERESTING SCALES" CROSS OUTSIDE "HORIZON" DURING A SMALL PORTION OF INFLATION

SLOW ROLL: $\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$

$$\dot{\phi} \approx -V'/3H \qquad N = \int H dt = \frac{8\pi}{m_{pl}^2} \int_{\phi}^{\phi_{end}} \frac{V(\phi)\, d\phi}{V'}$$

DENSITY PERT (SCALAR) $\delta\rho \sim \Delta\phi V' \sim HV'$

OUTSIDE HORIZON ($\lambda > H^{-1}$) $\quad \delta\rho/(\rho+p) \approx$ const

$\rho = +\frac{1}{2}\dot{\phi}^2 + V$

$p = +\frac{1}{2}\dot{\phi}^2 - V$

$$\left(\frac{\delta\rho}{\rho}\right)_{HOR} \approx \left(\frac{HV'}{\dot{\phi}^2}\right)_N \sim \frac{V_N^{3/2}}{m_{pl}^3 V_N'}$$

GRAV. WAVES (TENSOR) $h_{GW} \sim H/m_{pl}$

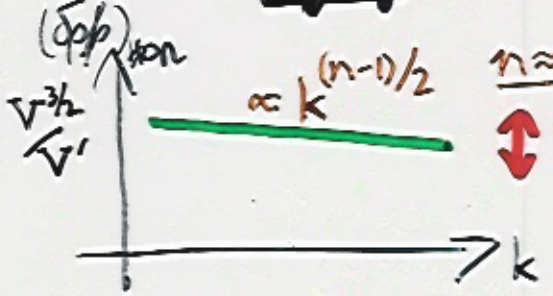
OUTSIDE HORIZON ($\lambda > H^{-1}$) $\quad h_{GW} \approx$ const

$$(h_{GW})_{HOR} \approx \left(\frac{H}{m_{pl}}\right)_N \sim V_N^{1/2}/m_{pl}$$

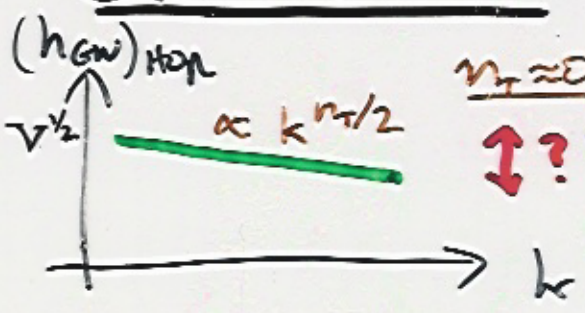
INFLATIONARY QUANTUM FLUC.

DEFINITE POTENTIAL MAKES DEFINITIVE PREDICTION

$\delta\rho/\rho$



Grav Wave



NEARLY SCALE-INVARIANT, MODEL-DEPENDENT AMPLITUDE

AMPLITUDES (measure by contribution to variance of CBR quadrupole)

$$S \equiv \frac{5\langle|a^S_{2m}|^2\rangle}{4\pi} \qquad T \equiv \frac{5\langle|a^T_{2m}|^2\rangle}{4\pi}$$

$$S \approx \frac{2\cdot 2\, V/m_{pl}^4}{(m_{pl}V'/V)^2} + \cdots \qquad T \approx 0.61\, V/m_{pl}^4 + \cdots$$

would like $\approx$ few $\times 10^{-11}$ to make structure

??? would like to measure!

SPECTRAL INDICES (scale invariant + small correction)

$$n = \underbrace{1}_{\text{S-I}} \underbrace{-\frac{1}{8\pi}\left(\frac{m_{pl}V'}{V}\right)^2 + \frac{m_{pl}}{4\pi}\left(\frac{m_{pl}V'}{V}\right)' + \cdots}_{\text{small correction} \sim (0.03\text{-}0.2)} \qquad n_T = \underbrace{0}_{\text{S-I}} \underbrace{-\frac{1}{8\pi}\left(\frac{m_{pl}V'}{V}\right)^2 + \cdots}_{\text{small correction}}$$

$$\frac{dn}{d\ln k} = -\frac{1}{32\pi^2}\left(\frac{m_{pl}^2V''}{V}\right)\left(\frac{m_{pl}V'}{V}\right) + \frac{1}{8\pi}\left(\frac{m_{pl}^2V''}{V}\right)\left(\frac{m_{pl}V'}{V}\right)^2 - \frac{3}{32\pi^2}\left(\frac{m_{pl}V'}{V}\right)^4$$

$\delta(10^{-3})$

NB: $n_T = -\frac{1}{5}\frac{T}{S}$ $\quad n_T < 0 \quad n_T \neq n-1$

DEVIATIONS FROM S-I: OPPORTUNITY NOT CURSE