

THE BEGINNING OF INFLATION

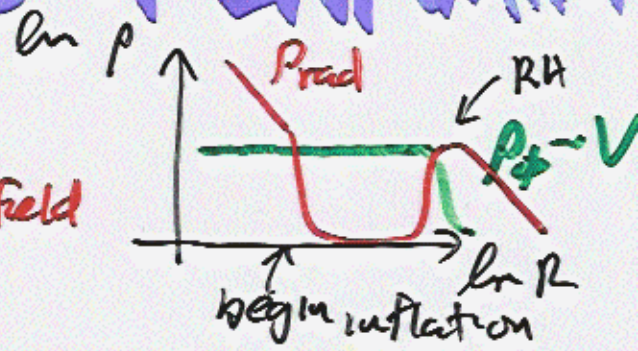
HOT BEGINNING

1: $\rho = \underbrace{aT^4}_{\text{rad}^2} + \underbrace{\rho_\phi}_{\text{scalar field}}$

2: when $aT^4 \approx V(\phi_i)$ inflation starts
thereafter, $T \sim e^{-Ht}$

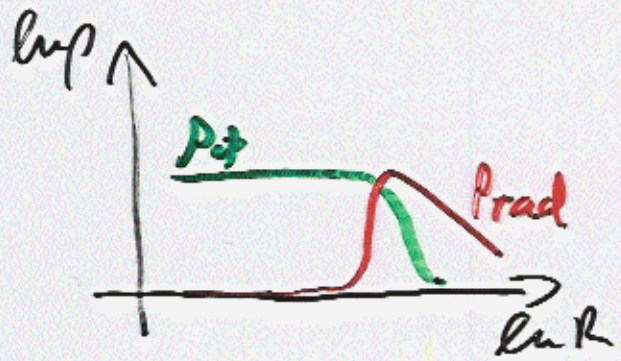
NB: $T_i \approx V(\phi_i)^{1/4}$

3: RHing: $T_f \approx V(\phi_i)^{1/4}$ (perfect)
actual: $T_{RH} = (\Gamma_\phi m_{pl})^{1/2}$




COLD BEGINNING

$\rho \approx \rho_\phi$ (no thermal bath)



SAME
OUTCOME

WORKED EXAMPLE



$$V(\phi) = \frac{1}{2} m^2 \phi^2 \quad V' = m^2 \phi$$

$$d \ln R = H dt \quad H^2 \approx \frac{8\pi}{3 m_{pl}^2} V(\phi) \quad \dot{\phi} = \frac{-V'}{3H}$$

$H \equiv \dot{R}/R$
Friedmann Eqⁿ
Slow Roll

$$\Rightarrow N \equiv \ln R_f / R = \frac{8\pi}{m_{pl}^2} \int_0^{\phi} \frac{V d\phi}{V'}$$

$$= 2\pi (\phi / m_{pl})^2$$

$$(1) N_{tot} > 50 \quad \Rightarrow \quad \phi_i > \left(\frac{50}{2\pi} \right)^{1/2} m_{pl} \approx 3 m_{pl}$$

$$(2) \left(\frac{\delta\rho}{\rho} \right)_{hor} \approx \left(\frac{V^{3/2}}{m_{pl}^2 V'} \right)_{N \sim 50} \approx 10^{-5}$$

$$\approx \left(\frac{m}{m_{pl}} \right) \left(\frac{\phi}{m_{pl}} \right)^2 = \left(\frac{m}{m_{pl}} \right) \left(\frac{N}{2\pi} \right)$$

$$\Rightarrow \quad m / m_{pl} \approx \frac{2\pi}{N} \cdot 10^{-5} \approx 10^{-6}$$

$$m \approx 2 \times 10^{13} \text{ GeV}$$

$$h_{GW} \approx H / m_{pl} \approx \frac{V^{1/2}}{m_{pl}^2} \approx \frac{m\phi}{m_{pl}^2} \approx 10^{-6}$$

(3) Initial Cond_x

Suppose $V(\phi_i) \approx m_{pl}^4$
"chaotic inflation"

$$\Rightarrow \frac{m^2 \phi_i^2}{m_{pl}^4} \sim 1 \quad \text{or} \quad \phi_i \approx \left(\frac{m_{pl}}{m}\right) m_{pl} \approx 10^6 m_{pl}$$

$$\Rightarrow N_{tot} \approx \frac{(\phi_i/m_{pl})^2}{2\pi} \approx 10^{11} \quad \text{WOW!}$$

EXAMPLE 2: $V = \lambda \phi^4 \dots \lambda = 10^{-14}$

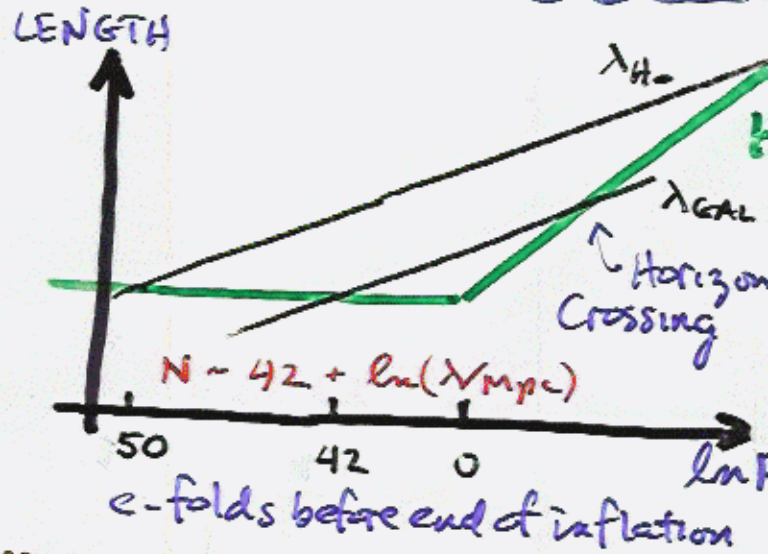
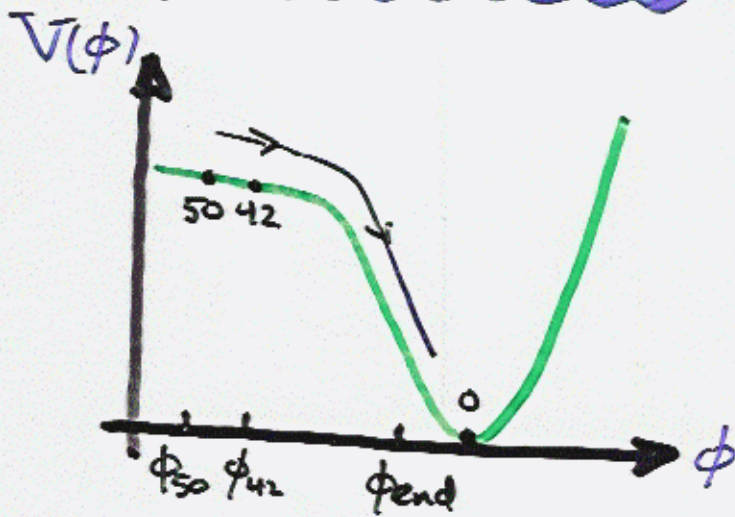
IN GENERAL:

SMALL PARAMETER $\lambda, m/m_{pl}$

SQUAT POTENTIAL



QUANTUM FLUCTUATIONS



"INTERESTING SCALES" CROSS OUTSIDE "HORIZON" DURING A SMALL PORTION OF INFLATION

SLOW ROLL:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$\dot{\phi} \approx -\frac{V'}{3H}$$

$$N = \int H dt = \frac{3\pi}{m_{pl}^2} \int_{\phi}^{\phi_{end}} \frac{V(\phi) d\phi}{V'}$$

DENSITY PERTURBATIONS (SCALAR)

$$\delta\rho \sim \Delta\phi V' \sim HV'$$

OUTSIDE HORIZON
($\lambda > H^{-1}$)

$$\delta\rho/(\rho+p) \approx \text{const}$$

$$\rho = +\frac{1}{2}\dot{\phi}^2 + \bar{V}$$

$$p = +\frac{1}{2}\dot{\phi}^2 - \bar{V}$$



$$\left(\frac{\delta\rho}{\rho}\right)_{\text{HOR}} \approx \left(\frac{HV'}{\dot{\phi}^2}\right)_N \sim \frac{V_N^{3/2}}{m_{pl}^3 V'_N}$$

GRAV. WAVES (TENSOR)

$$h_{\text{GW}} \sim H/m_{pl}$$

OUTSIDE HORIZON
($\lambda > H^{-1}$)

$$h_{\text{GW}} \approx \text{const}$$



$$(h_{\text{GW}})_{\text{HOR}} \approx \left(\frac{H}{m_{pl}}\right)_N \sim \frac{V_N^{1/2}}{m_{pl}}$$

MODELS

I.E., $V(\phi)$

GOOD NEWS: No lack of well motivated, viable models

CHAOTIC, NATURAL, NEW, SUSY, SUGRA, ELECTROWEAK, GUTS, NO-SCALE, EXTENDED, HYBRID, ...

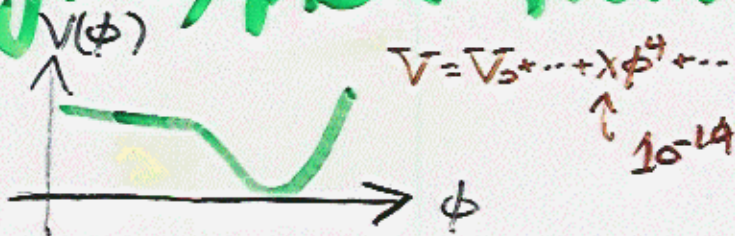
BAD NEWS: See above

I.E. NO "STANDARD" MODEL

(should be viewed in the light of our knowledge of physics at $E \gg 10^3 \text{ GeV}$)

Technical point: ^{TOGETHER $\phi \sim 10^5$} all have ^{VERY!} small number that begs explanation

"POTENTIALS" ARE VERY FLAT



MENU

Circa Aspen '95

★ SIMPLE ^{TOY} MODELS

SIMPLE POLYNOMIALS Steinhardt-MSS
LINDE'S CHAOTIC INFLATION $V = \lambda \phi^4$
POWER-LAW INFLATION $V = V_0 e^{-\beta \phi}$

★ COMPLICATED MODELS TIED TO PARTICLE PHYSICS

GUT Shafi-Vilenkin-Pi
SUSY GUT Holman Ramond Ross; Olive Phys Rep
Preons Pati et al
Electroweak Knox-MSS

★ "NATURAL" MODELS

Natural Inflation (Nambu-Goldstone boson)
Friedman-Olinto-Freeze

$$\lambda \approx \left(\frac{m}{M}\right)^n$$

Superstring inspired, modular, based
Banks et al
 $V(\phi) = \Lambda^4 f(\phi/M)$

Lo-energy SUSY, two-field Randall et al
Stewart et al

★ Non-STD GRAVITY

Extended (first-order) Inflation

Induced gravity Accetta-MSS-Zoller

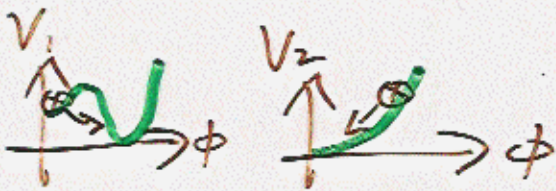
R^2 inflation Starobinski

Extra dimensions Shafi-Wetterich

Lu-Steinhardt
Kolb, Phys. Scripta



$\Omega < 1$
but not $\ll 1$



First-order (bubble), followed by "tuned" slow roll

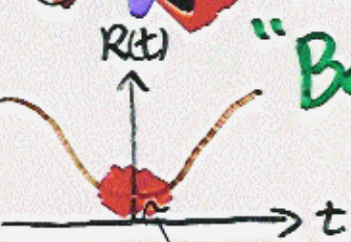
Bucher et al
Linde et al



"Pre big-bang" (aka kinetic) Inflation

"Bounce" (Universe is big & flat because it began big & flat)

Levin
Brustein et al
Veneziano et al



STRINGY
MAGIC



Exotica

Double(...) Inflation

Silk-MST
Starobinski
Kofman-Linde

TYPICALLY:

$V \approx (10^{14} - 10^{16} \text{ GeV})^4$
 $\Omega - 1 \ll 1$

$n \approx 0.95$, not quite scale invariant

EXCEPTIONS!

- $V \ll (10^{14} \text{ GeV})^4$ & exactly scale-invariant
- $s_{ph} \approx 0$ + "blue spectrum of GWs"
Pre big-bang
- $\Omega < 1$ Bucher et al; Linde

Knox-MST
Randall et al; Stewart et al

FOCUS ON

"ROBUST PREDICTIONS"

★ FLAT UNIVERSE

$$\Omega_0 \equiv \rho_{\text{TOTAL}} / \rho_{\text{CRITICAL}} = 1.0$$

$$\rho_{\text{TOT}} = \rho_B + \rho_{\text{CDM}} + \rho_{\text{VAC}} + \rho_{\text{UV}} + \rho_{\text{RAD}} + \dots$$

★ ALMOST SCALE-INVARIANT
SPECTRUM OF DENSITY PERTURBATIONS
(SCALAR)

★ ALMOST "GAUSSIAN"
SCALE-INVARIANT SPECTRUM OF GRAVITY WAVES
(TENSOR)

(+ sharply peaked spectrum of gravity waves)
in first-order inflation

ALL MODELS OF INFLATION BASED UPON SLOWLY ROLLING FIELD ϕ & $V(\phi)$

LANDAU-GINZBURG DESCRIPTION

THE OBSERVABLES: All given in terms of V

AMPLITUDE² OF DENSITY PERT

CONTRIBUTION² TO CMB QUADRUPOLE

$$S = \frac{2.9(V/m_{pl})}{(m_{pl}V'/V)^2} \approx 4 \times 10^{-11}$$

AMPLITUDE² OF GW₂

$$T = 0.56(V/m_{pl}^4) \approx ? < 10^{-10} \approx 10^{-13} ?$$

DEVIATION FROM SCALE INVARIANCE DENSITY PERT

$$n-1 = -\frac{1}{8\pi} \left(\frac{m_{pl}V'}{V}\right)^2 + \frac{1}{4\pi} \left(\frac{m_{pl}^2V''}{V}\right)' \approx \mathcal{O}(0.1)$$

DEVIATION FROM SCALE INVARIANCE GW

$$n_T = -\frac{1}{8\pi} \left(\frac{m_{pl}V'}{V}\right)^2 \approx \mathcal{O}(0.1)$$

"RUNNING" OF $n-1$

$$\frac{dn}{d\ln k} = -\frac{1}{32\pi^2} \left(\frac{m_{pl}^3V'''}{V}\right) \left(\frac{m_{pl}V'}{V}\right) + \frac{1}{8\pi^2} \left(\frac{m_{pl}^2V''}{V}\right) \left(\frac{m_{pl}V'}{V}\right)^2 - \frac{3}{32\pi^2} \left(\frac{m_{pl}V'}{V}\right)^3 \approx \mathcal{O}(10^{-3})$$

NB CONSISTENCY RELⁿ:

$$T/S = -5n_T$$

INFLATION SCORECARD

PREDICTIONS

WMAP

FLAT UNIVERSE

★ $\Omega_0 = 1.000$

NOW

1.02 ± 0.02

$\Omega_0 = 1.03 \pm 0.03$ ++*

GRADE

GOAL

± 0.001

* FOR DOING IT THE HARD WAY

DENSITY PERTS FROM QM FLUC

★ ADIABATIC

≥ 3 ACOUSTIC YEARS +

≥ 7

★ NEARLY SCALE-INVARIANT $(n-1) \sim \mathcal{O}(\pm 0.1)$

0.93 ± 0.03

$n = 1.05 \pm 0.09$ +

± 0.001

★ NEARLY POWER-LAW $dn/dlnk \sim \pm 10^{-3}$

-0.03 ± 0.02

$dn/dlnk = -0.02 \pm 0.04$ ✓

$\pm 10^{-3}$

★ GAUSSIAN

NO EVIDENCE AGAINST ✓



CDM

"HAS MUCH OF THE TRUTH" ++

GRAV WAVES FROM QM METRIC FLUC

★ $T/S \geq 10^{-3}$ (??)

$T/S \leq \mathcal{O}(1)$

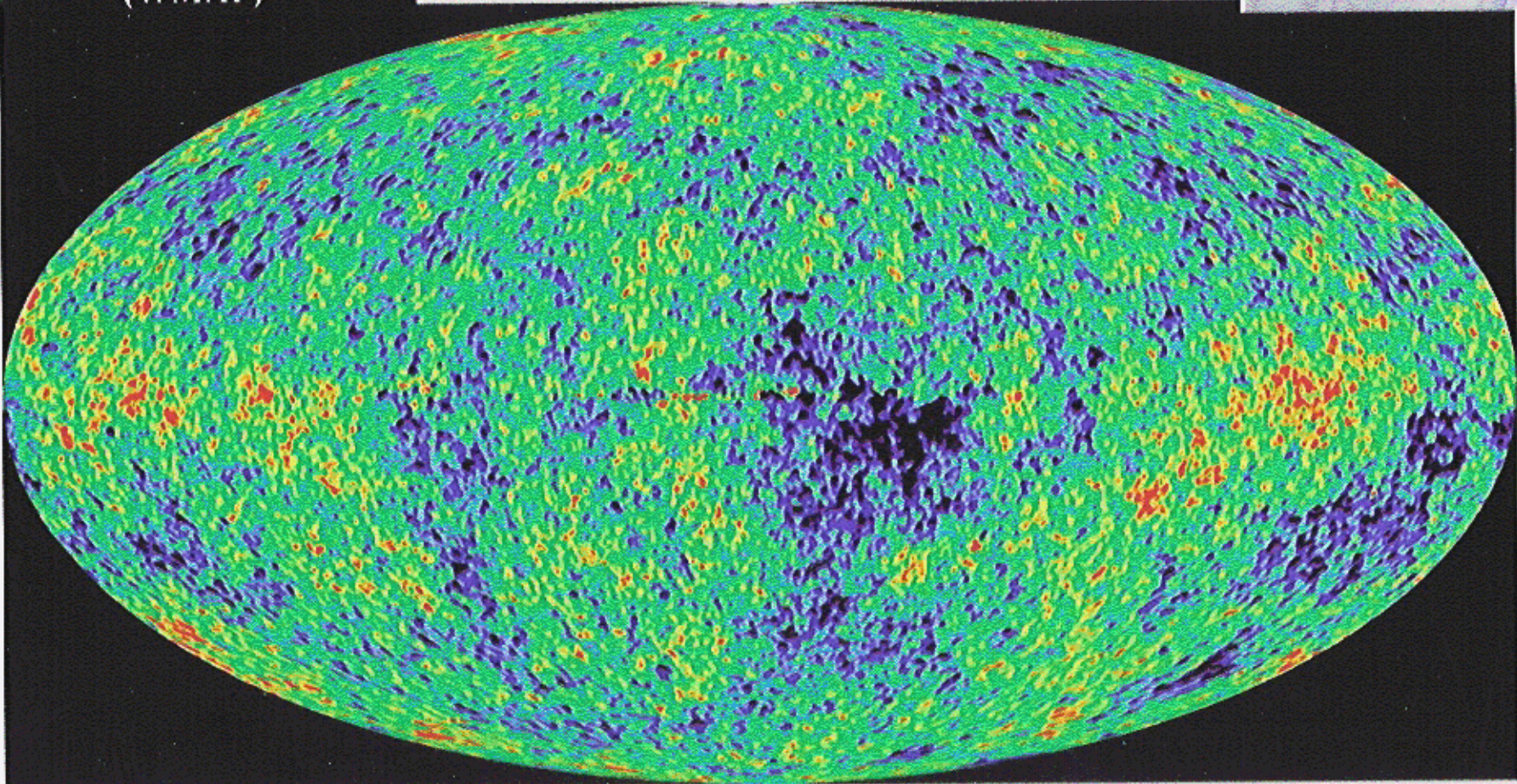
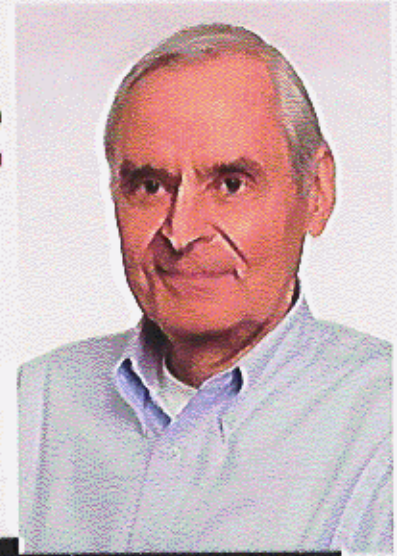
$10^{-3}/10^{-4}$

★ NEARLY SCALE INVARIANT $n_T = -\frac{1}{8} T/S$

0.71 (95% cl)

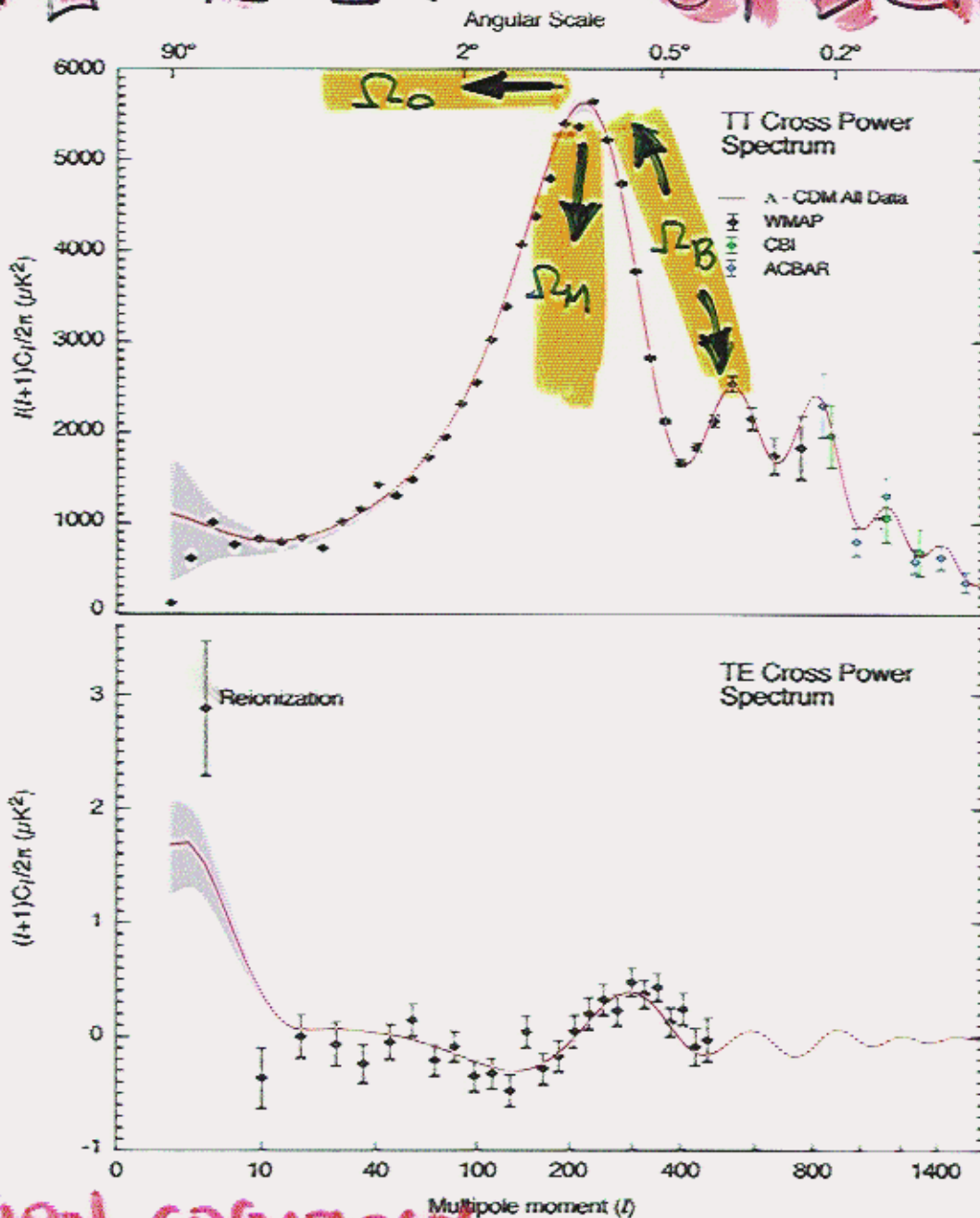
± 0.03

MAP OF THE UNIVERSE AT A SIMPLER TIME (400,000 YRS)



WMAP

POWER SPECTRUM



PRECISION COSMOLOGY

$$\Omega_0 = 1.02 \pm 0.02$$

$$t_0 = 13.7 \pm 0.2 \text{ Gyr}$$

$$* \Omega_M h^2 = 0.135 \pm 0.008$$

$$h = 0.71 \pm 0.035$$

$$* \Omega_B h^2 = 0.0224 \pm 0.001$$

$$\Omega_{DE} = 0.7 \pm 0.04$$

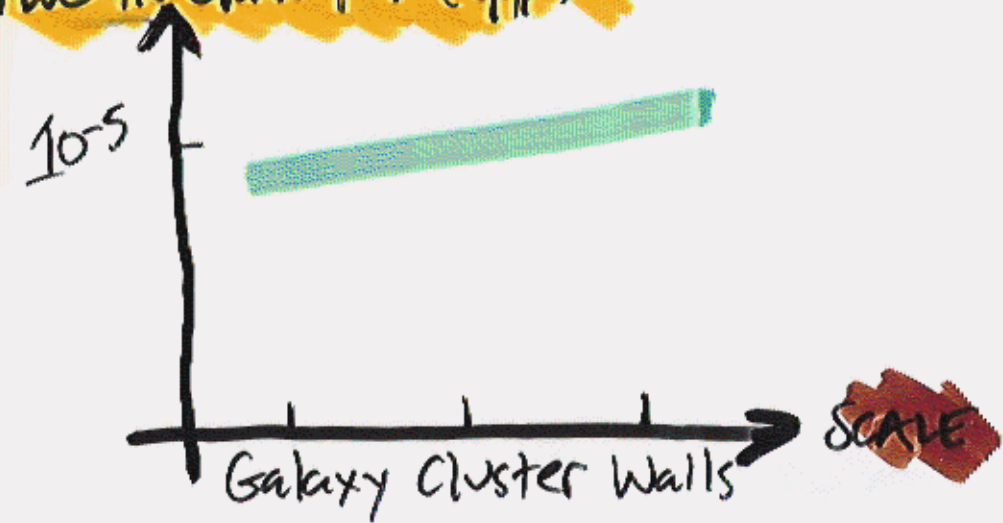
$$\Omega_M / \Omega_B = 6 \pm 0.4$$

$$* h^2 \approx \frac{1}{2}, h^{-2} \approx 2$$

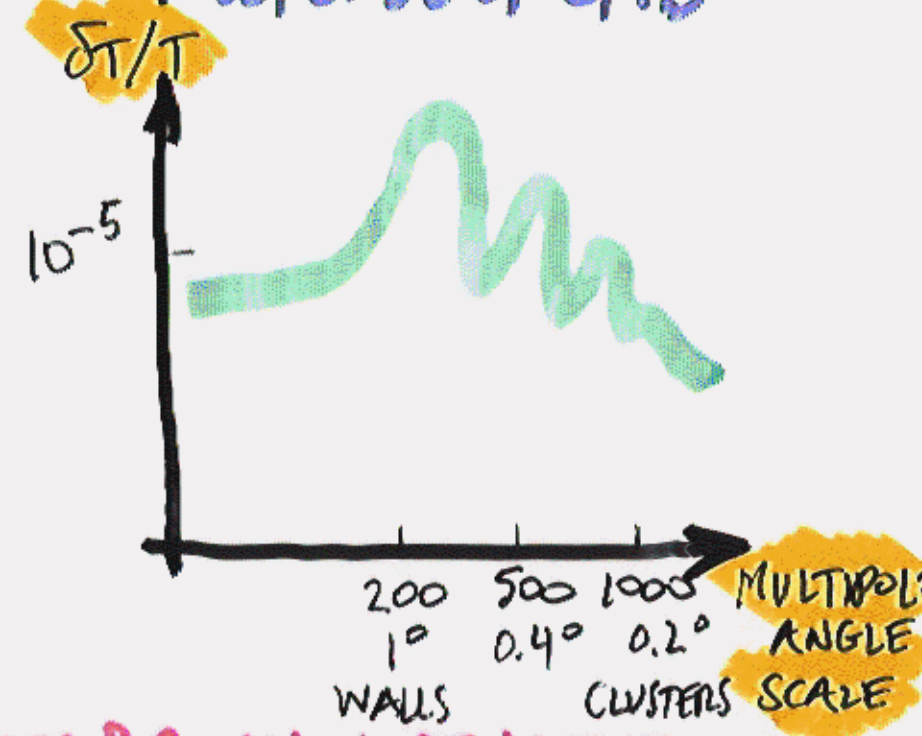
CMB IS A NON TRIVIAL PROBE OF MATTER DENSITY

UNDERLYING DENSITY PERTURBATIONS

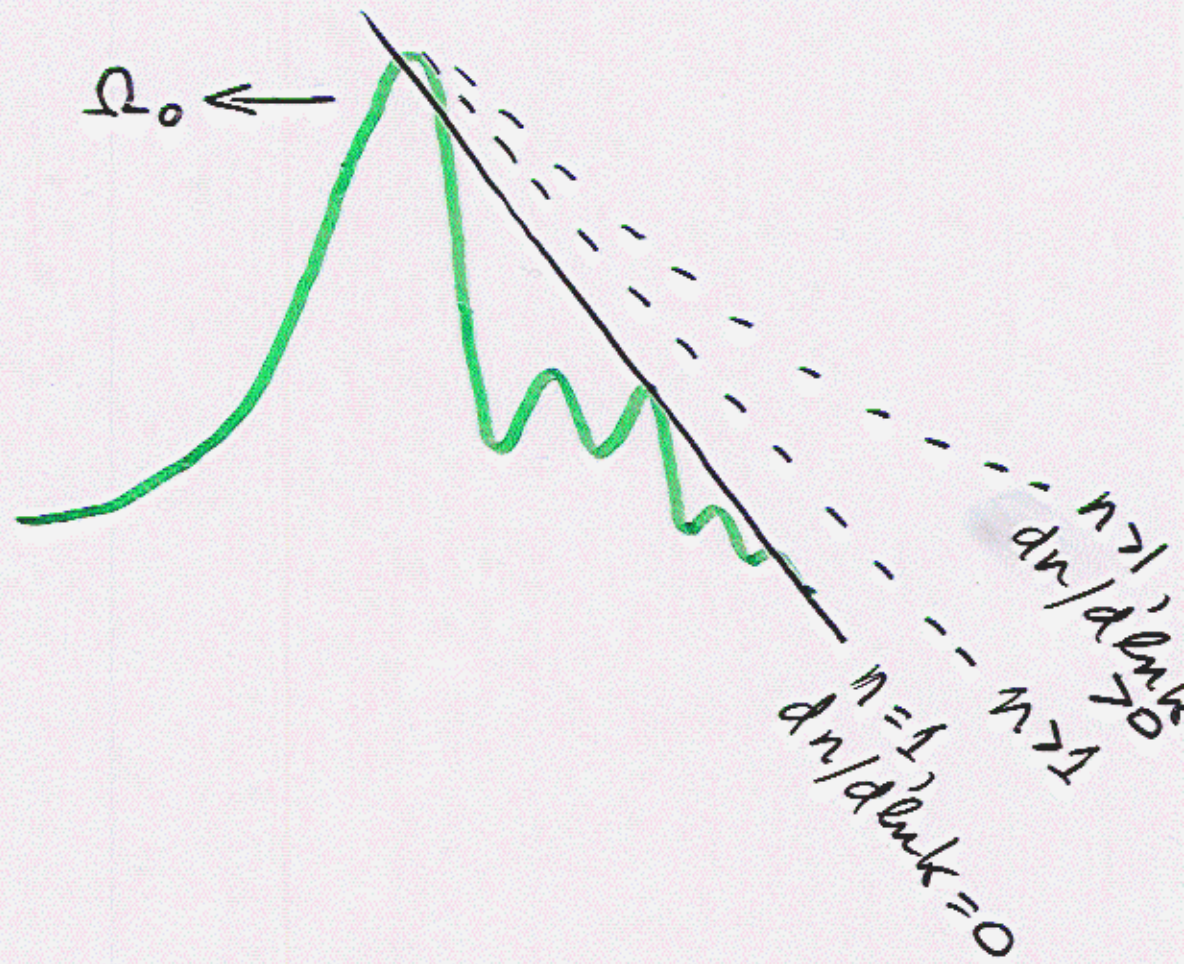
FLUC IN GRAV POT ($\delta\phi/a$)

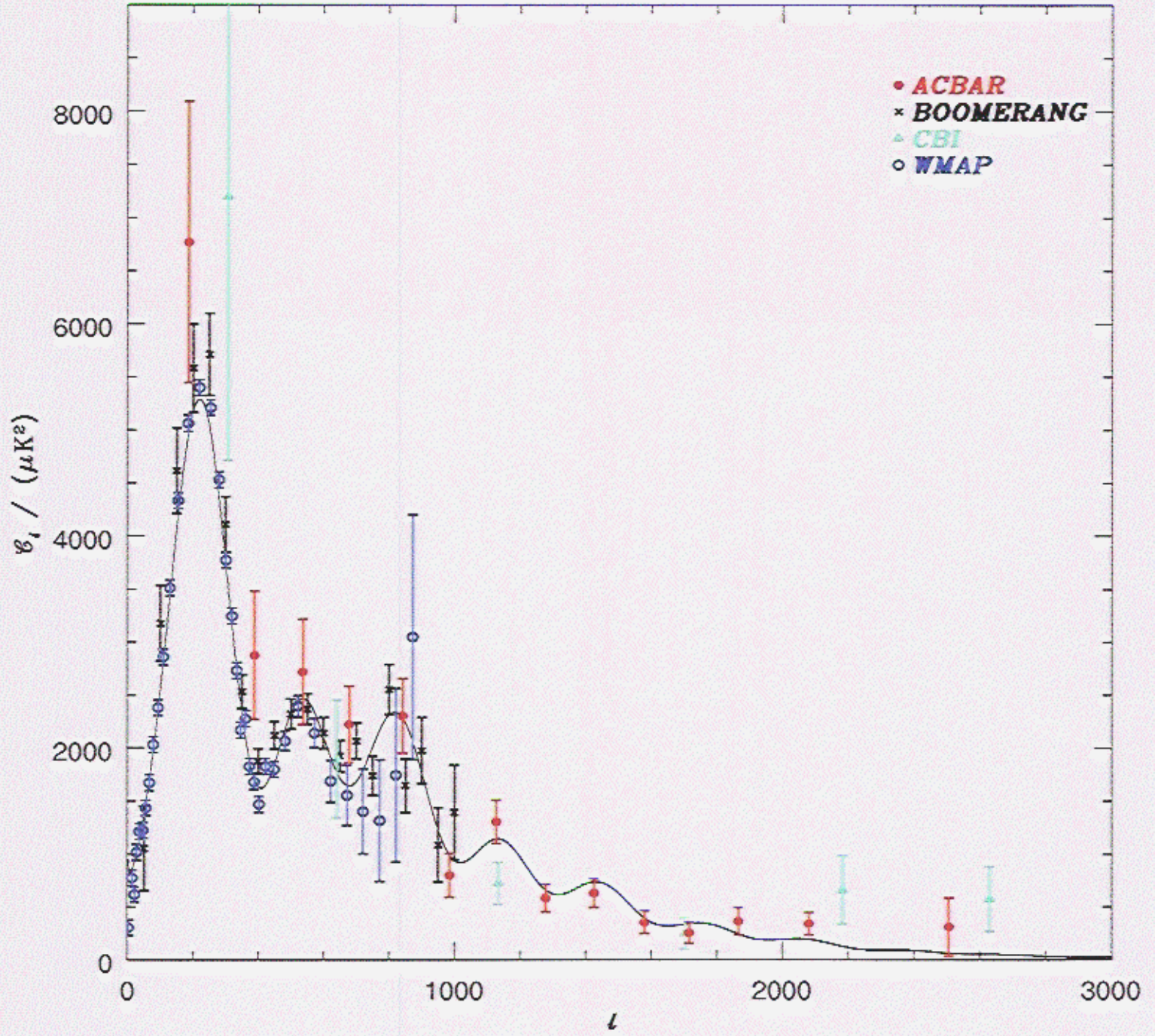


"RESPONSE OF CMB"

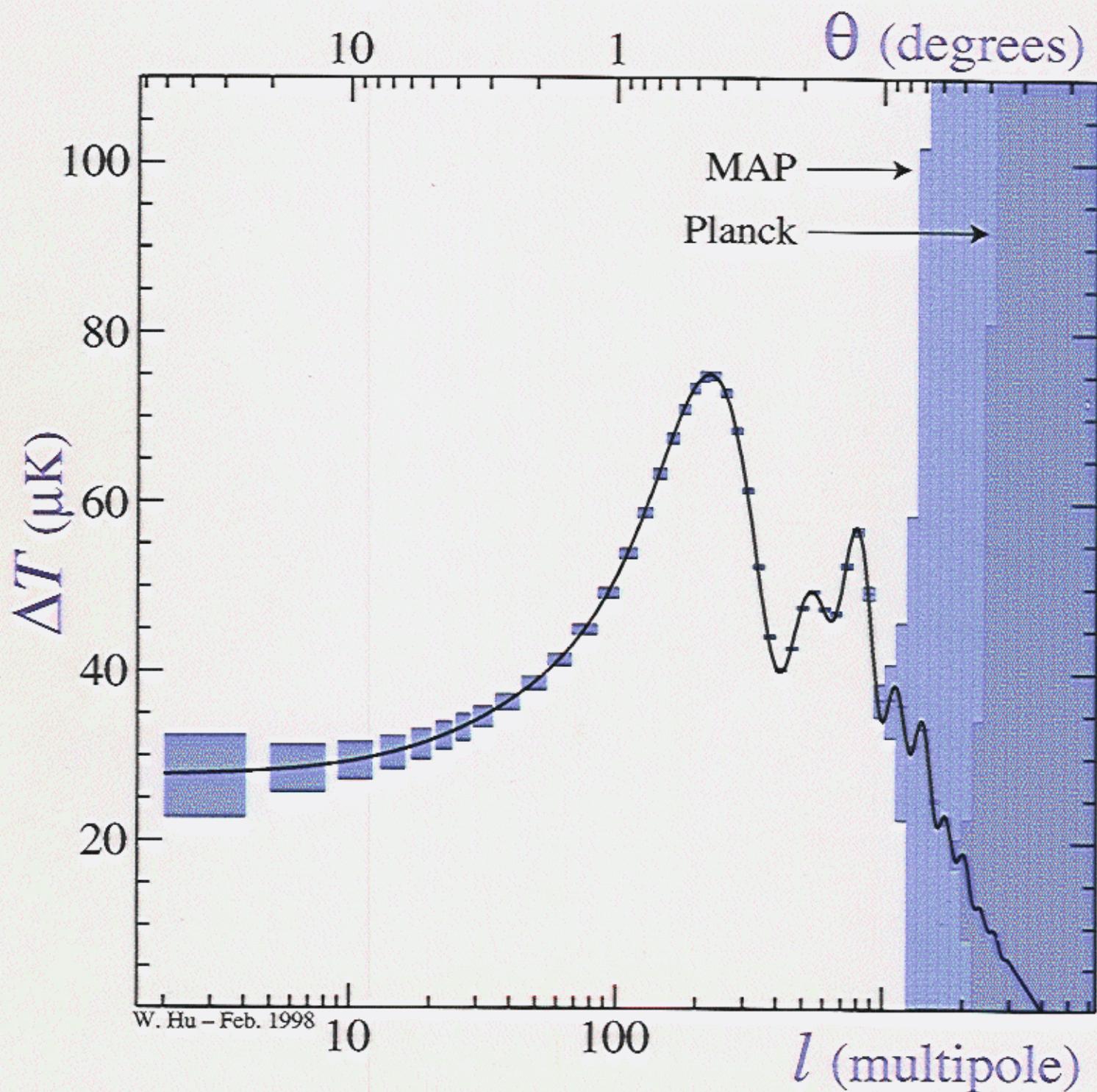


RESPONSE DEPENDS UPON: MATTER & BARYON DENSITIES, CURVATURE, DARK ENERGY ... & PRIMEVAL $\delta\rho/\rho$ → PROBES COSMOLOGY!





Projected Satellite Errors



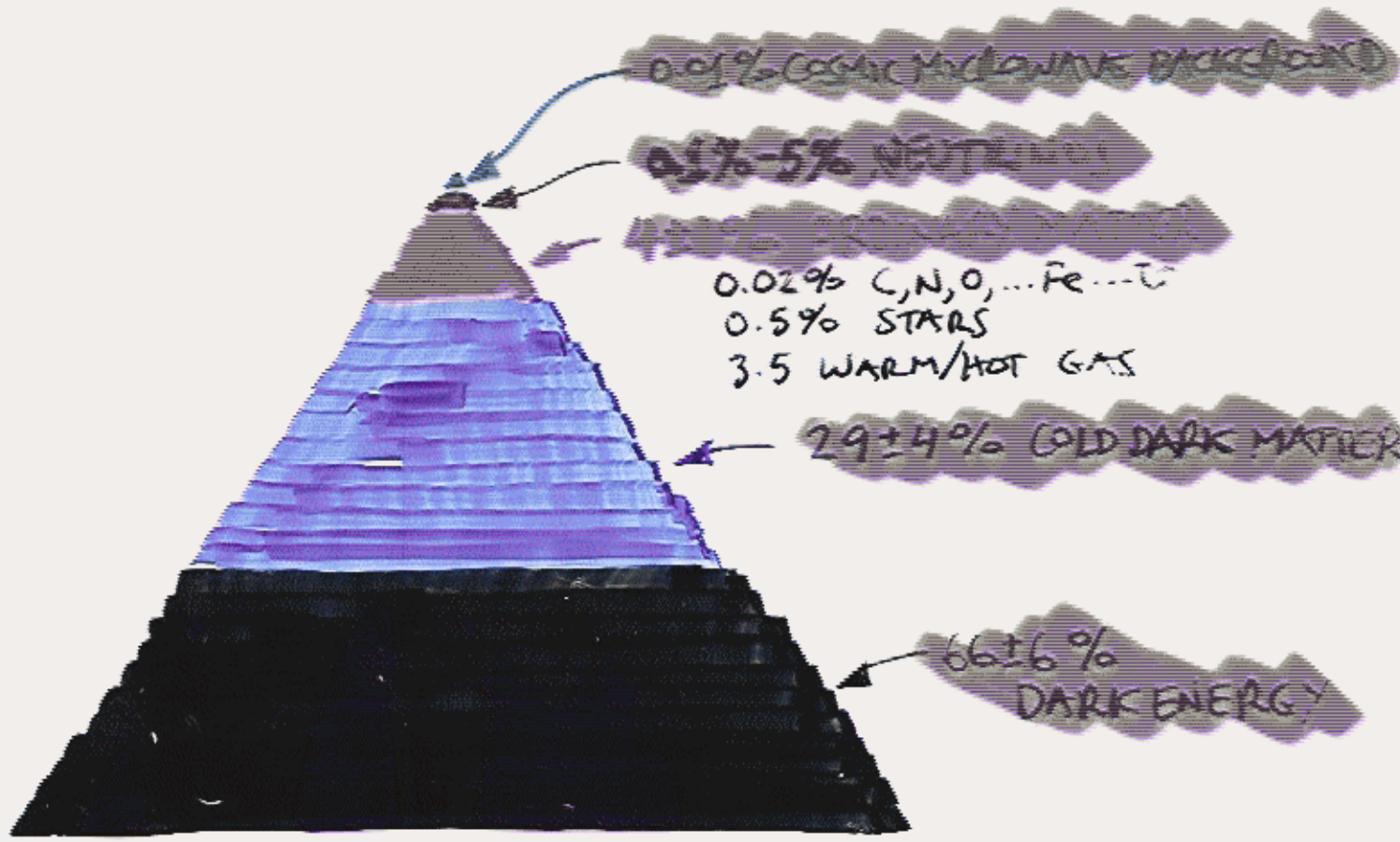
W. Hu - Feb. 1998

COSMIC RECIPE

FLAT, CRITICAL DENSITY UNIVERSE

0.5% STARS

33% DARK MATTER + 66% DARK ENERGY



MORE THAN 95% IN NEW FORMS OF MATTER & ENERGY!

CDM PREDICTIONS



"Bottom Up"

first stars	$z \sim 10-20$	✓
galaxies	$z \sim 2-5$	✓
clusters	$z \sim 0-1$	✓
superclusters	$z \sim 0$	



Large-scale Structure

voids, filaments, sheets



"Power Spectrum"

$$P(k) \propto k^{-2}$$

FROM CMB & REDSHIFT SURVEYS (2dF, SDSS)



Properties of clusters

number, distribution,
X-ray temp



"Lots of Recent Evolution"

merging, relaxing, ...

PROVIDED $\Omega_m h \approx 0.25 \pm 0.05 \Rightarrow \Omega_m \approx 0.4$

COLD DARK MATTER

EQ:

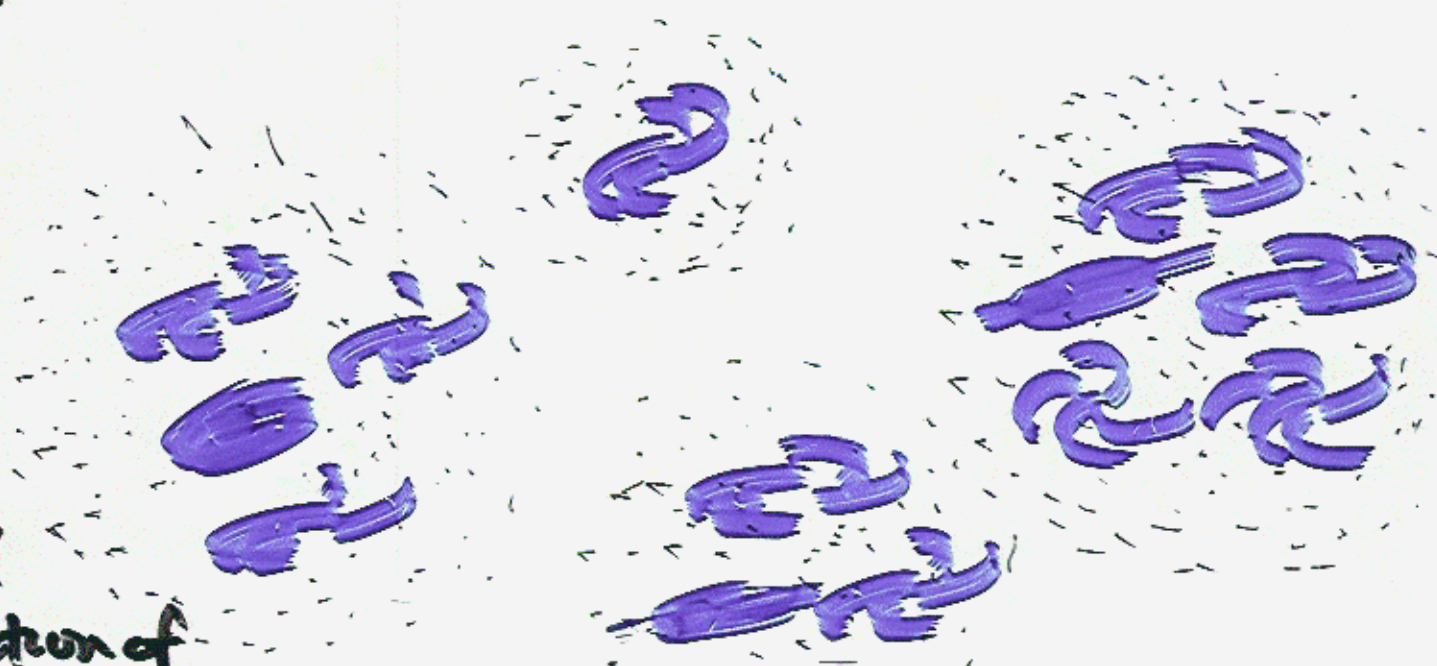
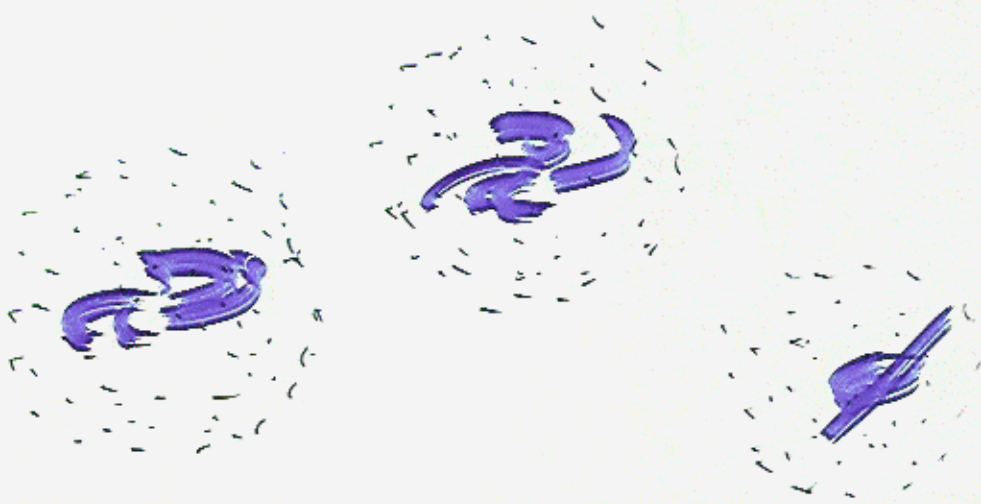
$R \sim 3 \times 10^5$
 $t \sim 100 \text{ yr}$
 $\rho \sim 10^{-25}$



$R \sim 1/3 - 1/2$
 $t \sim \text{few Gyr}$

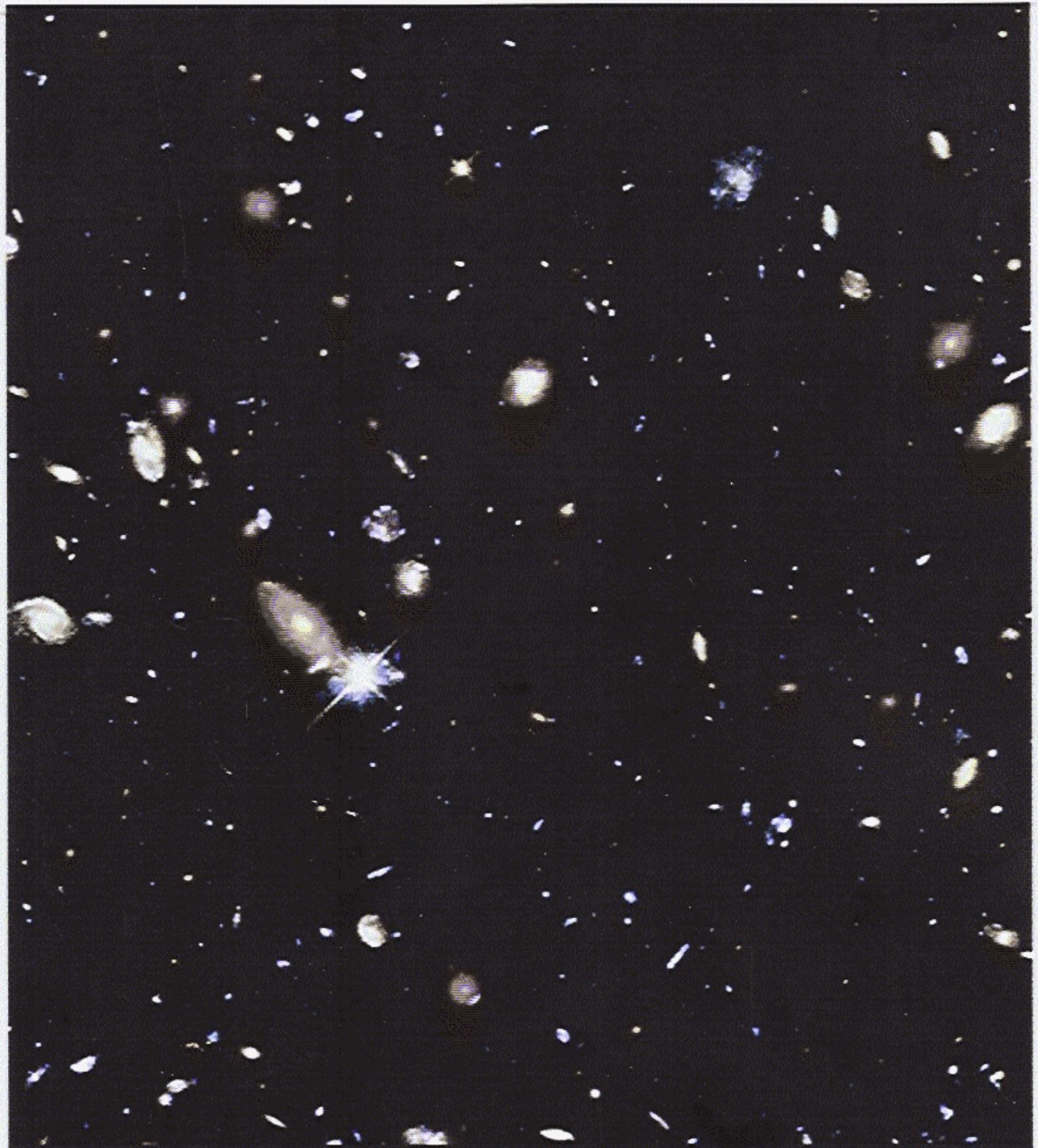
GALAXIES FORM

Dark halos, baryons
dissipate



TODAY

formation of
larger structures (superclusters) continues...



ubble Deep Field

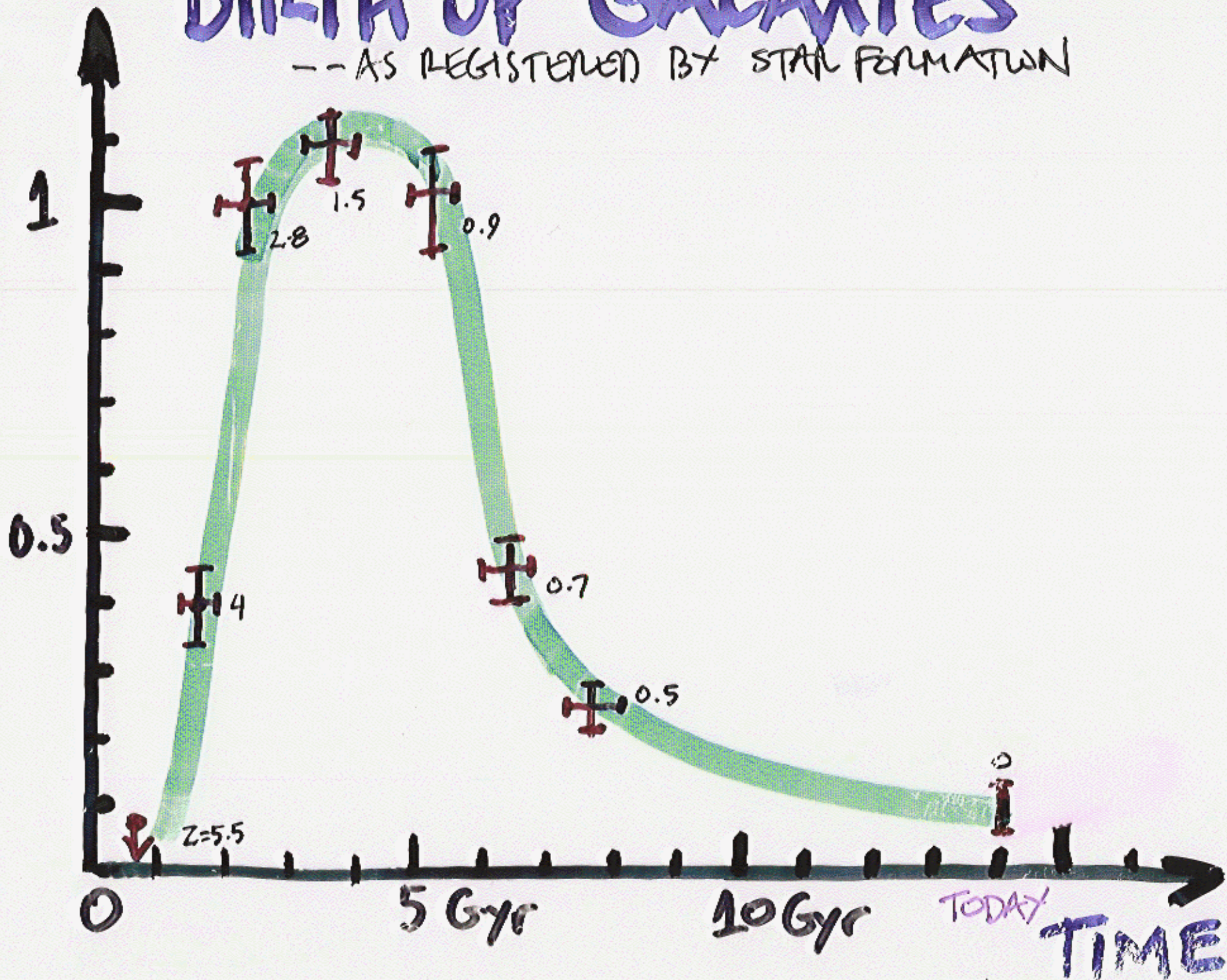
HST · WFPC

96-01a · ST ScI OPO · January 15, 1996 · R. Williams (ST ScI), NASA

BIRTH OF GALAXIES

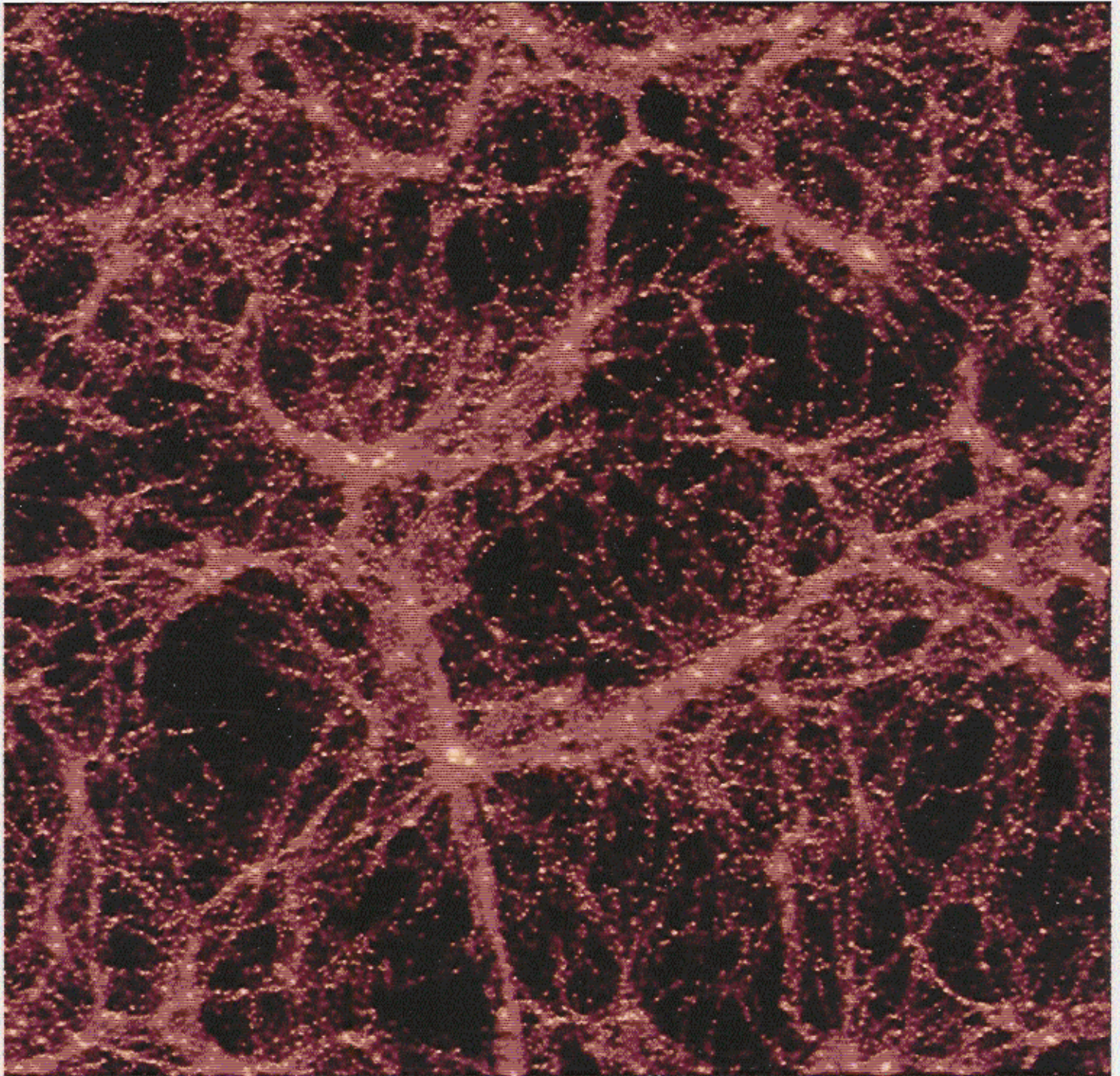
-- AS REGISTERED BY STAR FORMATION

STAR FORMATION RATE (RELATIVE SCALE)

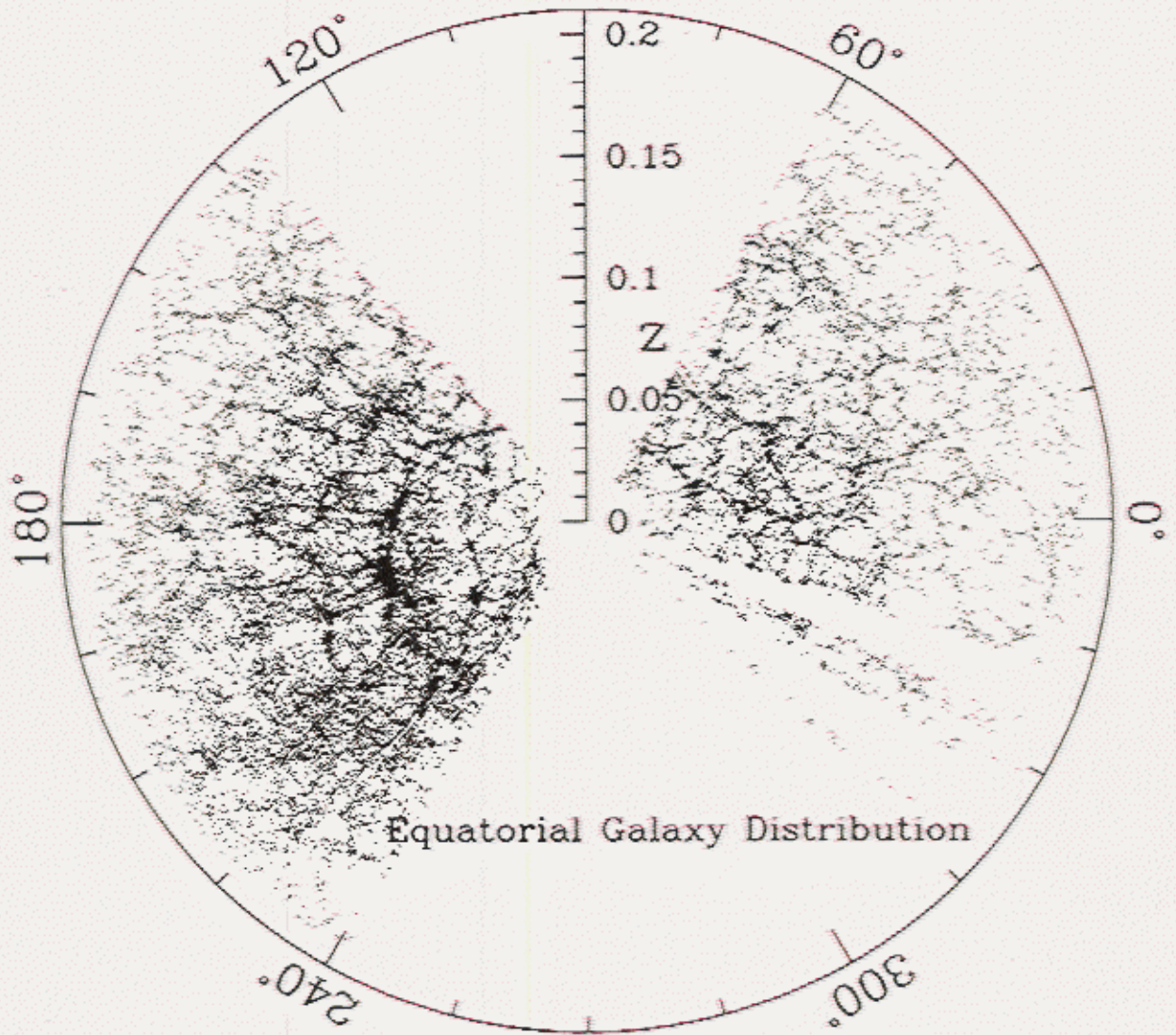


COSMIC WEB OF DARK MATTER

from Virgo Project



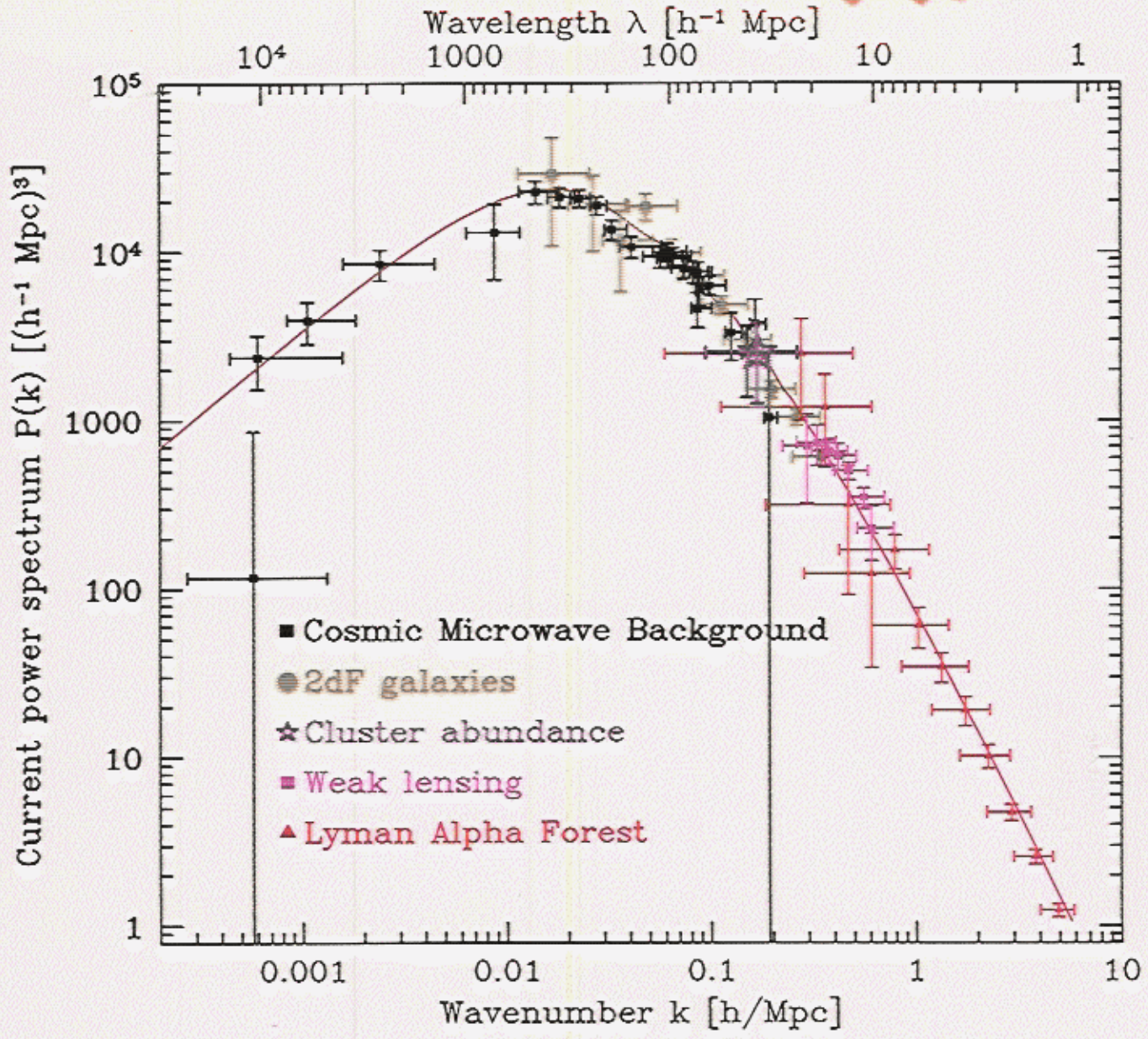
MAP of the UNIVERSE



Sloan Digital Sky Survey

CDM POWER SPECTRUM v. OBSERVATIONS

From the Ly- α Forest to the Hubble Scale
w/ a decade of overlap (CMB & LSS)



Tegmark-Zaldarriaga astro-ph/0207047 v3

NB: Baryon only, Λ CDM, ...
cannot make a similar claim

PRECISION TESTING HAS BEGUN

 FIRST MODEL RULED OUT!
 $V = \lambda \phi^4$

WMAP, 2dF, SDSS, DASI,
CAPMAP, BOOM-POL, ACT,
SPT, APEX, ... PLANCK,
LIGO, LISA, CMB-POL, BBO, ...

HOLY GRAIL: GW2

STATUS OF INFLATION:

EXCELLENT!



MS Turner / U. Chicago &
Fornicola

WHY the TENSORS MATTER

★ "SMOKING GUN" SIGNATURE

FLAT UNIVERSE & SCALE-INVARIANT q/p
PRE-DATE INFLATION

★ STRENGTH DIRECTLY TIED
TO $V(\phi)$ & $H_{\text{INFLATION}}$

★ RECONSTRUCTION OF $V(\phi)$

★ CONSISTENCY TEST

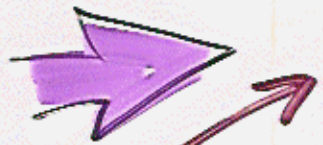
$$r/s = -5n_T$$

... BUT NO ROBUST PREDICTION
FOR AMPLITUDE

RECONSTRUCTION OF $V(\phi)$

measure $S, T/S, n-1$

CMB



$$V = 1.8 T m_{pl}^4$$

NB: $H_I^{-1} = 1.4 \times 10^{-34} \text{ sec}$
 $(T/10^{-10})^{-1/2}$

SCALE OF
INFLATION

$$V' = \pm \sqrt{\frac{8\pi T}{5}} \frac{V}{m_{pl}}$$

SHAPE
OF POTENTIAL

$$V'' = 4\pi \left[(n-1) + \frac{3}{5} \frac{T}{S} \right] \frac{V}{m_{pl}^2}$$

also measure n_T CMB + SUPER-USA

consistency test: $T/S = -5 n_T$

"PREDICTING" $n-1$ & T/S

REFORMULATE INFLATION EQNS
AS "FLOW EQNS" FOR $n-1, T/S$

KOFFMAN-MSC astro-ph/0006321

$$\frac{d(T/S)}{dn} = (n-1) \frac{T}{S} + \frac{1}{5} \left(\frac{T}{S} \right)^2$$

$$\frac{d(n-1)}{dn} = -\frac{1}{5} (n-1) \frac{T}{S} - \frac{1}{25} \left(\frac{T}{S} \right)^2 \pm \frac{dn}{dln k}$$

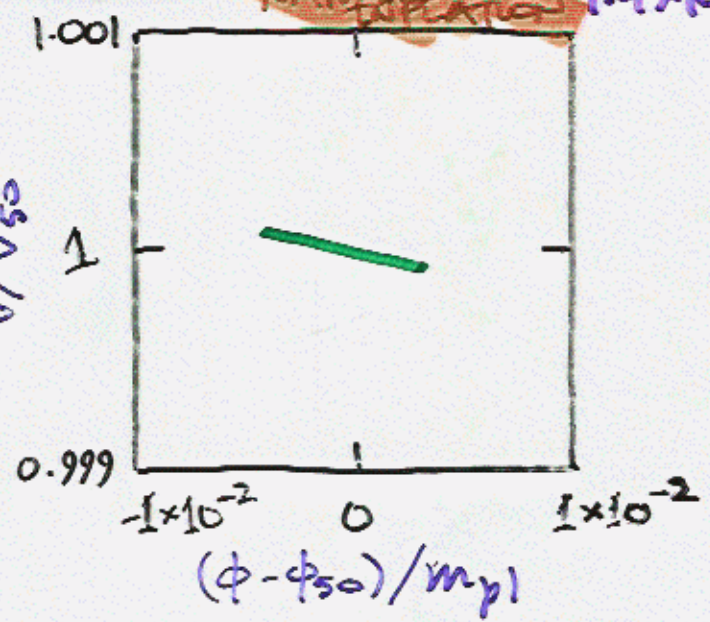
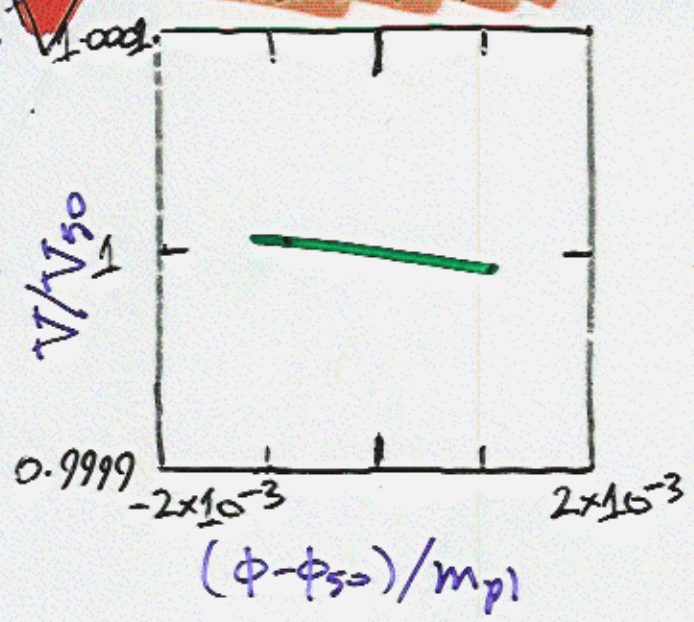
SCALE INVARIANT
LIMIT

MENU OF POTENTIALS

(SHAPE DETERMINED $n-1$ & $n_+ = -\frac{1}{2} \frac{T}{S}$) ASTRO-PH/9707035

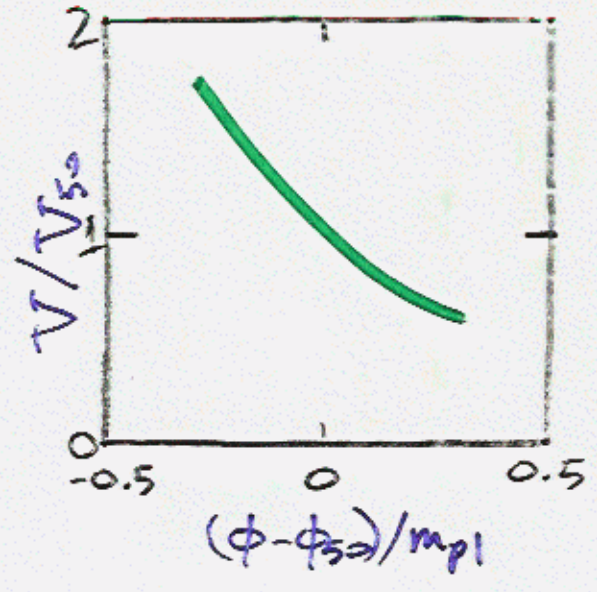
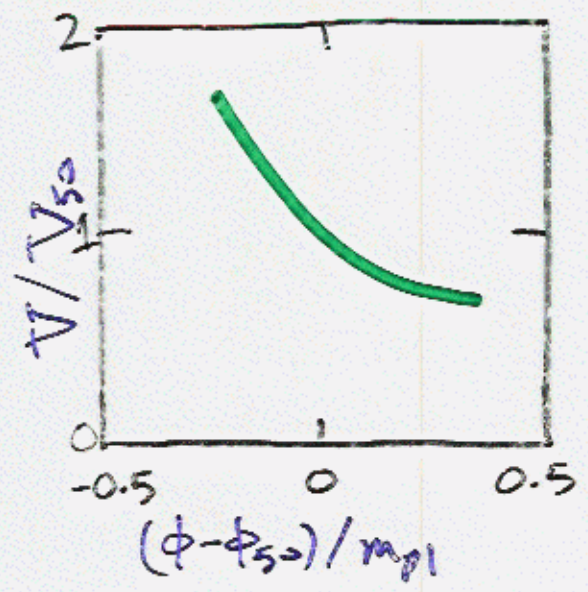
$(n-1) = -2 \times 10^{-6}, T/S = 1.4 \times 10^5$
NEW INFLATION

$(n-1) = -0.15, T/S = 1.4 \times 10^{-4}$
NATURAL INFLATION



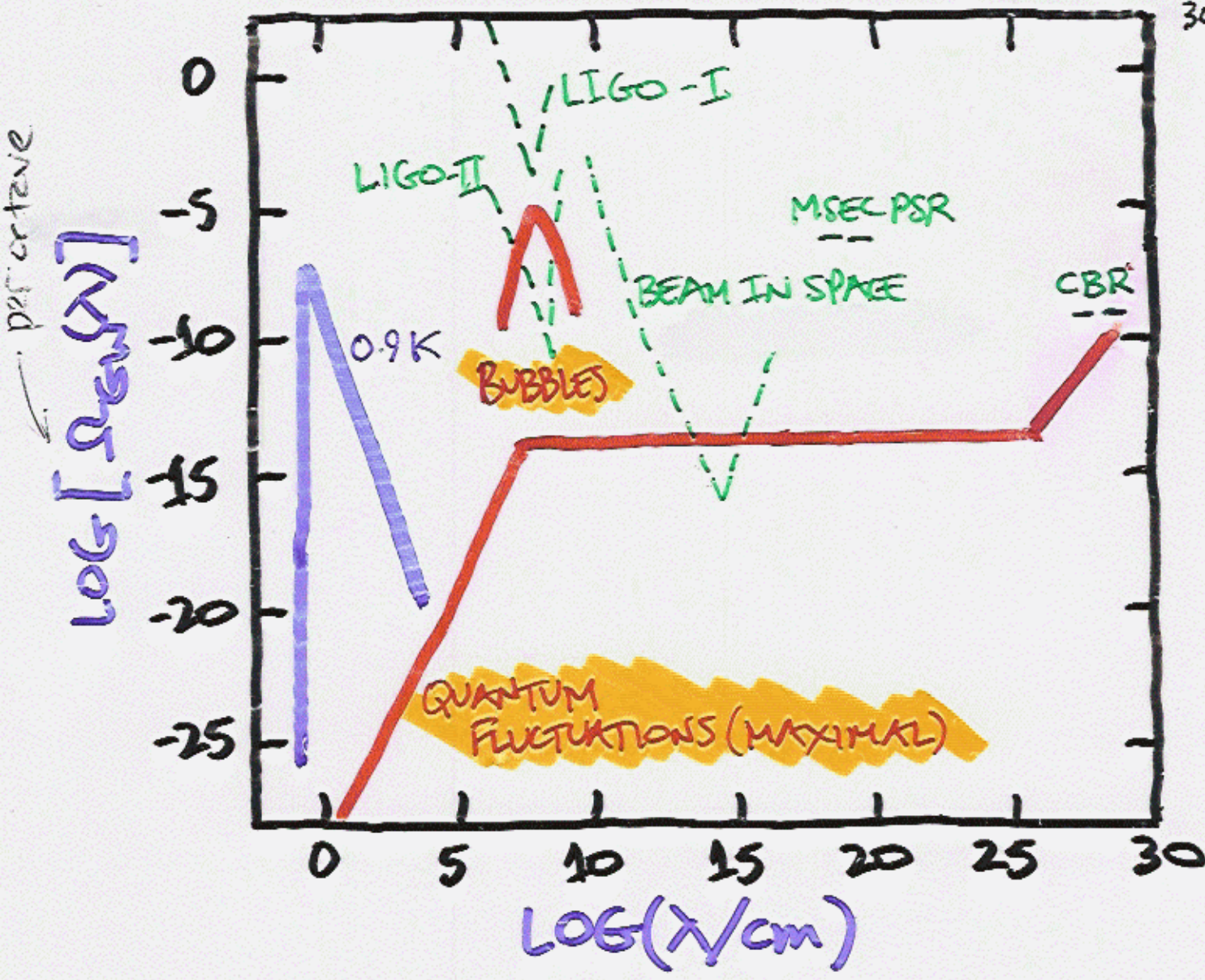
$(n-1) = 0, T/S = 1$
EG, CHAOTIC INFLATION

$(n-1) = -0.15, T/S = 1$
EG, EXTENDED INFLATION



GRAVITY WAVES FROM INFLATION

(MST-Wilczek PRL 62, 3089 (90))



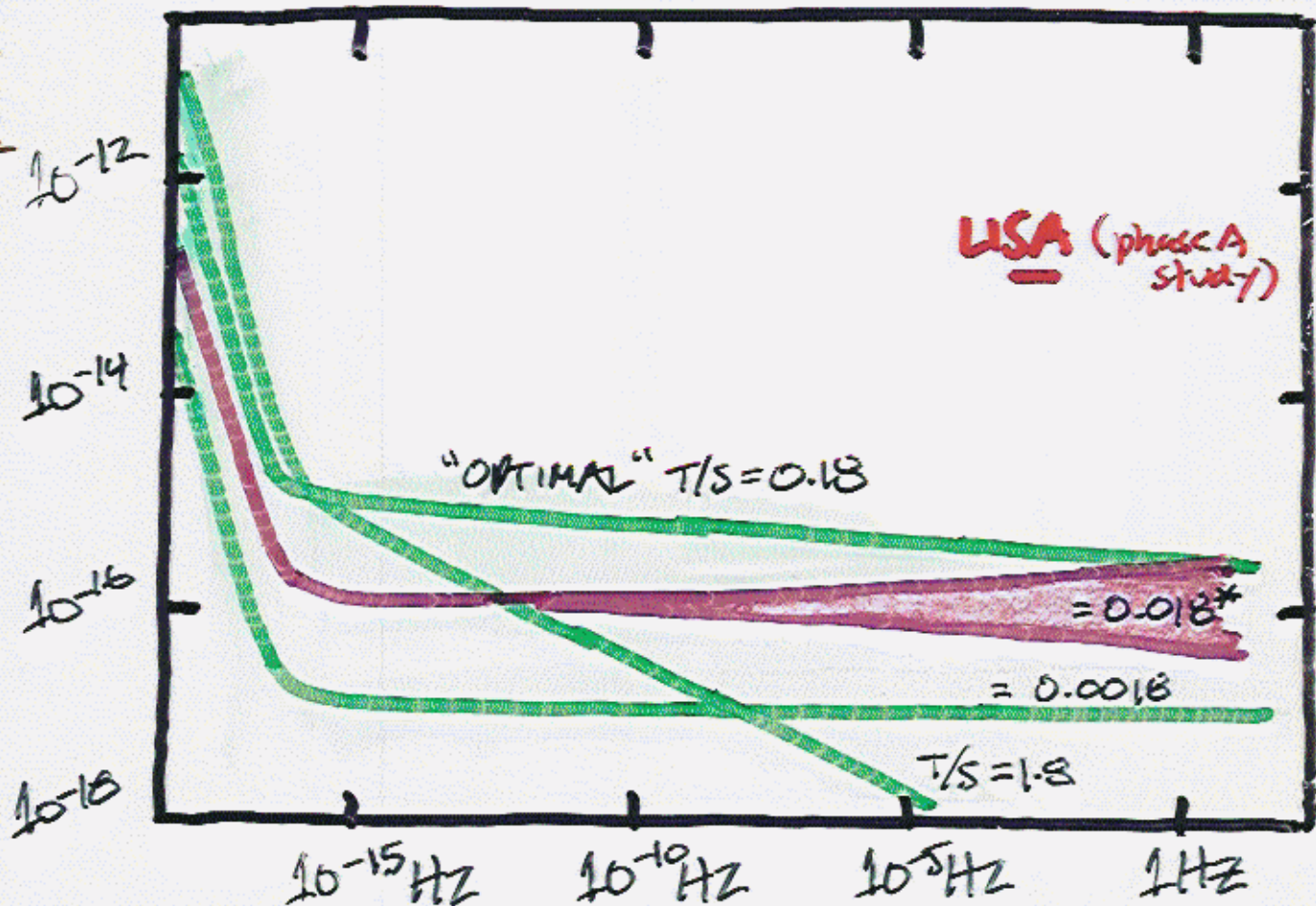
NB: "Astrophysical Background" Not Shown

ENERGY DENSITY IN INFLATION PRODUCED GWs

MST PRD55 (97)

LIGO-II

$\Omega_{gw}(f) h^2$ = fraction of critical density per dlnf



FREQUENCY f

LOBE normalized: $S+T = Q_{lobe} \approx 4H_0^{-1}$
 $\Rightarrow T = \frac{Q}{\sqrt{1+S/T}}$

* INCLUDES UNCERTAINTY DUE TO "RUNNING" OF POWER LAW INDEX

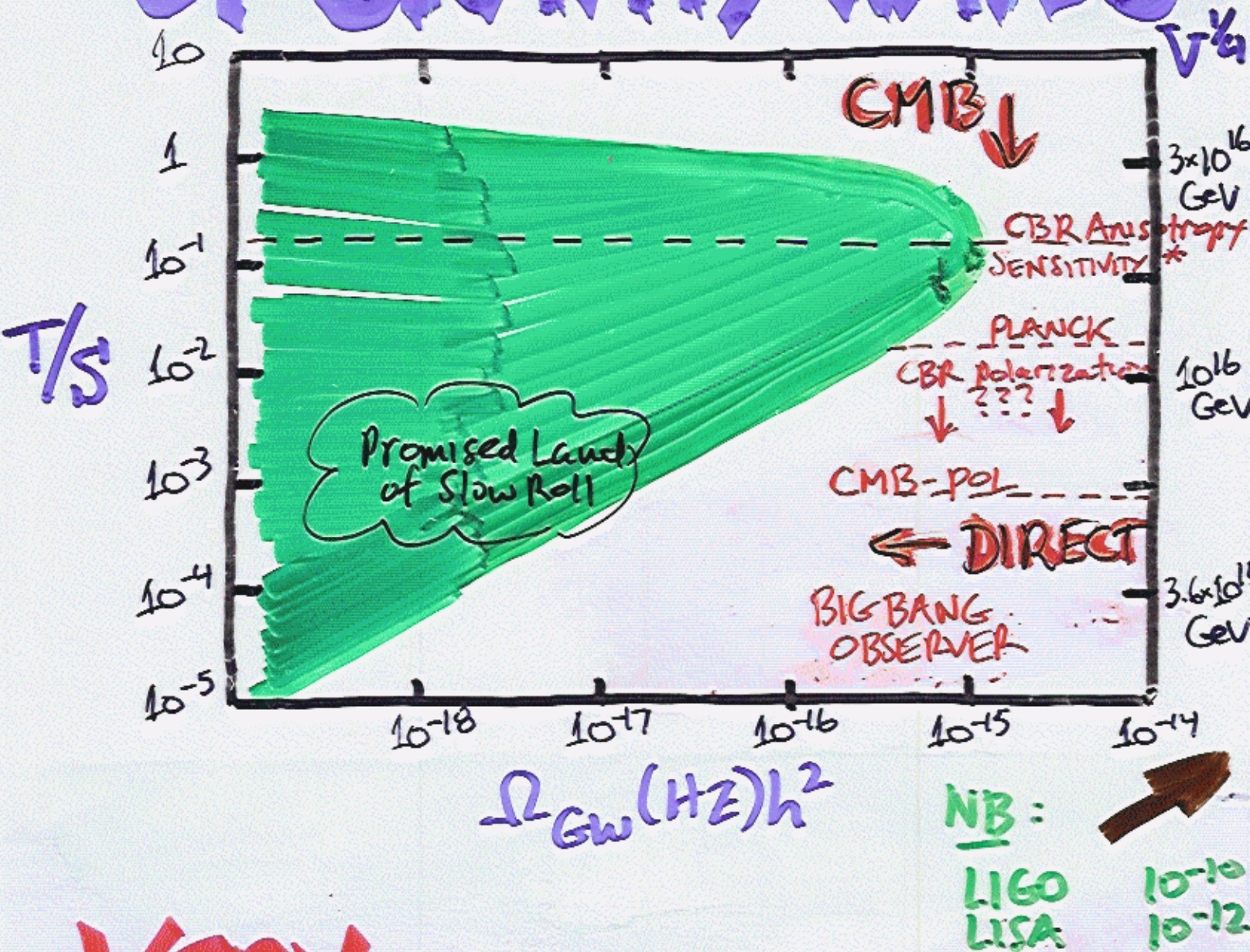
$$\Omega_{gw}(f) h^2 = 5.1 \times 10^{-15} \frac{n_T}{n_T - 1/2} \cdot e^{N n_T + \frac{1}{2} N^2 \frac{dn_T}{df}}$$

$$N = 33 + \ln(f/\text{Hz})$$

$$\equiv \frac{1}{\text{Per.} (H_0=100)} \times \frac{dln \omega}{dln f}$$

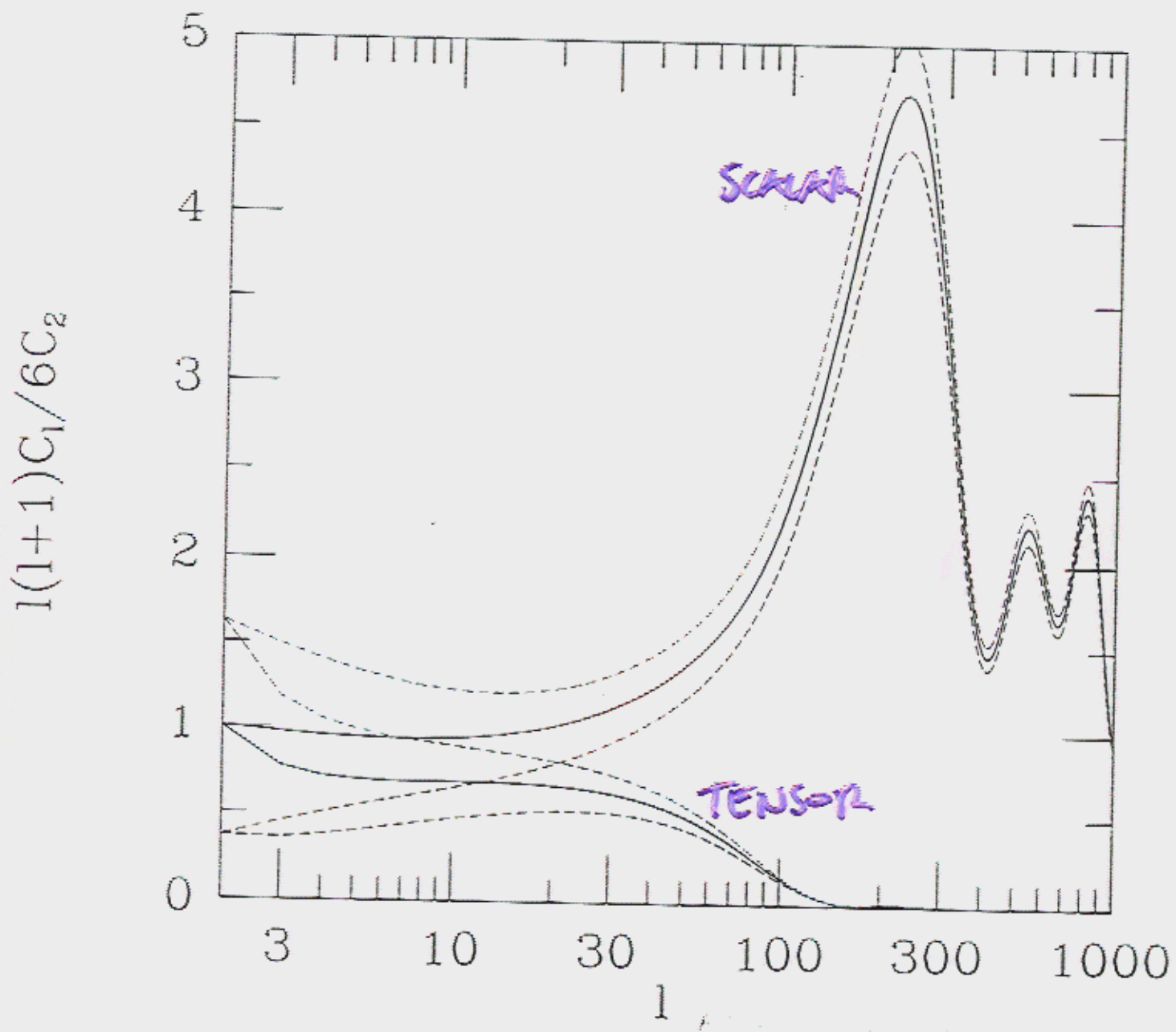
$$\frac{dn_T}{df} = -n_T [(n-1) - n_T] \approx \pm 10^{-3}$$

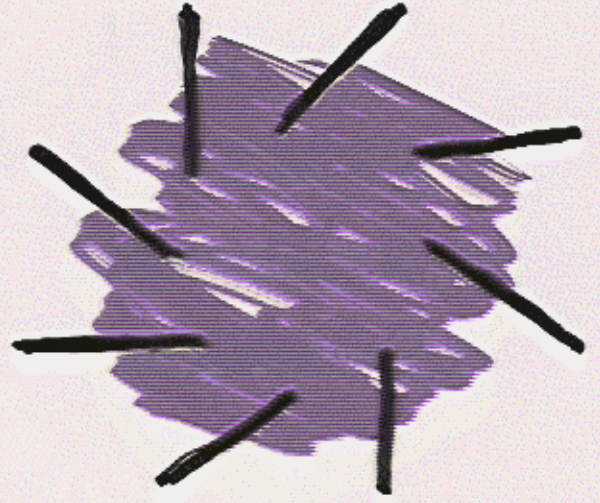
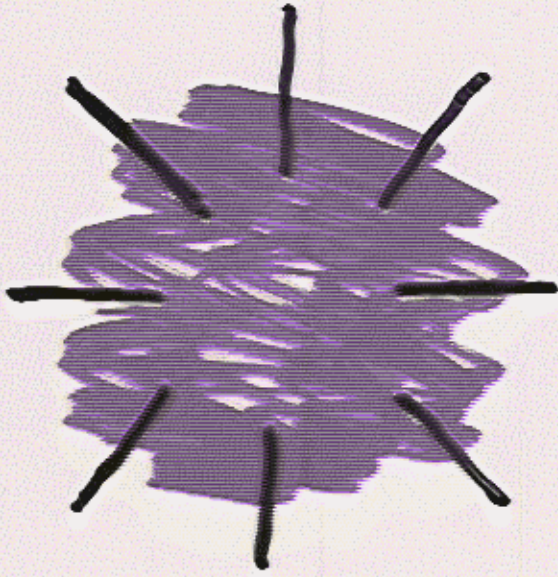
DETECTION OF GRAVITY WAVES



VERY CHALLENGING!

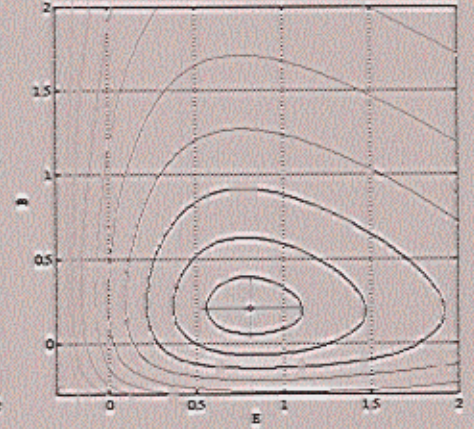
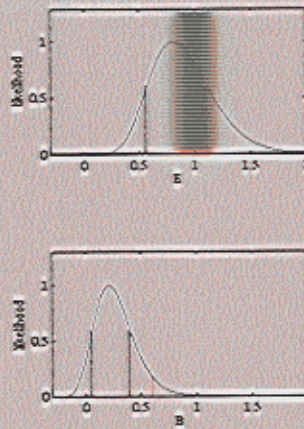
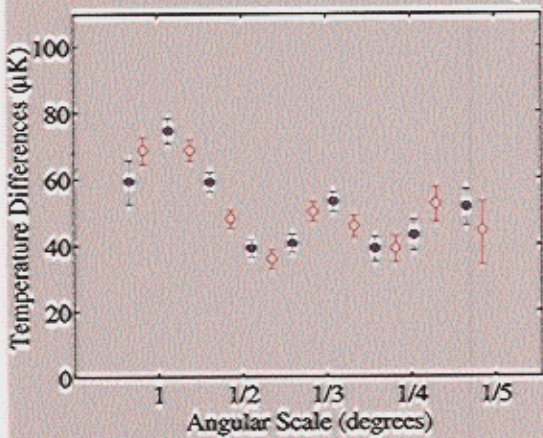
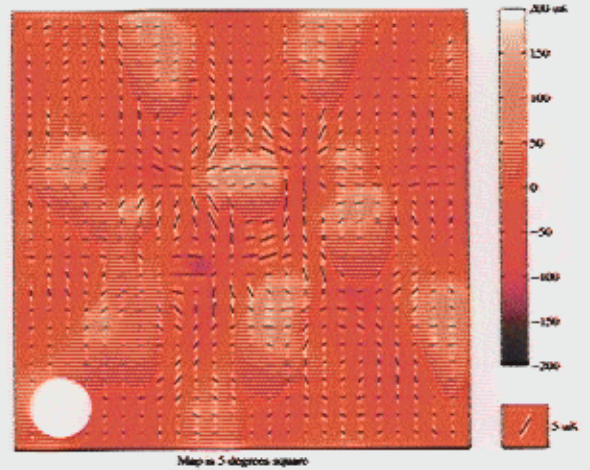
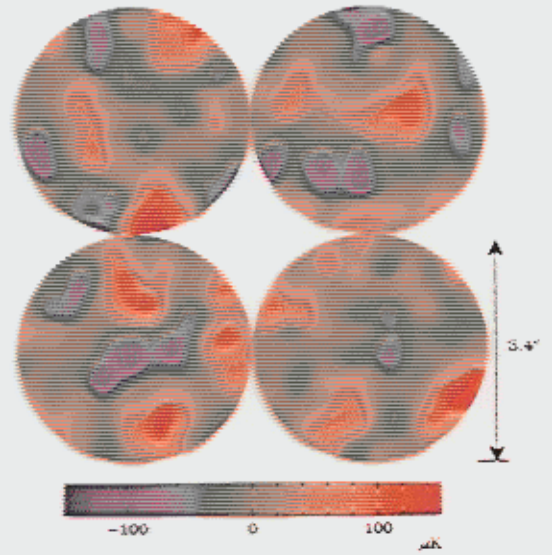
"DOUBLE DETECTION" $\Rightarrow n_T$ to ± 0.03





DASI

Acoustic Peaks and Polarization



MODEL-INDEPENDENT, "KINEMATIC" CONSTRAINTS TO T/S , n

SLOW ROLLOVER INFLATION

Hoffman - MST astro-ph/0006321 (PRD64, 023506(2001))

$dn/dlnk \geq 10^{-2}$

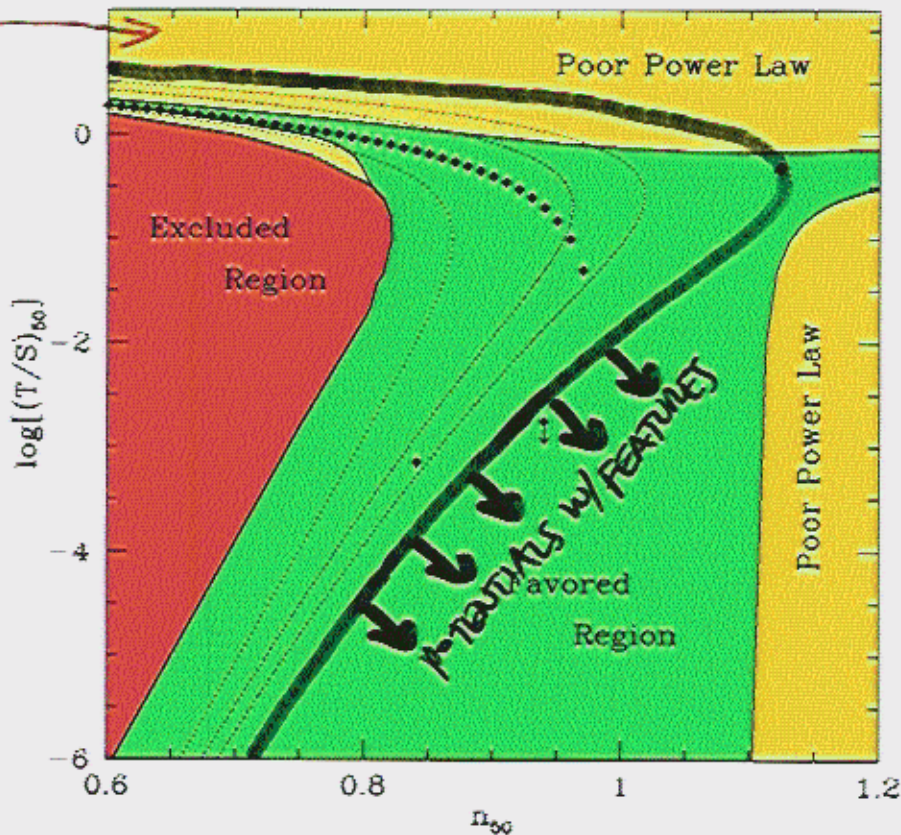


Figure 3: Same as Fig. 2, with a logarithmic scale for T/S .

For potential w/o features
 & $n > 0.85$ $T/S > 10^{-3}$

INFLATION RECAST AS FLOW EQNS:

$$d(T/S)/dN = (n-1)T/S + 1/5(T/S)^2$$

$$\frac{d(n-1)}{dN} = -\frac{1}{5}(n-1)T/S - \frac{1}{25}(T/S)^2$$

THE FUTURE



TEST FRAMEWORK

$\Omega_m = 1$, $(n-1) \sim \mathcal{O}(0.1)$, $dn/d\ln k \sim 10^{-3}$



PROBE UNDERLYING PHYSICS

precision measurements of $(n-1)$, $dn/d\ln k$ + TEASERS



FUNDAMENTAL MODEL



TRANS PLANCKIAN PHYSICS

... UNEXPECTED FEATURES



SOME CONNECTION TO LAB PHYSICS



THE MULTIVERSE DILEMMA

CAN IT BE MADE INTO SCIENCE



INFLATION IS A MODEST PROPOSAL

DOES NOT ADDRESS: INITIAL COND. FOR ϕ ,
SINGULARITY, DISTANT FUTURE



HIGH BAR TO ALTERNATIVES



SUPER HORIZON SIZE DENSITY
PERT ~ NOT TIED TO I.C.

REQUIRES: (1) Superluminal expansion
(2) Entropy production



THE TWO DEFINING
FEATURES OF INFLATION

astro-ph/
9302002

DOES THAT IMPLY THAT
ANY ALTERNATIVE
CAN BE DESCRIBED AS
SCALAR FIELD DYNAMICS?

"INFLATIONARY HEGEMONY"